

# Cosmological constraints from the double source plane lens SDSSJ0946+1006

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## ABSTRACT

We present constraints on the equation of state of dark energy,  $w$ , and the total matter density,  $\Omega_M$ , derived from the double-source-plane strong lens SDSSJ0946+1006, the first cosmological measurement with a galaxy-scale double-source-plane lens. By modelling the primary lens with an elliptical power-law mass distribution, and including perturbative lensing by the first source, we are able to constrain the cosmological scaling factor in this system to be  $\beta^{-1} = 1.404 \pm 0.016$ , which implies  $\Omega_M = 0.33^{+0.33}_{-0.26}$  for a flat  $\Lambda$  cold dark matter ( $\Lambda$ CDM) cosmology. Combining with a cosmic microwave background prior from *Planck*, we find  $w = -1.17^{+0.20}_{-0.21}$  assuming a flat  $w$ CDM cosmology. This inference shifts the posterior by  $1\sigma$  and improves the precision by 30 per cent with respect to *Planck* alone, and demonstrates the utility of combining simple, galaxy-scale multiple-source-plane lenses with other cosmological probes to improve precision and test for residual systematic biases.

**Key words:** gravitational lensing; strong – cosmological parameters.

## 1 INTRODUCTION

The current concordance cosmology of  $\Lambda$  cold dark matter ( $\Lambda$ CDM) gives a remarkably good fit to current observational data (Percival et al. 2010; Bennett et al. 2013; Planck Collaboration 2013), but a tension seems to be emerging between different cosmological probes. For example, assuming flat  $\Lambda$ CDM the recent Planck constraints on the Hubble constant (Planck Collaboration 2013) are significantly lower than those found by local measurements using supernovae (Riess et al. 2011; Freedman et al. 2012, but see Efstathiou 2014) and strong lens time delays (Suyu et al. 2014). Although this discrepancy may simply be due to the presence of unknown systematic errors, it might also signal physics beyond the flat  $\Lambda$ CDM model; *independent* cosmological probes are therefore needed to test the assumption that the Universe is spatially flat and that the dark energy is a cosmological constant.

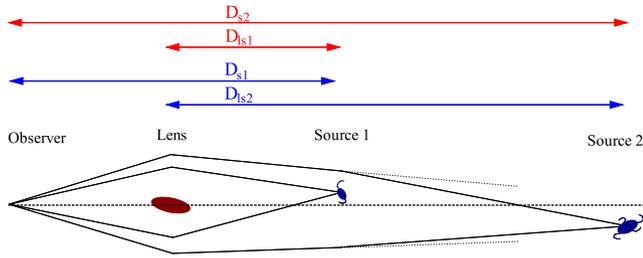
Strong gravitational lensing is potentially a powerful tool to test cosmological models (Witt, Mao & Keeton 2000; Kochanek 2002; Saha & Williams 2003; Schechter 2005; Oguri 2007; Gavazzi et al. 2008; Suyu et al. 2010, 2014; Collett et al. 2012; Oguri et al. 2012), due to its sensitivity on the distances between components of the lens system (e.g. the observer, the foreground massive lensing object and any background lensed sources). In principle, measurements of the Einstein radius and enclosed lens mass are sufficient to constrain cosmological parameters (Grillo, Lombardi & Bertin 2008; Biesiada, Piórkowska & Malec 2010), but robustly inferring the

lensing mass is degenerate with the choice of lens density profile. To make robust inference on cosmological parameters – without making strong assumptions about the lens mass distribution – additional information is required. Gravitational lens systems with two background sources at different redshifts (schematic shown in Fig. 1) provide sufficient information to make precision measurements of cosmology.

In Collett et al. (2012), we showed that double-source-plane lenses (DSPLs) can be a useful, complementary cosmological probe, allowing the dark energy equation of state to be constrained independently of the Hubble constant. Jullo et al. (2010) constrained cosmological parameters using 12 multiply-lensed sources behind the cluster Abell 1689, but the sparsity of lensed images and the clumpy mass distribution in clusters makes the measurement difficult; the systematic uncertainties are likely to be large (Zieser & Bartelmann 2012) and much of the information provided by the multiple background sources may need to be used to infer the complexity of the lensing mass distribution. Galaxy–galaxy strong lenses, on the other hand, tend to be very well fitted with simple mass distributions (e.g. Vegetti et al., in preparation) and therefore may be preferable objects for testing cosmology.

In this work, we present strong lensing models of SDSSJ0946+1006 (hereafter J0946), a DSPL serendipitously discovered by Gavazzi et al. (2008), and we use these lens models to estimate the cosmology-dependent scalefactor  $\beta$  that governs the relative lens strength acting on each of the two source planes. In Section 2, we outline the relevant theory of multiple-source-plane lensing. In Section 3, we develop a framework for fitting multiple-source-plane lenses with regularized pixelated sources and apply

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**Figure 1.** Sketch of a DSPL system. The cosmological scaling factor  $\beta$  is the product of  $D_{ls1}$  and  $D_{s2}$  (both in red) divided by the product of  $D_{ls2}$  and  $D_{s1}$  (both in blue). For a singular isothermal sphere, where the first source has no mass,  $\beta$  is the ratio of Einstein radii. Figure taken from Collett et al. (2012).

this technique to J0946. In Section 4, we convert our measurement of  $\beta$  into constraints on the cosmological parameters  $w$  and  $\Omega_M$ . We discuss and conclude in Section 5.

