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Highlights

- Inclusion of poverty principles within goal programming framework
- Novel solution of educational budget allocation model
- Expansion of scope of meta-goal programming technique

Incorporation of Poverty Principles into Goal Programming

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Abstract

This paper presents a methodology for the incorporation of poverty related concepts into the goal programming paradigm. The concepts of poverty line, absolute poverty and relative poverty are discussed in the context of their potential usage in the field of Operations Research. The synergy between goal programming and poverty lines is examined. Two additional meta-goals are proposed for inclusion in a meta-goal programming framework based around absolute and relative poverty respectively. The resulting poverty incorporating meta-goal programming model is compared against other goal programming variants over a test set of school budget allocation models. The results demonstrate how the incorporation of poverty principles allows for greater modelling flexibility in the goal programming framework. This in turn allows decision makers to avoid solutions that are below threshold levels for a subset of stakeholders in either a relative or absolute sense. Appropriate conclusions and suggestions for future research are given.

Keywords: Goal programming, multiple criteria decision making, poverty

Declarations of Interest: None
Conflicts of Interest: None

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1. Introduction

The original goal programming model, introduced by Charnes, Cooper and Ferguson (1955) and Charnes and Cooper (1961), minimises a function of unwanted deviations from a set of goals. This concept led to the first two goal programming variants. Lexicographic goal programming prioritises in a pre-emptive way the minimisation of unwanted deviations whereas weighted goal programming minimises a normalised, weighted sum of unwanted deviations. The underlying philosophy behind goal programming is satisficing (Simon, 1955), with the decision maker desiring to come as close as possible to achieving their desired set of goals. The weighted and lexicographic goal programming variants are based upon the minimisation of a single L_1 distance metric or a series of L_1 distance metrics respectively. This distance measure is associated with the concept of efficiency rather than that of equity (Romero *et al.*, 1998) as it considers the average rather than the worst case. Flavell (1976) introduces the Chebyshev goal programming variant which utilises the L_∞ distance metric, minimising the maximal weighted, normalised, unwanted deviation. This variant is hence associated with the Rawlsian (Rawls, 1971) concepts of equity, fairness and social justice. The post millennial development of goal programming variants has included a concentration on formulations that allow a blend of different underlying philosophies, each of which represents a focus of concern of the decision maker. These underlying philosophies are grounded in different socio-economic perspectives and often have a distinct correlation with one or more mathematical distance metrics that measure the difference between the achieved and goal target values over the set of objectives (Jones and Tamiz, 2010). In this vein, Romero (2001, 2004) introduces the extended goal programming variant, which minimises a parametric mix of L_1 and L_∞ distances, representing a range of solutions from optimisation to balance. This methodology is augmented by Jones and Jimenez (2013) to include an L_0 term in the achievement function, which represents the number of goals achieved, as well as a measure of compatibility with pairwise comparisons. Rodriguez *et al.*, 2002 develop the meta-goal programming (MGP) variant, which comprises minimisation of deviations from three meta goals based on the L_1 (optimisation), L_∞(balance) and L_0 (number of goals achieved) distance metrics. An interactive form of the MGP variant is proposed by Caballero *et al.*, 2006. MGP has subsequently been used in a variety of application areas including the design of human development indicators (Sayed *et al*, 2015) and cardinal preference aggregation in multi-criteria decision problems (Benitez-Fernandez and Ruiz, 2020). Chang (2007, 2008) introduces the revised multi-choice goal programming (RMCGP) variant, which allows for the simultaneous minimisation of deviations from a goal target and the improvement of that goal target value. Nayeri *et al.*, (2021) combines concepts from the MGP and RMCGP variants in the context of sustainable supply chain development. This combination is subsequently utilised by Gholizadeh, *et al.*, (2023) and Tahani *et al.* (2023) in the context of sustainable supply chains and inventory management respectively. The Robust Goal Programming (RGP) variant has been developed in order to provide a robust solution where model uncertainty exists. RGP was initially proposed by Kuchta (2006) with Ghahtarani and Najafi (2013) also combining goal programming and robust optimization in the context of a portfolio selection model. Further applications include Bojd and Koosha (2018), who utilise RGP in the context of capital budgeting and Hanks *et al.* (2020) who formulate an RGP model for a defence logistics problem. Recent usages of RGP for decision optimization include instances from the fields of sustainable development (Jia *et al.*, 2020), humanitarian logistics (Cheng *et al.*, 2021), healthcare network management (Hasani *et al.*, 2022) and renewable energy (Chen and Liu, 2023). Romero (2004) postulates that goal programming is an ideal multiple criteria methodology for the combination and comparison of different underlying philosophies due to its simplicity and flexibility. In this way, Gonzalez-Pachon and Romero (2016) examine the use of goal programming to derive and contrast the Bentham, Marx and Rawlsian based solutions to a multi-objective decision problem. Jones *et al*. (2016) examine the effect of centralisation as well as optimisation-equity trade-offs between objectives and stakeholders via an introduced extended network goal programming variant. The dual formulations of the sequence of weighted, Chebyshev, extended and extended network goal programmes are given by Merchant and Jones (2022).

Despite the above progress in incorporating different philosophies and socio-economic concepts into the goal programming paradigm, there are still many possibilities to enhance the power and flexibility of goal programming via further inclusions. One such possibility, which is the focus of this paper, is the inclusion of principles arising from the concept of poverty and measures for its minimization and hence alleviation.

The remainder of this paper is divided into five sections. Section 2 provides an overview of the key poverty concepts utilized in this paper and their relation to multiple criteria modelling. Section 3 details an extension of the meta goal programming variant in order to include absolute and relative poverty meta-goals. Section 4 details the development of a test set of educational budget allocation models that will be used to evaluate the performance of the formulation from Section 3. Section 5 then provides a comparative analysis of the poverty incorporating meta goal programming model against a number of other goal programming variants over the test set of educational planning models. Finally, Section 6 draws conclusions and presents suggestions for future research. This paper aims to make the following contributions to the multiple criteria literature:

- The explicit inclusion of absolute and relative poverty measures into the goal programming paradigm, achieved by the formulation of additional meta goals.
- The extension of the goal programming paradigm by including further underlying philosophies adapted from the field of social justice. This will increase the power of goal programming to model differential preferential requirements of decision makers.
- The development of a test set of budget allocation models which can be used to benchmark this research and future goal programming modelling advances.
- A brief commentary on the current state of goal programming modelling with respect to underlying decision maker utility functions and hence preferences arising from social concepts.

2. Poverty Definitions and Their Relationship to Multiple Criteria Modelling

Amongst the most fundamental and pragmatic concepts of poverty are the definitions of absolute and relative poverty (Decerf, 2017). **Absolute poverty** refers to falling below a certain level when measured against a pre-defined set of criteria, termed **poverty measures**. The level below which the poverty occurs is termed the **poverty line.** Simler and Arndt, 2007 point out some of the challenges and complexities related to the measurement of absolute poverty and hence the setting of absolute poverty lines, and hence present a method for estimating the variance of the poverty measures from survey data. Poverty measures can be single or multi-criterion in nature with commensurate and/or incommensurate units of measurement. Some recent examples of poverty measures include Dorn *et al.*, 2023 who consider income and leisure time in the context of Mexican households; Aristondo and Onaindia, 2023 consider energy poverty in a Spanish context and use incidence, intensity and inequality as measures. Beccaria *et al*, 2023 assess multiple poverty measures pertaining to deprivation of food, healthcare, housing services, housing equipment and social and educational opportunities in a Latin American context. As can be seen, often the measure(s) are financial, but can include a variety of criteria including possession of goods or access to services. **Relative poverty** refers to falling below an entity being below a percentage of average (mean or medium) of a set of comparable entities over the set of poverty criteria. The appropriateness of the two poverty concepts, absolute or relative, in given situations is a topic of significant debate in the literature (Foster, 1998). A pragmatic, but somewhat vague, consensus position is to say that the two measures are valid in different circumstances. When considering a set of entities, such as members of a population, the absolute and relative poverty levels are defined as the percentage of entities in absolute and relative poverty respectively.

