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The size of our causal Universe

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ABSTRACT

A Universe with finite age also has a finite causal scale. Larger scales cannot affect our local measurements or modelling, but far away locations could have different cosmological parameters. The size of our causal Universe depends on the details of inflation and is usually assumed to be larger than our observable Universe today. To account for causality, we propose a new boundary condition, that can be fulfill by fixing the cosmological constant (a free geometric parameter of gravity). This forces a cancellation of vacuum energy with the cosmological constant. As a consequence, the measured cosmic acceleration cannot be explained by a simple cosmological constant or constant vacuum energy. We need some additional odd properties such as the existence of evolving dark energy (DE) with energy-density fine tuned to be twice that of dark matter today. We show here that we can instead explain the current cosmic acceleration without DE (or modified gravity) as a the result of a primordial inflation with a causal scale smaller than the observable Universe today. Such scale corresponds to half the sky at $z = 1$ and 60 deg at $z = 1100$, which is consistent with the anomalous lack of correlations observed in the CMB.

Key words: cosmic background radiation – cosmological parameters – dark energy – early Universe – inflation.

1 INTRODUCTION

One of the most striking changes to Newton's gravity proposed by Einstein is that energy gravitates. Scientists have since been wondering if vacuum energy $\rho_{\rm vac}$ (vacuum fluctuations, zero-point fluctuations, quantum vacuum, dark energy (DE), or aether) could also gravitate. Measurements of cosmic acceleration (see e.g. Tutusaus et al. 2017; Planck Collaboration VI 2018; Abbott et al. 2019) point to a model with Λ that we refer to as lambda cold dark matter (ΛCDM) and could be interpreted as a consequence of the gravity of $\rho_{\rm vac}$. Even while the accuracy and precision of measurements have greatly improved in the last years, the mean values of cosmological parameters have remained similar for well over a decade (see e.g. Gaztañaga, Manera & Multamäki 2006; Gaztañaga, Miquel & Sánchez 2009). The Friedmann–Lemaitre–Robertson–Walker (FLRW) flat metric in comoving coordinates (t, χ)

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a(t)^2 (d\chi^2 + \chi^2 d\Omega^2) \quad (1)$$

is the exact general solution for a mathematically homogeneous and isotropic flat Universe. The scale factor, $a(t)$, describes the expansion of the Universe as a function of time. We can relate $a(t)$ to the energy content of the Universe for a perfect fluid by solving the field equations (see equations 9–10)

$$R_0^0 = R_{00} = -\left(\frac{3\ddot{a}}{a}\right) = 4\pi G(\rho + 3p) - \Lambda \quad (2)$$

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}, \quad (3)$$

where $\rho = \rho_{\rm m} a^{-3} + \rho_{\rm r} a^{-4} + \rho_{\rm vac}$ and $\rho_{\rm m}$ is the pressureless matter density today ($a = 1$), $\rho_{\rm r}$ corresponds to radiation (with pressure $p_{\rm r} = \rho_{\rm r}/3$) and $\rho_{\rm vac}$ represents vacuum energy ($p_{\rm vac} = -\rho_{\rm vac}$).[1] One can argue that Λ is indistinguishable from $\rho_{\rm vac}$, because field equations are degenerate to the combination

$$\rho_\Lambda \equiv \rho_{\rm vac} + \frac{\Lambda}{8\pi G}. \quad (4)$$

Here, we take Λ to be a fundamental (geometrical) constant, while $\rho_{\rm vac}$ depends on the actual energy content of our universe. The measured ρ_Λ is very small compared to what we expect for $\rho_{\rm vac}$. Moreover, $\rho_\Lambda \simeq 2.3\rho_{\rm m}$ today, which seems a remarkable coincidence. Possible solutions to this puzzle are: (i) $\Lambda = 0$ and ρ_Λ originates only from $\rho_{\rm vac}$ or some DE (Weinberg 1989; Elizalde & Gaztañaga 1990; Carroll, Press & Turner 1992; Huterer & Turner

[1] For easy of notation we focus on the flat case, but our results can easily be extended to the non-flat case or non-trivial topology.

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1999; Elizalde 2006), (ii) $\rho_{\rm vac}=0$ and we need to fix Λ or modified gravity (Gaztañaga & Lobo 2001; Gaztañaga et al. 2002; Lue, Scoccimarro & Starkman 2004; Nojiri, Odintsov & Oikonomou 2017), or (iii) there is a cancellation between Λ and $\rho_{\rm vac}$, as we will propose here.

Equations (1)–(3) are a mathematical extrapolation. A physical model requires a mechanism to produce homogeneity that respects causality. Note that the Lorentz invariance or covariance of the field equations in general relativity (GR) is not enough to warrant that a given solution has a causal structure (see Minguzzi & Sanchez 2006; Howard 2010 and references therein). This is not the case for the FLRW model, because ρ and p are the same everywhere at any fix cosmic time, and this cannot have causal explanation for a Universe that has a finite age. If we want a causal explanation for our Universe we cannot find the solution directly solving the Cauchy problem in GR, because we are not allowed to setup a causal initial conditions for such a problem. The only way to do this is to setup initial conditions that are random with no correlations. This is what happens in the realm of vacuum quantum fluctuations under Heisenberg uncertainty, which do not require a causal mechanism to exist. Inflation could them produce a large and homogeneous universe out of these initial quantum fluctuations. But even for such a case, there is a finite causal scale associated with inflation.

