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Spare parts classification in industrial manufacturing using the dominance-based rough set approach

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Abstract

Classification is one of the critical issues in the operations management of spare parts. The issue of managing spare parts involves multiple criteria to be taken into consideration, and therefore, a number of approaches exists that consider criteria such as criticality, price, demand, lead time, and obsolescence, to name a few. In this paper, we first review proposals to deal with inventory control. We then propose a three-phase multicriteria classification framework for spare parts management using the dominance-based rough set approach (DRSA). In the first phase, a set of 'if–then' decision rules is generated from historical data using the DRSA. The generated rules are then validated in the second phase by using both the automated and manual approaches, including cross-validation and feedback assessments by the decision maker. The third and final phase is to classify an unseen set of spare parts in a real setting. The proposed approach has been successfully applied to data collected from a manufacturing company in China. The proposed framework was practically tested on different spare parts and, based on the feedback received from the industry experts, 96% of the spare parts were correctly classified. Furthermore, the cross-validation results show that the proposed approach significantly outperforms other well-known classification methods. The proposed approach has several important characteristics that distinguish it from existing ones: (i) it is a learning-set based analysis approach; (ii) it uses a powerful multicriteria classification method, namely the DRSA; (iii) it validates the generated decision rules with multiple strategies; and (iv) it actively involves the decision maker during all the steps of the decision making process.

Keywords: Rough Sets, Spare parts, ABC classification, Multiple Criteria Inventory Classification, Dominance-based Rough Set Approach.

1. Introduction

Spare parts are common inventory stock items that are required for timely maintenance of industrial plant systems. A recent study [51] shows that the operational and maintenance support costs in a typical industrial plant account for more than 60% of the overall cost, where the spare parts related costs alone account for about 25% to 30%. This clearly indicates that better operations management of spare parts is required and has an important role in the availability of the plant at an optimal cost. An efficient and effective inventory management helps a firm maintain its competitive advantage [75]. In many large firms, it is not uncommon to hold tens of thousands of spare parts [32], e.g. the number of spares in a medium scale engineering business it may be in the tens of thousands while in a large scale chemical factory, it may be around hundreds of thousands. In such situations, it may become practically impossible to use human judgement alone to identify the appropriate stock control strategy of each spare part and hence inventory management

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becomes a great challenge. In order to facilitate spare parts management, one of the possible ways is to group the spare parts into specific categories by finding some similarities in their features and then, based on these common features, define a set of policies for each group.

One of the most well-known and commonly used classification techniques is the ABC classification, that uses the 80–20 rule (the Pareto principle). The ABC classification is particulary appropriate for the inventory management of materials that are fairly homogenous in nature and differ from each other mainly by unit price and demand volume. The ABC analysis has retained its popularity among practitioners in directing the control efforts and choosing the 'sufficient-enough' control parameters without the need of item-specific analyses [52].

The ABC classification technique has traditionally focused on a single criterion of price, which is usually measured in annual dollar usage. However, it is important to realize that optimizing the single objective of price is generally misleading, as several other criteria should be taken in consideration for better spare parts operations management. We contend that focusing on this single criterion ignores several other important criteria for classifying spare parts, such as criticality, lead time, demand, commonality, obsolescence and substitutability (see, e.g. [15][16][64]). The authors in [27], for instance, emphasize the role of the lead time criterion in analysing the competitiveness of companies. In terms of criticality, one can also argue that it is a function of the criticality of the spare parts for the machine as well as the criticality of the machine in the whole operational system [33][38][57]. Accordingly, the use of multiple criteria for spare parts classification has better justification as it attempts to consider all the operations management/control requirements of different types of items. The authors in [36][37] were amongst the first to state the importance of applying multiple criteria to ABC analysis, and since then, a number of Multiple Criteria Inventory Classification methods have been proposed in the literature. A discussion of some of the relevant and recent papers is presented in Section 3.