## 2 COSMOGRAPHY WITH DOUBLE-SOURCE-PLANE STRONG LENSES

In this section, we briefly review the theory of compound lensing, following Schneider, Ehlers & Falco (1992) and summarize how observations of multiple-source-plane strong lenses can be used to constrain cosmology. For a single-source-plane lens, the lens equation can be written as

$$\mathbf{y} = \mathbf{x} - \boldsymbol{\alpha}(\mathbf{x}), \quad (1)$$

where  $\mathbf{y}$  is the position of the source on the source plane and  $\mathbf{x}$  is the position in the image plane.  $\boldsymbol{\alpha}(\mathbf{x})$  is the scaled deflection caused by the lens, given by

$$\boldsymbol{\alpha}(\mathbf{x}) = \frac{1}{\pi} \int d^2x' \kappa(\mathbf{x}') \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^2}, \quad (2)$$

where  $\kappa(\mathbf{x})$  is the lens surface mass density,  $\Sigma(D_l \mathbf{x})$ , scaled by the lensing critical density,  $\Sigma_{\text{crit}}$ ,

$$\kappa = \frac{\Sigma}{\Sigma_{\text{cr}}(z_d, z_s)} \quad (3)$$

with

$$\Sigma_{\text{cr}}(z_d, z_s) \equiv \frac{c^2 D_s}{4\pi G D_l D_{ls}} \quad (4)$$

and  $D_{ij}$  are angular diameter distances between observer, lens and source. Changing the source redshift alters only the angular diameter distances in equation (4), whilst the other terms in equations (1)–(4) are unchanged. Hence, for two photons passing through the same point in the lens plane, but originating on different source planes, the ratio of scaled deflection angles is given by the cosmological scaling factor,  $\beta$ ,

$$\frac{\alpha_1}{\alpha_2} = \frac{D_{ls1} D_{s2}}{D_{s1} D_{ls2}} \equiv \beta. \quad (5)$$

In the special case of a singular isothermal sphere lens,  $\beta$  is simply the ratio of Einstein radii.

When multiple sources are present, the mass of the first source will lens the light coming from the second. Including this effect requires the use of the full multiple-lens-plane formalism, as given in Schneider et al. (1992). For a system with  $j - 1$  lens planes (any of which can be masses and/or sources), and a final source,

the position on each plane can be calculated by ray-tracing back iteratively, using

$$\mathbf{x}_j = \mathbf{x}_1 - \sum_{i=1}^{j-1} \beta_{ij} \boldsymbol{\alpha}_i(\mathbf{x}_i), \quad (6)$$

where the reduced deflections  $\boldsymbol{\alpha}_i$  are the physical deflections rescaled for the final source plane

$$\boldsymbol{\alpha}_i = \frac{D_{is}}{D_s} \hat{\boldsymbol{\alpha}}_i \quad (7)$$

and  $\beta_{ij}$  is the cosmological scaling factor given in equation (5) (from hereon we use  $\beta$  as shorthand for  $\beta_{12}$ ). In the case of a DSPL, the lens equation for photons originating on the first and second source planes are, respectively, given by

$$\mathbf{y}^{s1} = \mathbf{x} - \beta \boldsymbol{\alpha}_1(\mathbf{x}), \quad (8)$$

and

$$\mathbf{y}^{s2} = \mathbf{x} - \boldsymbol{\alpha}_1(\mathbf{x}) - \alpha_{s1}(\mathbf{x} - \beta \boldsymbol{\alpha}_1(\mathbf{x})). \quad (9)$$

Angular diameter distances, and hence  $\beta$ , are functions of redshift and the cosmological parameters,

$$D_{ij} = \frac{c/H_0}{(1+z_j)} \left( \frac{\text{sinn}\left(\sqrt{|\Omega_k|} \int_{z_i}^{z_j} \frac{dz}{E(z)}\right)}{\sqrt{|\Omega_k|}} \right), \quad (10)$$

where  $\text{sinn}(x) = \sin(x)$ ,  $x$ , or  $\sinh(x)$  for open, flat, or closed universes, respectively, and  $E(z) \equiv \frac{H(z)}{H_0}$  is the normalized Hubble parameter. For a  $w$ CDM cosmology,

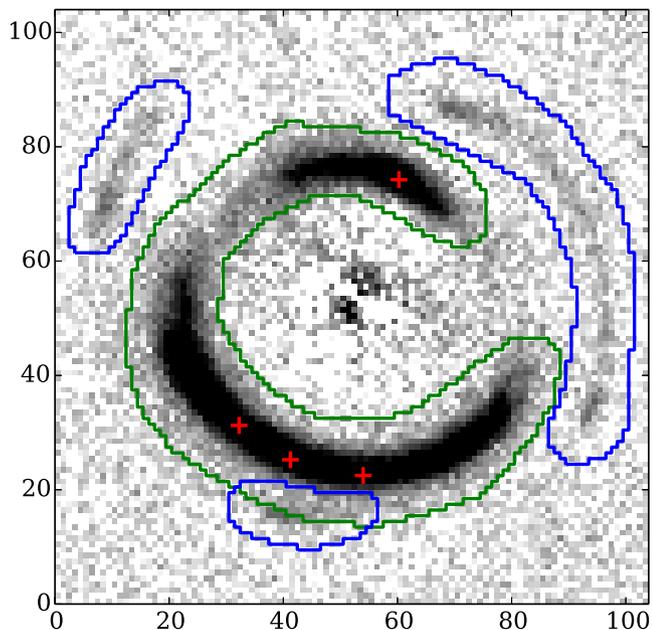
$$E^{w\text{CDM}} = \sqrt{\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + (\Omega_{\text{de}})(1+z)^{3(1+w)}}. \quad (11)$$

Equation (11) holds if the dark energy equation of state,  $w$ , is constant; fixing  $w = -1$  and  $\Omega_k = 0$  gives the concordance  $\Lambda$ CDM cosmology. Since angular diameter distances are inversely proportional to  $H_0$ , the ratio  $\beta$  is a function only of  $w$ ,  $\Omega_M$ ,  $\Omega_k$  and the redshifts of the lens and sources.

## 3 MODELLING THE DOUBLE-SOURCE-PLANE SYSTEM J0946

J0946 was discovered by Gavazzi et al. (2008) as part of the Sloan Lens ACS Survey (Bolton et al. 2006, 2008). The lens is a massive early-type galaxy at  $z_l = 0.222$  and the first source has a redshift of  $z_{s1} = 0.609$  (Gavazzi et al. 2008) while the second source has a photometric redshift of  $z_{s2} \approx 2.4$  (Sonnenfeld et al. 2012). Gavazzi et al. (2008) reported that for J0946 the Einstein radii are  $1.43 \pm 0.01$  and  $2.07 \pm 0.02$  arcsec for the inner and outer rings, hence  $\beta^{-1}$  is approximately 1.45. Subsequently, Sonnenfeld et al. (2012) have investigated the system in more detail, explicitly including the effects of the lower redshift source and using a two-component lensing model to constrain the dark and stellar mass distributions.

