Stochastic Multiattribute Acceptability Analysis: an application to the ranking of Italian regions

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Abstract

We consider the issue of ranking regions with respect to a range of economic and social variables. Departing from the current practice of aggregating different dimensions via a composite index, usually based on an arithmetic mean, we instead use Stochastic Multiattribute Acceptability Analysis (SMAA). SMAA considers the "whole space" of weights for the considered dimensions. The methodology is applied to the ranking of Italian regions, showing that although the North-South divide is definitely wider than the one measured simply in terms of GDP, there are Southern regions which perform generally better than those belonging to their broad region: a kind of Northern regions within the broader Southern region.

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1. Introduction

The measurement of regional socio-economic performance has become increasingly significant particularly in those countries characterised by persistent economic dualism such as Italy. Indeed, defining a comprehensive framework to assess regional performance is a crucial factor in both designing and evaluating regional policy. Moreover, the inclusion of the subjectivity in the evaluation exercise is crucial as measures taking into account subjectivity "have the potential to improve the policy making process, give citizens and decision makers more accurate information about quality of life and thereby significantly increase the toolkit of evidence-based measures" (Kroll and Delhey, 2013).

Despite the crucial importance of indicators for socio-economic performance to support effective regional policymaking taking into account also the subjective aspects of the evaluation, the actual measurement of regional socio-economic performance is far from being clear cut and unambiguously resolved. This is due to several problems founded on both technical and conceptual grounds. The most widely-used measures of economic performance are GDP, or alternatively Gross Value Added (GVA)[1]. However, there clearly remains long-standing general criticism about its validity as a measure of wellbeing dating back to 1934 (Kuznetz,1934) and more recently addressed, among others, by Kubiszewski et al. (2013), Costanza et al. (2009), and Stiglitz et al. (2009).

[1] GVA is equal to GDP plus subsidies less taxes on products. Of course, the choice between GDP and GVA does not affect comparison of regions within a country, because differences between regions are the same according to both measures.

Furthermore, once applied to a regional setting, important additional caveats also become manifest. Arguably, GDP is a reasonable measure if the scope of the analysis is more narrowly limited to the measurement of the regions' output. Nevertheless, it is not able to capture, for example, neither regions' income, nor regional productivity (Dunnell, 2009).

These observations on the validity of GDP and other one-dimensional indices to measure wellbeing pave the way for the use of composite indices to provide an overall evaluation through the aggregation of different dimensions (or 'criteria'). This rationale underpins, for example, the use of the Regional Competitiveness Index (RCI) (Annoni and Kozovska, 2010; Annoni, 2013) which builds upon the Global Competitiveness Index (GCI), published annually by the World Economic Forum (WEF) (Schwab, 2009; Schwab and Porter, 2007), and the World Competitiveness Yearbook by the Institute for Management Development (IMD, 2008).

Although composite indices such as the RCI give an overall evaluation of social, economic, and environmental conditions that are perhaps preferable to reliance on GDP and other one-dimensional indices, there still remain some methodological questions raised by their adoption. Ideally one would like different dimensions to be aggregated in a manner that achieves some desirable technical properties such as (i) neutrality (where, all ranked countries or regions are be treated equally), or (ii) monotonicity (where an improvement in performance should not result in a deterioration in ranking position). Nardo et al. (2008) suggest that, in the spirit of the well-known impossibility Arrow theorem (Arrow, 1951), there does not exist any perfect aggregation rule. Accordingly, two main pragmatic solutions can be considered:

- Following the Borda rule which assigns a score to each country or region according to the following procedure: Each unit (country or region) receives one point for each one of the *n* dimensions in which it is the last, two points for each dimension in which it is the last but one, and so on until the *n* points for the dimensions in which it is first. Finally, these points are then summed up;
- Following the Condorcet rule, which is based on the pairwise comparison between alternatives, counting the number of dimensions that are in favour of one alternative over another one.

All the proposed aggregation rules can be broadly condensed into these two basic approaches. Whichever aggregation procedure is actually adopted, a crucial issue remaining for any ranking (or evaluation) exercise generating a single index based on socio-economic characteristics, is the choice of weighting system. The WEF (1999)'s methodology considered in Lall (2001, p.98) contends

> the weighting system is *a priori;* the report says that "it was based on the economic literature", but which part of the literature yields the weights is left to imagination. Where in the literature, for instance, weight for finance as compared to technology come from? Can it be defined on economic grounds? The answers are not clear (p.1516).

Moreover, with regard to the aforementioned RCI the advocates of this measure explicitly admit that the RCI is "*the result of a long list of subjective choices*" (Dijkstra et al., 2011, p. 16). From a broader perspective, the central issue in ranking different entities is twofold:

(i) different attributes are considered;

(ii) different weights for the considered attributes are used.

The latter is perhaps the most pernicious problem. Indeed, with respect to the possibility of considering different dimensions, it is always possible to enlarge the set of considered dimensions to include all the aspects being relevant for almost anybody interested in the ranking. However, even if two individuals could agree on the set of considered dimensions, it is very rare, or even impossible, that they could completely agree on the weights to be assigned to those dimensions, due to, for example, fundamental differences in personal preferences. Let us remark that consideration of weights, or more, in general, of a given preference attitude cannot be avoided in aggregating different dimension. Indeed, putting together heterogeneous dimensions such as life expectancy, education, and per capita income in the Human Development Index, require always to answer to questions such as: given a certain level of each considered dimension and considering a certain decrement of one of them, how large should be the increase in one of the other two in order to maintain the overall level of satisfaction? These questions and other similar ones are unavoidable even if the dimension are measured in quantity terms *by the National Statistical Institutes* or similar institutions. In fact, in the context of the domain of the well-being measures, these questions are the analogous of the decision on how much to reduce the production of butter to increase the production of guns proposed by Samuelson in his famous text-book (Samuelson, 1948). Of course, if it is reasonable to admit that each individual has his/her own answer to this question so that each one could theoretically fix the quantity of butter that according to his/her preferences is worthwhile to be exchanged with a unit of gun valid, it is also pacific that this quantity of butter is not the same for each one. In the context of the well-being measure the ratio between the weight of life expectancy and education represents exactly how much "units of education" the considered individual is willing to sacrifice in order to increase of one "unit" life expectancy.

Therefore, considering a plurality of these weights amounts to consider a plurality of individuals having different preferences.

Indeed, despite the proliferation of composite socio-economic indicators (for a review considering more than 160 different indicators see Bandura, 2008), the weights set is clearly the manifest problem for composite indices such as, the popular Human Development Index (see, among others, Saisana et al. 2005; Permanyer, 2011; Cherchye et al. 2008, and Foster et al. 2009).

The Organisation for Economic Co-operation and Development (OECD) attempts to overcome the weighting issue by preferring to present a set of nine headline indicators[2] (OECD, 2014) for 362 OECD regions rather than a single composite indexto "*present the information in such a way that users can consider the relative importance of each topic and bring their own personal evaluations to the questions"* (OECD, 2014, p.8). Arguably a range of indicators is potentially even more difficult to communicate than a single metric.

This study argues that there is still some space for a more conceptually flexible approach ranking by composite index, where additionally one can more explicitly take into account the scope for attaching different weights to any considered dimensions (Helliwell, 2003; Helliwell and Barrington-Leigh, 2010). The Stochastic Multicriteria Acceptability Analysis (SMAA) (Lahdelma, Hokkanen and Salminen, 1998) method offers this possibility as it considers the whole set of possible weights (approximated through a very large sample of randomly extracted vectors of weights). In this way, it is possible to determine the probability by which each region is first, second, third etc. in performance ranking. Moreover, for each pair of regions it is possible to define the

[2] The considered dimensions are income, jobs, housing, education, health, environment, safety, civic engagement, and accessibility of services.

probability that one region is better than another or vice versa, in every possible pairwise comparison. Considering the whole set of possible vectors of weights, introduces a degree of subjectivity in the ranking and amounts to considering all the sensitivities, ranging from extreme values taking into account only one or few dimensions, to the more even-tempered, taking into account all the dimensions. Instead, the usual approach considering a single vector of weights levels out all the individuals collapsing them to an abstract and unrealistic set of "representative agents". Instead, considering a probability distribution of vector of weights can be interpreted as taking into account a population in which preference represented by each vector of weights are distributed according to the considered probability. For example, if it 35% the probability of weight vectors having the weight of dimension g_i, w_i, greater than the weight of dimension g_j, w_j, than this can be interpreted as consideration of a population in which fraction of individuals for which $w_i > w_j$ is 35%. Thus, in this perspective it is meaningful to consider the corresponding probability distribution of a ranking, rather than a single "representative" ranking that can amalgamate different preferences in the population hiding distribution of diversified ranking in the considered population. Observe also there is a second interpretation of the plurality of weight vectors. Indeed, each individuals can be imagined as composed by a plurality of selves (see e.g. Elster, 1988), so that the probability of a given ranking can be imagined as the fraction of multiple selves of a given individual for which holds the ranking at hand. In this perspective, the consideration of a multiplicity of weight vectors can be considered as the correspondent of a consideration of a multiplicity of priors in the economic models of decision under uncertainty (see e.g. Gilboa *et al*. 2010, Minardi and Savochkin, 2015). Of course the two interpretations of a plurality of weight vectors as representing a population of individuals or the multiple selves of a single individual are complementary rather than alternative, so that one can imagine a population of individuals each one having a plurality of selves. According to these interpretations, a plurality of vector of weights relieves robustness concerns compared to composite indices. In this respect, several techniques have already been popularised (see for example, Saisana et al., 2005; Nardo et al., 2008). Even non-academic institutions like the European Institute for Gender Equality have implemented such techniques for the construction of composite indices such as the European Gender Equality Index (EGEI, see especially chapter 3 in EGEI, 2013). However, in all these approaches the focus is on the stability of the obtained results without any systematic exploration of the whole range of possible weights. For example, in the EGEI report, *robustness analysis of the Gender Equality Index is performed considering* a certain number of scenarios (i.e. "models") drawn from the combination of 4 alternatives for weighting (2 kinds of equal weights, principal components analysis, AHP (Saaty, 1988)), 3 aggregation operators (arithmetic, geometric and harmonic mean) and missing data imputation (100 simulations). In this way *3,636 sets of scores were computed.* The median for each of the 27 States within these 3,636 possible scenarios has been computed and, then, the "best index" is *the one that minimises these differences and lies closest to the median.* Even this complex procedure does not systematically explore the whole spectrum of possible weighting schemes as the SMAA does, instead.

In this study we apply SMAA to the ranking of Italian regions with respect to social, economic, and environmental aspects. Despite the conspicuous methodological difference[3], this study closely aligns with the OECD initiative '*How's life in your region?*' (OECD, 2014) which aims to understand "…people's level of well-being and its determinants […] to gear public policies towards better achieving society's objectives." (OECD, 2014, p. 4).

In the light of this OECD claim and by using SMAA, we have directly been able to explore the full range of possible weight vectors, because we explicitly consider the whole spectrum of preferences and attitudes towards different aspects of well-being. Put crudely, a businessman might be more interested in economic performance aspects rather than in environmental performance aspects and a student might be more interested in social performance aspects. These diversified appreciations of various aspects of quality of life, determine a consequently different weighting of the considered criteria. Therefore, it could be reasonable to expect that some regions would be more preferred by some categories of individuals, while other regions would be preferred by others. This would be shown by some probability of being in the first rank positions despite their ranking based on GDP only. More specifically, with respect to the Italian North-South divide, one could expect that there could be some even small probability for the Mezzogiorno regions to be in the first positions for a given set of weights. Nonetheless, our research shows that this is not the case and this can be interpreted in the following manner. Southern regions of Italy are the less preferred for all the different categories of citizens, regardless of their relative preferences about the different dimensions of well-being. Essentially this is the core original contribution of our research to the discussion of the Italian regional dichotomy. Namely, our study shows that the strong performance of the North regions is widespread and generalized to

[3] As discussed in section 3 the OECD addressed the weighting issue by renouncing to the composite index approach in favour of a set of headline indicators.

all the categories of stakeholders. This conclusion is confirmed and reinforced using a class of multidimensional Gini indices and polarisation indices based on the ranking acceptability indices that measures both the concentration and the polarisation of the probability of obtaining a rank position not worse (or not better) than a given level. The interest of these multidimensional Gini and polarisation indices is related to the interpretation of the probability of an alternative *a* to attain a given ranking position *r* in terms of fractions of a population (in which the weight vectors of individuals are distributed accordingly to the considered probability) for which *a* attains ranking position *r*. Therefore, for example, a high multidimensional Gini or polarisation index for the first ranking position expresses a great concentration or polarisation in the population with respect to the alternatives considered the best. A similar conclusion can be drawn taking into account the interpretation of the multiple selves of a given individual. In any case, the fact that these multidimensional Gini and polarisation indices, originally proposed in this paper, confirm the gap between the North and South of Italy with more nuance than Gini indices and polarisation indices related to single indicators, must be interpreted in the sense that the perception among the population of concentration and polarisation of the best well-being (and conversely of the worst well-being) is stronger than the concentration and polarisation of single components of the same well-being.