The objective of this paper is to propose a data analytic approach for multiple criteria ABC classification of spare parts and demonstrate its usefulness by applying it to a real business problem in a manufacturing company. This approach relies on the Dominance-based Rough Set Approach (DRSA), which is a well-known multicriteria classification method that has been proposed by [41][42][77][78] to overcome the shortcomings of the conventional Rough Sets Theory (RST) [68][69] in multicriteria classification by allowing preference-oriented attributes and where the decision classes are defined in an ordinal way. The multicriteria classification is a fundamental problem of multicriteria decision making [84]. The multicriteria classification problem can be stated as follows: given a set of objects described by a set of criteria (attributes with preference-ordered domains), assign these objects to some pre-defined decision classes or categories, in such a way that each object is assigned to exactly one class. The DRSA has been successfully used in different real-world decision problems (see, e.g. [21][40][59])). The DRSA has some powerful capabilities that make it attractive for real-world decision problems (see [20]). Among the main characteristics of the DRSA is the use of a learning set as input to elicit and generalize the preferences of the decision maker, which minimizes the cognitive effort required from him/her. The use of a learning set as input is adopted in several multicriteria classification methods, including [3][4][17][31]. However, the main distinction of the DRSA compared to other multicriteria classification methods that are based on the use of a learning set as input is the simplicity and the easily understandable if–then decision rules provided as output, while other methods have no such straightforward interpretation [10].

The proposed approach is structured into three phases. The first phase uses a carefully selected set of spare parts as a learning set to generate a set of if–then decision rules that can be shown to the decision maker in a simple readable manner. These rules are generated by the DRSA. In the second phase, the decision rules are assessed and analysed by the decision maker as feedback for reinforced learning of the if–then rules. In addition to this, re-classification and cross-validation have also been used to further validate the generated decision rules. The third phase exploits the generated (and validated) decision rules in order to classify unseen spare parts.

We apply the proposed approach to a real-world case study and show its merit by comparing the results with other methods using ten-fold cross-validation. The dataset has been acquired from a manufacturing company in China. The company has been anonymized and renamed the Industrial Manufacturing Company (IMC) in this paper for reasons of data protection and business ethics. The company showed an interest in managing their stock items (spare parts) by gaining some useful insights through historical data collected over a period of time, and based on this learning, they were interested in classifying the new spare parts (for the newly purchased equipment). We used this case study to validate the results and illustrate the usefulness of the proposed approach. In the future, the approach can easily be extended to automatically analyse a large number of spare parts.

The approach proposed in this paper has several important characteristics: (i) it applies a learning-set based analysis, which is particularity useful in spare parts management for large firms; (ii) it uses a powerful multicriteria classification method, namely the DRSA, which is characterised by its simplicity and the easily understandable if–then decision rules provided as output; (iii) it includes a comprehensive collection of validation strategies enabling the decision maker to analyse the validity of the results; and (iv) it actively involves the decision maker in all the steps of the decision making process. A detailed discussion of these characteristics is given in Section 6.1.

This paper is organized as follows. Section 2 discusses related work. Section 3 proposes an ABC classification approach and methods of validation. Section 4 describes the case study. Section 5 provides a comparative study. Section 6 discusses some theoretical and practical aspects of the proposed approach. Section 7 concludes the paper by highlighting the merits and future challenge

2. Related work

In the past few decades, a number of approaches have been proposed to solve the multiple criteria inventory classification problem. In this section, we first characterize these approaches in terms of their classification criteria and methods used (Section 2.1) and then in terms of their application domains and validation strategies (Section 2.2). Lastly, we summarize the main aspects of the discussed approaches (Section 2.3)

2.1. Classification criteria and methods

The characteristics of the classification criteria considered and the classification methods used of about 37 approaches that we have identified in the relevant literature are summarized in Table 1. This table indicates also the nature of the output for each approach.

While the list of criteria used varies from one proposal to another, we can see that the first four criteria (viz. Criticality, Annual Cost Usage, Unit Price, and Lead Time) have been commonly used by most of the reported studies. In addition to these four criteria, the criterion Demand Rate has also been used but less frequently than the first four. Indeed, the demand rate can be derived from the Annual Cost Usage and Unit Price and hence it is redundant to use the demand rate alongside these two criteria. The other criteria have only been reported in a few (mostly one or two) studies related to the multiple criteria inventory classification problem, i.e. ordering cost [66][67], substitutability [19][47], replaceability [19][47], perishability [8], storage cost [8], current item status [26], severity of the impact of its running out [26], number of hits [53], average value per hit [53], payment terms [19], durability [54][55][73], limitations of warehouse space [49], last use date [54][55], supplier [54], and turnover rate [60]. We note that some proposed models offer more flexibility about the criteria to be included. For instance, the model proposed by [36] accepts any two criteria, while in the model proposed by [18], the user can include any criteria in the analysis. In the case study presented in Section 4, we have considered four criteria, namely Criticality, Annual Cost Usage, Unit Price, and Lead Time. However, the proposed approach is flexible enough and can be used with any number of criteria.

