Bedload transport and the stream power approach

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Abstract

Bedload transport is highly variable, and we address the issue of what value(s) of the transport rate a formula is expected to predict by incorporating two metrics of variance in an elementary scale correlation between the transport rate and stream power. The first captures uncertainty in the empirical relation we develop to describe the efficiency of the bedload transport process, and the second acknowledges that different patterns of transport are experienced under similar hydraulic conditions. Our simple, generalized expression explicitly applies to rivers in which there is always a high availability of mobile sediment on the bed, and we demonstrate that it provides a realistic portrayal of the observed range of transport rates in rivers with a sporadically mobile gravel armour and a characteristic bedload size that is nearly the same as that of the substrate. Accepting that the transport rate is influenced by the nature of the material available at the bed surface which is, in turn, moderated by antecedent flows, we suggest that if effort in the field is required to apply our methodology it should be expended on characterizing the sizes available on the bed *between* floods, rather than observing active transport during individual events.

Keywords: bedload transport efficiency, bedload transport formula, bedload transport rate, sediment availability, stream power

1. Introduction

The prediction of bedload transport rates is stubbornly paradoxical. Field observations, made over the past century, have consistently shown that bedload transport, the movement by rolling, sliding and/or saltating of sand, gravel and larger particles along the river bed, is highly variable and there are many possible transport rates for each measure of the driving force [1, 2]. At the same time, formulations of bedload transport envisage that there is a structural dependence between the bedload transport rate and variables that characterize the flow conditions and size of the sediment particles in motion [3]. The quest for accurate prediction has also failed to deliver satisfactory results because these equations compute bedload transport capacities and, by assuming an unlimited amount of sediment is available for transport, ignore the influence of changes in sediment supply or in-channel storage [3]. In rivers with a live bed most flows are able to transport all available size fractions, while in other rivers high transport rates are maintained, even though some size fractions on the bed are under-represented or not present in the bedload, by a persistent supply of sediment from upstream or proximal sources [4, 5, 6]. Such cases may provide the best opportunity for testing the effectiveness of predictive equations, because in these high availability rivers the transport rate is strongly related to some measure of the driving force (e.g., stream power) [4]. Nonetheless, many high availability rivers transport bedload at less than the maximum rate for the particle size in motion [4, 5, 6], so the overarching issue is how to compute a suboptimal transport rate. An inseparable matter is the question of what value(s) of the transport rate a formula is expected to predict.

Individual bedload samples or measurements of short duration obtained at different positions across the channel are typically combined into one composite sample to yield a rate that it is assumed applies to the entire river width and can be related to channel-wide (mean) flow conditions [7, 8, 9]. Accepting that rate variations are an ever-present feature of bedload transport records, we expect time-averaged, stream-wide data that extend across a range of flow conditions and cover a reasonable span of elapsed time to fluctuate about some empirically defined trend (see [2, 10]). Thus, it seems reasonable to expect that a bedload transport formula should be able to quantify this behaviour in the context of the expected deviations from it, and it has been suggested that a Bagnold-type stream power formula, which provides the most straightforward scale correlation, may be especially appropriate for developing knowledge of the magnitude of bedload transport from elementary hydraulic information [3, 11].

Recognizing that bedload transport is a highly variable process and its prediction is notoriously unreliable [3, 10], herein we develop and test a simple, generalized, stream power expression that accounts for the range of transport rates experienced in rivers with high sediment availability, a sporadically mobile armour and a characteristic bedload size that is similar to that of the substrate, under a range of flow conditions. We achieve this by reflecting on how best to represent the size of material in transit without making direct measurements, reimagining transporting efficiency as a variable driver of bedload transport and recognizing that transport rates relate not only to the flow conditions, but also to the prevailing condition of the bed, as moderated by antecedent events.

2. The stream power approach

Bagnold [12], like Gilbert [13] who first made the analogy between a river and transporting machine, perceived that bedload transport rates are measures of the work a river accomplished by moving bedload, computed as the immersed mass per unit width, i_b, which involves a proportion of the available power per unit bed area (specific stream power), $\omega = \rho QS/W$, expressed in mass units of kg m^{-1} s^{-1}: where ρ is the mass density of water; Q is the river discharge; S is the energy gradient approximated by the water surface or bed slope; W is the flow width; such that

$$i_b = \omega\ e_b / \tan \alpha \qquad 1)$$

where $e_b = i_b \tan \alpha / \omega$ is the efficiency of the bedload transport process, and tan α is approximated by the angle of internal dynamic friction of sand, ≈ 0.63 (note that Bagnold [12, 14] ignored the acceleration due to gravity, g). This formulation was derived without recourse to empirical reasoning [15, 16], and contains no threshold for transport and no explicit term for particle size. However, e_b cannot be determined directly and Bagnold [12, 14] resorted to developing empirical expressions to represent the transport efficiency.

Bagnold [14, p. 458] recognized that his empirical relation would not apply to circumstances in which 'the availability of the [bedload] sediment is limited'; a condition that was specified from the outset (see [15, p. 3]). Nonetheless, field data were invoked to show that it could be used to predict mean values in rivers with unimodal bed material and a high availability of sediment, and to demonstrate that, in rivers where the transport rate increases continuously and in direct proportion to stream power, efficiency is constant and proportional to the sediment calibre, D; usually specified as the median size of the sediment in transport, D_{50}: where D_{50} is the particle size for which 50% of the sampled bedload is finer by weight. In demonstrating that his stream power correlation adjusted by an empirical scaling of depth and grain size had wide applicability, Bagnold [17, p. 372] again emphasized that his formula applied 'to conditions of unlimited bedload availability'; an important caveat being that $D_{50} = D_{50\ bulk}$; where $D_{50\ bulk}$ is the particle size for which 50% of the bulk bed material is finer by weight.