There are multiple synergies between the above definitions from the field of poverty measurement and the field of multiple criteria decision making in general, and the technique of goal programming in particular. In general, both contain a set of multiple criteria by which a form of assessment, ranking or classification is required. Decisions must therefore be taken regarding the measurement and the relative importance of the measures or criteria. A key difference is that poverty measurement tends to be descriptive in nature, that is the output will be a set of statistics that describe the poverty situation and are intended to inform policy makers or other stakeholders of the situation. In this vein Zhou and Che, 2021 and Guo *et al.*, 2021 describe methodologies for poverty alleviation by agricultural subsidies and solar panel installation respectively. Multiple criteria decision making models tend, in the main, to be more prescriptive in nature. That is, definitive courses of action to optimize the decision situation under consideration in line with the specified preferences of the decision maker are given. The specific technique of goal programming has further synergies in that a numeric value on each criterion must be set, which is the desired level to be achieved. In goal programming this is the target level, whereas in poverty measurement it is an absolute or relative poverty line. The discussions on the setting of poverty lines (Simler and Arndt, 2007) hence, to an extent, mirror those in goal programming on the setting of target values (Jones and Tamiz, 2010). For the reason of this extra synergy, goal programming is chosen as the multiple criteria technique into which to incorporate poverty principles in this paper.

There exist other, more complex measures related to inequality that could also potentially be utilized in operational research for decision analysis. For instance, the Gini index (Gini, 1912) gives a statistical measure of dispersion within a set of entities that can be used to measure inequality. The Atkinson index (Atkinson, 1970) measures inequality specifically related to income and the effects of wealth re-distribution. The Theil index (Theil, 1967) gives an entropy based measure of overall inequality amongst a set of entities. Whilst the above three indices can potentially be used to derive poverty measures by considering individuals beyond a certain index level as being in poverty, the concepts of absolute and relative poverty are used in this paper as they are criteria based measures with, as described above, strong synergies and hence the most justifiably transferable to the field of multiple criteria decision making, and the technique of goal programming in particular.

3. Meta Goal Programming Model Incorporating Poverty

As detailed in the Section 1, there are multiple variants of goal programming, each containing one or more underlying distance metrics that represent a single, or multiple underlying decision maker philosophies. The meta-goal programming (MGP) variant is chosen as the vehicle to incorporate poverty concepts in this paper. This is because (i) The meta-goal variant already contains $L_1, L_\infty$and L_0 distance meta-goals for which the trade-offs with poverty reduction can hence be investigated and (ii) Meta goal programming allows for the addition of further meta-goals, in this case representing absolute and relative poverty. The fact that a meta-goal with a meta target exists is more natural for modelling poverty levels than, say, the parametric analysis of the extended goal programming variant.

The standard MGP model of Rodriguez *et al.*, 2002, modified in this paper to include two sided goals, is given in algebraic format as model (1)-(11). This has been expressed in a weighted form, it is hence assumed that normalised, unwanted weighted deviations from the meta-goals can be directly compared and the decision maker is interested in establishing the trade-offs between them. In order to facilitate the modelling, five types of sets comprising of unwanted deviational variables are first defined, which will be utilised throughout Section 3:

$S_k^{(1)}$: set of unwanted deviations associated with the k'th type 1 meta-goal, (the sum of deviations), $k = 1, \dots, \rho_1$

$S_k^{(2)}$: set of unwanted deviations associated with the k'th type 2 meta-goal, (the maximum deviation), $k = 1, \dots, \rho_2$

$S_k^{(3)}$: set of unwanted deviations associated with the k'th type 3 meta-goal, (number of unmet goals), $k = 1, \dots, \rho_3$

$S_k^{(4a)}$: set of unwanted deviations associated with the k'th type 4a meta-goal, (absolute poverty), $k = 1, \dots, \rho_{4a}$

$S_k^{(4r)}$: set of unwanted deviations associated with the k'th type 4r meta-goal, (relative poverty), $k = 1, \dots, \rho_{4r}$

$$Min\ a = \sum_{k=1}^{\rho_1} v_k^{(1)} \beta_k^{(1)} + \sum_{k=1}^{\rho_2} v_k^{(2)} \beta_k^{(2)} + \sum_{k=1}^{\rho_3} v_k^{(3)} \beta_k^{(3)} \qquad (1)$$

Subject to,

$$f_i(\underline{x}) + n_i - p_i = t_i, \qquad i = 1, \dots, q, \qquad (2)$$

$$g_j(\underline{x}) \le b_j, \qquad j = 1, \dots, m, \qquad (3)$$

$$\sum_{i \epsilon S_k^{(1)}} \left(\frac{u_i n_i}{|t_i|} + \frac{v_i p_i}{|t_i|} \right) + \alpha_k^{(1)} - \beta_k^{(1)} = Q_k^{(1)}, \qquad k = 1, \dots, \rho_1, \qquad (4)$$

$$\frac{u_i n_i}{|t_i|} + \frac{v_i p_i}{|t_i|} \le \lambda_k, \qquad i \epsilon S_k^{(2)}, k = 1, \dots, \rho_2, \qquad (5)$$

$$\lambda_k + \alpha_k^{(2)} - \beta_k^{(2)} = Q_k^{(2)}, \qquad k = 1, \dots, \rho_2, \qquad (6)$$

$$u_i n_i + v_i p_i - M y_{ik} \le 0, \qquad i \epsilon S_k^{(3)}, k = 1, \dots, \rho_3, \qquad (7)$$

$$\sum_{i \epsilon S_k^{(3)}} \frac{y_{ik}}{\left| S_k^{(3)} \right|} + \alpha_k^{(3)} - \beta_k^{(3)} = Q_k^{(3)}, \qquad k = 1, \dots, \rho_3, \qquad (8)$$

$$\underline{x} \ge \underline{0};\ \ n_i, p_i \ge 0, i = 1, \dots, q; \qquad \alpha_k^{(1)}, \beta_k^{(1)} \ge 0, k = 1, \dots, \rho_1, \qquad (9)$$

$$\alpha_k^{(2)}, \beta_k^{(2)} \ge 0, k = 1, \dots, \rho_2; \qquad \alpha_k^{(3)}, \beta_k^{(3)} \ge 0, k = 1, \dots, \rho_3, \qquad (10)$$

$$\lambda_k \ge 0, k = 1, \dots, \rho_2; \qquad y_{ik} = 0\ or\ 1, i \epsilon S_k^{(3)}, k = 1, \dots, \rho_3 \qquad (11)$$

Where the meta-goal achievement function (1) is formed of three terms. These three terms are the summations of unwanted deviations from the ρ_1 type 1 meta-goals, ρ_2 type 2 meta-goals and ρ_3 type 3 meta-goals respectively. The unwanted positive deviations from the k'th meta-goal of types 1,2 and

3 are termed $\beta_k^{(1)}$, $\beta_k^{(2)}$ and $\beta_k^{(3)}$ respectively. The associated relative importance weights are termed $\upsilon_k^{(1)}$, $\upsilon_k^{(2)}$ and $\upsilon_k^{(3)}$ respectively.