A number of techniques that take into account multiple criteria for the ABC classification problem have been proposed in the literature. These methods can be grouped into different categories as follows:

1. Clustering algorithms: distance functions, such as the C-means algorithm [24] and the k-means algorithm [61], Fuzzy C-Means Clustering [5];

2. approaches based on Data Envelopment Analysis (DEA), such as the FAHP-DEA [55] and the modified DEA-like model [83];
3. Optimization:
   - Optimization Models including linear programming approaches such as the Modified Linear Optimization Model [50], the Hybrid Weighted Linear Optimization method [58], R-model [72], ZF-model [88], Ng-model [64]; and non-linear programming models such as the Extended Ng-model [48];
   - Evolutionary Optimization including Simulated Annealing (SA) [63] and various evolutionary algorithms, such as Genetic Algorithms (GA) [47] and Artificial Neural Network (ANN) [66];
4. Multicriteria methods: Bi-criterion matrix [36][37], AHP [35][49][54][67], TOPSIS [8][23], DRSA [25] and UTADIS [80].
5. Other statistical methods: Exponential Smoothing Weights [53] and Peer-estimation approach [22].

Some of the approaches use fuzzy logic to take into account uncertainty and imprecision. Examples include Fuzzy AHP [18][19], Fuzzy classification [26], Fuzzy Logic [73] and Fuzzy C-Means Clustering [5]. Some other approaches apply mixed approaches, e.g. AHP and the k-means algorithm [61], FAHP and ANN [55], FAHP and DEA [49], CA and SAA [60].

Table 1: Overview of classification criteria and classification methods employed by the main reviewed contributions

	Criteria							
Ref.	Criticality	Annual Cost Usage	Unit price	Lead time	Demand Rate	Others	Used method	Output
[36]	Any two						Bi-criteria matrix	Classification
[37]	✓	✓					Bi-criteria matrix	Classification
[35]	✓	✓	✓	✓			AHP	Classification
[67]			✓	✓	✓	Ordering cost	AHP	Classification
[47]		✓		✓	✓	Substitutability, Replaceability	GA	Classification
[66]			✓	✓	✓	Ordering cost	ANN	Classification
[72]	✓	✓	✓	✓			R-model	Classification
[8]			✓	✓	✓	Perishability, Storage cost	TOPSIS	Classification
[64]		✓	✓	✓			Ng-model	Classification
[88]		✓	✓	✓			ZF-model	Classification
[18]	Any criteria						Fuzzy AHP	Classification
[24]	✓	✓	✓	✓			Case-based distance	Classification
[25]	✓	✓	✓	✓			DRSA	Classification + if-then rules
[26]	✓		✓	✓	✓	Current item status, Severity of the impact of its running out	Fuzzy classification	Classification
[53]						Number of hits, Average value per hit	Exponential Smoothing Weights	Classification
[19]			✓	✓	✓	Substitutability, Replaceability, Payment terms	Fuzzy AHP	Classification
[48]		✓	✓	✓			Extended Ng-model	Classification
[73]	✓			✓	✓	Durability	Fuzzy Logic	Classification + fuzzy rules
[22]	✓	✓	✓	✓			Peer-estimation approach	Classification
[49]		✓		✓		Limitations of warehouse space	FAHP-DEA	Classification
[86]	✓	✓	✓	✓			Artificial Intelligence	Classification
[5]	✓	✓	✓	✓			FCM	Classification
[23]	✓	✓	✓	✓			Two virtual items	Classification
[54]	✓		✓		✓	Durability, Last use date, Supplier	Fuzzy AHP	Classification
[83]	✓	✓	✓	✓			A modified DEA-like model	Classification
[63]	✓	✓		✓			Simulated annealing	Classification
[55]	✓		✓		✓	Durability, Last use date	FAHP and ANN	Classification
[50]	✓	✓	✓	✓			A modified linear optimization model	Classification
[61]	✓	✓	✓	✓			AHP and K-means algorithm	Classification
[65]		✓	✓	✓			CE-WLO	Classification
[80]		✓	✓	✓			UTADIS method	Classification
[60]		✓	✓	✓		Turnover rate	CA and SAA	Classification
[58]	✓	✓		✓			New hybrid weighted linear optimization model	Classification
This paper	✓	✓	✓	✓			DRSA	Classification + if-then rules + validation + generalization