The field data Bagnold [17] relied on pertain to rivers in which the observed efficiency is substantially less than the maximum for the D_{50} in motion (see [5, 6]); ranging between 0.7% and 2.6% in the Congo River, DR Congo, and East Fork River, USA, respectively. All the laboratory and field data exhibit a striking proportionality between the transport rate and stream power, that is modulated by flow depth and particle size (Figure 1a). This knowledge is encapsulated in a family of curves that apply to conditions where the bedload is of nearly uniform size and/or the available power is sufficient to move all accessible size fractions [18, 19]. The proportionality arises because in these rivers (and flumes) the amount of mobile sediment on the bed is effectively unconstrained by sediment supply [4]. Nonetheless, as Bagnold [12, 14] and Leopold and Emmett [19] appreciated, factors unrelated to the flow or its capacity to convey a given size of material also exert an influence on the transport rate. In the East Fork River, for example, the changing availability of transportable bed material gives rise to a different proportionality in the transport rate – stream power relation for rising, as opposed to other flow stages (Figure 1a). The effect is magnified in rivers in which the bed configuration is able to adjust to the imposed sediment supply, so bedload can be transported at a substantially higher rate [6]. In such rivers, which variously encompass channels where extreme sediment loading is a consequence of an event of extraordinary magnitude, related to natural or anthropogenic disturbances [20, 21, 22], or that are immune to natural constraints on the supply of sediment from riparian sources [23], transport rates approximate the maximum for the particle size in motion [5]. Accepting that a conventional bedload formula cannot be expected to accommodate changes in the availability of transportable bed material caused, for example, by a temporary enhancement to the sediment supply, the movement of bars, channel cross-section change, or the storage in and release of material from pools or behind channel obstructions, we find that, by specifying the characteristic efficiency for each river, as defined by Gomez [6], Bagnold's [12] elementary formulation represents the overall trend of the relation between the transport rate and stream power observed in diverse rivers in which there is a high, but not necessarily unlimited, availability of mobile sediment on the bed (Figure 1b; see [4]). That being the case, the remaining issue is how best to express this characteristic efficiency.

3. Expressing transport efficiency

It has been suggested that efficiency may depend on bed material characteristics [6, 23], and that efficiency and size of the sediment in motion are adjusted to the environmentally controlled rate at which sediment is supplied to a river [4]; which also helps to determine channel reach morphology [25, 26]. Although Bagnold's [12] original formula does not incorporate a sediment size term a measure of sediment properties is nonetheless required to compute the transport rate [11], and his empirical representation initially relied on a relation between the transport rate and relative roughness (the ratio of flow depth to sediment calibre, Y/D: where for unimodal, sand-sized sediment D is the mode size of the bulk bed material, $D_{m\ bulk}$, which is assumed to equate to D_{50}). However, Bagnold [14, 17] subsequently considered D to be an independent variable, because although the difference between $D_{m\ bulk}$ and D_{50} is often insignificant for sand this is not the case for gravel. Thus, whereas in sand-bed rivers the median particle size of the bedload normally approximates that of the bed material available for transport, in gravel-bed rivers, where there is often an imbalance between transport and supply, the size distribution of the surficial bed material typically differs from that of the bedload being transported over it [27, 28].

In order to maintain Bagnold's [12] dimensional balance, we introduce the parameter, D^* (the ratio of the representative size of the bedload to that of the material constituting the bed), to characterize the properties of the bedload (sediment in transport) and bed material (sediment available for transport). Like Bagnold [12, 14], in sand-bed rivers we generally expect $D^* = D_{50} / D_{50\ bulk}$. However, in gravel-bed rivers it is the largest particles on the bed surface that modulate the availability and mobility of bed material [27, 29], and we suggest D^* more likely satisfies the relation $D^* = D_{50} / D_{90\ surface}$: where $D_{90\ surface}$ is the particle size for which 90% of the surficial bed material is finer by weight or number.

To establish a relation between D^* and e_b we draw on data from rivers with live and armoured beds that are emblematic of two distinctive efficiency metapopulations [6]. In the former case most flows are able to transport all available size fractions and, in the latter case, even though there is a near-bankfull threshold for bed surface mobility [27, 29], high transport rates can be sustained by the supply of sediment from upstream or proximal sources [6]. The mode of sediment transport in our sample of 21 rivers ranges from incipient suspension to bedload, moving as multi-scale or low-angle bedforms, gravel and sand sheets, or individual particles. Transport rates were determined using diverse techniques, encompassing at-a-point sampling near the mouth of one of Earth's largest Tropical rivers, measurements, systematic sampling and passive monitoring in montane and lowland rivers, and estimates of the volumetric change in sediment deposits on the bed of a large dryland river. Collectively, as a plot of river slope as a function of relative smoothness, expressed as D/Y, shows (Figure 2a), our sample is representative of a wide range of alluvial rivers in which the bed material is relatively easily and frequently entrained by the flow. It should, however, be noted that the estimates of both bedload and bed material calibre we rely on are associated with large uncertainties; not the least of which is that, although the size range of the bedload is typically defined by samples obtained throughout a measurement period, the bed material is conventionally described by a single sample, and the actual bed surface size distribution, which varies over time [30, 31], is unknown.