Here, we fit a new lens model to simultaneously constrain the mass in the foreground lens, the velocity dispersion of the lower redshift source, and the cosmology-dependent term  $\beta$ . We use *Hubble Space Telescope* (HST) ACS imaging data in the F814W filter that have been drizzled to a 0.05 arcsec pixel scale and our point spread function model is taken from a bright, unsaturated star in the image. We first subtract the lensing galaxy by simultaneously fitting the foreground galaxy and both background sources in the same framework as described in Auger et al. (2011). Briefly, we assume that the lensing mass is a singular isothermal ellipsoid with external shear, and both sources are modelled with Sérsic profiles.



**Figure 2.** The galaxy-subtracted *HST* F814W image of J0946. We model only the data contained within the union of the green and blue masks. The first source is modelled as only contributing to the flux observed within the green mask, and the second source is assumed to only have non-zero flux within the blue mask. The red crosses mark the 4 pixels which we map back on to the first lens plane and use to determine the centroid of the first source’s mass.

The lensing galaxy has a complicated photometric profile and shows evidence for interactions (also see Sonnenfeld et al. 2012), but we find that it is well fitted using three Sérsic components. The goal of this modelling is to robustly characterize and remove the light from the lensing galaxy, and the result is shown in Fig. 2.

### 3.1 Lens model

We fit the lensed images shown in Fig. 2 with two pixelated sources lensed by an elliptical power-law mass distribution with external shear. We apply two overlapping masks (shown in Fig. 2) where we allow each source to place flux, leaving us a data vector of the  $N_D$  pixels within the union of the two masks, that we fit with two sources ( $s_1$  and  $s_2$ ) of  $N_{s1}$  and  $N_{s2}$  pixels, respectively.

Our primary lens model has six free parameters, two for position ( $x, y$ ), two for ellipticity ( $q, \theta_q$ ), the power-law index ( $\eta$ ) and the Einstein radius ( $b_1$ , the characteristic scale of deflections by the lens for the first source plane), plus two further parameters for external shear (the shear magnitude and position angle,  $\gamma_{\text{ext}}$  and  $\theta_\gamma$ , both defined in terms of their effect on the first source). We model the mass of the first source as an isothermal sphere with one free parameter, the Einstein radius ( $b_{s1}$ ). In our model, the centroid of this mass is a deterministic function of the primary lens model. Experience tells us that mass and light are closely aligned, but we do not know the unlensed light distribution for the first source a priori. To retain the prior that the mass and light are co-spatial, we map four of the brightest first source pixels in the image plane back on to the source plane according to the current primary lens model (see Fig. 2) and take the mean of these four positions as the centroid of the source mass. Because lensing conserves surface brightness, bright image plane pixels should always map on to bright source plane pixels, but in practice our inference is insensitive to

our choice of centroid as long as the mass is placed approximately near the light. The final free parameter in the model is the inverse of the cosmological scaling factor  $\beta$ , as defined in equation (5). In total our model has 10 free non-linear parameters, and we assume uninformative uniform priors for each of these.

### 3.2 Modelling DSPLs with pixelated sources

Our modelling follows the semilinear approach of Warren & Dye (2003), where we linearly solve for the optimal pixelated source during each iteration of non-linearly sampling over the mass model parameters.

#### 3.2.1 Modelling multiple pixelated sources with a fixed lens model

To solve for the source and lens parameters, we first pick a lens model and reverse ray-trace the image plane pixels back on to each source plane using the elliptical power-law deflection solver of Barkana (1998) and the multiple-lens-plane equation (equation 6). For each source, we then define a square grid that is sufficiently large to encompass the delensed position of each image pixel contained within the corresponding mask.<sup>1</sup> With the source grids now defined, and the delensed position of each image pixel now known, we can apply the bilinear interpolation method of Treu & Koopmans (2004) to generate the lensing matrix,  $l$ .

For a fixed lens model, we can write down the equation that relates the data vector (i.e. the observed pixels in the image plane) to the two sources as

$$\mathbf{d} = \mathbf{f}\mathbf{s} + \mathbf{n}, \quad (12)$$

where  $\mathbf{d}$  is the data vector,  $\mathbf{s}$  is the concatenated source vector ( $s_1, s_2$ ), and  $\mathbf{n}$  is the noise vector characterized by the covariance matrix,  $C_D$ .  $\mathbf{f}$  is the ( $N_D$  by  $N_{s1} + N_{s2}$ ) matrix that maps source flux on to the image plane and is the product of the point spread function operator ( $p$ ) and the lensing operator:  $\mathbf{f} = \mathbf{p}l$ .

Without any prior on the smoothness of the source (regularization), the merit function for this model is (Warren & Dye 2003)

$$\exp(-E_D), \quad (13)$$

where

$$E_D = \frac{1}{2} (\mathbf{f}\mathbf{s} - \mathbf{d})^T C_D^{-1} (\mathbf{f}\mathbf{s} - \mathbf{d}). \quad (14)$$

Warren & Dye (2003) and Suyu et al. (2006) show that the most likely source is thus given by

$$\mathbf{s}_{\text{ML}} = \mathbf{F}^{-1} \mathbf{D}, \quad (15)$$

where

$$\mathbf{F} = \mathbf{f}^T C_D^{-1} \mathbf{f} \quad (16)$$

and

$$\mathbf{D} = \mathbf{f}^T C_D^{-1} \mathbf{d}. \quad (17)$$

However, this may give sources that do not look like reasonable astrophysical objects, and when the number of source pixels is large

<sup>1</sup> This partially adaptive grid is designed so that choosing a fixed number of source grid pixels, i.e.  $N_{s1}$  and  $N_{s2}$ , does not bias the fit towards any particular lens model. For a non-adaptive source grid, models with high magnification would fill a smaller region of the source plane (and hence have fewer degrees of freedom) than those with low magnification.