Concerning the nature of the output shown in Table 1, most of the approaches generate a grouping of the spare parts into three classes: C, B and A. Unfortunately, the generated outputs in these cases cannot be used to classify new/unseen items. To classify unseen items, it is necessary to restart the process from scratch, which may be time consuming and may alter the already established classes. A possible solution to this issue is to use fuzzy logic (as described in [73]) in order to generate fuzzy rules permitting the classification of new items. A more advanced resolution of this issue is to use a case-based reasoning approach (as in [24]). However, the case-based reasoning methods fail to fully cope with all aspects of multiple criteria inventory classification problems, more specifically, with the presence of preference-ordered criteria. The use of the DRSA as proposed in this paper avoids the shortcomings of case-based reasoning approaches in the multiple criteria inventory classification problem. Indeed, in comparison to other classification methods and techniques (for instance in data mining, pattern recognition, and machine learning), the DRSA assumes that: (i) the decision classes are defined in an ordinal way; and (ii) the decision objects are evaluated over a set of criteria, meaning that the decision model should have some form of monotonic relationship with respect to the criteria.

2.2. Application domains and validation strategies

From a practical point of view, the studied approaches have been characterized with respect to the application domain considered, and the validation strategy used. Table 2 provides a summary of the main practical aspects of the proposals given in Table 1. This table shows, when appropriate, the main results of the comparative analysis included in the discussed papers. We briefly discuss each of these characteristics in what follows.

With respect to the application domain, Table 2 shows that most of the applications are related to healthcare, followed by the manufacturing industry. Other application domains include pharmaceuticals [66][67], followed by engineering [18][61], and then the energy sector [55]. There are a few applications to other fields, such as the automotive industry [5], distribution [19], port services [26], and university stationery inventory [47]. Furthermore, some of these applications have been made to spare parts management, while others have been applied to general stock keeping units. Additionally, there are only 13 papers out of 33 that carried out case studies with real-world applications, while some have used numerical examples (i.e. non-real data obtained, for instance, by simulation), and some others have used secondary data (extracted from other publications). In the present paper, the proposed approach has been applied to real-world data collected from a manufacturing company in China.

Regarding the validation techniques used, most of the previous studies rely on the re-classification strategy where the results of the classification methods used are compared to the initial results. The re-classification validation strategy has been used in, for instance, [22][36][37][49][67][80]. Some previous studies, including [8][65][67][80], use simulation as a validation strategy and a few of them (e.g. [19][47]) rely on a discussion with the decision maker to validate the results. Other validation techniques include the use of test data [55][66], experimental investigation [5][60], and clustering [61]. Some of the proposed approaches do not use any validation strategy, e.g. [54]. The approach proposed in this paper uses several complementary validation strategies: direct analysis of the obtained decision rules by the decision maker, re-classification analysis and cross-validation analysis.

Some of the discussed papers include a comparative study while others do not. The last two columns in Table 2 indicate, when appropriate, the methods that have been considered in the comparative studies and the main results of each comparative study. The approach proposed in this paper has been compared to several well-known classification techniques: fuzzy classification rule (FR), nearest neighbours (KNN), support vector machine (SVM), decision trees (DT), multi-layer perceptron network (MLPNN) and Naïve Bayes (NB).

2.3. Summary

Based on the previous discussion, it appears that only a few of the existing studies have been applied to spare parts classification. In addition, we can identify the following shortcomings:

1. Most of multicriteria methods that have been used in the previous literature are not adapted to deal with a large number of spare parts;
2. most of the multicriteria methods used require a large amount of information from the decision maker;
3. Apart from the re-classification, which is used most often as a validation strategy, existing proposals lack the use of appropriate and formal strategies to validate and exploit the results of the analysis;
4. Most of the existing literature does not carry out a real-world case study but relies on example analysis or data analysis using data extracted from other papers;
5. Several approaches lack an effective comparative study.