We used linear regression (in log-space) to extract a power-type relation between D^* and e_b and captured the uncertainty in the resulting coefficient by resampling (one million bootstrap samples) the residuals assuming a fixed exponent (Figure 2b). Inserting the resulting empirical quantity into Bagnold's [12] elementary formulation yields $i_b = \omega\, E_b / \tan \alpha$; where $E_b = 0.036 D^{*1.3}$. This allows us to quantify a river's behaviour over a given range of flows, and the confidence band (obtained by applying the percentile bootstrap method to the regression coefficient) provides a measure of the expected deviations from the overall trend. We employ bootstrapping, which is particularly useful when dealing with small sample sizes, to construct the confidence bands because it makes no assumptions about the normality (or otherwise) of the sample data that are used to estimate their distribution [33]. We also note in passing that, because Bagnold's [34] granular-fluid theory is incomplete [35], E_b accounts for factors other than e_b. Nevertheless, it is now possible to generate the family of curves that Reid and Frostick [24] postulated could be derived for rivers with different characteristic bed materials and bedload sizes.

4. Application to rivers

Accepting that ω can be approximated as $\rho QS/W$, the principal unknowns are the measures of $D_{50\,bulk}$, $D_{90\,surface}$ and D_{50}. Bed material size usually relates to bulk, point samples which it is presumed contain material representative of the sediment in motion, or random, areal samples that describe the bed surface. We assume that $D_{50\,bulk}$ and $D_{90\,surface}$ can be extracted from the bed gradations obtained in this manner. In rivers with a live, unstratified bed we assume that the median size of the material in transport, D_{50}, is no different to that of the substrate, $D_{50\ substrate}$ (the particle size for which 50% of the subsurface bed material is finer by weight), which ordinarily cannot be differentiated from $D_{50\ bulk}$. For rivers with a coarse surface layer maintained by successive periods of bedload transport during which essentially all sizes of the bed material move, and the bed is the source of much of the bedload, we assume D_{50} can also be approximated by $D_{50\ substrate}$. Like Parker *et al.* [27], we consider the similarity in the bedload and substrate particle size distributions to be a first approximation of reality that, for example, does not account for the selective transport of fine particles over a coarser, largely unbroken bed surface. Thus, to accommodate uncertainty in the relation between the particle size of the bedload and that of its source, we introduce an index term $1-P$; where P lies between 0 and 1 and which, interpolating on log units, gives $E_b = 0.036D^{*(1.3(1-P))}$; such that E_b varies between the values predicted for D^* (E_i) and $D^* = 1$. These are, in effect, limiting cases that prescribe the extent to which coarse sediment on the bed surface affects the mobility of the finer substrate.

In summary, we write

$$i_b = \omega\ E_b\ /\ \tan\alpha \qquad 2)$$

where, for a specified confidence level (Figure 2b), $E_b = 0.036D^{*(1.3(1-P))}$ and for sand-bed rivers $D^* = D_{50\ substrate}\ /\ D_{50\ bulk}$, and for gravel-bed rivers $D* = D_{50\ substrate}\ /\ D_{90\ surface}$.

We offer an independent test of this predictive tool's applicability by comparing its performance with observed rates in the poorly sorted, sandy, gravel-bed Arbúcies River, Spain, and armoured, gravel-bed Versilia River, Italy, where there is a relatively high availability of sediment in relation to runoff. The data encompass almost the entire range of water and sediment discharges experienced in each river and were obtained using a standard Helley-Smith sampler [36, 37]. Our formulation delimits both the upper and lower bounds to the range of transport rates experienced in both rivers (Figure 3). However, in making this observation we acknowledge that there are other possible ways of interpreting these data. One being that the transport rate does not always vary in direct proportion to stream power, because bedload transport occurs at a negligible rate below some threshold or critical value. Indeed, Bagnold [12, 14] introduced a threshold term, ω_o, to specify the lowest value of stream power at which the smallest amounts of bedload transport occur. Equations that address some of the deficiencies inherent in Bagnold's derivation of this threshold value have been advanced, but continual variations in the state of the bed surface make it difficult to specify ω_o with any confidence [11, 38], and we avoid introducing such a term for this reason. Moreover, although the bed surface is coarser than the subsurface in both rivers, differences in the size of the material in motion do not explain the variable nature of bedload transport (at a given ω) in these rivers; which is thought to be related to the migration of bedforms [36], and hysteretic behaviour caused by the formation/disintegration of the bed armour or particle clusters [37].

As we have already emphasized, we do not expect an elementary scale correlation between the transport rate and stream power to accommodate such variability; which is an important general characteristic of the transport process that field data can either capture or subdue [2]. It should also be noted that the methodology for evaluating the performance of a formula has remained unchanged for more than eight decades (see [39, 40]). That is, either observed and calculated relation(s) between the transport rate and some measure of the driving force for a given river, or observed values and those calculated with a given formula are compared, with a unique or one-to-one relation being the desired outcome. We suggest that it is unrealistic to expect this will ever be the case in systems which fluctuate over time and, to the extent that the success of our formulation relies on inferences about the size of mobile sediment available on the bed, rather than continuing to evaluate the performance of a formula on the basis of whether it simply over- or under-predicts observed transport rates we have redefined the meaning of predictability by incorporating two metrics of variance. One is prescriptive and captures uncertainty in the relation between D^* and e_b (Figure 2b). The other is descriptive and embraces the different patterns of transport experienced under similar hydraulic conditions (Figure 3). This is an important innovation that explicitly acknowledges there is no *a priori* requirement for a transport rate to be uniquely dependent on the flow conditions.