Particles separated by distances larger than the comoving Hubble radius $d_{\rm H}(t)=c/[a(t)H(t)]$ cannot communicate at time t. Distances larger than the horizon

$$\eta(a) = c\int_0^t \frac{{\rm d}t}{a(t)} = \int_0^a {\rm d}\ln(a)\, d_{\rm H}(a), \tag{5}$$

have never communicated. We know from the cosmic microwave background (CMB) and large-scale structure (LSS) that the Universe was very homogeneous on scales that were not causally connected (without inflation). This either means that the initial conditions where a causally smooth to start with or that there is a mechanism like inflation (Liddle 1999; Dodelson 2003; Brandenberger 2017) which inflates regions outside the Hubble radius. During inflation, $d_{\rm H}$ decreases which freezes out communication on comoving scales larger than the horizon $\chi_\S \simeq \eta(a_i)=d_{\rm H}(t_i)$ when inflation begins, at $a_i=a(t_i)$. Inflation also smoothed and stretches out any initial (quantum) inhomogeneities to scales that could be larger than our current horizon. This creates homogeneous and flat patches. When inflation ends, radiation from reheating makes $d_{\rm H}$ grow again. Thus, the scale $\chi_\S$ is fixed before inflation in comoving coordinates and is the same for all times, while the horizon η and $d_{\rm H}$ change with time. This is illustrated in Fig. 1. Inflation allows the full observable Universe to originate from a very small causally connected homogeneous patch, $\chi_\S$, which could be as small as the Planck scale. We usually assume that this region $\chi_\S$ is much larger than our observable universe today: $\simeq 3c/H_0$. As we approach our epoch (label 'now' in the figure), we believe that a mysterious DE produces a second inflation that makes $d_{\rm H}$ decrease again. Why a second inflation now? Are both inflations related?

Regardless of the details of inflation, a Universe of finite age will only be causally homogeneous for scales smaller than some cut-off $\chi<\chi_\S$. We need a boundary condition at $\chi=\chi_\S$ to account for the lack of causality (and therefore homogeneity) at larger scales. This results in a cancellation between Λ and $\rho_{\rm vac}$ and could be the cause of the current cosmic acceleration. In Section 2, we view this problem in Classical Physics, while in Section 3, we present a relativistic version. In Section 4, we estimate the size of the causal Universe and discuss the implications for inflation and CMB. We end with some discussion and conclusions.

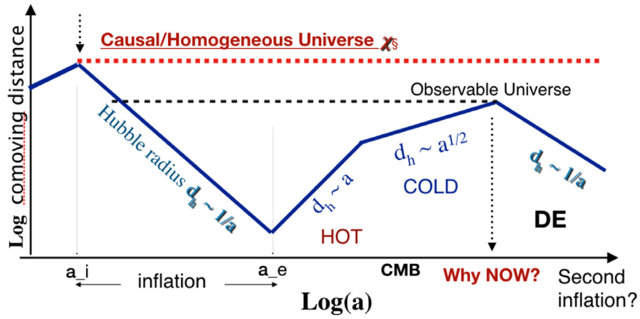

Figure 1. Comoving Hubble radius $d_{\rm H}=c/(aH)$ (blue line) as a function of the scale factor a. The causal Universe $\chi_\S$ is identified with the region inside the largest causally connected scale at the beginning of inflation (red dashed line). A mysterious DE produces a second inflation around our time (label 'now' in the figure) that makes $d_{\rm H}$ decrease again.

2 GAUSS'S LAW

A key property of Gravity is Gauss's law. The acceleration $\vec{\rm g}$ created by a point mass at distance $\vec{\rm r}$ is such that a spherical shell of arbitrary radial density $\rho(r)$ produces a field that is identical to a point source of equal mass m in its centre. This condition alone can be used to define gravity in classical mechanics, but the solution is more general than Newton's law (see Appendix A)

$$\vec{\rm g} \equiv -\vec{\nabla}\phi = -\left(\frac{Gm}{r^3}-\frac{\Lambda}{3}\right)\vec{r}$$
$$\nabla^2\phi = 4\pi G\rho_m - \Lambda. \tag{6}$$

Using Stokes theorem, the gravitational field produces a flux around 2D closed surface ∂V

$$\Phi = \oint_{\partial V} {\rm d}\vec{\rm r}\,\vec{\rm g} = -\int_V {\rm d}V\,(4\pi G\,\rho_m - \Lambda) \tag{7}$$

so the flux only depends on the total mass (and Λ) inside the boundary ∂V. Note the second term with Λ in equations (6)–(7) which corresponds to Hooke's law, i.e. proportional to distance. These of course are the same equations that come from GR in the Newtonian limit (see below). One can then argue that Λ is just part of the laws of gravity, as it is allowed by the symmetries of both GR and classical gravity. Current cosmological observations clearly indicate that $\Lambda\neq 0$. There is therefore no need for a search for some more exotic 'DE' explanation. For this to be confirmed, it would good if this interpretation can be used to predict the actual value measured for the cosmic acceleration, which is something that we do in Section 4.2.

Physicist assume that particles should be free at infinity, because of lack of causality. This is why boundary terms are usually neglected at infinity. In the same spirit, we will require here that test particles should be free ($\vec{\rm g}=0$) or more relevant for a fluid: that boundary terms should be zero (i.e. the flux $\Phi=0$), when outside causal contact, $r>r_\S$. For $r_\S \Rightarrow \infty$ this condition requires $\Lambda \Rightarrow 0$, as otherwise $\vec{\rm g}$ and Φ diverge. Observational evidence that $\Lambda\neq 0$ may then indicate that $r_\S$ is finite. This agrees with the finite age of the Universe. From equation (7), the boundary condition $\Phi(r>r_\S)=0$, implies

$$\Lambda = 4\pi G\rho_m(r<r_\S) \tag{8}$$

which is clearly related to the coincidence problem: $\rho_\Lambda \sim 2\rho_m$. Lets next explore this same argument in GR. For this we first need to see what is the relativistic version of equations (6)–(7).