To the best of our knowledge, this is the first time that SMAA method is applied to the performance ranking of regions and, more generally, for ex-post ranking of territorial entities according to their relative performance, instead of an ex-ante evaluation within a decision-making process. The proposed methodology can be adapted to study other geographic areas with likely different results. Accordingly, it would be valuable to investigate which categories of individuals tend to prefer one region over another. With respect to the Italian case, the most salient point is the stability in finding the south regions across all categories of individuals as the worst regions.

The paper is organised as follows. Section 2 positions the methodology with respect to the ranking of regions. Section 3 illustrates our proposal for a new ranking of Italian regions. Section 4 concludes.

2. From subjective objectivity to objective subjectivity in regional economic ranking

In Multiple Criteria Decision Analysis (MCDA) problems (Greco et al. 2016; Ishizaka and Nemery, 2013) a set of alternatives $A=\{a_1,...,a_m\}$ is evaluated based on a set of evaluation criteria $G=\{g_1,\dots,g_n\}$ in order to deal with decision problems such as choice of the best alternative or ranking of all the alternatives from the best to the worst. For example, in regional development ranking, the alternatives are the regions of the considered country (e.g., in the case of Italy, twenty regions) and the criteria are the dimensions with respect to which these regions should be evaluated (e.g., environment, cultural heritage, social capital and so on). The value function most commonly used to aggregate the evaluations of alternatives from A with respect to criteria from G is the weighted sum, which, after assigning a non-negative weight w_i to each criterion $g_i \in G$, $w_1+\dots+w_n=1$, gives to each alternative $a_k \in A$, the following overall evaluation:

$$u(a_k, w) = \sum_{i=1}^{n} w_i g_i(a_k). \qquad \text{eq. (1)}$$

It is worth noticing that different types of means can be expressed in terms of a weighted sum of some transformation of the evaluations $g_i(a_k)$. In greater detail, in the case of quasilinear means (see Aczel, 1948 and section 4.3.1 in Grabisch et al., 2009) we have

$$M\left(g_1(a_k),\ldots,g_n(a_k)\right) = f^{-1}\left(\sum_{i=1}^{q} w_i f\left(g_i(a_k)\right)\right) \qquad \text{eq. (2)}$$

with f being a strictly monotonic function. If $f(x)=\log(x)$, the quasilinear mean becomes the weighted geometric mean

$$G\left(g_1(a_k),\ldots,g_n(a_k)\right) = \prod_{i=1}^{n} g_i(a_k)^{w_i}. \qquad \text{eq. (3)}$$

Let us point out that, with respect to the arithmetic mean, the geometric mean has the advantage of not being completely compensatory because it does not permit to perfectly rebalance worse evaluations on one criterion with better evaluations on other criteria. Indeed, the non-compensatoriness of composite indices has been a largely discussed issue (see e.g. Munda and Nardo, 2009, Casadio Tarabusi and Guarini, 2013, Mazziotta and Pareto, 2016) and the most adopted aggregation function to avoid complete compensatoriness is the geometric mean that has been recently adopted for the Human Development Index (UNDP, 2010).

Very often one considers a simple arithmetic mean of the evaluations $g_i(a_k)$ that criteria $g_i \in G$ gives to alternatives $a_k \in A$, that is to assign an equal weight to each criterion. Two main questions arise: how is the ranking of an alternative a_k changing when the weights of considered criteria change? Given two alternatives a_k and a_h from A, is it larger the set of vectors of weights w_i for which a_k is preferred to a_h, or that one for which a_h is preferred to a_k?

Within MCDA these questions were addressed by the Stochastic Multiobjective Acceptability Analysis (SMAA) (Lahdelma, Hokkanen and Salminen, 1998; Lahdelma and Salminen, 2001; for two surveys see Tervonen and Figueira, 2008 and Lahdelma and Salminen, 2010). SMAA belongs to the family of MCDA methods aiming to provide recommendations on the problem at hand considering uncertainty or imprecision on the considered data and preference parameters.

In order to handle imprecision with respect to the weights assigned to the criteria and to the evaluations taken on criteria under attention, SMAA considers two probability distributions $f_W(w)$ and $f\chi(\xi)$ on W and χ, respectively, where

$$W = \{(w_1, \ldots, w_n) \in \mathbf{R}^n: w_i \geq 0,\ i=1,\ldots n, \text{ and } w_1 + \ldots + w_n = 1\} \qquad \text{eq. (4)}$$

and χ is the evaluation space, i.e. the space of the value that can be taken by criteria $g_i \in G$.

First of all, SMAA introduces a ranking function relative to the alternative a_k:

$$rank(k, \xi, w) = 1 + \sum_{h \neq k} \rho\left(u(\xi_h, w) > u(\xi_k, w)\right), \qquad \text{eq. (5)}$$

where ρ(false) = 0 and ρ(true) = 1.

Then, for each alternative a_h, for each evaluation of alternatives $\xi \in \chi$ and for each rank $r = 1, \ldots, l$, SMAA computes the set of weights of criteria for which alternative a_k assumes rank r:

$$W_k^r(\xi) = \{w \in W : rank(k, \xi, w) = r\}. \qquad \text{eq. (6)}$$

SMAA is based on the computation of the following indices:

• The rank acceptability index: it is the measure of the set of weight vectors and evaluations on considered criteria for which the alternative a_k gets rank r:

$$b_k^r = \int_{\xi \in \chi} f_\chi(\xi) \int_{w \in W_k^r(\xi)} f_W(w)\ dw\ d\xi; \qquad \text{eq. (7)}$$

b_k^r represents the probability that alternative a_k has the *r-th* position in the preference ranking. Let us remark that the rank acceptability index can be abridged to the Borda rule approach, because it is based on a scoring of each alternative. Moreover, the alternatives a_k for which $b_k^1>0$, i.e. the alternatives for which there exists at least one vector of weights for which they are the best, correspond to the efficient alternatives in the Data Envelope Analysis (Charnes, Cooper and Rhodes, 1987).

• The central weight vector: it is the barycentre of the set of weight vectors for which a_k is the best alternative and, consequently, it represents the preferences of the average individual giving to a_k the best position. It is formulated as follows:

$$w_k^c = \frac{1}{b_k^1} \int_{\xi \in \chi} f_\chi(\xi) \int_{w \in W^1(\xi)} f_W(w) w\ dw\ d\xi; \qquad \text{eq. (8)}$$

Of course, one can consider also the barycentre of the set of weight vectors for which a_k is the worst alternative, representing the preferences of the average individual giving to a_k the worst position.

• The confidence factor: it gives the frequency with which an alternative is the most preferred one using its central weight vector and it is given by:

$$p_k^c = \int_{\substack{\xi \in \chi : u(\xi_k, w_k^c) \geq u(\xi_h, w_k^c) \\ \forall h=1,\ldots,l}} f_\chi(\xi)\ d\xi. \qquad \text{eq. (9)}$$

Another interesting index in SMAA is the pairwise winning index (Leskinen et al., 2006), which gives the frequency that an alternative a_h is preferred or indifferent to an alternative a_k in the space of possible weight vectors and possible evaluations on single criteria:

$$p_{hk} = \int_{w \in W} f_W(w) \int_{\xi \in \chi : u(\xi_h, w) \geq u(\xi_k, w)} f_\chi(\xi) d\xi \; dw. \qquad \text{eq. (10)}$$

Therefore, the pairwise winning index is more in the line of the aforementioned Condorcet rule, because it is related to comparisons of couples of alternatives.

From a computational point of view, the multidimensional integrals defining the considered indices are estimated by using the Monte Carlo method. It is worth observing that in case the evaluations on criteria are known and therefore the only variability remains on the vectors of weights ($w_1, \ldots, w_n$), which are supposed to be uniformly distributed in the simplex W, then one can compute the pairwise winning indices p_{hk} using the exact formula given by Zheng and Zheng (2015). However, this formula cannot be used to compute the ranking acceptability indices b_r^k and, moreover, for the values p_{hk} the estimates supplied by the Monte Carlo method are surely acceptable (e.g. Tervonen and Ladhelma (2007) shows that 10,000 extractions are enough to get an error limit of 0,01 for b_r^k with a confidence of 95%).

In our application, for the sake of simplicity, we consider a distribution $f_W(w)$ on W that assigns to each criterion weights with a uniform probability on the interval [0,1] with a subsequent normalization returning a weight vector in W.. Moreover, again to remain as simple as possible, we have not considered the probability distribution $f_\chi(\xi)$ in a first computation in which imprecision in the data was not considered and, again, a specific uniform distribution in a second computation in which robustness with respect to errors in the measurement was tested. However, as explained in the following section, we have taken indirectly into account imprecision in the data through the normalization we have adopted.

We also use the rank acceptability index b_r^k to define a new multidimensional generalization of the Gini index developed as follows. First, for each r=1,…,n-1 let us consider the upward cumulative rank acceptability index of position l, l=1,…,n-1, as the probability that an alternative a_k has a rank position l or better (Angilella et al. 2016), that is

$$b_{\succeq l}^k = \sum_{s=1}^{l} b_s^k \, . \qquad \text{eq. (11)}$$

Now one can compute the Gini index of the upward cumulative rank acceptability index of position l, that is

$$G^{\succeq l} = \frac{\sum_{h=1}^{n}\sum_{k=1}^{n}\left|b_{\succeq l}^h - b_{\succeq l}^k\right|}{2n\sum_{h=1}^{n} b_{\succeq l}^h} \qquad \text{eq. (12)}$$

which, taking into account that $\sum_{h=1}^{n} b_r^h = 1$ for each r=1,…n, and, consequently, $\sum_{h=1}^{n} b_{\succeq l}^h = l$, gives

$$G^{\succeq l} = \frac{\sum_{h=1}^{n}\sum_{k=1}^{n}\left|b_{\succeq l}^h - b_{\succeq l}^k\right|}{2nl} \qquad \text{eq. (13)}$$

$G^{\succeq l}$ measures the concentration of probability to attain rank position l or better among the considered alternatives. The $G^{\succeq l}$ based on the rank acceptability indices b_r^k, takes into account all the possible vectors of weights and it is not based on a specific and to some extent arbitrary single vector of weights, as it is the case in the multidimensional concentration indices proposed in literature (for a review see e.g. Savaglio 2006 and

Weymark 2006). An index analogous to $G^{\succeq l}$ but measuring the concentration of probability to attain rank position l or worse, l=2,…,n, among the considered alternatives can be defined analogously as

$$G^{\preceq l} = \frac{\sum_{h=1}^{n}\sum_{k=1}^{n}\left|b_{\preceq l}^{h} - b_{\preceq l}^{k}\right|}{2n(n-l+1)} \qquad \text{eq. (14)}$$

where

$$b_{\preceq l}^{k} = \sum_{s=l}^{n} b_{s}^{k}\ , \qquad \text{eq. (15)}$$

is the downward cumulative rank acceptability index of position for alternative a_k (Angilella et al. 2016).

The two classes of inequality indices $G^{\succeq l}$ and $G^{\preceq l}$ are related as shown by the following result.