The approach proposed in this paper attempts to address these aspects, as discussed in Section 6.

Table 2: Overview of applications, validations and comparative studies for the main reviewed contributions

	Applications					Validation strategies			
Ref.	Field	In spare parts?	Real-world Case study?	Data from other papers?	Numerical example?	Reclassification	Other methods	Comparative study	Comments
[36]	Manufacturing firm	Yes	Yes	No	No	Yes	No	No	-
[37]	Service organization & manufacturing firm	Yes	Yes	No	No	Yes	No	Annual Dollar Usage (ADU)	The results indicate that ABC theory can be expanded rather simply to incorporate multiple criteria.
[35]	Hospital	No	No	Yes	Yes	Yes	No	ADU	AHP can improve the quality and completeness of the inventory analysis.
[67]	Pharmaceutical company	Yes	Yes	No	No	Yes	Simulation	ADU	AHP can improve the inventory control in this company.
[47]	University stationery inventory	Yes	Yes	No	No	No	Discussion with decision maker	AHP	GA performs much better than AHP.
[66]	Pharmaceutical company	Yes	Yes	Yes	Yes	Yes	Validation with test data	MDA	ANN models had higher predictive accuracy than MDA.
[72]	Hospital	No	No	Yes	Yes	Yes	No	ADU, AHP	The classification results using different methods were different.
[8]	Pharmaceutical company	No	Yes	No	No		Simulation	ADU	TOPSIS contributes to a low average inventory investment.
[64]	Hospital	No	No	Yes	Yes	Yes	-	ADU, R-model	The classification results using different methods were different.
[88]	Hospital	No	No	Yes	Yes	Yes	No	R-model	ZF-model provides a more reasonable and encompassing index than R-model.
[18]	Electrical appliances company	No	Yes	No	No	Yes	No	No	-
[24]	Hospital	No	No	Yes	Yes	Yes	No	No	-
[25]	Hospital	No	No	Yes	Yes	Yes	No	AHP	DRSA can provide a good approximation of the decision analysis conducted by AHP.
[26]	Keelung Port	Yes	Yes	No	No	Yes	No	ADU	ABC-FC analysis shows a high accuracy.
[53]	Hospital	No	No	Yes	Yes	Yes	No	ADU	The model is easy to understand by inventory managers.
[19]	Turkish distributor company	No	Yes	No	No	No	Discussion with decision maker	No	-
[48]	Hospital	No	No	Yes	Yes	Yes	No	Ng-model, ZF-mode	The proposed model provides a more reasonable and encompassing index than Ng-model.
[73]	Food manufacturing company	Yes	Yes	No	No	Yes	No	AHP	The proposed method provides a robust decision support system.
[22]	Hospital	No	No	Yes	Yes	Yes	No	ADU, R-model, ZF-model	Peer-estimation approach generates more reasonable and comprehensive performance.
[49]	Soft-drink production line	No	Yes	No	No	Yes	No	ADU	FAHP-DEA is simple enough, easy to use, and applicable.
[86]	Hospital	No	No	Yes	Yes	Yes	No	ADC, AHP, R-model, Ng-model, BPN, SVM, k-NN	SVM enables more accurate classification than other AI-based techniques.
[5]	Automotive company	Not specified	No	No	Yes	No	Experimental investigation	No	-
[23]	Hospital	No	No	Yes	Yes	Yes	No	ADU, R-model, ZF-model, Ng-model	It incorporates no subjectivity as well as provides a unique result for decision makers.
[54]	No	No	No	Yes	Yes	No	No	No	-
[83]	Hospital	No	No	Yes	Yes	Yes	No	ADU, AHP, R-model, ZF-model	The modified approach is more promising than other approaches.
[63]	Hospital	No	No	Yes	Yes	Yes	No	ADU, AHP	The new method exceeds all of them in minimizing both dissimilarity and total inventory value
[55]	Energypac Engineering Limited	No	No	Yes	Yes	No	Validation with test data	No	-
[50]	Hospital	No	No	Yes	Yes	Yes	No	AHP, R-model, Ng-model, ZF-model, Extended Ng-model, Two virtual items	The developed model improves discrimination power among inventory items
[61]	Engineering firm & hospital	Not specified	Yes	Yes	Yes	Yes	Clustering validation	R-model, ZF-model, Ng-model, Extended Ng-model, AHP, Peer-estimation	This veto system is an assurance against hidden problems but slightly worsens the clustering validity index.
[65]	Hospital	No	No	Yes	Yes	Yes	Simulation	R-model, ZF-method, Ng-model, Two virtual items and TOPSIS idea	CE-WLO enables more accurate classification and better inventory management cost effectiveness.
[80]	Hospital	No	No	Yes	Yes	Yes	Simulation	R-model, ZF-model, Case-based, linear utility function based approach	UTADIS has the smallest mean error rate. UTADIS gives the best fill rate. Case-based sorting gives the lowest inventory cost.
[60]	Sports apparatus manufacturer	No	Yes	No	Yes	No	Experimental investigation	GA, PSO	The proposed approach performs better than the others.
[58]	Hospital	No	No	Yes	Yes	Yes	No	ZF-model, Ng-model	The new model reduces the conflict in the classifications.
This paper	Industrial manufacturing company	Yes	Yes	No	No	Yes	Analysis by the decision maker, statistical analysis, cross-validation	Fuzzy Rule, KNN, SVM, DT, MLPNN, NB	The proposed approach outranks all the other approaches in terms of accuracy of classification.