3 RELATIVISTIC CASE

The symmetries of Einstein's field equations allow for a cosmological constant Λ term (Landau & Lifshitz 1971)[2]

$$R^{\nu}_{\mu} + \Lambda\delta^{\nu}_{\mu} = 8\pi\, G \left(T^{\nu}_{\mu} - \frac{1}{2}\delta^{\nu}_{\mu} T\right). \quad (9)$$

For a perfect fluid with density $\bar{\rho}$ and pressure $\bar{p} \equiv \omega\bar{\rho}$

$$T^{\nu}_{\mu} = (\bar{\rho} + \bar{p})u_{\mu}u^{\nu} - \bar{p}\delta^{\nu}_{\mu}, \quad (10)$$

where both $\bar{p}$ and $\bar{\rho}$ could change with space–time. For events comoving with the fluid we have $u^i = 0$ so that $u^{\mu}u_{\mu} = u^0 u_0 = 1$, so that the time–time component of the Ricci curvature is

$$R^0_0 = 4\pi G(\bar{\rho} + 3\bar{p}) - \Lambda \quad (11)$$

3.1 The generalized Gauss's law

Consider perturbations around Minkowski metric $\eta_{\mu\nu}$ (i.e. around empty space)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad (12)$$

where $h_{\mu\nu}$ are small corrections. To linear order in $h_{\mu\nu}$ we have $R_{00} = 1/2\, \Box h_{00}$ (Landau & Lifshitz 1971), so that we can define the gravitational potential as $\phi \equiv h_{00}/2$ to find

$$R_{00} = R^0_0 = \Box\phi \equiv -\nabla_{\mu}\nabla^{\mu}\phi = -\nabla_{\mu}\mathrm{g}^{\mu}, \quad (13)$$

where $\mathrm{g}^{\mu} \equiv \nabla^{\mu}\phi$ is the covariant gravitational acceleration. We can combine this relation with equation (11) to find

$$\Box\phi = 4\pi G\; (\bar{\rho} + 3\bar{p}) - \Lambda \quad (14)$$

which is the relativistic generalization of Poisson equation, equation (6). Thus the relativistic version of Gauss law in equation (7) is then

$$\Phi = \oint_{\partial M} \mathrm{d}x_{\mu}\; \mathrm{g}^{\mu} = -\int_M \sqrt{-g}\;\; \mathrm{d}^4x\;\; [4\pi G\; (\bar{\rho} + 3\bar{p}) - \Lambda], \quad (15)$$

where M is the 4D volume inside the 3D hypersurface ∂M. Traditionally, we take such boundary terms to be zero at infinity: $\Phi(\infty) = 0$ (see equation 4.7.8 in Weinberg 1972).

We can reach a similar expression without the weak field approximation by noticing that the equivalent of Poisson equation is the covariant time–time component of the field equations R^0_0 in equation (11). We can then identify the flux directly with

$$\Phi = -\int_M \sqrt{-g}\;\; \mathrm{d}^4x\;\; R^0_0 \quad (16)$$

which for a perfect fluid gives the same result as equation (15).

3.2 Causal boundary condition

As mentioned in the Section 1, scales larger than $\chi_{\S}$ can have no effect on the metric or the curvature of the universe around us. Mathematically, this appears as retarder Green functions ($\phi(\chi, t) = \phi(\chi - ct)$) as solutions to the wave equation equation (14) (or equation 11) with appropriate boundary conditions (typically $\chi_{\S} \to \infty$). This could result in a non-homogeneous solution for the metric of the Universe on very large scales (Gaztañaga 2020, in preparation). An observer situated at the edge of our causal boundary will find a similar solution, but could measure different cosmological parameters, because she sees a different patch of the initial conditions. There should be a smooth background across disconnected regions with an infrared cutoff in the spectrum of inhomogeneities for $\chi > \chi_{\S}$. Solutions in different regions could be matched as in Sanghai & Clifton (2015).

We usually assume that particles should be free at infinity, because of lack of causality: if there is no cause there should not be any effect. This is why boundary terms are usually set to zero at infinity. For example, to reproduce the field equations equation (9) in GR, from an action principle we need to neglect these boundary terms (e.g. Landau & Lifshitz 1971; Weinberg 1972). On scales $\chi < \chi_{\S}$, we have a homogeneous expanding Universe with $\bar{\rho} = \rho$. On larger scales, we require boundary terms to vanish. In particular, we will require $\Phi(\chi > \chi_{\S}) = 0$ in equation (16), so that there is no flux (i.e. no effects of gravity) beyond the causal scale. This implies

$$\frac{\Lambda}{8\pi G} = \frac{1}{2M_{\S}} \int_{M_{\S}} \sqrt{-g}\mathrm{d}^4x\; (\rho + 3p) \equiv \frac{<\rho>_{\S} + 3 <p>_{\S}}{2}, \quad (17)$$

where $M_{\S}$ is the volume inside the light-cone to the surface $\partial M_{\S}$, where $\chi = \chi_{\S}$. Note how this condition is similar to the one found by Lombriser (2019) and the mechanism for sequestering vacuum energy (Kaloper et al. 2016) from requiring an additional minimization of the Einstein–Hilbert action. Recall how here the scale $\chi_{\S}$ is fixed in comoving coordinates while the horizon η and d_{H} change with time (see Fig. 1). This is consistent with a constant value for Λ in equation (17).