Proposition. For any l=2,…,n, the following property hold

$$G^{\preceq l} = \frac{l+1}{n-l+1} G^{\succeq l-1}. \qquad \text{eq. (16)}$$

Proof. Since the probability of an alternative a_k to be ranked in position l or worse is the complement of the probability to be ranked in position l-1 or better, that is $b_{\preceq l}^{k} = 1 - b_{\succeq l-1}^{k}$, we get

$$G^{\preceq l} = \frac{\sum_{h=1}^{n}\sum_{k=1}^{n}\left|b_{\preceq l}^{h} - b_{\preceq l}^{k}\right|}{2n(n-l+1)} = \frac{\sum_{h=1}^{n}\sum_{k=1}^{n}\left|(1-b_{\succeq l-1}^{h}) - (1-b_{\succeq l-1}^{k})\right|}{2n(n-l+1)} = \qquad \text{eq. (17)}$$

$$\frac{\sum_{h=1}^{n}\sum_{k=1}^{n}\left|b_{\succeq l-1}^{h} - b_{\succeq l-1}^{k}\right|}{2n(n-l+1)} = \frac{l-1}{(n-l+1)} \frac{\sum_{h=1}^{n}\sum_{k=1}^{n}\left|b_{\succeq l-1}^{h} - b_{\succeq l-1}^{k}\right|}{2n(l-1)} = \frac{l-1}{(n-l+1)} G^{\succeq l-1}.$$

Besides Gini indices $G^{\succeq l}$ and $G^{\preceq l}$, upward and downward cumulative rank acceptability indices can be used to define a class of polarisation indices (Esteban and Ray 1994, Esteban *et al.* 2007, Wolfson 1994), measuring how much polarised is the probability to live in a region having a rank position l or better or the probability to live in a region having a rank position l or worse. Details are reported in appendix.
It is worth noticing here that the polarisation indices $EGR^{\succeq l}$ and $EGR^{\preceq l}$ have multidimensional nature so that they have to be evaluated also as a contribution to the literature on multidimensional polarization indices (see e.g. Gigliarano and Mosler, 2009; Sheicher, 2010; Aleskerov and Oleynik, 2016).
In what follows we apply the SMAA methodology to the ranking of Italian regions (spatial alternatives $A=\{a_{1,\ldots,}\ a_m\}$) using a set of socio-economic and environmental variables as evaluation criteria ($G=\{g_1,\ldots,g_n\}$) to be evaluated according to the set of vectors of weights W.

3. Application to performance ranking of Italian regions

Building upon Guerrieri and Iammarino (2006)[4] we apply the aforementioned SMAA to rank the 20 Italian regions according to a set of 65 indicators belonging to the newly introduced 'BES[5]: Equitable and Sustainable Well-being' database (ISTAT, 2015). The ranking related to these variables contains a large amount of information on many

[4] It is worth noticing that Guerrieri and Iammarino (2006) already provided an analysis more comprehensive than the one based on a single indicator. Nonetheless, the methodological approach is substantially different. Guerrini and Iammarino (2006) adopt the Principal Component Analysis (PCA) to "obtain new summary-variables to encapsulate all the information available through linear combinations, while at the same time identifying the interdependencies among the original variables" (p. 170).
[5] From the Italian *Benessere Equo e Sostenibile*. Website: http://www.istat.it/en/archive/180526. The data used in the analysis refer to year 2014 as it represents the most recent year for which a balanced dataset can be extracted. See Appenidx A for details.

aspects of regional development; one that goes well beyond the mainstream measure(s) of regional economic output (e.g. GVA or GDP). This choice is in line with the idea of the multi-dimensionality of quality of life widely accepted in the literature (Stiglitz et al., 2009; OECD, 2011). As it is well known, Italy has a long history of economic dualism dating back to the unification process in 1861 (Del Monte and De Luzenberger, 1989; Spadavecchia, 2007; Torrisi et al. 2015). Our results confirm such a socio-economic dualism along with the several dimensions here considered. Building upon Pike et al. (2012, pp. 17-18) we preliminary explore the spatial disparities considering the issue of concentration and polarisation of the selected indicators, separately. Indeed, it is well known that inequality measures could be low despite the presence of a strong polarisation (Esteban and Ray, 1994). Table 1 reports measures of concentration (Gini index) and polarization EGR (Esteban, Gardìn, and Ray, 2007) index for each of the 65 variables.

INSERT TABLE 1 ABOUT HERE

From Table 1 it is worth stressing that there are variables showing levels of concentration and polarization much higher than the Households Disposable Income (HDI)[6] (Gini index of 0.10 and an EGR index of 0.06). Overall, the inequality measures range from 0.02 (SOC3) to 0.47 (ENV3). Furthermore, two key aspects - Employment and Social Conditions - show Gini indices as high as about 0.26 (WORK2) and 0.21 (SOC7) and an EGR index of 0.15 and 0.10, respectively.

In the following we investigate the dualism between Southern and Northern Italian regions taking into account a synthesis of the above dimensions. Details about the normalisation procedure are reported in appendix.

[6] As shown in Table A.1 the variable *Econw1* refers to HDI.

For illustrative purposes, we begin with the evaluation according to the usual arithmetic mean (equal weights) of the performances normalized on the interval having as extreme the minimum and the maximum evaluations.

INSERT TABLE 2 ABOUT HERE

After the data were normalised , performances of Italian regions were aggregated using a weighted geometric mean, because it allows one to avoid complete compensatoriness as discussed in Section 2. Both the resulting composite indices are shown in Table 2. As expected, in both cases Northern regions have overall a better performance than Southern ones. For example, in both rankings Trentino Alto Adige achieves the first position followed by Friuli-Venezia Giulia. As for the bottom five positions, Calabria ranks 16th, followed by Basilicata, Apulia, Sardinia, and Campania for the composite index based on the arithmetic mean, while for the composite index based on the geometric mean, in the same rank position there are Basilicata, Calabria, Apulia, Campania and Sicily. The most striking differences between ranking regards Sardinia and Sicily, with the former being 19th in the ranking based on arithmetic mean and 13rd in the ranking based on geometric mean, and the latter being 13rd and 20th, respectively. The Kendall Tau of the two rankings is 0.811.

Afterwards, to carry out further analysis of this ranking, we used the SMAA approach on the composite index based on geometric mean with the aim of exploring the whole space of possible weight vectors considering the whole spectrum of possible individual preferences. In this perspective one could expect that some region could be more preferred by some categories of individuals, while other regions could be preferred by others. This would be proved by some probability of being in the first rank positions also for regions that usually are at the last positions of the usual rankings. More specifically, with respect to the Italian North-South divide one could expect that there could be some even small probability for the Mezzogiorno regions to be in the first positions. Our research shows that this is not the case and this can be interpreted as showing that Southern regions of Italy are the less preferred by all the different categories of citizens. This is the key original contribution of our research to explaining the Italian dichotomy. That is, our study shows that the prevalence of the North regions is widespread and generalized to all the categories of stakeholders. In fact, our approach is based on the answer to the following question: how robust is the observed dualism with respect to the relative importance granted to each dimension? Despite its crucial relevance, indeed, the above question can have only limited or no answer according to the mainstream approach based on weighted arithmetic mean of an opportune transformation of considered dimensions. This approach is followed, for example, by the EU to build the EU Regional Competitiveness Index[7] (Annoni and Kozovska, 2010; Dijkstra et al., 2011). Indeed, the weighting issue is still controversial and even sophisticated attempts to achieve a common weighing framework to be applied to composite wellbeing measures have not been fully convincing (for a general discussion about the weighting issue as applied to well-being measures see, for example, Decancq and Lugo, 2008). Nonetheless, mainstream composite indices of regional socio-economic performance do not allow for differences in the weighting system and are thus effectively maintaining an unwarranted mask of objectivity. They implicitly assume equal weighting which may not be justified with respect to the preferences of different groups of individuals. The equal weighting assumption runs counter to a policy world

[7] Although we acknowledge that the cited index does perform a sensitivity analysis to test the robustness of the weighting vectors, it is worth stressing that it limits the analysis to a given interval (Dijkstra et al., 2011) with range lower or equal to 0.2 according to the development stage. Similarly, with respect to the UK case, Huggins (2010) tests the robustness of the UK Competitiveness Index by means of alternative single values for the chosen weights.

that values local preferences, and hence runs counter to the seminal contributions founded on their importance. These relate to different preferences for sets of local public goods according to the Tiebout (1956) model and further developments in fiscal federalism building upon the work of Oates (1972).

The OECD proposed overcoming the weighting issue by presenting a set of nine headline indicators[8] rather than a single composite index (OECD, 2014) for 362 OECD regions. Arguably, this approach is potentially even more difficult to communicate to the public and decision-makers alike.

The SMAA approach can make a substantial contribution to achieve a better balance in the debates regarding the trade-off between a composite index and a range of indicators. On the one hand, SMAA allows for maximum variety in the relative evaluation of each dimension of wellbeing. On the other hand, in principle, it does not prevent computation of a composite index based on a set of regional characteristics. Therefore, it seems reasonable to apply SMAA as a method offering a broader methodological perspective in tackling the measurement of regional well-being.

Following the SMAA approach, we considered a uniform sampling of 1,000,000 of weights vectors. In order to take into account differences in the weighting of each characteristic (concerning dimensions of regional social, economic, and environmental performance) – potentially reflecting differences in preferences - we explicitly highlight the unavoidable subjectivity behind any ranking exercise simply through applying the SMAA approach. Table 3 reports the resulting ranking.

INSERT TABLE 3 ABOUT HERE

[8] The considered dimensions are income, jobs, housing, education, health, environment, safety, civic engagement, and accessibility of services.

For the sake of clarity, rather than reporting Rank Acceptability Index (RAI), i.e. the ratio between the occurrences a region achieves a given rank and the total number of cases considered, in Table 3 we preferred to show the Rank Frequency (RF). Therefore, Table 3 reports the number of occurrences, out of the 1 million cases, a region achieves each possible ranking from 1 to 20, depending on different weights assigned to each of the 65 considered dimensions. Indeed, numerical approximations could assign a misleading null probability to some RAI in cases in which, even if with a small number of occurrences, RF is not null. However, when there is no risk of these misleading conclusions, we refer to RAI rather than to RF (because, of course, RAI=RF/1,000,000). In Table 3, for example, one can see that Piedmont never ranks 1st or 2nd and it ranks 3rd in 10 times out of the 1 million cases considered. Furthermore, it never ranks 12th or worse (i.e. the related RF is null). Furthermore, for each extraction, the set of vectors of weights generating a given ranking can be stored. Hence, an interesting by-product of the analysis is represented – for each region – by the set of weight generating its best, that is the central weight vector recalled in section 2, and worst performance in terms of ranking. Table 4 reports the five criteria with greatest average weights in the set of vector of weights assigning the best position to the corresponding region. Table 5, vice versa, reports the five criteria with greatest average weights corresponding to the worst position of considered region.

INSERT TABLE 4 ABOUT HERE

INSERT TABLE 5 ABOUT HERE

The analogous tables with the weights of all criteria can be found in electronic appendix. The information about the weights giving the best and the worst position supplies interesting elements to analyse the key factors determining good and bad evaluations by citizens. For example, Trentino-South Tyrol is the most preferred region for almost all the vector weights and thus it is not surprising that the average vector of weights assigning it the first position is giving substantially an equal weight to all criteria. However, it is interesting to investigate which is the average vector of the weights for which Trentino-South Tyrol is attaining its worst position (being the 4th) when the greatest weights are taken by criteria HEALTH3 (0.03), WORK8 (0.03), WORK6 (0.029), HEALTH5 (0.025) and POL2 (0.025), that, therefore, are the criteria more important for the average individual appreciating Trentino-South Tyrol to a lesser extent. We have also tested the stability of the central weight vectors for the four regions for which is not null the probability to be the most preferred by computing the relative confidence factor (details are reported in appendix) and we can conclude that the indications supplied by the central weight vector are quite stable.