In goal set (2), $f_i(\underline{x})$ represents the achieved value of the i'th goal, which is a function of a decision variable set $\underline{x}$. t_i is the desired target value of the i'th goal. There are q goals in total. n_i and p_i represent the negative and positive deviations from the i'th target value respectively. Constraint set (3) imposes feasibility restrictions on the decision variable set $\underline{x}$ assuming, without loss of generality, that these are represented in the form of m inequalities, where the j'th inequality constrains the function $g_j(\underline{x})$ to be less than a right hand side value b_j.

Equation set (4) represents the type 1 meta-goals. u_i and v_i are the preferential weights associated with the negative and positive deviations from the i'th goal. These preferential weights are set equal to zero if the decision maker does not wish to penalise the corresponding deviation. The percentage normalisation method has been assumed, that is deviations from the $i'th$ goal are divided by the absolute value of their corresponding target value, $|t_i|$. This implies that $t_i \neq 0, i = 1, \ldots, q$. In the case that a zero target value is required, a different normalisation method should be used, as detailed in Jones and Tamiz, 2010. The aim is to keep the sum of unwanted, normalised deviations from goals contained in the set, $S_k^{(1)}$ associated with the $k'th$ type 1 meta-goal below a target level $Q_k^{(1)}$. The negative deviation from this value is termed $\alpha_k^{(1)}$.

Inequality set (5) and equation set (6) enforce the type 2 meta-goals, where λ_k is the maximum unwanted, normalised deviation from amongst the deviations from goals in the set, $S_k^{(2)}$, of goals associated with the k'th type 2 meta-goal. Note that it is goals rather than deviations that are associated with a meta-goal. This assumes that, in the case that both deviations are unwanted, these are not separated in the meta-goal. It is, however, possible for deviations from a goal to be represented in multiple meta-goal sets. The aim is to achieve a level of λ_k no greater than a target value $Q_k^{(2)}$. The negative deviation from this goal is represented by $\alpha_k^{(2)}$.

Inequality set (7) and equation set (8) model the type 3 meta-goals. A binary variable y_{ik} is associated with each goal contained in the set $S_k^{(3)}$, which relates to the k'th type 3 meta-goal. This takes the value zero if all unwanted deviations from this goal are zero (i.e. the goal is met) and the value one otherwise (i.e. the goal is not met). Note that this formulation uses one binary variable for each goal in each type 3 set. An alternative formulation is to use one binary variable for each of the q goals in the model, as per the original formulation of Rodriguez, et al, 2002. It is recommended, particularly for larger scale models, that the formulation which uses the least binary variables is chosen as this is a key factor in determining the solution time of the resulting meta goal programme. M is an arbitrarily large positive constant. The aim is to penalise the positive deviation, $\beta_k^{(3)}$, from the target proportion, $Q_k^{(3)}$, of unmet goals in the k'th type 3 meta-goal set, $S_k^{(3)}$. The corresponding negative deviation $\alpha_k^{(3)}$is not penalised in the achievement function (1). Sign restriction sets (9), (10) and (11) impose the appropriate sign restrictions on the model's decision and deviational variables. Note that the decision variable set $\underline{x}$ is specified, as per mathematical programming standard convention, as non-negative and continuous but other specifications such as integer, binary, and unrestricted in sign are also possible.

3.1 Incorporation of Absolute Poverty

The concept of absolute poverty is discussed in Section 2. In the context of a goal programming model, a goal is defined to be in absolute poverty if its normalised deviation is equal to or beyond a proportional level AP_k in an unwanted direction. This is incorporated into the MGP via the $k'th$ type 4A meta-goal, which is modelled via the addition of equation sets (12) and (13):

$$\frac{wa_i^- n_i}{|t_i|} + \frac{wa_i^+ p_i}{|t_i|} - Mz_{ik} \leq AP_k, \qquad i\epsilon S_k^{(4a)}, k = 1, \ldots, \rho_{4a} \ , \qquad (12)$$

$$\sum_{i\in S_k^{(4a)}} \frac{z_{ik}}{\left|S_k^{(4a)}\right|} + \alpha_k^{(4a)} - \beta_k^{(4a)} = Q_k^{(4a)}, \qquad k = 1, \ldots, \rho_{4a} \qquad (13)$$

Where wa_i^- and wa_i^+ are weights associated with the negative and positive deviations in the measurements of absolute poverty. The value of wa_i^- and wa_i^+ depends on how strictly the poverty line concept is to be enforced when it is transferred to the goal programming paradigm. In all cases, if a deviation is not penalised by the decision maker then its corresponding (wa_i^- or wa_i^+) weight is set equal to zero. For unwanted deviations, if the poverty concept is to be strictly interpreted, then the corresponding (wa_i^- or wa_i^+) weight should be set equal to one in all cases. This will ensure equal treatment for all goals in relation to poverty, irrespective of their perceived importance to the decision maker. In the case that a less strict poverty interpretation is required, then the decision maker preference weights can be incorporated and the values set at $wa_i^- = u_i$, $wa_i^+ = v_i$,$\forall\ i\epsilon S_k^{(4a)}$. This model assumes that there are ρ_{4a} distinct subsets of goals within which absolute poverty is to be measured. Note that a strict interpretation of absolute poverty would require $\rho_{4a} = 1$ as the poverty line should be consistently measured across the whole set of relevant goals. However, the cardinality of the single set does not have to be set equal to the number of objectives q, as not all the objectives may pertain to a criteria relevant to poverty measurement in the context of the application area of the meta goal programme. z_{ik} is a binary variable that takes the value one if the unwanted deviation is on or beyond the absolute poverty line and zero otherwise.

Equation set (13) models the deviations from the target value of $Q_k^{(4a)}$ for the proportion of the set of goals $S_k^{(4a)}$ from the k'th absolute poverty meta-goal that are beyond the poverty line. $\beta_k^{(4a)}$ represents the unwanted positive deviation from this target and $\alpha_k^{(4a)}$the negative deviation from the target, which is not penalised.

3.2 Incorporation of Relative Poverty

The concept of relative poverty is discussed in Section 2. In the context of a goal programming model, a goal is defined to be in relative poverty with respect to the k'th relative poverty meta-goal if its unwanted, normalised deviation is equal to or beyond a proportional level AR_k below the average of all the unwanted, normalised deviations in the corresponding k'th relative poverty meta-goal set, termed $S_k^{(4r)}$. This is modelled by the inclusion of equations (14) – (16) in the meta goal programme:

$$\frac{1}{\left|S_k^{(4r)}\right|}\sum_{i\in S_k^{(4r)}}^{q}\frac{wr_i^{-}n_i+wr_i^{+}p_i}{|t_i|}-\frac{wr_j^{-}n_k+wr_j^{+}p_k}{|t_j|}+nr_j-pr_j=0,\ \ j\in S_k^{(4r)}, k=1,\dots,\rho_{4r},\quad (14)$$

$$nr_j-Ms_{jk}\le AR_k\ , j\in S_k^{(4r)},\qquad k=1,\dots,\rho_{4r}\ ,(15)$$

$$\sum_{i\in S_k^{(4r)}}\frac{s_{jk}}{\left|S_k^{(4r)}\right|}+\alpha_k^{(4r)}-\beta_k^{(4r)}=Q_k^{(4r)},\qquad k=1,\dots,\rho_{4r}\qquad (16)$$