there may be unreasonably many degrees of freedom. This problem can be overcome by the introduction of a regularization term (Wallington, Narayan & Kochanek 1994; Warren & Dye 2003) that prefers the source to be smooth. This regularization adds additional terms to the merit function that penalize spiky sources, such that the full merit function is given by

$$\exp(-E_D - \lambda_1 E_{s1} - \lambda_2 E_{s2}), \quad (18)$$

where

$$E_{si} = \frac{1}{2} (s_i^T R_i s_i) \quad (19)$$

and  $R_i$  is the regularization matrix. This regularization modifies equation (15) to

$$s_{MP} = \mathbf{A}^{-1} \mathbf{D}, \quad (20)$$

where, the matrix  $\mathbf{A}$  is defined as

$$\mathbf{A} = F + \begin{pmatrix} \lambda_1 R_{s1} & 0 \\ 0 & \lambda_2 R_{s2} \end{pmatrix}. \quad (21)$$

Suyu et al. (2006) showed how to optimize the strength of this regularization,  $\lambda$ , for a single source. Under certain assumptions (Suyu et al. 2006), the optimal value of  $\lambda$  is found by solving the equation

$$\frac{d}{d \log \lambda} \log P(\mathbf{d}|\lambda, f, r) = 0, \quad (22)$$

where  $r$  represents the choice of regularization.

When there are multiple sources, equation (22) can be generalized to find the optimal regularization for each,

$$\frac{\partial}{\partial \log \lambda_i} \log P(\mathbf{d}|\{\lambda\}, f, r) = 0. \quad (23)$$

Generalizing equation 19 of Suyu et al. (2006) to include two sources, it can be shown that

$$\begin{aligned} \log P(\mathbf{d}|\{\lambda\}, f, r) = & -\lambda_1 E_{s1} - \lambda_2 E_{s2} - E_D - \frac{1}{2} \log(\det \mathbf{A}) \\ & + \frac{N_{s1}}{2} \log \lambda_1 + \frac{N_{s2}}{2} \log \lambda_2 \\ & + \text{terms independent of } \{\lambda_1, \lambda_2\}. \end{aligned} \quad (24)$$

Hence, the optimal value for each regularization constant is given by

$$2\lambda_i E_{si} = N_{si} - \lambda_i \frac{\partial}{\partial \lambda_i} (\log(\det \mathbf{A})), \quad (25)$$

which must be solved iteratively since both  $\mathbf{A}$  and  $E_{si}$  are functions of  $\lambda_1$  and  $\lambda_2$ .<sup>2</sup>

By picking a specific coarseness for the source pixel grid, we are implicitly assuming the source to be smooth on scales smaller than the pixel size. By using a fine pixel grid, we can minimize this assumption and allow the Bayesian evidence to choose the regularization rather than an ad hoc choice of pixel size. To balance the need for a fine grid and computational demands, we use source grids that have 80 pixels on a side for the first source and 50 pixels on a side for the second source. Fewer pixels are needed for the second source since the signal to noise of the first source's arc is much greater than the second's, hence the optimal regularization for the second source is likely to be significantly stronger than for the

first source. Our results do not change significantly if the number of source pixels is changed.

### 3.2.2 Determining the mass model

Whilst solving for the source is a linear problem, we determine the lens model parameters by using the parallel tempered ensemble sampler of Foreman-Mackey et al. (2013) to sample this 10-dimensional space, solving equation (20) at each iteration in order to evaluate the merit function of equation (18). By using the parallel tempered ensemble sampler, we are able to have high confidence that we are sampling the full posterior and not a single island of high likelihood. Ideally, we would optimize the regularization for each iteration of the sampler; however, this is computationally expensive; we find (as also found by Vegetti et al. 2010) that it is sufficient to optimize the two source regularizations at the most probable lens model<sup>3</sup> and fix these values throughout the chain.

### 3.3 Modelling results

Applying the techniques developed above, our best fit lens model is shown in Fig. 3, along with the noise normalized residual and the optimal reconstruction of the lensed sources. The marginalized parameter constraints are given in Table 1 and we show 2D projected sample distributions for  $\eta$ ,  $\theta_{\text{lens}}^E$ ,  $\theta_{s1}^E$  and  $\beta^{-1}$  in Fig. 4; the results of our modelling can be summarized as follows.

(i) The mass distribution of J0946 in the region probed by the two Einstein rings is very close to that of an isothermal sphere with a projected logarithmic density slope of  $\eta = 1.027_{-0.025}^{+0.023}$  (an isothermal sphere has  $\eta = 1$ )

(ii) The Einstein radius of the primary lens is very precisely determined. The primary lens has an Einstein radius of  $\theta_{\text{lens}}^E = 1.397_{-0.001}^{+0.001}$ , similar to the values found by previous models of J0946 (Bolton et al. 2008; Gavazzi et al. 2008; Vegetti et al. 2010; Sonnenfeld et al. 2012).<sup>4</sup>

(iii) By modelling the full arcs we have been able to make precise inference on the Einstein radius of the first source, and we find that  $\theta_{s1}^E = 0.161_{-0.021}^{+0.025}$ , strongly excluding the zero mass case. This implies a velocity dispersion of  $\sim 97 \pm 7 \text{ km s}^{-1}$ .