Going back to Table 3 – considering all the variations in weights – the analysis confirms the North-South divide according to the wider perspective at hand. Based on a rather comprehensive set of indicators, including but not confined to Income, and a comprehensive set of possible weights, Northern and Centre regions perform generally better than Southern regions. On this regard, it is worth stressing here three main elements. First, only Centre-Northern regions (Trentino-South Tyrol, Friuli-Venezia Giulia, Emilia Romagna, and Tuscany) ranked first at least once. Second, only Southern regions (Campania, Apulia, Calabria, and Sicily) ranked last at least once. Third, their best rank is as low as 16th (Campania), 16th (Apulia), 9th (Calabria), and 17th (Sicily). However, within the group of the four above regions under consideration, Calabria achieves its highest rank of 9 in just 2 cases out of the million cases here considered. Within this big picture, Sardinia represents a notable exception. Indeed, its best rank is

5th (though in just 30 out of the million cases considered), its lowest rank is 17^{h}, and it achieves with its highest frequency the 13th rank in 318,317 cases out of the 1 million cases considered, hence, in about 1/3 of cases. The contrast with the other main island is sharp. Indeed, Sicily, as already mentioned, never ranks better than 17th and its highest RAI of about 66% (i.e. about 2 out of 3 cases) corresponds to the last rank.

On the same premise, although Table 3 reports the RF for all ranks, in what follows the analysis will focus on the highest RF for each region. The argument for this is that the rank related to the highest RF for each region is the rank the region achieves with the highest probability, and, therefore, with the highest level of robustness. Table 3 shows that the region with the highest RF in the first position is Trentino Alto Adige (with a RAI of 99.96%). Friuli-Venezia Giulia achieved the highest RF in the second position (with a RAI of 45.63%). Tuscany, Emilia Romagna, and Aosta Valley, achieved the highest rank in the 3rd, 4th, and 5th position with a RAI of 43.37%, 50.06%, and 31.64%, respectively. That is to say, Trentino Alto Adige achieves the first position in this ranking exercise with a rather massive degree of robustness to the choice of different weighting vectors. On the same premise, the data related to the other four aforementioned Centre-North regions achieve the subsequent four ranks with a substantially high robustness (at least in 30% of cases).

Piedmont shows a datum of similar magnitude with its highest RAI of 45.94% referring to the 5th position. The remaining positions show a quite high degree of variation with maximum RAIs between 25.34% (Lazio, 12th position) and 91.51% (Calabria, 17th). Nonetheless, the Southern highest RAIs lay in the area characterised by a rank of 13 or worse, yet, with the already mentioned exception of Sardinia; furthermore, Southern highest RAIs are never below the threshold of 30%.

From a slightly different angle, as far as the bottom five positions are concerned, our analysis confirms that the general wisdom concerning the Southern generalised low performance has a robust basis. Indeed, Basilicata, Calabria, Apulia, Campania, and Sicily show their own highest RAI in the 16th, seventeen17th, 18th, 19th, and 20th rank. with RAIs of 89.64%, 91.50%, 70.71%, 55.21%, and 66.46%, respectively.

The above results do confirm that the North-South divide is definitely wider than the one measured simply in terms of Income. Moreover, the geographical divide is robust to a massive variety of weighting choices. In other words, it is not reasonable to imagine a set of weight vectors able to result in a different overall picture in terms of regional disparities.

To further address this issue, building upon Angilella et al. (2013), Table 6 reports the upward cumulated RAIs $b_{\geq l}^{k}$ for each rank.

INSERT TABLE 6 ABOUT HERE

Therefore, for any rank, values in Table 6 show the probability of achieving at least that rank. For example, while Piedmont achieves a rank of 4 or above[9] with probability 0.002, Aosta Valley ranks 2nd or better with probability 0.003, and so on so forth.

From Table 6, it is worth noticing that 4 regions out of 20 have a probability of (or very close to) 1, to be ranked 5th or better. Namely, Trentino Alto Adige, Emilia Romagna (probability of 0.996), Friuli-Venezia Giulia, and Tuscany. Conversely, there are regions like Marche and those from Abruzzo to Sardinia (in the order they appear in Table 6), with a null probability of belonging to the group of top five regions. In order to provide an even more intuitive representation of this evidence, Graph 1 shows a map of the cumulated RAIs reported in Table 6.

[9] In that precise case the number represents exactly the probability to achieve rank 4 as the probabilities related to higher ranks are null.

INSERT GRAPH 1 ABOUT HERE

The Italian dualism is apparent with only Northern regions having a chance to belong to the group of top five regions according to different weighting vectors. A complementary[10] Graph 2 below reports the probability of belonging to the group of bottom 5 regions.

INSERT GRAPH 2 ABOUT HERE

Graph 2, while confirming from a different perspective the evidence reported in Graph 1, offers interesting elements of differentiation between Southern and Islands regions. First, a white area emerges in the heart of the darkness of Southern regions competing in the Italian regional "relegation zone": it refers to the Basilicata datum (probability of only 0.078). Similarly, Abruzzo has a 0.03 probability of belonging to the same group. Sardinia even shows a null probability of belonging to the group of bottom five regions. To some extent, therefore, according to this peculiar perspective, Abruzzo, Basilicata, and Sardinia represent a kind of Northern regions within the broader Southern region. Put differently, in a broader Southern region generally lagging behind the Northern one, Abruzzo, Basilicata, and Sardinia perform generally better than the regions belonging to their broad region.

The RAI approach allows the comparison of regional performance along the cross-sectional dimension. Thus, by comparing RAIs we are able to compare the overall probability of achieving a given rank between regions. For example, as noted above, the 4th position is achieved by Piedmont in about 0.2% of cases, while Aosta Valley achieves the same position in about 6% of cases. Nonetheless, RAIs fail to provide a direct comparison of the two regions. RAIs tell us that, overall, Piedmont performed

[10] Data reported in Graph 2 come from applying the complement rule to probabilities related to rank 16 reported in Table 8 4.

better than 15 regions and worse than four other regions in about 0.2% of cases. Or, in the cumulated case, the same region (Piedmont) performed at least better than 16 other regions in about 0.2% of the cases. However, neither the simple RAIs nor the cumulated ones are able to give information about the direct comparison between two regions. For example, what is the probability of Piedmont achieving a rank higher than the neighbour Lombardy? Or, with regard to the previous case, what is the probability of Piedmont achieving a rank better than Aosta Valley?

Clearly, an answer to this kind of questions is crucial in both policy design and policy evaluation as they provide information on the relative performance of potentially similar jurisdictions. In order to answer this kind of questions, we provide in Table 7 the Pairwise Comparison Index (PCI) for each couple of regions.

INSERT TABLE 7 ABOUT HERE

Table 7 shows the pairwise winning indices p_{hk} that gives the region a_h the probability to obtain a better score than region a_k. Thus, figures reported in each row represent relative frequencies of the region in that row achieving a score higher than regions reported in columns according to the rule '*row wins against column*'. Hence, regarding the previously mentioned direct comparison Piedmont vs Lombardy, Piedmont achieved a better score than Lombardy in about 78% of cases. Of course, symmetrically Lombardy performed better than Piedmont in about 22% of cases. The last column of Table 7 reporting the Average PCI (APCI) aims to provide a synthetic measure of the overall performance of each region with respect to other region. Thus for a region a_k , the corresponding APCI, denoted q_k, is given by the arithmetic mean of the PCI p_{kh} of region a_k with respect to other regions a_h, that is

$$q_k = \frac{\sum_{h \neq k} p_{hk}}{n-1}. \qquad \text{eq. (18)}$$

Of course, the APCI ranges from zero (i.e. the region achieves a lower score than the remaining 19 in all cases considered) to 1 (i.e. the region achieves a better score than all the "opponents" in all cases). Therefore, Trentino Alto Adige (APCI of 1[11]), Tuscany (API of 0.908), and Emilia Romagna (APCI of 0.876) confirm to be "champions" also according to this peculiar perspective. On the other edge, Sicily with an APCI of only 7% confirms all its weakness in this context. Furthermore, in terms of North-South divide, Table 7 shows that from Abruzzo to Sicily, in only very minor occurrences a Southern region achieves a better score than regions belonging to the Centre-North broad region. Noteworthy, Sardinia has a better performance than the Southern Campania, Apulia, Basilicata, Calabria, and Sicily in all the cases here considered. In 12,5% of cases it performs even better than the Northern Veneto.

For the sake of conciseness, we do not analyse all the pairwise comparisons reported in Table 7. Nonetheless, it is worth stressing here that our approach allowing the direct comparison of pairs of regions unveils patterns of both similarity and dissimilarity even within the same broad region. In so doing, it makes a substantial contribution aiming to go a step further the already widely researched North-South divide.

Finally, building upon Pike et al. (2012)let us apply to this analysis indices $G^{\succeq l}$ and $G^{\preceq l}$ that are shown in Table 8 along with the polarisation indices $\text{EGR}^{\succeq l}$ and $\text{EGR}^{\preceq l}$ to explore spatial inequality and polarisation according to the proposed multidimensional perspective They confirm a great concentration, especially for the best rank positions, as shown by the very high values of $G^{\succeq l}$ for small l, and for the worst rank positions, as

[11] The exact value being equal to 0.99997475.

shown by the very high values of $G^{\preceq l}$ for great l. Let us observe that the levels of concentration are not only much higher than the Households Disposable Income, but also of all the inequality measures of single indicators shown in Table 1. The same evidence overall applies to EGR indices.

This further proves that the comprehensive North-South divide is exacerbating the concentration present in the considered attributes taken singularly and, moreover, it is not related to a given vector of weights assigned to the considered criteria, because the RAI on which $G^{\succeq l}$ and $G^{\preceq}$ are based take into account the whole variety of all possible vectors of weights. Table 8 reports the whole set of Gini indices.

INSERT TABLE 8 ABOUT HERE

Since each variable may be affected by measurement error, (see e.g. LeSage 1999), we have further taken in consideration perturbations in the values assigned to each region by the 65 variables of the BES dataset. Details about the procedure are reported in appendix. The results show that our ranking exercise is robust to substantial differences in measurement.

To summarise: the existence of the North-South divide in Italy is empirically robust to a detailed consideration of a wide variety of dimensions, weighting choices, and measurement errors.

4. Concluding remarks

The SMAA technique has been justified, explained and applied to the performance ranking of Italian regions. This involved a set of socio-economic and environmental indicators, including but not confined to GDP. To the best of our knowledge, this is the first attempt to explore differences in local development using such an approach permitting to take into consideration different preferences of different class of individuals corresponding to different weight vectors. In the Italian regional context characterised by a strong and persistent dualism, this exercise has two main features. First, it allows for a validation of computational results based on prior knowledge of both quantitative and qualitative aspects the Italian regions built over decades of research involving the *questione meridionale* (Southern question). To some extent the analysis at hand confirms that (i) the North-South divide is definitely wider than if measured simply in terms of GDP and that (ii) the presence of uneven patterns of regional development seem robust to an extensive massive variety of weighting choices and perturbations of values on the dimensions considered (thus taking into account also measurement errors).

Second, our approach based on SMAA methodology is able to unveil patterns of spatial disparities more clearly than seems present in the extant empirical literature on the Italian North-South divide. Our analysis finds clear-cut and robust evidence of a generalised better performance of Sardinia with respect to the other big island (Sicily) and, overall, with respect to the broader Southern region. This study has also proposed a class of original multidimensional concentration and polarisation indices. With regard to concentration we propose Gini indices that measure the concentration of the probability of attaining good or poor ranking positions. Similarly, we propose a novel multidimensional extension of the EGR index to analyse the polarisation of the above probabilities. These indices measure a gap between the North and South of Italy that is even more severe than the indices related to single dimensions would indicate.

The implementation of more advanced techniques to unveil and highlight the subjectivity involved in any ranking of territorial units is open for future research attention. Specifically, more advanced models could be developed to take into consideration the interaction between criteria (Angilella, Corrente and Greco, 2015) and the hierarchy of criteria (Angilella, Corrente, Greco and Slowinski, 2015). Nonetheless, our exploratory analysis demonstrates the utility of the SMAA approach – which is even potentially applicable in cross-national comparisons. It is able to make a substantial contribution to achieve robust evaluation of the relative socio-economic performance moving from 'subjective objectivity' and towards more 'objective subjectivity'. Essentially, the SMAA approach can objectively take into consideration the 'inner subjectivity' of all evaluation derived from aggregation of different dimensions with the full spectrum of different weighting choices.