3.3 Vacuum energy does not gravitate

Inside $\chi < \chi_{\S}$, we can use equations (1)–(3) with $\rho = \rho_{\mathrm{m}} + \rho_{\mathrm{r}} + \rho_{\mathrm{vac}}$ and $p = \rho_{\mathrm{r}}/3 - \rho_{\mathrm{vac}}$, so that we can write equation (17) as

$$\frac{\Lambda}{8\pi G} = \frac{<\rho_m>_{\S}}{2} + <\rho_r>_{\S} - \rho_{\mathrm{vac}} \equiv \rho_{\S} - \rho_{\mathrm{vac}}, \quad (18)$$

where $\rho_{\S}$ is the mean matter and radiation contribution in the integral of equation (17). The values of ρ_{m} and ρ_{r} evolve with space–time, so that $\rho_{\S}$ is the average contribution inside the volume $M_{\S}$, while the vacuum density contribution is constant (by definition). We can combine equation (4) with equation (18)

$$\rho_{\Lambda} \equiv \frac{\Lambda}{8\pi G} + \rho_{\mathrm{vac}} = \rho_{\S} - \rho_{\mathrm{vac}} + \rho_{\mathrm{vac}} = \rho_{\S}, \quad (19)$$

which shows that vacuum energy cancels out and cannot affect the observed value of ρ_{Λ}. In this respect, we can conclude that vacuum does not gravitate. This result is independent of the value of $\chi_{\S}$ or the value ρ_{vac}, which could both be infinite (as it follows from quantum field theory in the case of ρ_{vac}).

4 THE SIZE OF OUR CAUSAL UNIVERSE

Sometimes in our past, at time t_{i}, inflation (or a similar mechanism) blow the initial quantum fluctuations and create a large homogeneous patch for our universe. In terms of our comoving coordinates, its size is $\chi_{\S} = c/(a(t_{\mathrm{i}})H(t_{\mathrm{i}}))$, where $H(t_{\mathrm{i}})^2 = 8\pi/3G\rho(t_{\mathrm{i}})$ is given by the (potential) energy of inflation $\rho(t_{\mathrm{i}}) \sim V(\phi)$. This comoving scale has remain constant and outside causal contact throughout the evolution of the Universe. As we do not know the values of a_{i} or $\rho(t_{\mathrm{i}})$ it seems impossible to estimate how large $\chi_{\S}$ is from current observations or first principles.

But imagine that DE does not exist. Then, the horizon of our expanding universe (that emerged after inflation ended) will

[2]We use the sign conventions in Landau & Lifshitz (1971), but use greek letters for 4D space–time indexes and latin for 3D spatial indexes.

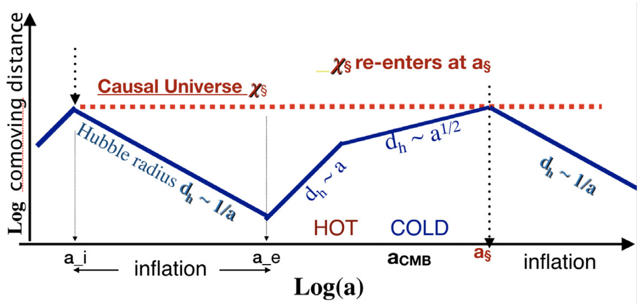

Figure 2. Same as Fig. 1, but here the causal universe $\chi_\S$ (set by inflation) is responsible for the late-time cosmic acceleration.

eventually reach $\chi_\S$ at some time $a_\S$. This is illustrated in Fig. 2. If we assume that vacuum energy does not evolve after inflation (i.e. $\omega_{\rm vac} = -1$), we can use equations (17)–(19) to estimate

$$\rho_\Lambda = \rho_\S = \frac{\int_{M_\S} \sqrt{-g} d^4x (\rho_m + 2\rho_r)}{\int_{M_\S} \sqrt{-g} d^4x}. \tag{20}$$

We can actually only do this calculation at time $a_\S$ because (from current observations) we only know the full content of the Universe at that time. At earlier times, part of the causal region is outside our horizon. Thus, we can use our measurements of ρ_Λ, $\rho_{\rm m}$, and $\rho_{\rm r}$ to estimate $a_\S$ and therefore $\chi_\S$.

The horizon after inflation (see equation 5) is

$$\chi(a) = \eta(a) - \eta(a_{\rm e}), \tag{21}$$

where $a_{\rm e}$ represents the end of inflation. We then have $\chi_\S = \chi(a_\S) = \eta(a_{\rm i})$ where $a_\S$ is the time when the causal boundary enters the horizon after inflation and $a_{\rm i}$ the beginning of inflation. Fig. 2 illustrate this. We calculate $\rho_\S$ in equation (20) as the integral to $\chi_\S$ in the light-cone

$$\rho_\S = \frac{\int_0^{\chi_\S} {\rm d}\chi\ \chi^2\ a^3\ (\rho_m a^{-3} + 2\rho_r a^{-4})}{2\int_0^{\chi_\S} {\rm d}\chi\ \chi^2\ a^3}, \tag{22}$$

where $a = a(\chi)$ in equations (21) and (5). For $H(a)$ we use equation (3) with $\Omega \equiv \rho/\rho_{\rm c}$, $\rho_{\rm c} = 3H_0^2/8\pi G$, and $\Omega_{\rm r} = 4.2 \times 10^{-5}$ (Planck Collaboration VI 2018) for a flat Universe $\Omega_m = 1 - \Omega_\Lambda - \Omega_r$. We find $\chi_\S$ from equation (22) numerically using $\Omega_\Lambda = \Omega_\S \equiv \rho_\S/\rho_{\rm c} \simeq 0.69 \pm 0.01$:

$$\chi_\S = (3.149 \pm 0.006)\ \frac{c}{H_0} \tag{23}$$

$$a_\S = 0.933 \pm 0.006. \tag{24}$$

to be compared to $a_0 = 1$ and $\chi_0 = 3.200 \frac{c}{H_0}$ today. So we can see that the scale of our causal universe is slightly smaller than our observable universe today. Because $\chi_\S$ is smaller than 2π times our observable horizon, we should be able to see this horizon in our past light-cone at $\theta_\S(z) = \chi_\S/\chi(z)$. At $z \simeq 1$ about half of the sky ($\theta_\S \sim 180$ deg) is causally disconnected. At larger redshifts this boundary tends to a fix value $\theta_\S \simeq 60$ deg. depending on $\chi_\S$ (and therefore Λ). This has implications for CMB observations (see Section 4.2). In Appendix B, we discuss how these results change in the presence of DE. But our proposal is that DE is not needed to explain cosmic acceleration. If we set $\Omega_{\rm r} = 0$ we find $a_\S = 0.86$ and $\chi_\S = 3.081$, so our results are not very sensitive to the details of the early Universe after inflation.

4.1 Inflation and the coincidence problem

Do the results in previous section, e.g. equation (23), depend on the observer? An astronomer in a galaxy at $z = 9$, when $a = 0.1$, will measure $\Omega_{\rm m}$ to be $\Omega_{\rm m}(z = 9) = \Omega_{\rm m}(z = 0)a^{-3}H_0^2/H^2 \simeq 0.997$ and $\Omega_\Lambda \simeq 0.002$. If she could measure these values with the accuracy that we measure then today she will get the exact same result for equations (23)–(24) as we do today. She will conclude that $\chi_\S$ is larger than the observable universe at $z = 9$ and will predict that that cosmic acceleration will happen in the future when the horizon reaches $\chi_\S$. So in that respect, there is no coincide problem in these results. They do not depend on when we estimate them.

Equation (20) indicates that when the causal boundary is close to re-entering the Horizon the expansion becomes dominated by ρ_Λ. This is because $\rho_m(a_\S) < \rho_\S = \rho_\Lambda$, as density decreases with the expansion. This results in another inflationary epoch at $a = a_\S$ which keeps the Causal Universe frozen (see Fig. 2). We are trap inside our causal horizon $\chi_\S$: causality can only play a role for comoving scales $\chi < \chi_\S$. This warrants the constancy of Λ and also explains cosmic acceleration as a consequence of (the first) primordial inflation.

We can now recast the coincidence problem (why $\rho_\Lambda \simeq 2.3\rho_{\rm m}$?) into a new question: why do we live at a time which is close to $a_\S$? or why the scale/energy of inflation is close to our horizon/energy today and not larger or smaller? In terms of anthropic reasoning (Weinberg 1989; Garriga & Vilenkin 2003), at earlier times the Universe is dominated by radiation and there are no stars or galaxies to host observers. Closer to $a \simeq a_\S$, the Universe is dominated by matter and there are galaxies and stars with planets and potential observers. At later times $\Omega_\Lambda \simeq 1$ and galaxies will be torn apart by the new inflation. Moreover, $a_\S$ has the largest Hubble radius (see Fig. 2) with the highest chances to host observers like us. So, there is nothing too special about this coincidence. Ultimately, the reason why $\chi_\S \sim 3c/H_0$ resides in the details of inflation: when inflation begins $a_{\rm i}$ and ends $a_{\rm e}$ (see Fig. 2). This recasts the coincidence problem into an opportunity to better understand inflation and the origin of homogeneity. We propose here to identify $\chi_\S = \eta(a_i)$ with the comoving horizon before inflation begins at time $t_{\rm i}$

$$a_{\rm i} H_{\rm i} = c\chi_\S^{-1} \simeq (0.3176 \pm 0.0006)\ H_0, \tag{25}$$

where $H_{\rm i} = H(t_{\rm i})$ or $a_{\rm i} = a(t_{\rm i})$. This shows how $a_{\rm i}$ determines $\chi_\S$, while $a_{\rm e}$ determines when it re-enters the horizon (see Fig. 2), and therefore how large $\rho_\S$ is. The Hubble rate during inflation H_I is proportional to the energy of inflation. During reheating this energy is converted into radiation: $H_I^2 \simeq \Omega_{\rm r}\ H_0^2\ a_{\rm e}^{-4}$, with $a_{\rm e} \equiv e^N a_{\rm i}$. We can combine with equation (25) to find

$$a_{\rm i}\chi_\S = \frac{H_{\rm i}}{H_I} {\rm e}^{-2N}\ \Omega_r^{1/2}\ \left(\chi_\S^2 H_0/c\right) \simeq 4 \times 10^8\ l_{\rm Planck}, \tag{26}$$

where for the second equality we have used the canonical value of $N \simeq 60$ and $H_{\rm i} \simeq H_I$, which also yields $a_i \simeq 1.56 \times 10^{-53}$ and $H_{\rm I} \sim 10^{10}$ GeV. The condition $a_{\rm i}\chi_\S > l_{\rm Planck}$ requires $N < 70$, close to the value found in Dodelson & Hui (2003). Thus, the whole causal size of our Universe $\chi_\S$ could result from a quantum fluctuation at the Planck scale $l_{\rm Planck}$. Such vacuum fluctuation could generate an inflationary expansion. After $N \simeq 70$ e-folds inflation ends with reheating, which results into matter and radiation today. Thus, this model links the cosmological constant scale to the Planck scale via inflation.