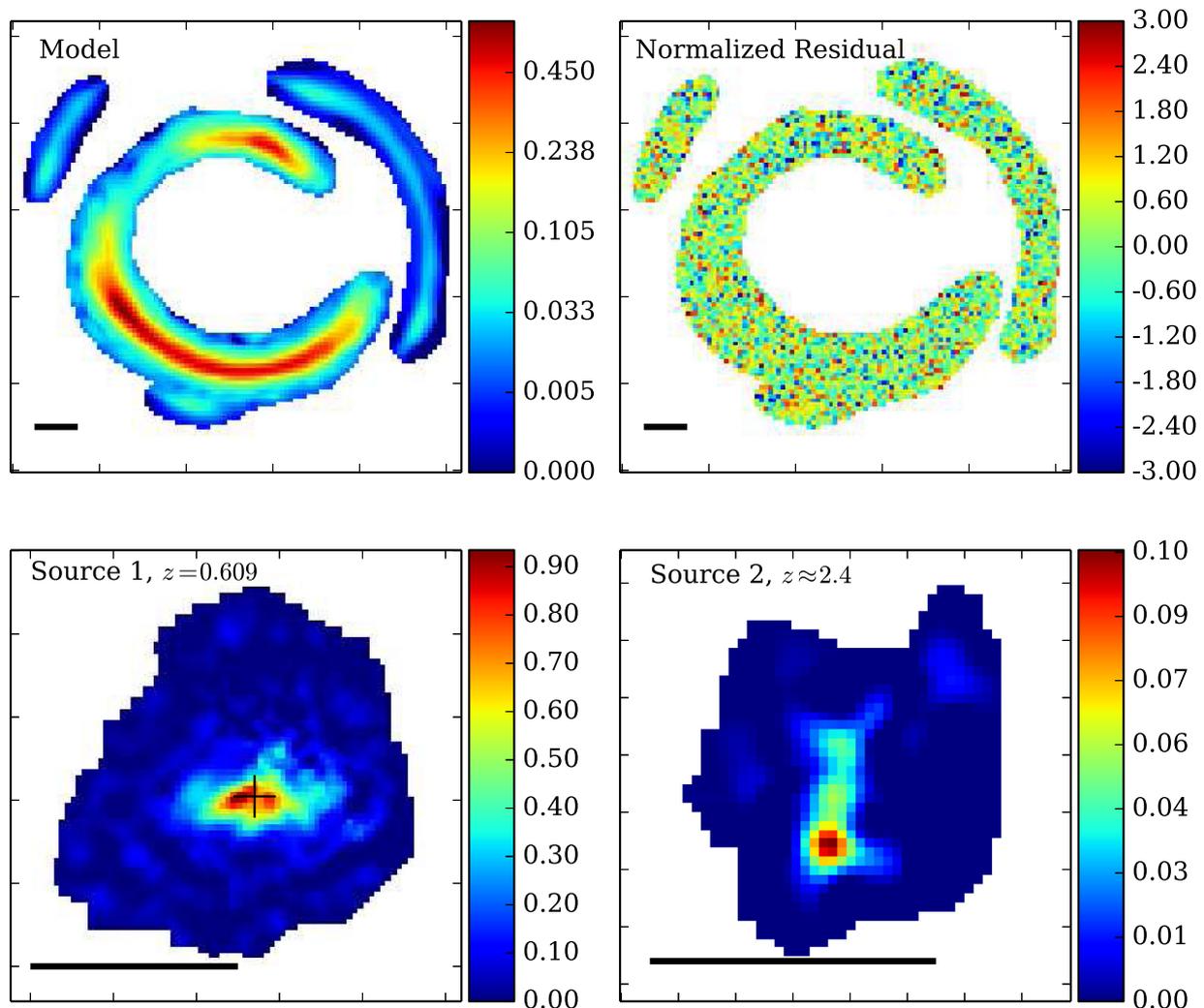
(iv) The cosmological scaling factor  $\beta^{-1}$  is  $1.405_{-0.016}^{+0.014}$ , and the distribution is well approximated by a Gaussian centred at 1.404 with width 0.016.

Gavazzi et al. (2008) find that there are two families of solutions for the lens model that lie in a degeneracy space between  $\eta$ ,  $\theta_{s1}^E$  and  $z_{s2}$ , although Sonnenfeld et al. (2012) break this degeneracy by modelling the stellar velocity dispersion profile of the primary lensing galaxy. However, the lensing data alone break this degeneracy if the full surface brightness distributions of the sources are reconstructed: if we start our model at the 'Family II' posterior position from Gavazzi et al. (2008), we find that the higher redshift source

<sup>3</sup> The most probable lens model can be solved iteratively, positing large regularizations, optimizing the lens model then solving for the optimal regularizations and repeating. One iteration is typically sufficient (Vegetti et al. 2010).

<sup>4</sup> We note that here the Einstein radius  $\theta_{\text{lens}}^E$  is the circularized (i.e. intermediate axis) radius within which the mean surface mass density is equal to the lensing critical density (i.e. the mean convergence is unity). Other authors may report Einstein radii measured along the major axis of the elliptical mass distribution or defined as a lens strength in some other way, but when we apply this definition they are all in close agreement.

<sup>2</sup>  $\frac{\partial}{\partial \lambda_i} (\log(\det \mathbf{A}))$ , can be efficiently solved numerically using the Cholesky root of  $\mathbf{A}$ ;  $\log(\det \mathbf{A}) = 2 \sum_j \log[(\mathbf{A}^{1/2})_{jj}]$



**Figure 3.** From top left clockwise. (1) Most probable model for the  $F814W$  image of J0946, the colour scale is non-linear. (2) Normalized residual, (image-model)/ $\sigma$ , where  $\sigma$  is the uncertainty in each pixel. (3) Most probable first source, the centroid of this source’s mass is shown by the black cross. (4) Most probable second source. In all images the black bar shows a 0.5 arcsec scale.