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Stochastic Multiattribute Acceptability Analysis: an application to the ranking of Italian regions

TABLES AND FIGURES

Table 1 – Disparities in social, economic, and environmental indicators

Disparities in SOC8al, economic, and environmental indicators											
Health			Education			Working Conditions			Economic Welfare		
Variable	Gini	EGR	Variable	Gini	EGR	Variable	Gini	EGR	Variable	Gini	EGR
Health1	0.06	0.03	Edu1	0.06	0.03	Work1	0.10	0.05	Econw1	0.10	0.06
Health2	0.05	0.02	Edu2	0.09	0.05	Work2	0.26	0.15	Econw2	0.13	0.06
Health3	0.12	0.06	Edu3	0.16	0.08	Work3	0.19	0.09	Econw3	0.29	0.15
Health4	0.17	0.08	Edu4	0.17	0.08	Work4	0.22	0.11	Econw4	0.29	0.14
Health5	0.16	0.07	Edu5	0.11	0.05	Work5	0.07	0.03	Econw5	0.16	0.07
			Edu6	0.16	0.08	Work6	0.04	0.02	Econw6	0.29	0.15
						Work7	0.02	0.01	Econw7	0.28	0.13
						Work8	0.13	0.06			
						Work9	0.12	0.05			
Social Capital			Politics			Safety			Social Welfare		
Variable	Gini	EGR	Variable	Gini	EGR	Variable	Gini	EGR	Variable	Gini	EGR
Soc1	0.10	0.04	Pol1	0.09	0.04	Sfty1	0.22	0.11	Swel1	0.10	0.05
Soc2	0.10	0.05	Pol2	0.05	0.02	Sfty2	0.38	0.19	Swel2	0.04	0.02
Soc3	0.02	0.01	Pol3	0.04	0.02	Sfty3	0.39	0.18	Swel3	0.06	0.03
Soc4	0.12	0.05	Pol4	0.05	0.02	Sfty4	0.12	0.05	Swel4	0.07	0.03
Soc5	0.06	0.03	Pol5	0.07	0.03	Sfty5	0.12	0.06			
Soc6	0.19	0.08	Pol6	0.02	0.01	Sfty6	0.13	0.07			
Soc7	0.21	0.10	Pol7	0.02	0.01	Sfty7	0.08	0.04			
Soc8	0.11	0.05									

Source: Authors' elaboration on ISTAT (2015)

Table 1 – Disparities in SOC8al, economic, and environmental indicators (cont.)

Disparities in SOC8al, economic, and environmental indicators (cont.)											
Land Use			Environment			R&D			Quality of Life and SOC8al conditions		
Variable	Gini	EGR	Variable	Gini	EGR	Variable	Gini	EGR	Variable	Gini	EGR
Land1	0.20	0.10	Env1	0.38	0.18	Rd1	0.05	0.02	Ql1	0.39	0.21
Land2	0.15	0.07	Env2	0.11	0.05	Rd2	0.06	0.03	Ql2	0.20	0.10
			Env3	0.47	0.23				Ql3	0.11	0.05
			Env4	0.06	0.03				Ql4	0.22	0.12

Source: Authors' elaboration on ISTAT (2015)

Table 2 – Social, economic and environmental performance index (SEEPI)

Region	Aritmetic mean on original values normalized on the interval [min,max]	Rank	Geometric mean of z values normalized on the interval [M-3σ, M+3σ]	Rank
Piedmont	0.528	7	0.515	5
Valle d'Aosta	0.552	5	0.513	6
Lombardy	0.530	6	0.510	7
South Tyrol - Trentino	0.644	1	0.597	1
Veneto	0.525	8	0.486	10
Friuli-Venezia Giulia	0.566	2	0.545	2
Liguria	0.508	9	0.491	8
Emilia-Romagna	0.560	3	0.538	4
Tuscany	0.557	4	0.544	3
Umbria	0.507	10	0.489	9
Marche	0.500	12	0.479	12
Lazio	0.504	11	0.480	11
Abruzzo	0.468	15	0.451	15
Molise	0.475	14	0.459	14
Campania	0.398	20	0.357	19
Apuglia	0.400	18	0,367	18
Basilicata	0.445	17	0.421	16
Calabria	0.446	16	0.404	17
Sicily	0.486	13	0.349	20
Sardinia	0.400	19	0.463	13

Source: authors’ elaboration on ISTAT (2015)

Table 3 – Rank Frequency

Rank	PI	VA	LO	TR	VE	FR	LI	ER	TO	UM	MA	LA	AB	MO	CM	PU	BA	CA	SI	SA
1	0	0	0	999575	0	37	0	97	291	0	0	0	0	0	0	0	0	0	0	0
2	0	2898	1	354	1	456340	0	175472	364935	0	0	0	0	0	0	0	0	0	0	0
3	10	13685	14	62	28	302865	0	249538	433796	0	0	2	0	0	0	0	0	0	0	0
4	2406	60622	551	9	234	239232	12	500637	196262	11	1	21	0	1	0	0	0	0	0	0
5	459397	316363	130609	0	7424	1514	2830	70271	4617	4798	315	1783	0	50	0	0	0	0	0	30
6	417799	198102	320399	0	16417	11	15872	3729	90	20923	1409	4862	0	190	0	0	0	0	0	199
7	106735	231232	457922	0	46031	1	65449	234	7	60977	6272	22980	1	786	0	0	0	0	0	1370
8	13160	87659	72638	0	225310	0	303111	22	2	183239	29574	68277	29	2918	0	0	1	0	0	14063
9	425	44262	13960	0	140473	0	252711	0	0	277223	99798	137621	137	8449	0	0	0	2	0	24939
10	63	28263	3349	0	135533	0	174880	0	0	248078	188519	164327	839	19395	0	0	9	3	0	36743
11	5	11950	485	0	162381	0	131240	0	0	142344	263167	175134	5328	44753	0	0	7	8	0	63196
12	0	4405	65	0	141248	0	42665	0	0	49526	263389	253360	21152	95754	0	0	45	29	0	128360
13	0	488	7	0	73239	0	9892	0	0	11924	109913	121075	115996	238797	0	0	264	89	0	318317
14	0	67	0	0	37033	0	1245	0	0	926	32041	43764	290271	359241	0	0	1920	602	0	232891
15	0	4	0	0	13849	0	86	0	0	25	5543	6639	541594	228176	0	0	23479	3984	0	176619
16	0	0	0	0	609	0	7	0	0	6	58	150	21371	1393	1	2	896435	76815	0	3153
17	0	0	0	0	189	0	0	0	0	0	1	5	3282	97	617	2725	77825	915063	76	120
18	0	0	0	0	1	0	0	0	0	0	0	0	0	0	193856	707119	15	3292	95717	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	552132	208143	0	111	239614	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	253394	82011	0	2	664593	0

Source: authors' elaboration on ISTAT (2015).

Table 4 – Criteria with the five greatest average weights in the set of vector of weights assigning to the corresponding region the best position

	HEALTH1	POL5	WORK4	SFTY2	LAND2
Abruzzo	0.03	0.03	0.029	0.029	0.029
	WORK3	SQ3	WORK8	SOC7	WORK5
Basilicata	0.033	0.032	0.031	0.031	0.03
	WORK8	SFTY6	SFTY2	WORK2	LAND2
Calabria	0.031	0.031	0.03	0.027	0.026
	POL4	LAND2	SQ1	WORK5	SQ2
Campania	0.034	0.033	0.033	0.032	0.032
	WORK8	HEALTH3	HEALTH1	POL1	POL7
Emilia-Romagna	0.027	0.026	0.024	0.023	0.023
	WORK8	WORK6	SFTY3	HEALTH3	SFTY6
Friuli-Venezia Giulia	0.027	0.026	0.024	0.023	0.023
	ECONW3	HEALTH3	POL3	SFTY1	SOC3
Lazio	0.03	0.029	0.029	0.028	0.027
	EDU2	SFTY1	HEALTH2	POL3	RD2
Liguria	0.026	0.026	0.024	0.024	0.024
	SFTY1	ECONW7	SWEL2	SWEL4	SQ1
Lombardy	0.035	0.034	0.034	0.034	0.034
	SFTY6	SFTY1	RD1	HEALTH3	LAND1
Marche	0.035	0.034	0.033	0.032	0.032
	POL1	LAND2	HEALTH3	SFTY5	WORK2
Molise	0.038	0.037	0.035	0.033	0.032
	SFTY4	POL7	SFTY1	HEALTH5	ECONW5
Piedmont	0.029	0.026	0.026	0.025	0.024
	POL5	WORK5	SQ2	POL7	POL3
Apuglia	0.031	0.03	0.028	0.027	0.026
	SFTY1	EDU3	ECONW3	SWEL3	SQ1
Sardinia	0.024	0.023	0.023	0.023	0.023
	WORK4	ECONW5	LAND1	HEALTH5	POL6
Sicily	0.026	0.026	0.024	0.022	0.022
	HEALTH3	WORK6	POL2	WORK8	ENV2
Tuscany	0.026	0.024	0.024	0.023	0.023
	HEALTH1	HEALTH2	HEALTH3	HEALTH4	HEALTH5
South Tyrol - Trentino/Südtirol	0.015	0.015	0.015	0.015	0.015
	SFTY1	WORK6	ECONW3	HEALTH5	SQ1
Umbria	0.03	0.027	0.026	0.024	0.024
	ENV3	SQ3	WORK5	SFTY7	SFTY1
Aosta Valley	0.023	0.022	0.021	0.021	0.02
	HEALTH2	HEALTH5	POL7	SWEL3	RD1
Veneto	0.035	0.034	0.034	0.034	0.034

Source: Authors' elaboration on data from ISTAT (2015)

Table 5 – Criteria with the five greatest average weights in the set of vector of weights assigning to the corresponding region the worst position

	SFTY6	SFTY5	SFTY4	HEALTH2	SOC3
Abruzzo	0.025	0.024	0.023	0.02	0.02
	POL6	POL7	HEALTH5	POL4	HEALTH3
Basilicata	0.026	0.026	0.025	0.025	0.024
	EDU6	WORK4	POL5	ENV2	HEALTH4
Calabria	0.032	0.03	0.029	0.028	0.025
	SFTY5	HEALTH2	ECONW7	SOC1	SOC2
Campania	0.018	0.017	0.017	0.017	0.017
	SFTY1	SFTY4	SFTY6	WORK6	ECONW5
Emilia-Romagna	0.031	0.027	0.025	0.022	0.021
	WORK3	ECONW3	SOC5	SQ3	POL3
Friuli-Venezia Giulia	0.031	0.031	0.03	0.03	0.029
	SFTY2	HEALTH2	WORK8	SFTY4	SFTY7
Lazio	0.029	0.025	0.025	0.025	0.025
	SWEL4	HEALTH1	LAND2	SFTY2	SWEL3
Liguria	0.031	0.027	0.027	0.026	0.025
	WORK3	LAND2	WORK8	WORK6	SFTY7
Lombardy	0.027	0.027	0.025	0.024	0.024
	SOC1	SWEL4	LAND2	WORK9	POL4
Marche	0.032	0.032	0.032	0.031	0.031
	ENV1	SFTY6	WORK6	SFTY4	WORK8
Molise	0.029	0.026	0.025	0.023	0.021
	ENV2	HEALTH1	EDU2	ENV3	RD2
Piedmont	0.031	0.028	0.028	0.026	0.026
	SFTY3	SOC3	HEALTH5	SQ3	SQ4
Apuglia	0.023	0.019	0.018	0.018	0.017
	HEALTH1	EDU1	EDU2	WORK3	ECONW7
Sardinia	0.022	0.022	0.022	0.022	0.022
	ECONW2	WORK9	ECONW4	ENV1	SQ2
Sicily	0.018	0.016	0.016	0.016	0.016
	SFTY4	LAND1	WORK7	SOC5	POL7
Tuscany	0.032	0.03	0.029	0.027	0.027
	HEALTH3	WORK8	WORK6	HEALTH5	POL2
South Tyrol - Trentino/Südtirol	0.03	0.03	0.029	0.025	0.025
	WORK5	ECONW3	SFTY1	ENV3	HEALTH2
Umbria	0.03	0.025	0.025	0.025	0.024
	POL7	LAND2	SQ1	HEALTH3	ECONW3
Aosta Valley	0.03	0.03	0.027	0.026	0.026
	SFTY1	WORK2	SFTY2	HEALTH3	POL2
Veneto	0.037	0.036	0.036	0.035	0.035