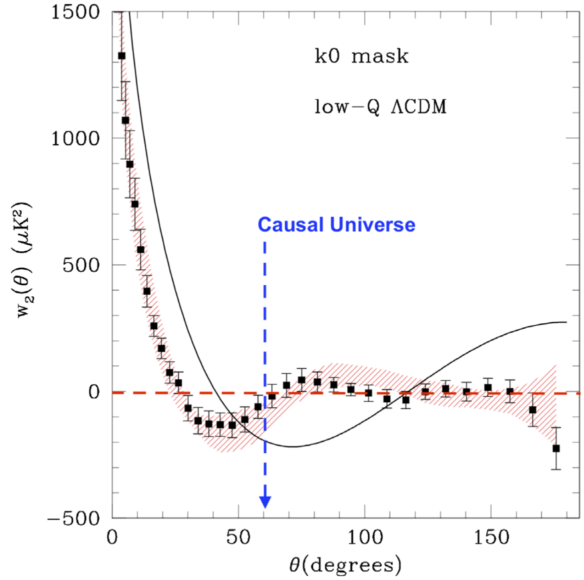

Figure 3. Two-point correlation function of measured CMB temperature fluctuations in WMAP (points with errorbars) as a function of angular separation (from Gaztañaga et al. 2003). The black continuous line shows the ΛCDM prediction for an infinite Universe. Shaded region shows ΛCDM simulations where we suppress the large-scale modes (multipoles $l < 5$).

4.2 Implications for CMB

The (look-back) comoving distance to the surface of last scattering $a_* \simeq 9.2 \times 10^{-4}$ (Planck Collaboration XXIII 2018) is $\chi_{\rm CMB} = \eta(1) - \eta(a_*) \simeq 3.145 \frac{c}{H_0}$. This is similar to our estimate for $\chi_§$ in equation (23). Thus, we would expect to see no correlations in the CMB on angular scales $\theta > \theta_§ \equiv \frac{\chi_§}{\chi_{\rm CMB}} \simeq 60$ deg for $\Omega_\Lambda = \Omega_§ \simeq 0.7$. The lack of structure seen in the CMB on these large scales is one of the well-known anomalies in the CMB data, see Schwarz et al. (2016) and references therein. This lack of correlations above 60 deg has been interpreted as a universe with non-trivial topology (Luminet et al. 2003). Fig. 3 shows a comparison of the measured CMB temperature correlations (points with error-bars) with the ΛCDM prediction for an infinite Universe (continuous line). There is a very clear discrepancy that Copi et al. (2009) estimates to happen in only 0.025 per cent of the realizations of the infinite ΛCDM model. The significance of this discrepancy is model dependent. If errors are estimated from the data (and not from the model), ΛCDM is strongly ruled out by this measurement (Gaztañaga et al. 2003). Even assuming a ΛCDM model, the lack of large-scale correlations in $w_2(\theta)$ represent an odd alignment of lower order multipoles of the angular power spectrum c_l (Schwarz et al. 2016).

A small causal Universe could explain other CMB anomalies (Planck Collaboration XXIII 2014) and result in significant variations of cosmological parameters (or observables) when averaged over causally disconnected regions of the sky (Fosalba & Gaztanaga 2020, in preparation). Early indications for such variations in the CMB power spectrum (Gaztanaga, Fosalba & Elizalde 1998) were interpreted as non-Gaussian initial conditions.

We can also predict Ω_Λ from the lack of CMB correlations. From Fig. 3 we roughly estimate $\theta_§ \simeq 60 \pm 3$ deg to find (using equation (22)) $\Omega_\Lambda = 0.7 \pm 0.1$. (the larger the angle the smaller Ω_Λ). But note that this rough estimate does not take into account the foreground (late) ISW and lensing effects (Fosalba, Gaztañaga & Castander 2003; Das & Souradeep 2014), which add non-primordial correlations to the largest scales. This requires further investigation.

Also note that this estimate for Ω_Λ corresponds to the size of disconnected regions at the location of the CMB, which might be slightly different to the value near us, as we see a different patch of the primordial Universe (see below). Note also that there are temperature differences on scales larger $\theta_§$, but they are not correlated, as expected in causality disconnected regions. Nearby regions are connected which creates a smooth transition across disconnected regions.

5 DISCUSSION AND CONCLUSIONS

ΛCDM in equations (1)–(3) assumes that ρ is constant everywhere at a fixed comoving time. This requires a causal initial conditions (Brandenberger 2017) unless there is inflation, where a tiny homogeneous and causally connected patch, the causal Universe $\chi_§$, was inflated to be very large today. Regions larger than $\chi_§$ are out of causal contact. Here, we require that test particles become free (or the relativistic flux is zero) as we approach $\chi_§$. No cause should produce no effect. This leads to equation (17), which is the main result in this paper. If we ignore the vacuum, this condition requires: $\Lambda = 8\pi G\rho_§$, where $\rho_§$ is the matter and radiation inside $\chi_§$ (equation 20). For an infinite Universe ($\chi_§ \to \infty$) we have $\rho_§ \Rightarrow 0$ which requires $\Lambda \Rightarrow 0$. This is also what we find in classical gravity with a Λ term, because Hooke's term diverges at infinity (see equation 6). So the fact that $\Lambda \neq 0$ could indicate that $\chi_§$ is not infinite. Adding vacuum $\rho_{\rm vac}$ does not change this argument because $\rho_\Lambda \equiv \Lambda/8\pi G + \rho_{\rm vac} = \rho_§$ turns out to be independent of $\rho_{\rm vac}$ (see equation (19)). Thus, whether the causal size of the Universe is finite or not, $\rho_{\rm vac}$ cannot gravitate! The cancellation between Λ and $\rho_{\rm vac}$ is a direct consequence of the boundary condition and it also implies that deSitter Universe (empty with a cosmological constant) is not causal (it produces curvature even when empty) and therefore not a physical model.