To illustrate this point, we refer to data collected, using standard Helley-Smith samplers, in two armoured, gravel-bed rivers with high sediment loadings that were a consequence of a dam-break flood and large amounts of rain falling on hillslopes destabilized by timber harvesting [41, 42]. As the sediment supply waned, bedload transport rates in the Fall River, USA, declined by an order of magnitude (Figure 4a). Conversely, in Redwood Creek, USA, the elevated transport rates were sustained over a period of years by a persistently high in-channel sediment supply (Figure 4b). Both rivers transport bedload more efficiently than their counterparts in the reference metapopulations from which our relation between D^* and e_b was derived (Figure 2b; [6]), but the envelope that defines the upper, size-dependent limit to transport can be delineated using an analogous empirical expression ($e_b = 0.0115\ D_{50}^{-0.51}$, and substituting $D_{50\,substrate}$ for D_{50}) that applies to rivers in which the sediment supply regime gives rise to transport rates that are at or near the maximum rate for the particle size in motion [5].

5. Value(s) of the transport rate

We have shown that a simple generalized expression can provide a realistic portrayal of the range of transport rates experienced during a series of flows in high availability rivers with a sporadically mobile gravel armour and a characteristic bedload size that is nearly the same as that of the substrate (Figures 3 and 4). The residual challenge is how to use this knowledge to estimate the rate of bedload transport for a range of hydraulic conditions in a location with no observations. Present practice is to predict the transport capacity, but such a result is of limited practical value because, even in rivers in which mobile sediment is readily available on the bed, the transport rate (at a given ω) varies by an order of magnitude and the transport rate–flow relation may be nonstationary (Figure 4a). In some cases, the variability may be linked to the movement of bedforms [43, 44], and will necessarily be smoothed out or subdued in data that reflect measurements integrated over the entire cross-section and made over a relatively lengthy period of time (see [45]), or amplified if sampling times are short. However, in most cases the source(s) of the variability and reason(s) for the time-dependency are unknown and, to the extent that they are an inherent feature of the bedload transport process, estimates of the transport rate should allow that there is no unique value for a given flow condition and that they incorporate a measure of uncertainty that is transferred to the resulting transport rate–flow relation.

Accepting that the size and amount of sediment in motion are unknowns, our methodology permits a continuum of trends, elaborated by $1–P$, with distinct extremes, and the uncertainty surrounding them to be delineated. The latter is an essential requirement for applications in which the cumulative transport is of interest and involve propagating uncertainty over time [46], and the former opens the possibility of developing time-dependent relations. As an example, we compare our estimates of the bedload yield of Fall River with Pitlick's [42] time-dependent, integral measures for 1983, when dunes were migrating over the bed, and 1987, by which time the channel had nearly recovered from the effects of the dam-break flood and less of the bed was mobile. Pitlick [42] estimated that snowmelt runoff transported 15.5×10^3 and $0.4 \times$

10^3 Mg (dry mass) of bed load between 1st June and 30th July in 1983 and 1987, respectively. Based on the mean daily flow for the time periods in question and assuming $P = 1$ (1983) and, on the basis of trial and error, $P = 0.25$ (1987), we compute the (dry) mass transported for 1983 and 1987 as 11.42×10^3 and 0.39×10^3 Mg, respectively (with both Pitlick's [42] estimations within the bootstrapped 95% confidence bands for our derived estimates). In the Fall River, as the sediment supply waned, the armour progressively reformed on the undifferentiated bed surface created after the dam-break flood. That is, in 1983, the larger discrepancy (0.74) between our and the original computations is due to the greater influence exerted by sediment supply (which our methodology does not account for) immediately after the dam-break flood, whereas the equivalency realized in 1987 is indicative of the reestablishment of the connection between the availability of mobile sediment sizes on the bed surface and the transport rate.

For more than sixty years it has been acknowledged that there are large discrepancies between the results obtained from the application of bed-load transport formulae and field data [47]. In situations where our methodology is employed to obtain an estimate of the transport rate in the absence of data, P indexes the availability of those (measured) mobile sizes in the surficial bed material or substrate to the (unmeasured) size of the bedload. $P = 0$ applies to a bed that reacts as if it is composed of its coarsest components, as may be the case during a flood event that occurs after a long period of inactivity, and values of $P > 0$ apply to the loose beds that may, for example, be created by infrequent, high-magnitude events or floods that occur closely together (e.g., [48, 49]). Inferences about the time dynamics of bed-surface material size have been made by analyzing bedload sediments and sediment transport rates (e.g., [18, 50]), and one way of capturing this behaviour is to calibrate a formula using a small number of observations of transport rates; even though this would require significant effort and may still not achieve acceptable accuracy [51]. Recognizing that the transport rate is influenced by the nature of the material available at the bed surface which is, in turn, moderated by antecedent flows [31, 52–56], we suggest that if effort in the field is required to apply our methodology it should be expended on characterizing the sizes available for transport within the channel boundary and on the bed surface between floods, rather than observing active transport during individual events. This would involve obtaining representative samples of the surface and subsurface bed material at different points in time; procedures that involve established methodologies and are relatively easy to accomplish, but are rarely repeated in field conditions [30, 31].