Where wr_i^- and wr_k^+ are weights associated with the negative and positive deviations in the measurements of relative poverty respectively. Similar comments regarding the setting of these weights as those of the absolute poverty weights wa_i^- and wa_i^+ in the previous subsection can be made. That is, they can be set to value one for all unwanted deviations and zero for all deviations not penalised by the decision maker if a strict interpretation of relative poverty is required. Alternatively, they can be set as $wr_i^- = u_i$, $wr_i^+ = v_i$, $\forall\ i\epsilon S_k^{(4r)}$ if the incorporation of decision maker preferential weights is desired. nr_k and pr_k represent the negative and positive deviations of the $j'th$ unwanted, normalised deviation from the average unwanted, normalised deviation from the set of goals associated with the $k'th$ relative poverty meta-goal, $S_k^{(4r)}$. The binary variable s_{jk} takes the value one if the unwanted deviation from j'th goal in the k'th relative poverty meta goal set is on or beyond the relative poverty line, and zero otherwise. Equation (16) aims to keep the proportion of goals in the k'th relative poverty meta goal set that are beyond the poverty line for that set (AR_k) below a target level $Q_k^{(4r)}$. $\beta_k^{(4r)}$ represents the unwanted positive deviation from this target and $\alpha_k^{(4r)}$the negative deviation from the target, which is not to be penalised. Similarly to the absolute poverty case, a strict interpretation of the concept of relative poverty requires a single AR_k value, that is a universal definition of poverty level across the sets of goals to be compared. A further tightening of the interpretation could include setting ρ_{4r}=1, which enforces the condition that all goals relevant to poverty must be considered when determining whether an individual goal falls below the poverty line or not.

3.3 Formulation Incorporating Absolute and Relative Poverty

The above considerations lead to an augmented meta goal programming model with five principal types of meta-goals, termed a meta-goal programming with poverty incorporation (MGPPI). The algebraic formulation of the MGPPI is given by (2)-(11), (12)-(13), (14)-(16) and (17)-(20):

$$Min\ a = \sum_{k=1}^{\rho_1} \upsilon_k^{(1)}\beta_k^{(1)} + \sum_{k=1}^{\rho_2} \upsilon_k^{(2)}\beta_k^{(2)} + \sum_{k=1}^{\rho_3} \upsilon_k^{(3)}\beta_k^{(3)} + \sum_{k=1}^{\rho_{4a}} \upsilon_k^{(4a)}\beta_k^{(4a)} + \sum_{k=1}^{\rho_{4r}} \upsilon_k^{(4r)}\beta_k^{(4r)} \quad (17)$$

Subject to,

Equations and sign restrictions (2)-(16)

$$\alpha_k^{(4a)}, \beta_k^{(4a)} \geq 0, k = 1, \dots, \rho_{4a}, \qquad \alpha_k^{(4r)}, \beta_k^{(4r)} \geq 0, \qquad k = 1, \dots, \rho_{4r}, \quad (18)$$

$$s_{jk} 0\ or\ 1, j \in S_k^{(4r)}, k = 1, \dots, \rho_{4r}, \qquad z_{ik} = 0\ or\ 1, i \epsilon S_k^{(4a)}, \qquad k = 1, \dots, \rho_{4a}, \quad (19)$$

$$nr_j, pr_j \geq 0, \qquad j \in S_k^{(4r)} \quad (20)$$

The MGPPI achievement function (17) now contains five terms, each representing a penalisation of the sum of unwanted deviations relating to a type of meta-goal. The relative importance of each meta-goal is given by its set of related weights, defined as follows:

- $\upsilon_k^{(1)}$ is the relative weight given to penalisation of the unwanted positive deviation, $\beta_k^{(1)}$, from the k'th meta-goal pertaining to the minimisation of the sum of unwanted deviations from the goals in set $S_k^{(1)}$, $k = 1, \dots, \rho_1$
- $\upsilon_k^{(2)}$ is the relative weight given to penalisation of the unwanted positive deviation, $\beta_k^{(2)}$, from the k'th meta-goal pertaining to the minimisation of the maximal, unwanted deviation from amongst the goals in set $S_k^{(2)}$, $k = 1, \dots, \rho_2$

- $\upsilon_k^{(3)}$ is the relative weight given to penalisation of the unwanted positive deviation, $\beta_k^{(3)}$, from the k'th meta-goal pertaining to the minimisation of number of unmet goals in set $S_k^{(3)}$, $k = 1, \dots, \rho_3$
- $\upsilon_k^{(4a)}$ is the relative weight given to penalisation of the unwanted positive deviation, $\beta_k^{(4a)}$, from the k'th meta-goal pertaining to the percentage of goals not meeting the absolute poverty level in set $S_k^{(1)}$, $k = 1, \dots, \rho_{4a}$
- $\upsilon_k^{(4r)}$ is the relative weight given to penalisation of the unwanted positive deviation, $\beta_k^{(4r)}$, from the k'th meta-goal pertaining to the percentage of goals not meeting the relative poverty level in set $S_k^{(1)}$, $k = 1, \dots, \rho_{4r}$

The above five sets of relative weights can be set in an *a priori* manner using preferential information elicited from the decision maker or in an *a posteriori* manner using a weight sensitivity analysis algorithm such as the one given by Jones, 2011 in order to develop a set of solutions sufficient to inform the decision maker of the set of trade-offs between meta-goals.

4. Formation of Budget Planning Test Set

In order to assess the effectiveness of the MGPPI model (17)-(20) against other goal programming variants, it is necessary to develop a test-set of goal programmes. The field of financial budget allocation is chosen for the test set as it involves the fair distribution of financial resource amongst a set of entities, who are assumed to be of equal importance which leads to a clear definition of poverty levels. Examples of applications of goal programming for financial budget allocation are given by Bojd and Koosha, 2018 who use robust goal programming in order to solve the capital budgeting problem and Turgay and Taskin, 2015 who use fuzzy goal programming to allocate resources to departments of a hospital under a limited overall budget.

The financial budget allocation model involves the assignment of funds to an entity dependent on its scores over a set of n attributes. Each entity i has a goal of not receiving lower than its current budget, t_i. There is also an overall budget, B, to be respected. The above considerations lead, in its simplest form, to the following goal programme:

$$Min\ a = \sum_{i=1}^{q} \frac{n_i}{t_i} \tag{21}$$

Subject to,

$$\sum_{j=1}^{n} a_{ij}x_j + n_i - p_i = t_i, \qquad i = 1, \dots, q \ , \qquad (22)$$

$$\sum_{i=1}^{q}\sum_{j=1}^{n} a_{ij}x_j = B \ , \qquad (23)$$

$$n_i, p_i \geq 0, \qquad i = 1, \dots, q; \quad x_j \geq 0, \qquad j = 1, \dots, n \qquad (24)$$

Where a_{ij} is the j'th attribute score of the i'th entity. n_i and p_i are the respective negative and positive deviations from the budget target, t_i, of the i'th entity. x_j is the j'th decision variable which represents the per unit funding of the j'th attribute, which is required to be non-negative in the context of the problem. The weighted goal programme (21)-(24) is a form of constrained least absolute value regression. It can also be reformulated as any other goal programming variant and further modified to impose hard restrictions on the level of funding received by an entity or the level of per unit funding allowed for an attribute. The achievement function (21) is normalised, to allow for consideration of percentage deviations from each budget, however the normalisation could be omitted if comparison between absolute budgets is required as all deviations are in the same (monetary) units.