Source: Authors' elaboration on data from ISTAT (2015)

Table 6 – Cumulated Rank Acceptability Index

	Rank																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
PI	0.000	0.000	0.000	0.002	0.462	0.880	0.986	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
VA	0.000	0.003	0.017	0.077	0.394	0.592	0.823	0.911	0.955	0.983	0.995	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
LO	0.000	0.000	0.000	0.001	0.131	0.452	0.909	0.982	0.996	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
TR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
VE	0.000	0.000	0.000	0.000	0.008	0.024	0.070	0.295	0.436	0.571	0.734	0.875	0.948	0.985	0.999	1.000	1.000	1.000	1.000	1.000
FR	0.000	0.456	0.759	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
LI	0.000	0.000	0.000	0.000	0.003	0.019	0.084	0.387	0.640	0.815	0.946	0.989	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
ER	0.000	0.176	0.425	0.926	0.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
TO	0.000	0.365	0.799	0.995	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
UM	0.000	0.000	0.000	0.000	0.005	0.026	0.087	0.270	0.547	0.795	0.938	0.987	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
MA	0.000	0.000	0.000	0.000	0.000	0.002	0.008	0.038	0.137	0.326	0.589	0.852	0.962	0.994	1.000	1.000	1.000	1.000	1.000	1.000
LA	0.000	0.000	0.000	0.000	0.002	0.007	0.030	0.098	0.236	0.400	0.575	0.828	0.949	0.993	1.000	1.000	1.000	1.000	1.000	1.000
AB	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.027	0.143	0.434	0.975	0.997	1.000	1.000	1.000	1.000
MO	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.012	0.032	0.077	0.172	0.411	0.770	0.999	1.000	1.000	1.000	1.000	1.000
CM	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.194	0.747	1.000
PU	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.710	0.918	1.000
BA	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.026	0.922	1.000	1.000	1.000	1.000
CA	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.005	0.082	0.997	1.000	1.000	1.000
SI	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.096	0.335	1.000
SA	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.016	0.041	0.077	0.141	0.269	0.587	0.820	0.997	1.000	1.000	1.000	1.000	1.000

Source: Authors' elaboration on ISTAT (2015).

Table 7 – Pairwise comparison index

	PI	VA	LO	TR	VE	FR	LI	ER	TO	UM	MA	LA	AB	MO	CM	PU	BA	CA	SI	SA	APCI
PI	1	0.579	0.787	0	0.974	0	0.995	0.005	0	0.993	0.999	0.997	1	1	1	1	1	1	1	1	0.76645
VA	0.421	1	0.583	0	0.941	0.006	0.898	0.074	0.017	0.896	0.976	0.94	0.999	0.998	1	1	1	1	1	1	0.73745
LO	0.213	0.417	1	0	0.962	0	0.954	0.001	0	0.948	0.989	0.989	1	0.998	1	1	1	1	1	0.998	0.72345
TR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
VE	0.026	0.059	0.038	0	1	0	0.395	0.001	0	0.426	0.664	0.606	0.966	0.894	1	1	0.999	0.999	1	0.875	0.5474
FR	1	0.994	1	0	1	1	1	0.692	0.529	1	1	1	1	1	1	1	1	1	1	1	0.91075
LI	0.005	0.102	0.046	0	0.605	0	1	0	0	0.564	0.806	0.83	0.999	0.982	1	1	1	1	1	0.942	0.59405
ER	0.995	0.926	0.999	0	0.999	0.308	1	1	0.295	1	1	1	1	1	1	1	1	1	1	1	0.8761
TO	1	0.983	1	0	1	0.471	1	0.705	1	1	1	1	1	1	1	1	1	1	1	1	0.90795
UM	0.007	0.104	0.052	0	0.574	0	0.436	0	0	1	0.843	0.732	1	0.98	1	1	1	1	1	0.926	0.5827
MA	0.001	0.024	0.011	0	0.336	0	0.194	0	0	0.157	1	0.482	0.978	0.896	1	1	1	1	1	0.829	0.4954
LA	0.003	0.06	0.011	0	0.394	0	0.17	0	0	0.268	0.518	1	0.981	0.886	1	1	1	1	1	0.829	0.506
AB	0	0.001	0	0	0.034	0	0.001	0	0	0	0.022	0.019	1	0.294	1	1	0.977	0.994	1	0.241	0.32915
MO	0	0.002	0.002	0	0.106	0	0.018	0	0	0.02	0.104	0.114	0.706	1	1	1	0.998	0.999	1	0.407	0.3738
CM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.231	0	0.001	0.71	0	0.0971
PU	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.769	1	0	0.003	0.859	0	0.13155
BA	0	0	0	0	0.001	0	0	0	0	0	0	0	0.023	0.002	1	1	1	0.921	1	0.004	0.24755
CA	0	0	0	0	0.001	0	0	0	0	0	0	0	0.006	0.001	0.999	0.997	0.079	1	1	0	0.20415
SI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.29	0.141	0	0	1	0	0.07155
SA	0	0	0.002	0	0.125	0	0.058	0	0	0.074	0.171	0.171	0.759	0.593	1	1	0.996	1	1	1	0.39745

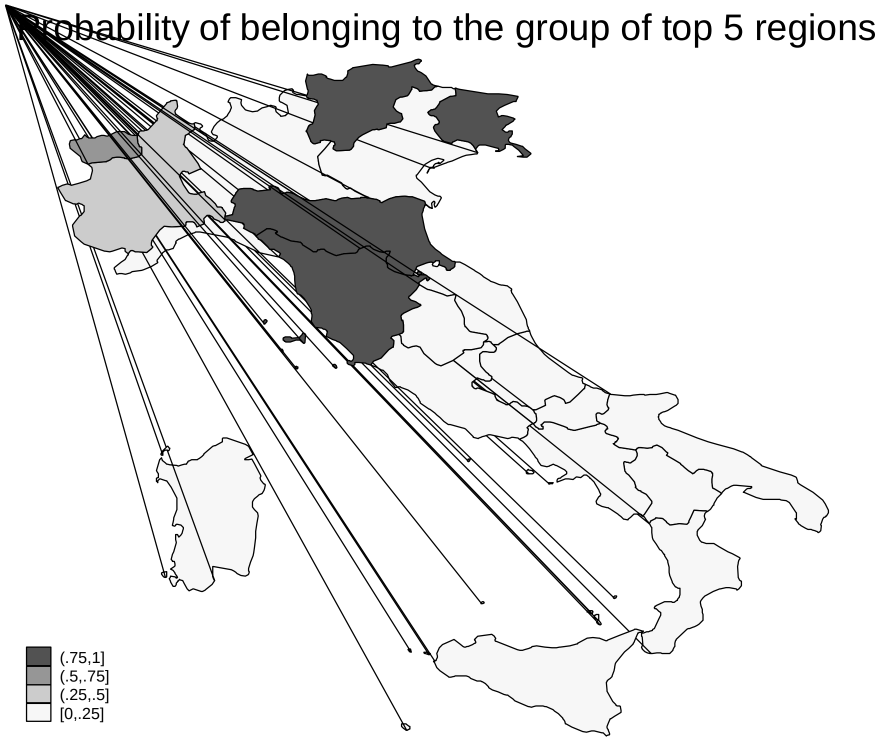
Source: authors' elaboration on ISTAT (2015).

Table 8 – Multidimensional inequality G-indices and Polarisation EGR-indices

Rank (l)	$G^{\geq l}$	$G^{\leq l}$	$EGR^{\geq l}$	$EGR^{\leq l}$
1	0.9999	-	0.9663	-
2	0.9092	0.0526	0.8441	-
3	0.8716	0.1010	0.9015	0.0868
4	0.8397	0.1538	0.9209	0.1434
5	0.7741	0.2099	0.6025	0.1468
6	0.7225	0.2580	0.4864	0.2145
7	0.6728	0.3096	0.5232	0.3432
8	0.6117	0.3623	0.4261	0.3382
9	0.5555	0.4078	0.3500	0.3303
10	0.5048	0.4545	0.3992	0.3466
11	0.4576	0.5048	0.3153	0.3904
12	0.4107	0.5593	0.3431	0.5225
13	0.3605	0.6160	-	0.5853
14	0.3106	0.6695	-	-
15	0.2628	0.7247	-	-
16	0.2100	0.7885	-	-
17	0.1579	0.8398	-	-
18	0.1030	0.8946	-	-
19	0.0503	0.9270	-	-
20	-	0.9561	-	-

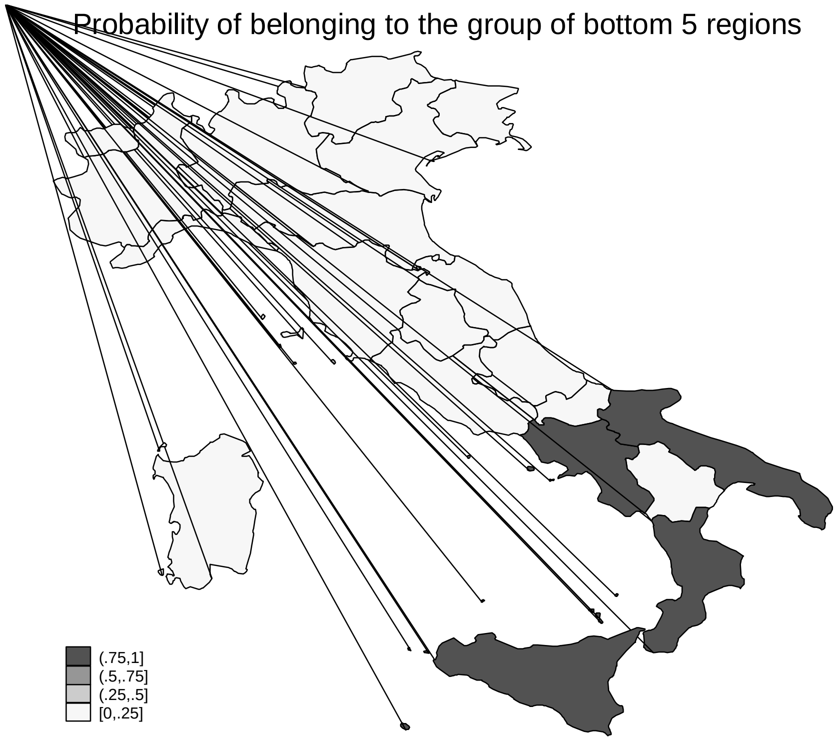
Source: Authors' elaboration on data from ISTAT (2015). EGR weighted for population, alpha=1; beta=1.