**Table 1.** The median and 68 per cent confidence bounds on the 10 marginalized parameters of our lens model.

$x_{\text{lens}}$	$y_{\text{lens}}$	$q_{\text{lens}}$	$\theta_q$	$\eta_{\text{lens}}$	$\theta_{\text{lens}}^E$	$\theta_{s1}^E$	$\gamma_{\text{ext}}$	$\theta_\gamma$	$\beta^{-1}$
$51.885^{+0.057}_{-0.038}$	$51.429^{+0.040}_{-0.036}$	$0.946^{+0.009}_{-0.005}$	$30.6^{+4.0}_{-4.8}$	$1.027^{+0.023}_{-0.025}$	$1.397^{+0.001}_{-0.001}$	$0.161^{+0.025}_{-0.021}$	$0.069^{+0.002}_{-0.003}$	$-27.20^{+0.75}_{-0.56}$	$1.405^{+0.014}_{-0.016}$

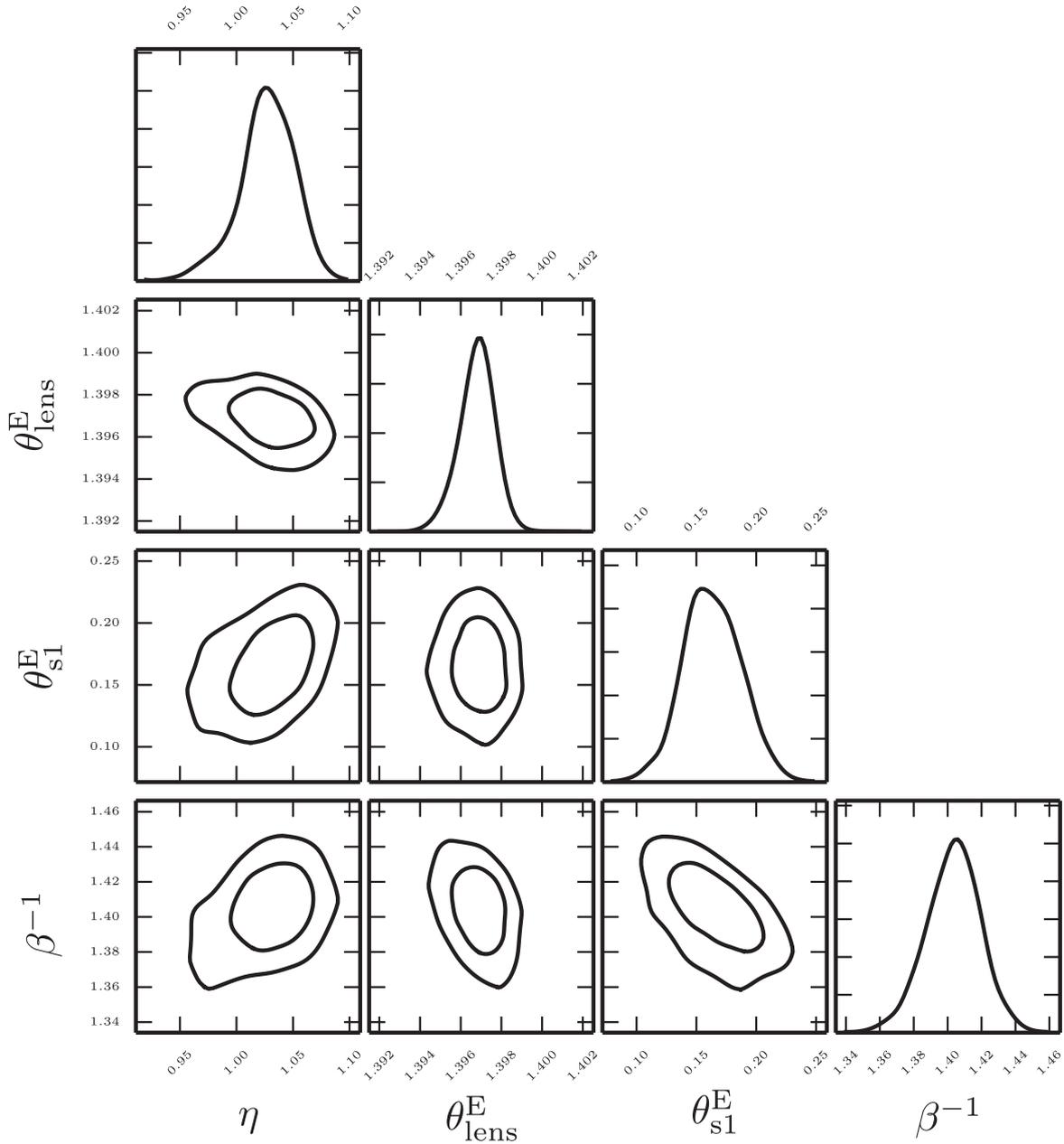
is not well focused, but subsequently optimizing the model leads to our optimal solution which agrees well (in terms of the parameters  $\eta$  and  $\theta_{s1}^E$ ) with the lensing-and-dynamics model of Sonnenfeld et al. (2012).

Both of these models find a nearly isothermal mass distribution that is in tension with the power-law slope of  $\eta = 1.196$  inferred by Vegetti et al. (2010) using only the first source. Once again, we have started our sampling from their best-fitting model and find that this model does not focus the second source; the posterior again converges to the solution presented in Table 1 and Fig. 4. The reason for the discrepancy is unclear, but we note that otherwise both models are quite similar, including the magnitude and orientation of the external shear, the near-circularity of the mass density profile, and the reconstructed surface brightness distribution of the lower redshift source; further investigation comparing our code with the code used in Vegetti et al. (2010) is currently ongoing.

#### 4 COSMOLOGICAL PARAMETER CONSTRAINTS

Because it is a ratio of four angular diameter distances,  $\beta$  is a function of only the matter content of the Universe, spatial curvature, the equation of state of dark energy, and the redshifts of the lensing and source galaxies. The lens and first source redshifts for J0946 are known from the SDSS spectroscopy (Gavazzi et al. 2008), and Sonnenfeld et al. (2012) used five-band *HST* imaging to determine a photometric redshift for the second source of  $z_{s2} = 2.41^{+0.04}_{-0.21}$  (68 per cent CL, Fig. 6). Assuming a flat  $\Lambda$ CDM cosmology,  $\beta$  only depends on these redshifts and  $\Omega_M$ , which we infer to be  $\Omega_M = 0.33^{+0.33}_{-0.26}$ , with the full probability density function shown in Fig. 5.