Reformulating as a MGPPI with $\rho_i = 1$ and a single set $S_k^{(i)}$ comprising of all negative deviations from funding targets, $i = 1,2,3,4a,4r$ gives the following algebraic formulation:

$$Min\ a = v_1\beta_1 + v_2\beta_2 + v_3\beta_3 + v_{4a}\beta_{4a} + v_{4r}\beta_{4r} \qquad (25)$$

Subject to,

$$\sum_{j=1}^{n} a_{ij}x_j + n_i - p_i = t_i, \qquad i = 1, \dots, q, \quad (26)$$

$$\sum_{i=1}^{q}\sum_{j=1}^{n} a_{ij}x_j = B \ , \qquad (27)$$

$$\sum_{i=1}^{q}\left(\frac{u_i n_i}{|t_i|}\right) + \alpha_1 - \beta_1 = Q_1 \ , \qquad (28)$$

$$\frac{u_i n_i}{|t_i|} \leq \lambda, \qquad i = 1, \dots, q \ , \quad (29)$$

$$\lambda + \alpha_2 - \beta_2 = Q_2 \ , \qquad (30)$$

$$u_i n_i + v_i p_i - M y_i \leq 0, \qquad i = 1, \dots, q \ , (31)$$

$$\sum_{i=1}^{q} \frac{y_i}{q} + \alpha_3 - \beta_3 = Q_3, \qquad (32)$$

$$\frac{wa_i^- n_i}{|t_i|} - Mz_{ik} \le 0, \qquad i = 1, \dots, q, \qquad (33)$$

$$\sum_{i=1}^{q} \frac{z_i}{q} + \alpha_{4a} - \beta_{4a} = Q_{4a}, \qquad (34)$$

$$\frac{1}{q}\sum_{i=1}^{q} \frac{n_i}{|t_i|} - \frac{n_k}{|t_j|} + nr_j - pr_j = 0, \qquad j = 1, \dots, q, \qquad (35)$$

$$nr_j - Ms_j \le AR, \qquad j = 1, \dots, q, \qquad (36)$$

$$\sum_{i=1} \frac{s_j}{q} + \alpha_{4r} - \beta_{4r} = Q_{4r}, \qquad (37)$$

$$x_j \ge 0, j = 1, \dots, n; \qquad n_i, p_i \ge 0, \qquad i = 1, \dots, q, \qquad (38)$$

$$\alpha_1, \beta_1, \alpha_2, \beta_2, \alpha_3, \beta_3, \alpha_{4a}, \beta_{4a}, \alpha_{4r}, \beta_{4r} \ge 0, \qquad (39)$$

$$\lambda \ge 0; \qquad y_i = 0 \; or \; 1, \qquad i = 1, \dots, q, \qquad (40)$$

$$s_j, z_j = 0 \; or \; 1, \quad j = 1, \dots, q; \quad nr_j, pr_j \ge 0, \qquad j = 1, \dots, q \qquad (41)$$

Where the weight set $v_1, v_2, v_3, v_{4a}, v_{4r}$ can be elicited *a priori* from the preferences of the decision maker, with some instances given below. The test set involves 30 instances of a 100-school budget setting problem, based around a transition to a common funding approach as is currently proposed in the United Kingdom. There are 14 attributes by which the funding can be allocated defined as follows:

- Number of pupils in the school (x_1)
- School catchment area type(Rural (x_2), Suburban(x_3), Urban (x_4))– 3 binary attributes defining if the school belongs to the catchment area type category or not
- School area median income (Low(x_5), Medium(x_6), High(x_7)) - 3 binary attributes defining if the school belongs to the catchment area income category or not

- Percentage of pupils that that do not achieve their required educational targets at age 14 (x_8).
- Predicted population growth in the catchment school's area (x_9).
- Percentage of pupils for whom English is not their first language (x_{10})
- Percentage of students who receive free school meals (sometime used as a proxy for deprivation in the United Kingdom) (x_{11})
- Economic status of the local economy to which the school belongs (growing(x_{12}), stable(x_{13}), declining(x_{14})) – 3 binary attributes defining if the school falls into a category or not.

The target values for each school (t_i) are set equal to pre-funding formula levels in order to minimise budgetary turbulence. The data is synthetic, generated by statistical sampling, as actual schools data is sensitive and hence confidential. The data for all instances can be found at the following DOI link: 10.17029/da2c808e-464f-4905-b747-84a4945fcd91 in Microsoft Excel format. The poverty level is set at 80% of the target level (absolute) and 80% of the average achieved level (relative). This is set at this level as any deviation beyond 80% is likely to cause considerable challenge to the institution involved. The total budget B is set at 95% of the current cumulative budgets, to reflect pressure on educational financial budgets. The meta-goal targets are set at strict levels of $Q_1 = Q_2 = Q_{4a} = Q_{4r} = 0.01$ in order to ensure trade-offs between the achievements of the meta-goals are generated.

5. Experimental Results and Discussion

In order to assess the effectiveness of the MGPPI formulation, all instances from the test-set are formulated as the following goal programming variants and solved to optimality using the LINGO 17.0 package on a 3.1GHz PC with 4GB of RAM.

- WGP: A standard weighted goal programming model (21)-(24).
- WGP-PF: A weighted goal programming model with a penalty function (Jones and Tamiz, 1995) that doubles the per unit penalty for negative deviations beyond 80% of the target value.
- CGP: A Chebyshev goal programming model
- MGP: A standard meta goal programming model with equal weighting of the three meta goals
- MGPPI-EW: An MGPPI model with meta weights $v_1 = 0.25, v_2 = 0.25, v_3 = 0, v_{4a} = 0.25, v_{4r} = 0.25$
- MGPPI-AP: An MGPPI model with meta weights $v_1 = 0.0667, v_2 = 0.0667, v_3 = 0, v_{4a} = 0.80, v_{4r} = 0.0667$

- MGPPI-RP: An MGPPI model with meta weights $v_1 = 0.0667, v_2 = 0.0667, v_3 = 0, v_{4a} = 0.0667, v_{4r} = 0.80$

The three MGPPI are designed to represent an emphasis for equal-weighting of meta-goals (MGPPI-EW), an emphasis on the reduction of absolute poverty (MGPPI-AP) and an emphasis of the reduction of relative poverty (MGPPI-RP). The respective emphases on absolute and relative poverty are implemented by setting the corresponding weight (v_{4a} or v_{4r} respectively) to the value of 0.8. The remaining 0.2 of weight is divided equally between the other three relevant weights v_1, v_2 and either v_{4r} or v_{4a} respectively, each thus taking the value 0.0667. These weighting scheme is chosen in order to investigate the raising of one meta-goal to a sufficient level (0.8) in order to give it a heavy degree of importance relative to the other meta-goals, but not at so high a level that the problem effectively degenerates into a single criterion model. The MGPPI-AP and MGPPI-RP weighting schemes are designed to show sufficient variance from the equally weighted MGPPI-EW scheme and each other. The results in the last three columns of Table 1 demonstrate the effectiveness of this weighting scheme design, with three different levels recorded for each of the measures given by the rows. Further analysis of the individual differences between the MGPPI-AP, MGPPI-RP and MGPPI-EW solutions and their implications is given later in this Section. All thee MGPPI models do not consider the number of goals achieved meta-goal, i.e. $v_3 = 0$, as it is not relevant in the context of the application. The resulting solutions are measured with respect to the following criteria:

- ABSPOV: The proportion of schools that are in absolute poverty with respect to the 80% criterion.
- RELPOV: The proportion of schools that are in relative poverty with respect to the 80% criterion
- WORSTCASE: The proportional change in budget for the worst-affected school
- BESTCASE: The proportional change in budget for the best-affected school
- SUMSHORTFALL: The cumulative change in budget for all schools.
- SOLTIME: The solution time (in seconds)

Table 1 gives the average values over the 30 instances of each of the above criteria for the seven goal programming variants considered, rounded to three decimal places.