6. Conclusion

In a wide range of rivers the interaction between the flow and coarse sediment moving as bedload exerts a major control on channel morphology [25, 26, 32], and decisions about river management that involve making changes to the channel boundary or flow conditions would be better informed by knowledge of the rate at which bedload is transported [45]. Nonetheless, even though the available data show bedload transport is highly variable (see [1, 2]), a common complaint is that the accuracy with which bedload transport rates can be predicted is much less than desired and typically only to within one order of magnitude [10]. Our solution to this predicament is to present a methodology, based on accepted theory [15], that embraces this inherent variability. In so doing, we emphasize that, as previously stated [2], we expect our (or any other) method of predicting transport rates to apply to a specific population of rivers. Hence, our methodology explicitly applies to rivers in which there is always expected to be a high, but not necessarily unlimited, availability of mobile sediment on the bed (see Figures 1 and 3).

The foundation of our methodology is Bagnold's [15] affirmation that only a proportion of stream power is expended on bedload transport and the material in transport is derived from the bed. Using field data that are representative of alluvial rivers in which the bed sediments are relatively easily and frequently entrained by the flow (Figure 2), to determine the quantity involved we develop a relation, by comparing the ratio of the representative size of the bedload to that of the material constituting the bed, D^*, with the efficiency of the bedload transport process. For practical application and to avoid the problematic situation of having to sample or measure the bedload in order to compute its transport rate, for sand and gravel-bed rivers we suggest $D^* = D_{50\,substrate} \,/\, D_{50\,bulk}$ and $D_{50\,substrate} \,/\, D_{90\,surface}$, respectively, and we assume that all three measures can be determined from bed material samples. We account for uncertainty in our empirical relation by specifying confidence limits, and introduce an index term, $1-P$, to accommodate time variations in bed material texture. $P = 0$ applies to a bed that reacts as if it is composed of its coarsest components, and values of $P > 0$ apply to loose beds. That is, our methodology (Equation 2) explicitly acknowledges there is no *a priori* requirement for the transport rate to be uniquely dependent on the flow conditions. However, there is no definitive answer to the question of which value(s) of P to exploit, the choice of which rests with the user. This introduces an additional degree of uncertainty, but decisions about parameterization are inevitably required when models are used to simulate the physical environment, and using any bedload formula to delineate an applicable range of transport rates necessarily involves an element of trial and error (see [57]).

We demonstrate that our simple generalized expression provides a realistic portrayal of the range of transport rates experienced in high availability rivers with a sporadically mobile gravel armour and a characteristic bedload size that is nearly the same as that of the substrate

(Figures 3 and 4). Other contemporary formulae have also embraced the idea that bedload transport depends on the material in direct contact with the flow and it is the bed surface that modulates bed mobility (e.g., [27, 58]). That said, the cause of the time-variations that occur in the surficial bed material and their relation to flow and transport and conditions are incompletely understood and attempts to study them have mostly been undertaken in laboratory flumes (e.g., [59].) Nevertheless, given that the 'success in predicting bed load transport rates hinges to a large extent on the availability of mobile sediment sizes within the channel boundary' [57, p. 3], and the relative ease of sampling the bed material as compared to the bedload, we propose that it is time for a paradigm shift. Specifically, rather than attempting to obtain estimates of the transport rate for representative flows, effort should routinely be directed towards characterizing the bed surface layer and substrate and the textural changes that occur within the channel boundary *between* floods.

Data accessibility. The information relied upon is catalogued in the electronic supplementary material.

Authors' contributions. B.G.: conceptualization, formal analysis, writing—original draft; P.S.: conceptualization, formal analysis, writing—original draft. All authors gave final approval for publication and agreed to be held accountable for the work performed therein.