For constant vacuum ($\omega \equiv p/\rho = -1$), we find $\chi_§ \simeq 3c/H_0$ for $\Omega_§ = \Omega_\Lambda \simeq 0.7$. We can also estimate $\chi_§$ as $c/(a_i H_i)$ when inflation begins, see equation (25). After inflation $\chi_§$ freezes out until it re-enters causality at $a_§ \simeq 0.93$, close to now ($a = 1$). This starts a new inflation (as $\rho_\Lambda = \rho_§ > \rho_m$) that keeps the causal boundary frozen. Thus a finite $\chi_§$ explains why $\rho_\Lambda \simeq 2\rho_m$. It also predicts that CMB temperature should not be correlated above $\theta > \theta_§ \simeq 60$ deg. A prediction that matches observations (see Fig. 3). This is one of the well-known anomalies measured in the CMB. One would also expect the CMB spectrum to be anisotropic on the largest scales, which is another measured anomaly (see Planck Collaboration XXIII 2014). One can reverse this argument to use the lack of CMB correlations above $\theta_§ \simeq 60$ deg, to estimate $\chi_§ \simeq \theta_§ \chi_{\rm CMB}$. Together with condition $\rho_§ = \rho_\Lambda$, this provides a prediction of $\Omega_\Lambda \simeq 0.7 \pm 0.1$, which is independent of other measurements for Ω_Λ. More work is needed to account for the late ISW and lensing and to interpret the CMB measurements with a metric that is not homogeneous (Gaztañaga 2020, in preparation).

Note that because the Universe is not strictly homogeneous outside a causal region, the causal boundary for observers far away from us could be slightly different from ours, because they see a different patch of the Universe which could have slightly different energy content. Continuity across nearby disconnected regions forces these differences to be small, but it is impossible to quantify this without a model for the initial conditions and a better understanding of the process that generates the primordial homogeneity. In general such differences could affect structure formation, galaxy evolution, and CMB observations. The fact that we can measure a concordance picture from different observations with the ΛCDM model indicates that these differences must be small. But tensions between measurements of cosmological parameters (or fundamental constants of nature) from very different redshifts (e.g. between CMB and local measurements) or different parts of the sky at high redshifts (such as dipolar variation of fine structure constant in Webb et al. 2011) could be related to such in-homogeneities, rather than to evolution of the DE equation of state or other more exotic explanations.

For $\chi_{\S} >> 3c/H_0$ we cannot explain cosmic acceleration with $\omega = -1$, because the resulting ρ_Λ in equation (20) would be very small. We need evolving DE with equation of state $\omega_{\rm DE} > -1$ and $\rho_\Lambda = \rho_{\rm DE}$ today (see Appendix B). But DE gives no clue as to why $\rho_{\rm DE} \simeq 2\rho_{\rm m}$ today and cannot explain the anomalous lack of CMB correlations at large scales. We can apply Occam's razor to argue that there is no need for DE or modify gravity. Comparing Fig. 1 with Fig. 2, we can see that the measured cosmic acceleration today can be explained by a finite causal scale $\chi_{\S} \simeq 3c/H_0$, which is expected for Universe with a finite age and a mechanism like inflation.

We have speculated that the finite causal size of our universe could result from a quantum fluctuation at the Planck scale $l_{\rm Planck}$ which produces an inflated expansion by a factor $\simeq e^{70}$ leading to a reheating into matter and radiation today. This primordial inflation eventually results into late-time cosmic acceleration. The existence of such finite causal scale can be tested further with observations of the variation of cosmological or fundamental parameters over cosmological scales.

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APPENDIX A: HOOKE'S LAW

In Newton's gravity, a point mass m generates a radial acceleration $\vec{g} = -\vec{\nabla}\phi = -Gm\vec{r}/r^3$, where ϕ is the Newtonian potential. The Λ term in GR's field equations, equation (9), corresponds here to an additional term that is linear in r, as in Hooke's law

$$\vec{g} = -\vec{\nabla}\phi = \vec{g}_{\rm Newton} + \vec{g}_{\rm Hooke} = -\left(\frac{G}{r^3} + G_2\right) m\,\vec{r}. \tag{A1}$$

Hooke's constant, G_2, can be related to Λ, as we will see below. This Hooke's term is unique in that it is the only distance dependence (other than the inverse square law) that has a key property for gravity: that a spherical mass shell of arbitrary density $\rho(R)$ produces a gravitational field which is identical to a point source of equal mass in its centre

$$\vec{g}_{\rm Hooke} = -G_2 \int_{\rm shell} {\rm d}^3R\,(\vec{r} - \vec{R})\,\rho(R) = -G_2 m\,\vec{r}, \tag{A2}$$

where $\vec{R}$ covers the shell and $\vec{r}$ is some position outside. This is a key property of gravity, needed to treat the Universe as a whole and sustain Gauss's and Birkhoff's theorems. Thus in classical mechanics the above law of gravity equation (A1) is consistent with the symmetries and observations, as long as G_2 is small enough. In fact Newton, and other scientist, had already noticed this, but did not consider Hooke's term for lack of observational evidence (and references therein Calder & Lahav 2008).