Table 1: Average values of criteria over 30 instances of 100-school budget setting problem

Criterion	WGP	WGP - PF	CGP	MGP	MGPPI - EW	MGPPI - AP	MGPPI - RP
ABSPOV	0.094	0.088	0.106	0.102	0.068	0.045	0.083
RELPOV	0.062	0.057	0.073	0.068	0.044	0.048	0.036
WORSTCASE	-0.299	-0.298	-0.278	-0.279	-0.289	-0.304	-0.293
BESTCASE	0.390	0.389	0.317	0.318	0.329	0.413	0.349
SUMSHORTFALL	-6.670	-6.673	-6.985	-6.894	-6.959	-6.863	-7.089
SOLTIME(s)	0.129	0.158	0.152	0.193	10.182	12.361	11.391

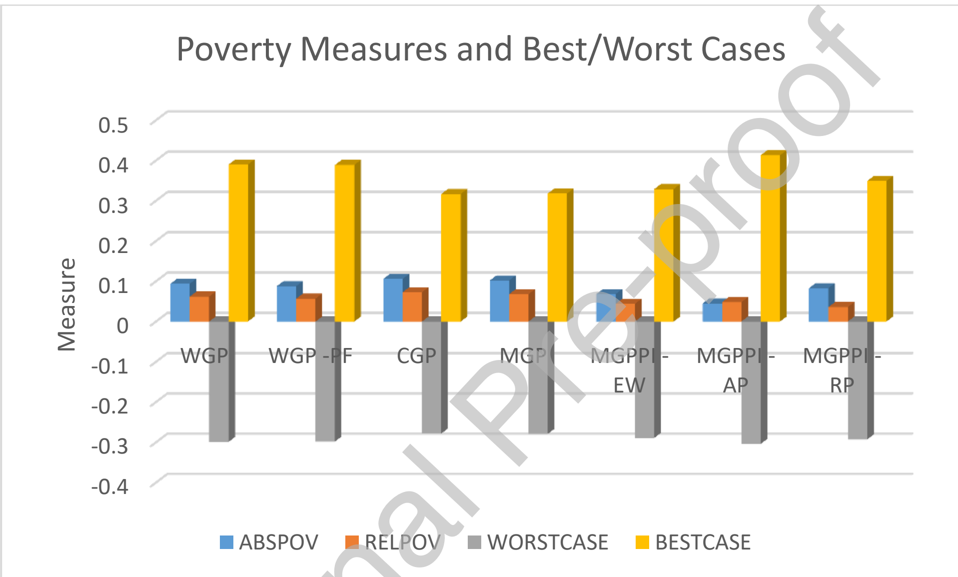

Figure 1: Graphical Representation of Absolute and Relative Poverty and Best/Worst Cases

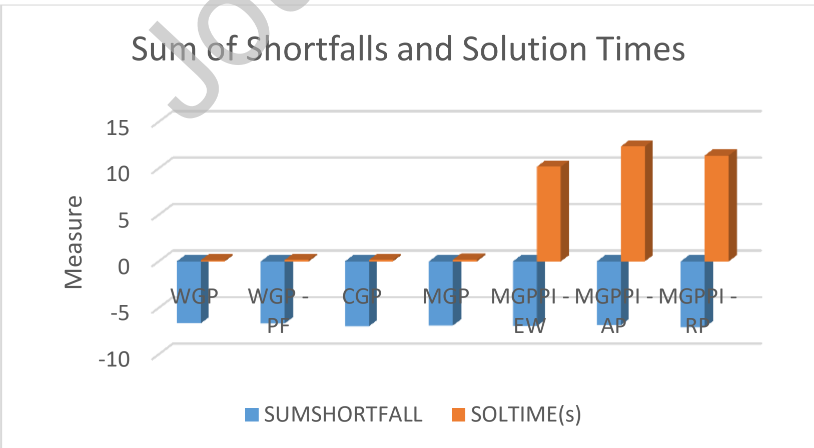

Figure 2: Sum of Budget Shortfalls and Solution Times

Firstly, it can be observed that the level of schools in absolute and relative poverty when using classic weighted goal programming (WGP) is 9.4% and 6.2% respectively. The lower figure for relative as opposed to absolute poverty is explained by the fact that the average school is slightly below its goal (as shown by the SUMSHORTFALL row), thus making the achievement of the relative poverty level easier that of its absolute poverty counterpart. Utilising a penalty function (WGP-PF) somewhat improves these values, by 6.4% for absolute and 12.4% for relative poverty respectively, however the improvement is not of the level shown by the poverty-incorporating goal programming variants in the three rightmost columns of Table 1. Utilising Chebyshev goal programming (CGP) is worse than the classical weighted (WGP) variants in respect to the poverty measures, showing a degradation of 8.5% and 17.7% of the ABSPOV and RELPOV values respectively. This is traded off against a 7.2% relative improvement of the worst affected school (WORSTCASE), from -0.298 to -0.278. This indicates that concentrating on the worst affected entity, i.e. utilizing the L_∞ distance metric, is not in itself an effective measure for poverty reduction amongst a set of entities. In fact, in the 100-school instance test-set a trade-off between the improvement of the worst entity and the achievement of poverty targets for entities between the 3rd and 21st funding loss percentiles (the minimum and maximum percentage of entities reaching a poverty target across all entities) has been demonstrated. Utilising the standard meta-goal programming variant (MGP), with its mix of average (L_1) and extreme case (L_∞) meta-goals does not improve on the standard WGP model for the poverty measures either, although the degradation is less than in the CGP case, at 8.5% and 9.7% for absolute and relative poverty respectively.

The MGPPI-EW variant gives a substantive reduction in the levels of the entities not achieving the poverty targets as compared to any of the four non poverty-incorporating variants to the left of Table 1. Compared to the classic WGP model, it achieves a 27.7% and 22.6% improvement on the ABSPOV and RELPOV targets respectively. Against the best performing non-poverty incorporating variant with respect to the poverty targets (WGP-PF), it achieves a 22.7% and 22.8% improvement on the ABSPOV and RELPOV targets respectively. The trade-offs for this poverty improvement are a 4.3% worsening in the overall shortfall (SUMSHORTFALL) and an increase in solution time of approximately two orders of magnitude. Concentrating on absolute poverty reduction via the MGPPI-AP variant leads to a further 33.8% reduction in the ABSPOV level, however at the cost of a 9.1% worsening in the RELPOV level. The total reduction in ABSPOV over the standard WGP variant is 52.1%. Concentrating on relative poverty reduction via the MGPPI-RP variant leads to a further 18.2% reduction in the RELPOV level, however at the cost of a 22.1% worsening in the ABSPOV level. The total reduction in RELPOV over the standard WGP variant is 41.9%. Thus, whilst all three poverty-incorporating variants improve both poverty levels as compared to any of the four non poverty incorporating variants, a definite trade-off between the achievement of ABSPOV and RELPOV can be seen amongst the three poverty incorporating models, controlled by the setting of the v_{4a} and v_{4r} meta-weights.