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Figure captions

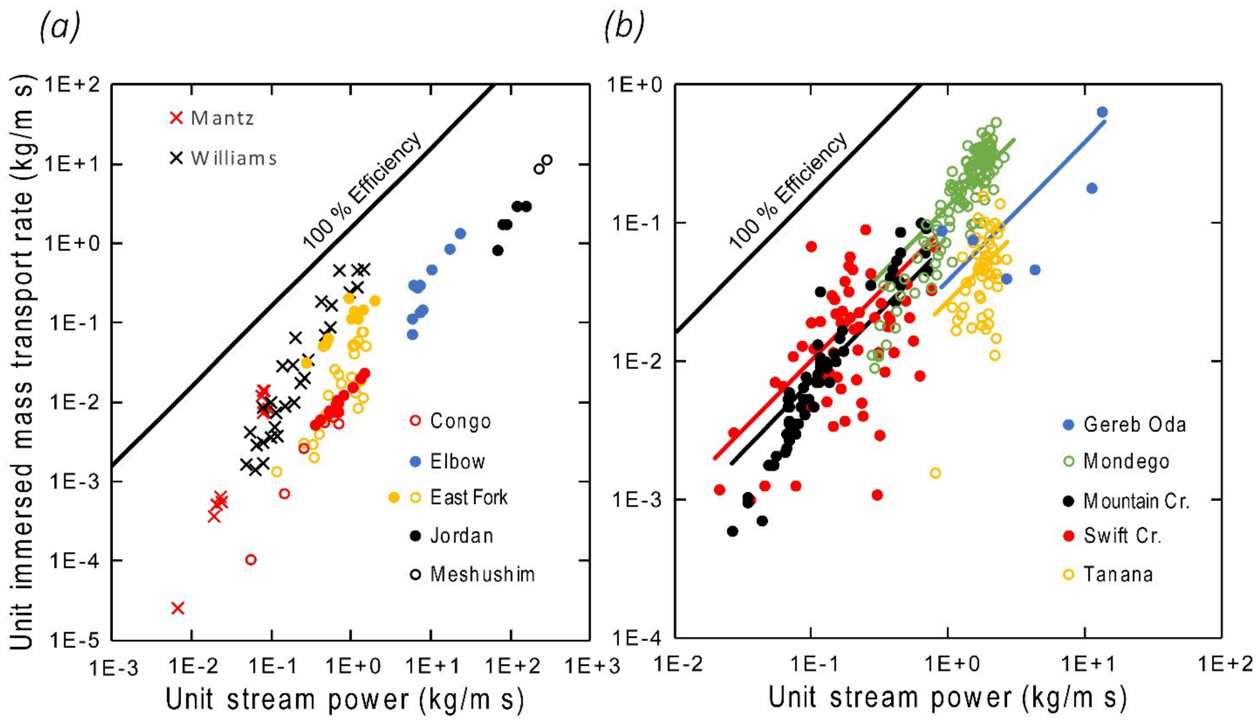
Figure 1. (a) Graphical representation of the data Bagnold [17] relied on to demonstrate the close empirical correlation of bedload transport rates with stream power in laboratory flumes and natural rivers (with 100% efficiency line indicated). Note the different relations for rising (solid circles) and other (open circles) flow stages in the East Fork River, USA. See text for discussion. (b) Relation of bedload transport to stream power in high availability rivers (as defined by Gomez and Soar [14]). Solid lines delineate the relation $i_b = \omega\, e_b / \tan \alpha$ for each river: where e_b is the characteristic efficiency (as defined by Gomez [6]).

Figure 2. (a) Classification of alluvial river channels according to the dominant mode of sediment transport (grey open symbols, after Church [32]; with selected lines of constant τ_* indicated: where the Shields Number, $\tau_* = \rho g YS/g(\rho_s - \rho)D$; ρ_s is the sediment density and the other terms are as defined in the text), displayed on a graph of slope versus relative smoothness (D/Y). Solid, labeled symbols illustrate the diversity of the rivers we rely on to establish a relation between the ratio of the representative size of the bedload to that of the material constituting the bed and the efficiency of the bedload transport process. Note that for the open, unlabeled symbols sediment calibre, D, is defined as the median size exposed on the bed surface, and for the solid, labeled symbols it is the median size of the material transport. (b) Relation between the ratio of the representative size of the bedload to that of the material constituting the

bed, D^*, and the efficiency of the bedload transport process (with selected confidence bands indicated). The data used to characterize these 21 rivers are catalogued in the electronic supplementary material. See text for discussion.

Figure 3. Observed transport rates and limiting cases predicted by Equation 2 for the high availability (a) Arbúcies River, Spain and (b) Versilia River, Italy (with the selected confidence band and 100% efficiency line indicated). The measures of $D_{50\ substrate}$ and $D_{90\ surface}$ employed are catalogued in the electronic supplementary material. See text for discussion.

Figure 4. Application of Equation 2 to rivers with high sediment loadings which consequently transport bedload at a higher rate than predicted; (a) Fall River, USA, Station 1 and (b) Redwood Creek, USA, Station 1 (with 100% efficiency line indicated). The measures of $D_{50\ substrate}$ and $D_{90\ surface}$ employed are catalogued in the electronic supplementary material, and the upper, size-dependent limit to transport in these rivers is delineated (dashed line) by Gomez's [5] empirical expression. See text for discussion.



(a)
(b)
Mantz
Williams
100 % Efficiency
Congo
Elbow
East Fork
Jordan
Meshushim
Gereb Oda
Mondego
Mountain Cr.
Swift Cr.
Tanana
Unit immersed mass transport rate (kg/m s)
Unit stream power (kg/m s)
1E+2
1E+1
1E+0
1E-1
1E-2
1E-3
1E-4
1E-5
1E+3