Graphs 1 – Probability of belonging to the group of top five regions



Source: authors' elaboration on ISTAT (2015)

Graph 2 – Probability of belonging to the group of bottom 5 regions



Source: authors' elaboration on ISTAT (2015)

Stochastic Multiattribute Acceptability Analysis: an application to the ranking of Italian regions

APPENDIX A

A.1 Multidimensional polarisation indices

The multidimensional Gini index has been developed as follows. We build upon the polarisation index proposed by Esteban and Ray (1994) corrected as proposed by Esteban *et al.* (2007). More precisely, with respect to the upward cumulative rank acceptability index $b^k_{\succeq l}$, l=1,…,n-1, we computed the mean value $b^M_{\succeq l}$ of the upward cumulative rank acceptability indices $b^k_{\succeq l}$, k=1,….,n, that is

$$b^M_{\succeq l} = \frac{\sum_{k=1}^{n} P_k b^k_{\succeq l}}{\sum_{k=1}^{n} P_k}, \qquad \text{eq. (A.1)}$$

With P_k, being the population of the k-th region, k=1,….,n. After we calculated the normalized upward cumulative rank acceptability indices $\tilde{b}^k_{\succeq l}$, that is

$$\tilde{b}^k_{\succeq l} = \frac{b^k_{\succeq l}}{b^M_{\succeq l}}. \qquad \text{eq. (A.2)}$$

On the basis of values $\tilde{b}^k_{\succeq l}$, we defined the cumulative distribution $F^{\succeq l}:[0,1]\rightarrow[0,1]$ such that for all $x\in[0,1]$

$$F^{\succeq}(x) = \frac{\sum_{k:\tilde{b}^k_{\succeq l}\geq x} P_k}{\sum_{k=1}^{n} P_k}. \qquad \text{eq. (A.3)}$$

Following the methodology proposed by Aghevli and Mehran (1981) and Davies and Shorrocks (1989), we found also an optimal partition $\rho^{\succeq l}$ of the distribution $F^{\succeq l}$ in r groups minimise the Gini index value of within-group inequality, r≤n, that is

$$\rho^{\succeq l} = (z_0^{\succeq l}, z_1^{\succeq l}, ..., z_{r-1}^{\succeq l} \leq z_r^{\succeq l} = 1; y_1^{\succeq l}, ..., y_r^{\succeq l}; p_1^{\succeq l}, ..., p_r^{\succeq l}) \qquad \text{eq. (A.4)}$$

with $0 = z_0^{\succeq l} \le z_1^{\succeq l} \le ... \le z_{r-1}^{\succeq l} \le z_r^{\succeq l} = 1$ and $y_1^{\succeq l}$ and $p_1^{\succeq l}$ being the average value of the normalized cumulative rank acceptability indices $\tilde{b}_{\succeq l}^{k}$ and the population shares in the interval $[z_{i-1}^{\succeq l}, z_i^{\succeq l}]$ of $\tilde{b}_{\succeq l}^{k}$ values.

Finally we computed the polarization index $EGR^{\succeq l}$ as follows:

$$EGR^{\succeq l}(F, \alpha, \beta, \rho^{\succeq l}) = \sum_{i=1}^{r}\sum_{j=1}^{r}\left(p_i^{\succeq l}\right)^{1+\alpha} p_j^{\succeq l}\left|y_i^{\succeq l} - y_j^{\succeq l}\right| - \beta\left[G(F) - G\left(\rho^{\succeq l}\right)\right] \qquad \text{eq. (A.5)}$$

with $\alpha \in [1,1.16]$ is the sensitivity to polarization and $\beta \ge 0$. In our application to the study of Italian regions we considered 2 groups in the partition $\rho^{\succeq l}$, $\alpha=1$ and $\beta=1$. Analogous polarization indices $EGR^{\preceq l}$, $l=1,\ldots,n-1$, can be defined with respect to the downward cumulative rank acceptability index $b_{\preceq l}^{k}$.

A.2 Data and normalisation procedure

Table A.1 reports variables description along with summary statistics. Please note also that the last column of Table A.1 reports the categorization of each variable according the good/bad nature of the considered criteria.

Taking inspiration from Mazziotta and Pareto (2016), to make comparable variables expressed on different metric we normalised them according to the following formula that assigns to each value x on a “good criterion”, that is a criterion with a preference increasing with respect to the assigned value (e.g. gross domestic product), the normalized value

$$\bar{x} = \begin{cases} 0 & if\ z \le -3 \\ \dfrac{z+3}{6} & -3 < z < 3 \\ 1 & z \ge 3 \end{cases} \qquad \text{eq. (A.6)}$$

where z is the z-score

$$z = \frac{x - M}{\sigma} \qquad \text{eq. (A.7)}$$

with M and σ being the mean and the standard deviation of the considered criterion, respectively, so that

$$\bar{x} = \begin{cases} 0 & \text{if } x \le M - 3\sigma \\ \dfrac{x - M + 3\sigma}{6\sigma} = 0.5 + \dfrac{z}{6} & \text{if } M - 3\sigma < x < M + 3\sigma \\ 1 & \text{f } x \ge M + 3\sigma \end{cases} \qquad \text{eq. (A.8)}$$

In case of a “bad criterion”, that is a criterion with a preference decreasing with respect to the assigned value (e.g. the social exclusion), the normalized value $\bar{x}$ of x is given by

$$\bar{x} = \begin{cases} 1 & if\ z \le -3 \\ \dfrac{3 - z}{6} & -3 < z < 3 \\ 0 & z \ge 3 \end{cases} \qquad \text{eq. (A.9)}$$

that is,

$$\bar{x} = \begin{cases} 1 & \text{if } x \le M - 3\sigma \\ \dfrac{M - x + 3\sigma}{6\sigma} = 0.5 - \dfrac{z}{6} & \text{if } M - 3\sigma < x < M + 3\sigma \\ 0 & \text{f } x \ge M + 3\sigma \end{cases} \qquad \text{eq. (A.10)}$$

The idea is to consider as extreme of the normalization scales the values M-3σ and M+3σ within which lie 99,73% of values in case of normal distribution and, by the Chebyshev's inequality, 89% of values for any distribution for which an average and standard deviation are defined.

For illustrative purposes, we begin with the evaluation according to the usual arithmetic mean (equal weights) of the performances normalized on the interval having as extreme the minimum and the maximum evaluations, that is

$$\widetilde{x_i} = \frac{x_i - x_{min}}{x_{max} - x_{min}};$$ eq. (A.11)

in case of a "good criterion", or

$$\widetilde{x_i} = \frac{x_{max} - x_i}{x_{max} - x_{min}}$$ eq. (A.12)

in case of a "bad criterion".

A.3 Robustness checks

Stability of central weights.

We have tested the stability of the central weight vectors for the four regions for which is not null the probability to be the most preferred by computing the relative confidence factor. We proceeded as follows. We generated perturbed evaluations on considered criteria for all the regions by extracting random values in the interval

$$[g_i(a)\text{-}0.25\sigma_i, g_i(a)+ 0.25\sigma_i]$$

for the evaluations of each region *a* on considered criteria g_i, where σ_i is the standard deviation of the criterion g_i, i=1,….,65. Taking the central weight vector of the region *a** for which we test the stability of the weight vector giving it the best position, we computed the new ranking corresponding to the perturbed evaluations. We repeated this procedure 1,000,000 times and we got an estimation that the region *a** remains the best. This probability is 100% for Trentino Alto Adige, 87.2% for Toscana, 84,5% for Emilia Romagna and 80% for Friuli-Venezia Giulia.

Measurement error.

To test the robustness to measurement error we have taken in consideration perturbations in the values assigned to each region by the 65 variables of the BES dataset. More precisely, we considered an interval of variation

$$[g_i(a)-k\sigma_i,\ g_i(a)+k\sigma_i]$$

for the evaluations of each region *a* on considered criteria g_i, where σ_i is the standard deviation of the criterion g_i, i=1,….,65 and $k \geq 0$. The case k=0 corresponds to the absence of any perturbation, that is, the case of RF in above Table 2. We further considered the case k=0.25, k=0.5 and k=1. In each one of these case and in each one of 1,000,000 of iterations we randomly extracted not only a vector of weights for the 65 criteria, but also a perturbed evaluation $\tilde{g}_i(a)$ in the considered range for each region *a* on each criterion g_i, i=1,….,65. On the basis of the perturbed values, for each one of the 65 criteria considered by BES, we computed the "perturbed mean" and the "perturbed standard deviation" and we normalized according to equations (8) and (9) the perturbed evaluations $\tilde{g}_i(a)$. The RF and the PWI corresponding to k=0.25 are shown in Table A.3 and Table A.4, respectively. The analogous tables for k=0.5 and k=1 can be found in the electronic appendix.

In order to assess the consistency and reliability of the resulting ranking, the Intraclass Correlation Coefficient (ICC) has been computed considering the above k=0.25, 0.5, and 1 as resulting from alternative evaluation exercises performed by 3 additional raters with respect to the actual measurement released by the Italian National Institute of Statistics. To this end, the consistency-of-agreement ICC (CA-ICC) has been used. The rationale for adopting the CA-ICC is that different measurements are considered consistent if the scores from any two measurements (or *raters*) give the same ranking to all the regions (Shrout and Fleiss, 1979; McGraw and Wong, 1996a, 1996b). The results reported in Table A.4 show that our ranking exercise is robust to the substantial differences in measurement here hypothesised. Indeed, both the individual and the average coefficients are in no occasion[1] lower than 0.60 with 15 out of the 20 ranking here considered showing a ICC higher than 0.80.

[1] All the ICC are statistically significant according to related F-test.

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APPENDIX TABLES

Table A.1 - Variables description and descriptive statistics

Variable	Description	Categorisation	Mean	Std. Dev.	Min	Max
	Health					
HEALTH1	Life expectancy without limitations in activities at age 65	Good	44.69	4.532909	38.6	52.6
HEALTH2	Rate of mortality from dementia and diseases of the nervous system	Bad	19.795	1.616519	16.5	22.3
HEALTH3	People aged 14 and older who have at least one risk behaviour in alcohol consumption	Bad	16.83	3.635946	10.7	24.3
HEALTH4	People aged 14 and over who do not practice any physical activity	Bad	39.135	11.79966	17.6	60.4
HEALTH5	People aged 3 years and older who consume at least 4 daily servings of fruits and / or vegetables	Good	17.62	5.246713	6.2	28.3
	Education					
EDU1	People 25-64 who have completed at least the secondary school	Good	59.565	6.690155	47.2	70.1
EDU2	People aged 30-34 who have completed a university degree	Good	24.2	4.166533	17.4	31.6
EDU3	Graduates who enrol for the first time at the university in the same year in which they graduated	Good	13.995	4.328665	8.4	24
EDU4	People 15-29 years neither in education, employment or training (Neet)	Bad	25.115	7.691915	14.3	40.3
EDU5	People aged 25-64 who participated in education and training in the 4 weeks preceding the interview	Good	8.275	1.695776	5.1	12
EDU6	People aged 6 and over who have practiced three or more cultural activities in the preceding 12 months	Good	26.295	7.74627	14.8	42.1
	Working Conditions					
WORK1	Employment rate of the population aged 20-64 years	Good	60.08	10.70015	42.4	73.6
WORK2	Rate of non-participation in the work of the population aged 15-74 years	Good	22.92	11.26805	9	42.7
WORK3	Fixed-term employees and employees who started their current job for at least five years	Good	20.205	7.301008	10.1	38

Variable	Description	Categorisation	Mean	Std. Dev.	Min	Max
WORK4	Rate of low-pay employees	Bad	11.28	4.651497	6.8	22.4
WORK5	Rate of overeducated employees	Bad	23.16	2.860327	17.6	29.4
WORK6	Employment rate of women aged 25-49 with at least one child aged 6-14 over total number of women	Good	81.13	6.629844	67.3	92
WORK7	Work satisfaction	Good	7.245	0.213923	6.9	7.7
WORK8	Employment insecurity perception	Good	10.74	2.627466	5.9	17
WORK9	Share of involuntary part-time to total employment by gender	Bad	11.845	2.518244	7	16.7
	Economic Welfare					
ECONW1	Average disposable income of households	Good	17223.9	3245.781	12343	21286
ECONW2	Index of inequality in disposable income	Bad	5.16	1.302386	3.6	8.9
ECONW3	People at risk of poverty	Bad	19.36	10.29913	7.7	40.1
ECONW4	People living in households with severe material deprivation	Bad	10.96	6.194684	3.1	26
ECONW5	People living in overcrowding situations, in dwellings without some services and with structural problems	Bad	9.44	2.845569	5.5	15.7
ECONW6	People under 60 years living in households with very low work intensity	Bad	12.09	6.605811	4.6	25
ECONW7	Subjective evaluation index of economic difficulty	Bad	16.61	9.040896	4.6	38.4
	Social Capital					
SOC1	People aged 14 and over who declare themselves very satisfied of family relationships	Good	33.73	6.012452	20.9	46.3
SOC2	People aged 14 and over who declare themselves very satisfied with the friendly relations	Good	24.02	4.593657	15.4	35.3
SOC3	People aged 14 and over who have relatives, friends or neighbours you can count on	Good	81.88	3.134461	74.5	88
SOC4	People aged 14 and over who during the last 12 months they have played at least one social participation activities	Good	24.015	5.632825	14.9	40
SOC5	People aged 14 and over who talk about politics or that you inform policy at least once a week, who participated in online consultations or vote on social or political issues or	Good	66.915	7.447166	53.7	75.7