As pointed out in Collett et al. (2012), the main benefit of using DSPLs to constrain cosmological parameters is that they have novel parameter degeneracies compared to other cosmological probes; by



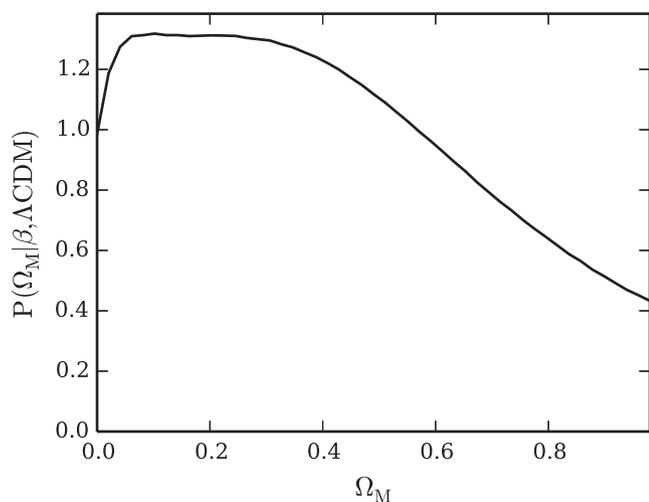
**Figure 4.** The posterior for  $\beta^{-1}$  from our model, and the three model parameters that are most degenerate with it: the logarithmic slope of the projected density for the first lens ( $\eta = 1$  corresponds to isothermal), and the Einstein radii of the primary lens and the first source ( $\theta_{\text{lens}}^E$  and  $\theta_{s1}^E$ , respectively) in arcseconds. The contours show the 68 and 95 per cent confidence regions.

combining our DSPL with a cosmic microwave background (CMB) prior we make significantly more precise inference than with either probe individually. Adding the measurement of  $\beta$  is particularly powerful for constraining non- $\Lambda$  cosmological models, where the equation of state,  $w$ , is not fixed to  $w = -1$ . Our constraints on the  $w$ CDM model (assuming a constant equation of state for dark energy and a flat cosmology) are shown in Fig. 7. With a uniform prior on  $w$  and  $\Omega_M$ , J0946 alone is only particularly powerful at ruling out cosmologies with both low  $\Omega_M$  and very negative  $w$ ; however this is part of the region favoured by the CMB. J0946 plus a Planck prior (where we have importance sampled the constraints derived from the Planck low- $l$ , high- $l$  CMB temperature measurement, *Wilkinson Microwave Anisotropy Probe (WMAP)* polarization measurement and included CMB lensing; Planck Collaboration 2013)

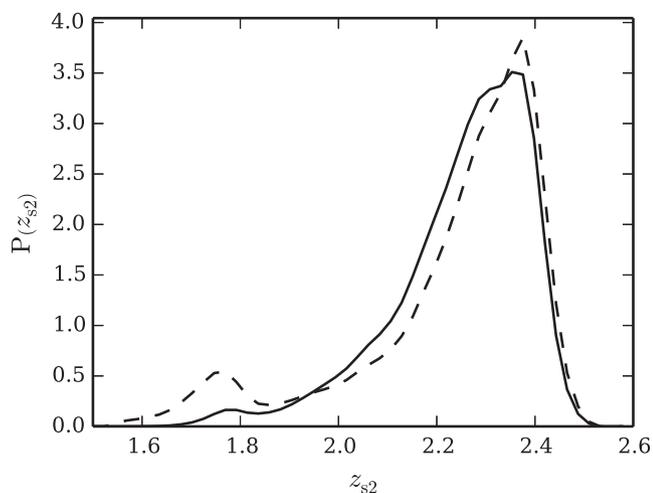
gives  $w = -1.17^{+0.20}_{-0.21}$  when marginalizing over  $\Omega_M$ , and we note that the Planck prior alone ( $w = -1.49^{+0.36}_{-0.27}$ ) has a 50 per cent larger uncertainty if J0946 is not included. The J0946 constraint plus a *WMAP9* prior (Bennett et al. 2013) gives  $w = -0.99^{+0.27}_{-0.25}$  compared to  $w = -0.98^{+0.44}_{-0.54}$  for *WMAP* only.

In principle,  $\beta$  can be used to constrain evolving models of the dark energy equation of state, but we leave this to later work since a sample of several DSPLs is required to give interesting constraints on these models (Collett et al. 2012). Furthermore, we have not investigated non-flat cosmologies, since at fixed  $w = -1$ ,  $\Omega_M = 0.3$ ,  $z_1 = 0.2$ ,  $z_{s1} = 0.6$ ,  $z_{s2} = 2.3$ ,

$$\frac{\partial \beta}{\partial \Omega_k} \sim 0.02\beta \quad (26)$$



**Figure 5.** The probability distribution function for  $\Omega_M$  given our measurement of  $\beta$ , marginalized over the photometric redshift for the second source. A uniform prior has been assumed for the range  $0 < \Omega_M < 1$ .



**Figure 6.** The probability distribution function for the second source in J0946,  $z_{s2}$ . The prior is shown dashed and is taken from the photometric redshift estimate of Sonnenfeld et al. (2012). The posterior is shown solid derived using our measurement of  $\beta$  and assuming  $\Lambda$ CDM with a uniform prior on  $\Omega_M$ .

and  $\Omega_k$  is already constrained at the per cent level (e.g. Planck Collaboration 2013).

## 5 DISCUSSION AND CONCLUSION

We have derived the first cosmological constraints from a galaxy-scale DSPL. Our measurement of  $\beta$  is completely independent of other cosmological probes, and can hence easily be combined with other data sets to produce tighter cosmological parameter estimates, lift parameter degeneracies, and test for the presence of unknown systematics. Because of the complementarity of DSPLs with the CMB, our measurement with just a single DSPL improves the precision of the inference on  $w$  by approximately one third. More precise inferences have been made by combining the CMB with, e.g. baryon acoustic oscillation (BAO) measurements (combining *Planck* with the BAO results from Percival et al. 2010 yields  $w = -1.12^{+0.10}_{-0.11}$ ), but we note that the number of DSPLs that will be useful for cosmo-