Figure 1

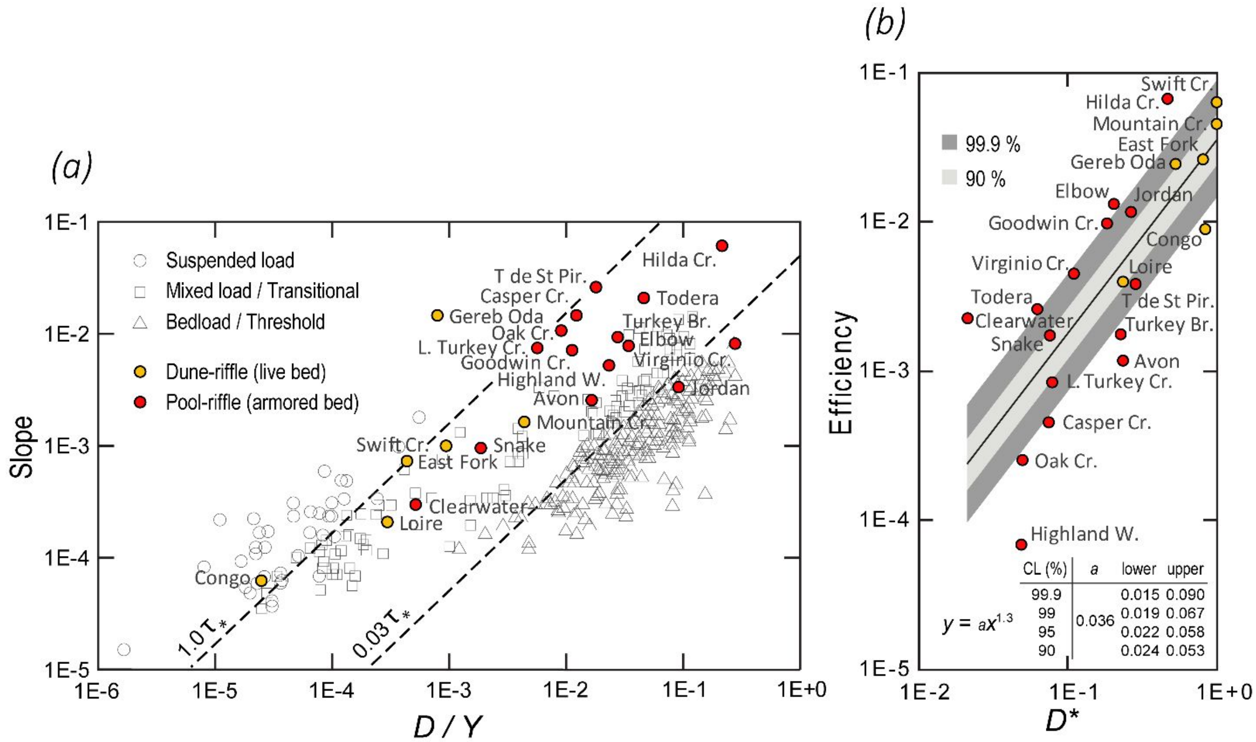

(a)
Suspended load
Mixed load / Transitional
Bedload / Threshold
Dune-riffle (live bed)
Pool-riffle (armored bed)
Slope
D / Y
1.0 τ*
0.03 τ*
Hilda Cr.
T de St Pir.
Casper Cr.
Todera
Gereb Oda
Turkey Br.
Oak Cr.
Elbow
L. Turkey Cr.
Goodwin Cr.
Virginio Cr.
Highland W.
Jordan
Avon
Mountain Cr.
Swift Cr.
Snake
East Fork
Clearwater
Loire
Congo
(b)
Efficiency
D*
99.9 %
90 %
Swift Cr.
Hilda Cr.
Mountain Cr.
East Fork
Gereb Oda
Elbow
Jordan
Goodwin Cr.
Congo
Virginio Cr.
Loire
Todera
T de St Pir.
Clearwater
Turkey Br.
Snake
Avon
L. Turkey Cr.
Casper Cr.
Oak Cr.
Highland W.
y = ax^1.3
CL (%) a lower upper
99.9 0.015 0.090
99 0.036 0.019 0.067
95 0.022 0.058
90 0.024 0.053