Variable	Description	Categorisation	Mean	Std. Dev.	Min	Max
	have read and posted opinions on social and political problems on the web in recent 3 months					
SOC6	People aged 14 and over who during the last 12 months have carried out unpaid work for organizations or volunteer groups	Good	10.255	3.782296	5	21.9
SOC7	People aged 14 and over who during the last 12 months have funded associations	Good	14.91	5.775802	5.7	28.8
SOC8	People aged 14 and over who trust others	Good	23.1	4.450429	17.3	33.6
	Politics					
POL1	People aged 18 and over who have voted in the European Parliament elections	Good	57.46	9.097912	42	70.5
POL2	People aged 14 and over who express confidence in the Italian Parliament	Good	3.41	0.305907	2.8	4
POL3	People aged 14 and over who express confidence in the judicial system	Good	4.215	0.297843	3.4	4.8
POL4	People aged 14 and over who express confidence in parties	Good	2.355	0.230503	1.9	2.8
POL5	People aged 14 and over who express confidence in the regional government, the provincial government or in their own town	Good	3.72	0.503253	2.9	4.9
POL6	People aged 14 and over who express confidence in the police and fire brigade	Good	7.055	0.24165	6.6	7.5
POL7	Average age of MPs - XVII Legislature - January 2014	Bad	49.925	1.560322	45.9	52.5
	Safety					
SFTY1	Rate of theft in dwelling	Bad	16.125	6.641724	4.9	31.9
SFTY2	Rate of pickpocketing	Bad	5.96	4.291289	0.9	16.5
SFTY3	Rate of robbery	Bad	1.56	1.132487	0.1	4.6
SFTY4	Women 16-70 years old who have experienced physical violence in the last five years	Bad	7.015	1.468359	4.3	9.3
SFTY5	Women 16-70 years old who have suffered sexual violence	Bad	6.165	1.361994	3.9	9.1
SFTY6	Women 16-70 years old who have suffered physical or sexual violence in the last five years by region and division	Bad	4.885	1.227867	2.4	7.6

Variable	Description	Categorisation	Mean	Std. Dev.	Min	Max
SFTY7	People aged 14 and over who feel safe walking alone at night in the area where they live	Good	61.225	8.897361	49.8	82.6
	Social Welfare					
SWEL1	People aged 14 and over who have expressed a satisfaction score for life between 8 and 10	Good	36.55	7.224993	20.6	54
SWEL2	People aged 14 and over who say they are very or fairly satisfied Leisure	Good	65.24	5.196497	56.3	75.7
SWEL3	People aged 14 and over who feel that their situation will improve in the next five years	Good	26.455	3.034273	22.3	34.6
SWEL4	People aged 14 and over who feel that their situation will worsen over the next five years	Bad	18.605	2.487331	13.5	22.9
	Land Use					
LAND1	People aged 14 and over who feel the landscape of the place in which they live is suffering from obvious degradation	Bad	18.61	6.7765	7.8	32.6
LAND2	People aged 14 and over who declare among the 5 environmental problems for which express their concern for the landscape there is the ruin caused by excessive building activity	Bad	16.665	4.363397	9.4	24.2
	Environment					
ENV1	Municipal waste landfilled	Bad	39.83	28.26601	6.1	111
ENV2	People aged 14 and over believe that the extinction of plant species or animal is between 5 their concerns	Bad	17.385	3.36816	12.6	24
ENV3	Electricity consumption covered by renewable sources	Good	62.24	72.42566	10.4	310.2
ENV4	People aged 14 and over who are very or fairly satisfied the environmental situation of the area in which they live	Good	74.72	8.568276	57.2	89.6

Variable	Description	Categorisation	Mean	Std. Dev.	Min	Max
	R&D					
RD1	Employed with university education in Scientific or Technological professions	Good	14.97	1.622896	12.4	20
RD2	People of 16-74 years who have used the Internet at least once a week in the preceding 3 months	Good	58.95	6.595174	48.4	66.5
	Quality of Life and social conditions					
Ql1	Households reporting that the house is connected to the natural gas network	Good	9.35	7.47068	1.4	30.7
Ql2	Recycled municipal waste	Good	43.77	16.16729	12.5	67.6
Ql3	Index of overcrowding of prisons	Bad	105.03	21.4091	56.8	138
Ql4	Households by great difficulty in reaching at least 3 essential services	Bad	6.835	2.894328	3.2	12.3

Source: Authors' elaboration on ISTAT (2015).

Table A.3 - Rank Frequency (robustness test, k=0.25)

Rank	PI	VA	LO	TR	VE	FR	LI	ER	TO	UM	MA	LA	AB	MO	CM	PU	BA	CA	SI	SA
1	0	0	0	998815	0	166	0	341	678	0	0	0	0	0	0	0	0	0	0	0
2	1	4108	0	1002	2	445152	0	184418	365317	0	0	0	0	0	0	0	0	0	0	0
3	190	17410	129	171	64	307539	1	261237	413255	1	0	4	0	1	0	0	0	0	0	0
4	11131	68928	2805	12	584	242603	89	462256	211434	58	9	83	0	3	0	0	0	0	0	3
5	435991	293305	153725	0	9297	4424	5045	78754	8784	7382	651	2501	0	86	0	0	0	0	0	57
6	393362	204409	312966	0	20750	112	21629	11129	469	24965	2504	7068	3	293	0	0	0	0	0	340
7	136168	218117	409982	0	53052	4	75953	1672	57	67130	9556	25305	7	1061	0	0	0	0	0	1937
8	21202	92997	90856	0	198641	0	282672	185	6	181407	39316	73612	68	3858	0	0	1	0	0	15178
9	1626	48897	21816	0	137070	0	243843	8	0	261397	109332	137741	332	10556	0	0	2	1	0	27379
10	296	30792	6292	0	130486	0	176477	0	0	235060	190695	163269	1736	23101	0	0	8	8	0	41781
11	30	14480	1192	0	146759	0	129479	0	0	144362	254416	178354	8370	50893	0	0	30	13	0	71621
12	3	5601	206	0	133506	0	49066	0	0	58667	240715	231382	32046	108336	0	0	105	46	0	140321
13	0	784	29	0	80402	0	13395	0	0	17196	108877	119802	129500	233520	0	0	550	174	0	295772
14	0	158	2	0	51442	0	2109	0	0	2281	36081	49578	295192	330359	0	0	3897	1040	0	227859
15	0	14	0	0	33640	0	230	0	0	88	7734	10836	500080	233752	0	0	35021	5925	0	172680
16	0	0	0	0	3139	0	11	0	0	6	107	441	28293	3850	8	18	862737	96525	3	4862
17	0	0	0	0	1150	0	1	0	0	0	7	24	4372	331	1527	6393	97577	888164	244	210
18	0	0	0	0	12	0	0	0	0	0	0	0	1	0	226384	673688	69	7717	92129	0
19	0	0	0	0	3	0	0	0	0	0	0	0	0	0	555747	235253	3	367	208629	0
20	0	0	0	0	1	0	0	0	0	0	0	0	0	0	216334	84648	0	20	698995	0

Source: Authors’ elaboration on ISTAT (2015).

Table A.4 - Pairwise Comparison Index (robustness test, k=0.25)

	PI	VA	LO	TR	VE	FR	LI	ER	TO	UM	MA	LA	AB	MO	CM	PU	BA	CA	SI	SA	APCI
PI	1.000	0.577	0.743	0.000	0.967	0.001	0.989	0.019	0.001	0.986	0.997	0.995	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.764
VA	0.423	1.000	0.574	0.000	0.932	0.010	0.885	0.086	0.022	0.884	0.969	0.933	0.999	0.997	1.000	1.000	1.000	1.000	1.000	0.999	0.736
LO	0.257	0.426	1.000	0.000	0.950	0.000	0.936	0.005	0.001	0.934	0.983	0.983	1.000	0.997	1.000	1.000	1.000	1.000	1.000	0.997	0.723
TR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
VE	0.033	0.068	0.050	0.000	1.000	0.000	0.384	0.001	0.000	0.413	0.622	0.581	0.934	0.858	1.000	1.000	0.996	0.998	1.000	0.833	0.539
FR	0.999	0.990	1.000	0.000	1.000	1.000	1.000	0.679	0.526	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.910
LI	0.011	0.115	0.064	0.000	0.616	0.000	1.000	0.000	0.000	0.559	0.789	0.808	0.998	0.976	1.000	1.000	1.000	1.000	1.000	0.934	0.594
ER	0.981	0.914	0.995	0.000	0.999	0.321	1.000	1.000	0.314	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.876
TO	0.999	0.978	0.999	0.001	1.000	0.474	1.000	0.686	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.907
UM	0.014	0.116	0.066	0.000	0.587	0.000	0.441	0.000	0.000	1.000	0.809	0.720	0.999	0.972	1.000	1.000	1.000	1.000	1.000	0.917	0.582
MA	0.003	0.031	0.017	0.000	0.378	0.000	0.211	0.000	0.000	0.191	1.000	0.486	0.968	0.882	1.000	1.000	1.000	1.000	1.000	0.817	0.499
LA	0.005	0.067	0.017	0.000	0.419	0.000	0.192	0.000	0.000	0.280	0.514	1.000	0.969	0.871	1.000	1.000	0.999	1.000	1.000	0.814	0.507
AB	0.000	0.001	0.000	0.000	0.066	0.000	0.002	0.000	0.000	0.001	0.032	0.031	1.000	0.311	1.000	1.000	0.969	0.992	1.000	0.253	0.333
MO	0.000	0.003	0.003	0.000	0.142	0.000	0.024	0.000	0.000	0.028	0.118	0.129	0.689	1.000	1.000	1.000	0.995	0.999	1.000	0.410	0.377
CM	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.266	0.000	0.002	0.745	0.000	0.101
PU	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.734	1.000	0.000	0.007	0.862	0.000	0.130
BA	0.000	0.000	0.000	0.000	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.031	0.005	1.000	1.000	1.000	0.900	1.000	0.006	0.247
CA	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.001	0.998	0.993	0.100	1.000	1.000	0.001	0.205
SI	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.255	0.138	0.000	0.000	1.000	0.000	0.070
SA	0.000	0.001	0.003	0.000	0.167	0.000	0.066	0.000	0.000	0.083	0.183	0.186	0.747	0.590	1.000	1.000	0.994	0.999	1.000	1.000	0.401

Source: Authors’ elaboration on ISTAT (2015).

Table A.4 – Intraclass correlation coefficients

	Rank																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
individual	1.00	0.99	0.98	0.88	0.92	0.94	0.89	0.91	0.94	0.94	0.90	0.84	0.91	0.79	0.62	0.70	0.67	0.80	0.77	0.65
Average	1.00	1.00	0.99	0.97	0.98	0.98	0.97	0.97	0.98	0.98	0.97	0.96	0.97	0.94	0.87	0.90	0.89	0.94	0.93	0.88
F- test (19, 19)	803.94	354.44	196.93	29.82	45.34	58.79	32.39	39.81	61.10	63.29	36.18	22.47	39.28	16.46	7.56	10.13	9.19	16.73	14.53	8.37
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Source: authors' elaboration on data from ISTAT (2015)

Table A.4 - Regions abbreviations and macro-areas composition

Nord-West	
Piedmont	PI
Aosta Valley	VA
Liguria	LI
Lombardy	LO
Nord-East	
South Tyrol -Trentino	TR
Veneto	VE
Friuli-Venezia Giulia	FR
Emilia-Romagna	ER
Centre	
Tuscany	TO
Umbria	UM
Marche	MA
Lazio	LA
South	
Abruzzi	AB
Molise	MO
Campania	CM
Apuglia	PU
Basilicata	BA
Calabria	CL
Islands	
Sicily	SI
Sardinia	SA