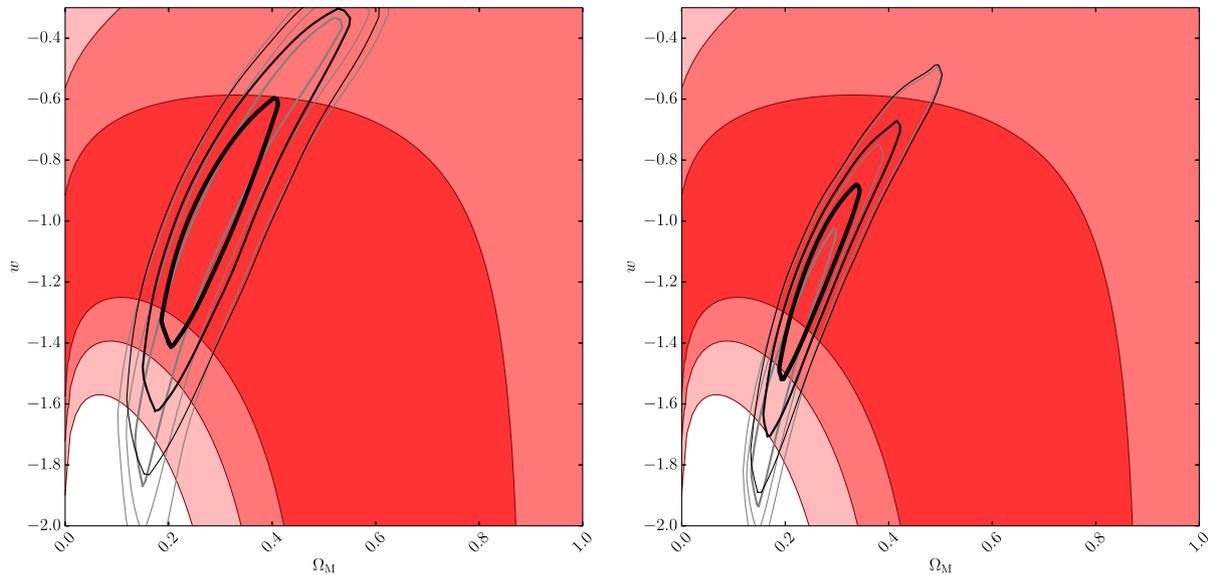
logical inference will increase by orders of magnitude with Euclid (Collett et al., in preparation), dramatically improving the precision but also helping to uncover systematic biases. For example, combining *Planck* with either J0946 or BAO measurements causes the inference on  $w$  to shift closer to  $-1$  by around  $1\sigma$ .

The lens model that we have presented is robustly constrained, but our inference on  $\beta$  depends on the assumption that the observed lensing is entirely due to an elliptical power-law mass distribution at  $z = 0.222$  and an isothermal mass distribution at  $z = 0.609$ . The latter point is not a significant concern here, since the highest redshift source has an impact parameter with respect to the  $z = 0.609$  source that is more than three times larger than the Einstein radius  $\theta_{s1}^E$ . Similarly, the power-law description for the central total mass density distribution is motivated by the absence of any correlation between the power-law indices and radii of strong lenses (e.g. Koopmans et al. 2006, 2009; Auger et al. 2010), as well as the power-law behaviour of the total mass distribution over a large range of scales from the ensemble weak lensing mass profile of lenses (Gavazzi et al. 2007) and mass profiles of massive X-ray-bright galaxies (Humphrey & Buote 2010). Furthermore, Suyu et al. (2014) have explicitly modelled one time delay gravitational lens, RX J1131–1231, with both a power-law and a stars-plus-dark matter model and find no significant difference in the cosmographic inference between the two models when stellar kinematics are included in the modelling; a similar analysis for J0946 is under way.

The mass-sheet degeneracy (Falco, Gorenstein & Shapiro 1985) will also impact our inference on  $\beta$ . Although multiple-source-plane lenses largely break this degeneracy for true mass sheets, Liesenborgs et al. (2008) have shown that a ring of mass (in addition to the mass from the power-law model) between the Einstein radii of the two sources can mimic the mass-sheet degeneracy even for multiple-source-plane lenses. However, it is not clear what physical process would be responsible for significant ring-like projected over- (or under-) densities and we therefore neglect this possibility. Lensing by line-of-sight structures is also not included in our model, and if these objects introduce a positive external convergence then our estimate of  $\beta$  will be low. Treu et al. (2009) found no evidence that this line of sight is atypical, but even in the absence of a bias the unmodelled external convergence will lead to an artificially low uncertainty. We estimate that ignoring the external convergence results in an  $\sim 1$  per cent systematic uncertainty on  $\beta$  (i.e. comparable to the statistical uncertainty) which would degrade the precision of our inference on  $w$  by  $\approx 25$  per cent. However, directly modelling the line of sight using the existing SDSS and *HST* imaging (e.g. Wong et al. 2012; Collett et al. 2013) and including the velocity dispersion profile from Sonnenfeld et al. (2012) will significantly decrease this systematic uncertainty. Furthermore, modelling the strong lensing with all of the available *HST* data will reduce our statistical uncertainty while allowing us to further test for residual systematics by comparing our inference on  $\beta$  between the different *HST* filters.

Although there is still room for improvement of our measurement of  $\beta$  for J0946, the most significant obstacle for DSPL cosmological constraints is the scarcity of simple multiple-source-plane lenses. Gavazzi et al. (2008) suggest that one in 40–80 galaxy-scale strong lenses should be a DSPL, and tentative *Euclid* forecasts of  $\sim 100\,000$  galaxy–galaxy strong lenses<sup>5</sup> imply  $\sim 2000$  DSPLs and  $\sim 40$  triple-source systems. Although, it is not clear how many of these systems

<sup>5</sup> E.g. <http://euclidfrance.sciencesconf.org/file/21610>.



**Figure 7.** The  $w$  and  $\Omega_M$  plane. Red shows the 68, 95 and 99.7 per cent confidence constraints derived from our measurement of the cosmological scalefactor in J0946. In the left-hand panel, grey shows the *WMAP9* constraints whilst the Planck 2013 constraints are shown in the right-hand panel. In both panels, black shows the combined constraint from J0946 and the CMB prior.

will be useful for cosmography, including favourable (and well-measured) lens and source redshifts (e.g. Collett et al. 2012), our analysis of J0946 demonstrates the significant degeneracy-breaking power of even a single DSPL.

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