Notice how equation (A1) diverges for $r \Rightarrow \infty$. We expect instead that $\vec{g} \Rightarrow 0$, as particles should be free at infinity (as there could be no causal connection). This explains why on theoretical grounds we would expect $G_2 = 0$ even if this relation is allowed by the symmetries of the problem. On the other hand, if causality ends at some finite distance $r_{\S}$, as the Universe has a finite age, then we see that Newton's law alone cannot describe a causal gravitational interaction. We need to also have Hooke's law. Requiring that gravitational forces $\vec{g}$ in equation (A1) vanished at the causal boundary $r_\S$

$$\vec{g}(r = r_\S) = 0 \quad \Rightarrow \quad G_2 = -\frac{4\pi G}{3V_\S} \tag{A3}$$

so that Hooke's gravitational force is repulsive and only becomes comparable to Newton's gravity for separations comparable to $r_\S$.

We can estimate the flux using equation (A1) to find

$$\Phi = \oint_{\partial V} \mathrm{d}\vec{r}\, \vec{g} = -4\pi G m_V - 3V G_2 m_\S, \tag{A4}$$

where m_V is the mass inside V and $m_\S$ is all the mass in all the (causally connected) universe. As expected, $\Phi = 0$ in equation (A4) reproduces equation (A3) for $V = V_\S$. For the Newtonian term, masses outside V do not contribute to Φ because for a small cone centre outside and crossing V the in going flux exactly cancels with the outgoing flux (as the inverse square law in $\vec{g}$ compensates the increase in the area crossed then the cone goes inside to when is going outside). This is not the case for the Hooke's law, where the difference in flux is just proportional to the volume inside V. As all the masses contribute equally, we have

$$\Phi_{\mathrm{Hooke}}(V) = -G_2 \sum_m m(\vec{r}) \oint_{\partial V} (\vec{R} - \vec{r})\, \mathrm{d}\vec{A} = -3V G_2 m_\S \quad , \tag{A5}$$

where the total mass is $m_\S = \sum_{\mathrm{m}} m$. This result is also true in arbitrary number of dimensions (see Wilkins 1986). This explains why Hooke's law behaves like vacuum energy (i.e. like negative pressure: $\omega = -1$) or Λ in GR: the density remains constant as we increase the volume! Comparing equation (A4) with equation (7) we have

$$\Lambda = -3G_2 m_\S = 4\pi G \rho_\S, \tag{A6}$$

where in the last step we have also used equation (A3), or equivalently $\Phi = 0$ in equation (A4). This of course agrees with equation (8), which shows that equation (A1) and equation (6) are equivalent.

This result provides a Classical Physic's interpretation of the cosmological constant: it is just related to Hooke's constant G_2 and the total mass $m_\S$ in the (causal) universe. Note how Λ can be zero if either $G_2 = 0$ or if $m_\S = 0$. Note also how $G_2 = 0$ for infinite volume and in this case $\Lambda = 0$ because $m_\S/V_\S$ also goes to zero. This relation establishes a clear connection between the value of Λ and the matter–energy content in the Universe, as required by causality. It also gives new light into Mach's principle.

APPENDIX B: EFFECTIVE DE

Here, we generalize the results of Section 4 to the case with DE. If vacuum energy suffers a phase transition or changes with time, as could have happened during inflation, the cancellation presented in Section 3.3 will not happen (because $\omega \neq -1$) and an evolving ρ_{vac} (which we usually call DE) could contribute to the effective value of ρ_Λ. Consider the general case of DE after inflation

$$\rho_{\mathrm{DE}}(a) = \rho_{\mathrm{vac}} + \rho_{\mathrm{DE}}\, a^{-3(1+\omega)}$$
$$p_{\mathrm{DE}}(a) = -\rho_{\mathrm{vac}} + \omega\, \rho_{\mathrm{DE}}\, a^{-3(1+\omega)}, \tag{B1}$$

where (by definition) only one component of DE is evolving. We then have from equations (17) and (4)

$$\rho_\Lambda = \rho_\S + \rho_{\mathrm{DE}} \left[1 + \frac{1 + 3\omega}{2} \hat{a}_\S^{-3(1+\omega)} \right]. \tag{B2}$$

where $\hat{a}_\S$ is some mean value of a in the past light-cone of $a_\S$ in equation (22). This reduces to $\rho_\Lambda = \rho_\S$ for $\omega = -1$. For $a_\S \Rightarrow \infty$, we have $\rho_\S \Rightarrow 0$ because $\rho_{\mathrm{m}}(a)$ and $\rho_{\mathrm{r}}(a)$ tend to zero as we increase $a_\S$. The same happens with $\hat{a}_\S^{-3(1+\omega)}$ for $\omega > -1$, so that:

$$\rho_\Lambda = \rho_{\mathrm{DE}} \quad \text{for} \quad \mathrm{a}_\S \Rightarrow \infty \quad \& \quad \omega > -1. \tag{B3}$$

So evolving DE could produce the observed cosmic acceleration in an infinitely large Universe. This solution does not explain why $\rho_\Lambda = \rho_{\mathrm{DE}} \simeq 2\rho_{\mathrm{m}}$. The original motivation to introduce DE was to explain how vacuum energy ρ_{vac} could be as small as the measured ρ_Λ (Weinberg 1989; Huterer & Turner 1999). But we have shown in Section 3.3 that the causal boundary condition explains why ρ_{vac} does not contribute to ρ_Λ and also results in $\rho_\Lambda \simeq 2\rho_{\mathrm{m}}$. This removes the motivation to have DE, as it represents an unnecessary complication of the model (Occam's razor).

If observations find ω to be significantly larger than $\omega = -1$ this will indicate that something like DE exist. The actual measured value of ω will not directly tell us the size of $\chi_\S$ unless we also have some model for ρ_{DE}. But very accurate measurements for the evolution of ρ_Λ might actually be able to separate a component that is constant (i.e. $\rho_\S$) from a component that is evolving (i.e. DE).

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