With regard to solution times, similar comments apply to MGPPI-AP and MGPPI-RP as for MGPPI-EW. The solution time for the three poverty incorporating models on average increased by a factor of 71.5 as compared with the four non-poverty incorporating variants. This is caused by the addition of two binary variables per entity in order to model absolute and relative poverty. It is not an issue in the case of the strategic budget allocation models reported in this paper but may be a matter for consideration if the field of application is (i) operational, (ii) related to harder combinatorial models such as vehicle routing or scheduling or (iii) contains significantly larger numbers of entities than the 100-school instances in this paper.

Table 2: Pairwise dominance levels for the ABSPOV measure

ABSPOV	WGP	WGP -PF	CGP	MGP	MGPPI - EW	MGPPI - AP	MGPPI - RP
WGP	-	0	20	18	*0*	*0*	4
WGP-PF	15	-	21	20	*0*	*0*	6
CGP	5	1	-	2	*0*	*0*	2
MGP	5	2	12	-	*0*	*0*	2
MGPPI-EW	26	24	29	29	-	*0*	18
MGPPI-AP	30	30	30	30	29	-	29
MGPPI-RP	19	12	24	22	*2*	*0*	-

Tables 2 and 3 give the level of dominance of one goal programming model over another across the set of 30 instances for the ABSPOV and RELPOV measures respectively. Dominance is defined as when the row model achieves a strictly better value for the statistic than the column model. For instance, the value of 20 in the WGP row and CGP column of Table 2 indicates that the WGP model achieves a better ABSPOV level for 20 out of the 30 instances. Conversely, the CGP row and WGP column entry of Table 2 shows that the CGP model achieves a strictly better ABSPOV value in 5 of the 30 instances. By implication, the WGP and CGP models achieved the same ABSPOV value in the remaining 5 instances. For the poverty incorporating models, values in bold indicate the number of instances that follow the expected dominance trend suggested by the average values of Table 1 whereas values in italics indicate the number of instances that are contrary to that trend. The ABSPOV values in Table 2 show less instances that are trend contrary and more that are trend conforming than the RELPOV values in Table 3. In particular, the MGPPI-AP model that specifically concentrates on absolute poverty reduction shows better ABSPOV values for all 30 instances compared to the non-poverty models and 29 out of 30 instances for the other poverty-reducing models, with zero instances contrary to the trend. Table 3 shows that the MGPPI-RP model that specifically concentrates on relative poverty reduction also achieves a good performance against the non-poverty models (27-29 instances out of 30 with better RELPOV values with zero instances contrary to trend) but slightly less against the other poverty reducing models (18-20 instances out of 30 with better RELPOV values with 3 instances contrary to trend). The weakest result concerning the poverty-incorporating models is the RELPOV pairwise comparison between MGPPI-EW and MGPPI-AP with 11 instances following the trend, 11 instances equal for both models and 8 instances contrary to trend. This indicates the framework is less strong at achieving relative poverty targets when relatively lower weight (v_{4r}) is placed on the relative poverty meta-goal. The above discussions show that whilst achieving better levels of entities achieving relative poverty targets is shown in Table 1 to be easier than for absolute poverty targets, care must be taken to assign sufficient weight to relative poverty if this measure is of concern to the decision maker due to the deterioration of the measure at equal-weight and lower values.

Table 3: Pairwise dominance levels for the RELPOV measure

RELPOV	WGP	WGP -PF	CGP	MGP	MGPPI - EW	MGPPI - AP	MGPPI - RP
WGP	-	4	18	15	*2*	3	*0*
WGP-PF	10	-	20	18	*1*	4	*0*
CGP	8	5	-	4	*0*	2	*0*
MGP	8	4	12	-	*0*	2	*0*
MGPPI-EW	22	20	27	26	-	11	*3*
MGPPI-AP	24	21	24	23	*8*	-	*3*
MGPPI-RP	28	27	29	29	20	18	-

Table 4 gives the per unit funding x_j values for all seven models for instance 3 (chosen as it broadly follows the average trends), i.e. the solutions in decision space. The complete set of values for all models and instances in decision and objective space can be found at the following DOI link: 10.17029/da2c808e-464f-4905-b747-84a4945fcd91.

Table 4: Decision variable values for instance 3.

Variable	WGP	WGP-PF	CGP	MGP	MGPPI-EW	MGPPI-AP	MGPPI-RP
x_1	3.98	3.98	3.92	3.97	3.97	3.78	3.96
x_2	0.00	0.00	50.45	0.00	0.00	107.41	0.00
x_3	79.04	76.89	0.00	0.00	0.00	0.00	7.97
x_4	0.00	0.00	98.38	93.76	93.76	60.13	40.68
x_5	19.06	19.64	187.04	206.04	206.04	184.70	208.14
x_6	0.00	0.00	0.00	0.00	0.00	0.00	0.00
x_7	15.71	15.10	0.00	0.00	0.00	0.00	0.00
x_8	0.00	0.00	0.00	0.00	0.00	1.12	0.91
x_9	0.00	0.00	1.68	0.99	0.99	0.00	0.45
x_{10}	10.55	11.52	2.51	1.39	1.39	1.27	3.15

x_{11}	0.00	0.00	0.00	0.00	0.00	4.95	0.00
x_{12}	0.00	0.00	0.00	0.00	0.00	0.00	0.00
x_{13}	52.08	49.79	67.67	39.27	39.27	275.16	77.88
x_{14}	48.59	0.00	0.00	0.00	0.00	0.00	0.00

Table 4 demonstrates that the models have allocated the funding according to significantly different rationales. The Table should be interpreted by considering the change in magnitude of the values across variants rather than their absolute magnitude as the units of measurement differ. All variables except for x_6 (medium income area) and x_{12} (growing local economy) are used by one of the models. The poverty incorporating models allocate significantly more funding into schools via the low income area x_5 and urban area x_4 categories as compared with the standard WGP model. The MGPPI-AP and MGPPI-RP models are the only two that allocate some funding according to the number of students not reaching the required educational standard x_8 variable. Conversely, the WGP allocates more funding than the poverty incorporating models in the x_3(suburban), x_7(high income), x_{10} (free school meal number) and x_{14} (declining local economy) categories.

Table 5, which illustrates the percentage differences in budgets for all schools, also shows a significance difference in the budgetary allocations between the WGP and the poverty incorporating models and also within the three poverty incorporating models themselves. Although mostly the movement (negative or positive) from the target is in the same direction for all models, its magnitude can differ significantly. Consider the first school, for example, its budget is exactly met for the WGP model but has an increase of up to 37% in the MGPPI-AP model. In general, the MGPPI-AP model allows for greater positive outliers than its MPPPI-RP counterpart, as the positive outliers can make the relative poverty targets harder to achieve by contributing towards an increased target.

Table 5: Percentage deviation from budgets for all schools (Ent).