Figure 2

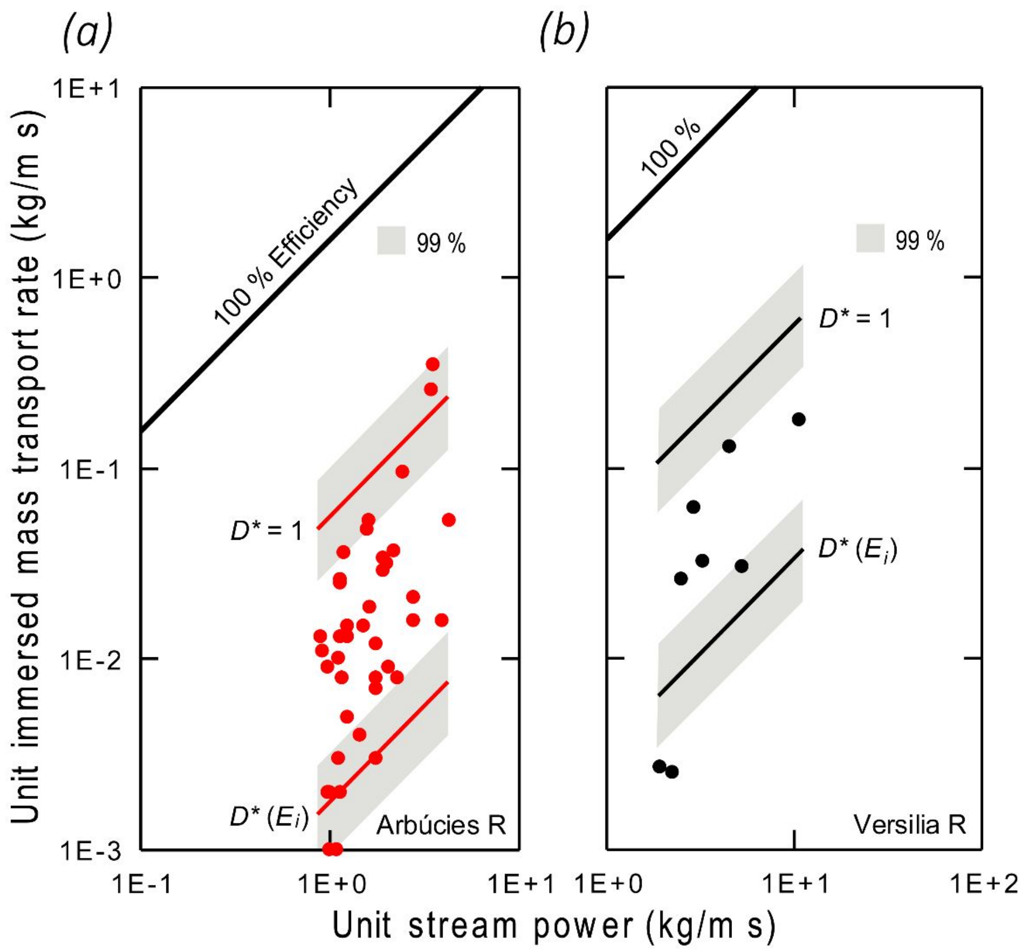

(a)
(b)
Unit immersed mass transport rate (kg/m s)
1E+1
1E+0
1E-1
1E-2
1E-3
100 % Efficiency
99 %
D* = 1
D* (Ei)
Arbúcies R
100 %
99 %
D* = 1
D* (Ei)
Versilia R
1E-1
1E+0
1E+1
1E+0
1E+1
1E+2
Unit stream power (kg/m s)

Figure 3

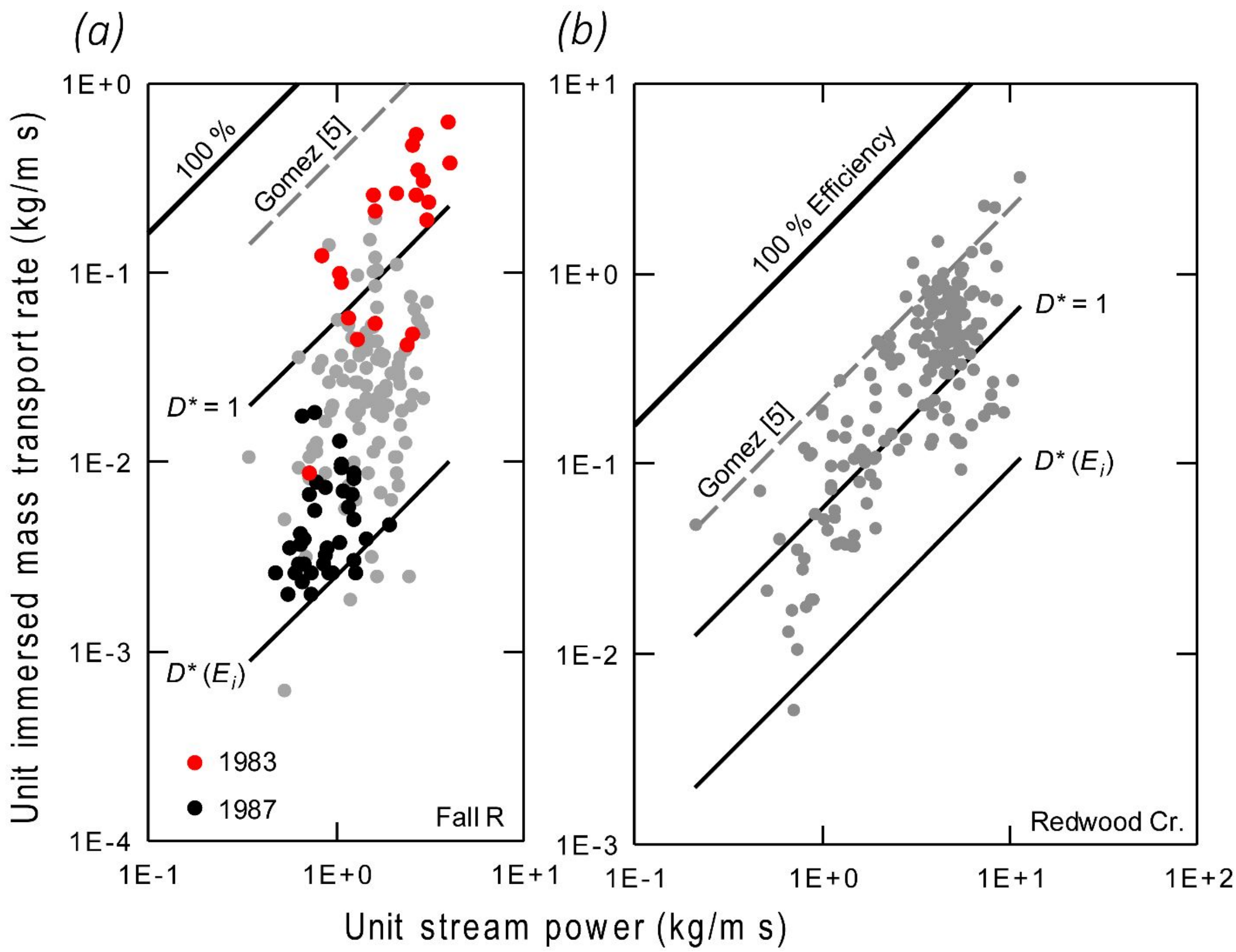

(a)
(b)
Unit immersed mass transport rate (kg/m s)
Unit stream power (kg/m s)
100 %
Gomez [5]
D* = 1
D* (E_i)
1983
1987
Fall R
100 % Efficiency
Redwood Cr.
1E+0
1E-1
1E-2
1E-3
1E-4
1E+1
1E+0
1E-1
1E-2
1E-3
1E-1
1E+0
1E+1
1E+2

Figure 4