Ent	WGP	EW	AP	RP	Ent	WGP	EW	AP	RP
1	0.00	0.15	0.37	0.09	51	0.04	0.02	0.00	0.03
2	-0.02	-0.10	-0.09	-0.08	52	0.05	0.04	0.06	0.06
3	0.00	0.01	0.07	0.01	53	0.26	0.19	0.18	0.24
4	-0.06	-0.10	-0.12	-0.08	54	0.00	-0.02	-0.03	-0.01
5	0.04	-0.01	-0.01	0.01	55	0.09	0.09	0.13	0.08
6	-0.03	-0.06	-0.08	-0.07	56	-0.15	-0.17	-0.18	-0.16
7	-0.15	-0.08	-0.08	-0.10	57	-0.13	-0.13	-0.11	-0.13
8	-0.10	0.02	0.02	-0.03	58	-0.20	-0.19	-0.20	-0.19
9	0.07	0.10	0.07	0.07	59	-0.16	-0.15	-0.16	-0.14
10	-0.23	-0.19	-0.20	-0.19	60	-0.15	-0.13	-0.14	-0.15
11	0.03	0.03	0.02	0.02	61	-0.29	-0.23	-0.18	-0.24
12	-0.21	-0.23	-0.24	-0.22	62	-0.10	-0.13	-0.15	-0.11
13	-0.09	-0.09	-0.15	-0.13	63	-0.06	-0.03	0.03	-0.04
14	0.18	0.26	0.19	0.20	64	0.01	0.14	0.12	0.00
15	-0.11	-0.16	-0.14	-0.15	65	-0.09	-0.07	-0.10	-0.12
16	-0.02	-0.07	-0.07	-0.06	66	-0.15	-0.14	-0.15	-0.14

17	0.11	0.13	0.21	0.11	67	0.00	0.07	0.04	-0.01
18	0.12	0.06	0.06	0.09	68	-0.16	-0.06	-0.07	-0.15
19	0.09	0.10	0.11	0.13	69	-0.28	-0.20	-0.20	-0.24
20	-0.03	0.00	-0.03	-0.03	70	0.09	0.26	0.25	0.12
21	-0.08	-0.12	-0.13	-0.10	71	0.08	0.10	0.08	0.08
22	-0.01	-0.06	-0.06	-0.04	72	-0.18	-0.16	-0.17	-0.17
23	0.02	0.12	0.13	0.10	73	0.05	0.10	0.09	0.09
24	-0.01	-0.07	-0.08	-0.03	74	-0.13	-0.16	-0.19	-0.16
25	0.01	-0.02	-0.04	0.00	75	0.00	-0.08	-0.06	-0.07
26	-0.22	-0.21	-0.20	-0.19	76	-0.13	-0.14	-0.17	-0.15
27	-0.24	-0.19	-0.20	-0.24	77	-0.28	-0.26	-0.29	-0.27
28	-0.20	-0.20	-0.19	-0.18	78	0.09	0.09	0.07	0.09
29	-0.01	0.02	0.09	-0.01	79	-0.15	-0.12	-0.14	-0.15
30	-0.11	-0.14	-0.03	-0.13	80	-0.13	-0.14	-0.11	-0.14
31	0.13	0.11	0.09	0.14	81	-0.15	-0.17	-0.18	-0.16
32	0.12	0.08	0.05	0.07	82	0.00	0.06	0.05	0.00
33	-0.13	-0.13	-0.16	-0.13	83	0.06	0.08	0.07	0.05
34	-0.10	-0.12	-0.14	-0.12	84	0.05	0.16	0.09	0.10
35	0.00	0.26	0.39	0.27	85	-0.01	-0.03	-0.06	-0.04
36	0.08	0.04	0.04	0.06	86	-0.05	-0.10	-0.10	-0.07
37	-0.03	-0.08	-0.10	-0.05	87	0.14	0.14	0.13	0.14
38	0.06	0.09	0.20	0.04	88	0.09	0.04	0.04	0.07
39	0.02	0.18	0.17	0.03	89	-0.06	-0.03	0.01	-0.03
40	0.12	0.09	0.05	0.07	90	-0.21	-0.22	-0.20	-0.23
41	-0.25	-0.25	-0.27	-0.24	91	0.32	0.26	0.23	0.25
42	0.12	0.11	0.08	0.11	92	-0.31	-0.19	-0.19	-0.24
43	0.00	-0.05	-0.05	-0.02	93	0.08	0.02	0.07	0.03
44	0.04	0.11	0.23	0.12	94	0.00	0.22	0.25	0.13
45	0.00	-0.08	-0.09	-0.04	95	0.11	0.12	0.15	0.11
46	-0.06	-0.07	-0.06	-0.04	96	-0.21	-0.26	-0.26	-0.24
47	-0.18	-0.19	-0.18	-0.18	97	-0.22	-0.23	-0.20	-0.23
78	-0.09	-0.20	-0.15	-0.20	98	0.01	0.06	0.05	0.02
49	-0.27	-0.20	-0.20	-0.22	99	0.18	0.14	0.11	0.12
50	-0.12	-0.12	-0.11	-0.10	100	0.17	0.16	0.14	0.17

6. Conclusions

This paper has shown that a concept from the field of poverty measurement and reduction, the existence of relative and absolute poverty targets, can be transferred into the Operational Research paradigm and used to add modelling flexibility to the technique of goal programming. In doing so, this paper follows the long tradition of utilisation of social science concepts in the multiple criteria decision making field, as detailed in Sections 1 and 2. Of course, the definition of "poverty" is dependent on the field of application and should not be confused in moral equivalence terms with the alleviation of actual physical and material poverty, unless the field of application concerns these concepts. Nevertheless, the concepts of absolute and relative poverty thresholds have shown to be valuable in lowering the number of low achieving entities in goal programming models pertaining to distribution and allocation of resources, such as the school budget problem detailed in Section 3. The poverty-incorporating models developed in this paper are shown in Section 4 to prove more effective in reducing absolute (ABSPOV) and relative (RELPOV) measures than a range of other current goal programming variants that do not explicitly consider poverty. Meta-goal programming is found to be a good variant within which to incorporate poverty measures as it allows for the inclusion of target-based meta-goals that can model advanced concepts arising from the standard set of goals. The usage of corresponding meta-weights is shown in Section 4 to be effective in controlling and elaborating the trade-off between absolute and relative poverty. The goal programming weights in this paper have been chosen with the aim of demonstrating equal treatment between entities whilst the meta-weights were set as appropriate to demonstrate an equal mix or an emphasis towards a particular meta-goal. However, in some situations further weight sensitivity analysis may be required, in which case the use of a weight sensitivity algorithm such as Jones (2011) in either weight or meta-weight space, or both, is recommended

The natural synergies between goal programming and the poverty targets considered in this paper have facilitated the construction of the models in Section 2. However, there exist further possibilities of further research into the usages of different forms of multiple criteria decision making and different poverty measures in order to strengthen the modelling flexibility of the multi-criteria field. Furthermore, this paper should act as an encouragement for continued research into the incorporation of traditional and arising techniques from the field of social science into the specific technique of goal programming and the more general field of multiple-criteria decision making.

It is also hoped that the open access test set of 30 instances of the 100-school education model can be used by researchers wishing to test new goal programming or multi-objective models and algorithms. As the underlying statistical assumptions are the same, the instances can be combined to form a smaller set of larger instances if required.

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Author Statement

DF Jones: Development of goal programming models, R Treloar: Development of educational test model set; D Ouelhadj: Co-design and solution of mathematical models, A Glampedakis: Integration of poverty principles, P. Bartmeyer: co-design and solution of models.

Conflicts of Interest

Conflicts of Interest: None