3. The proposed approach

The proposed approach can be divided into three main phases: (1) learning, (2) validation, and (3) generalization. Figure 1 shows the flowchart of the proposed approach wherein the first phase aims to use a representative set of spare parts data combined with the expertise and experience of the decision maker in order to generate a collection of if–then decision rules summarizing the preference of the decision maker. The second phase is to validate the output of the first phase through a re-classification of the learning set and a cross-validation analysis. The validation process can be further strengthened by taking input from the decision maker that may help in revising the learning set. Once the if–then rules are validated, the third and final phase is to use these rules for the classification of unseen spare parts. The main advantage of the proposed approach is that these rules can be applied to new or unknown spare parts in the stock.

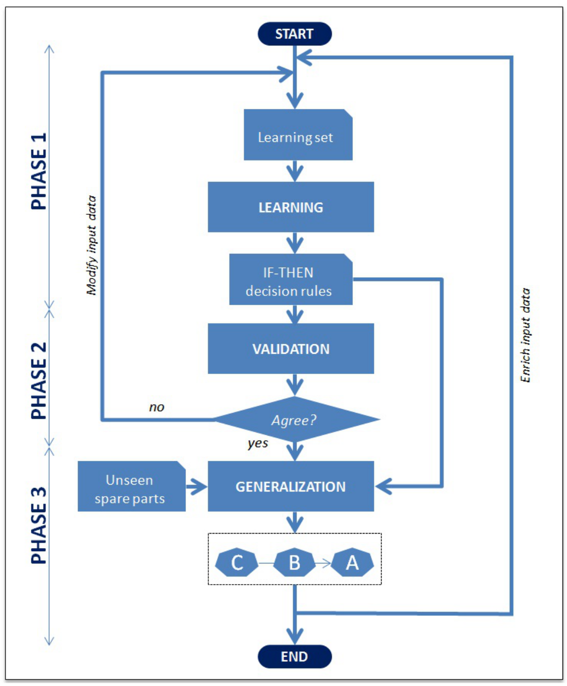

Figure 1: General schema of the proposed approach

3.1. Phase 1—Learning

The objective of this phase is to use a collection of carefully identified spare parts to generate a set of if–then decision rules indicating the priority level of each spare part based on its scores in terms of several evaluation criteria. The assessment of the input data is a crucial step in this phase because the quality and representativeness of the decision rules obtained depends largely on the quality and representativeness of the learning set.

The learning phase, which relies on the DRSA, contains three steps: data structuring, approximation, and inference of decision rules.

3.1.1. Data structuring

In rough sets theory, information regarding the *decision objects* is often structured in a 4-tuple *information table* $\mathbf{S} = \langle U, Q, V, f \rangle$, where U is a non-empty finite set of objects and Q is a non-empty finite set of attributes such that $q : U \rightarrow V_q$ for every $q \in Q$. The V_q is the domain of attribute q, $V = \bigcap_{q \in Q} V_q$, and $f : U \times Q \rightarrow V$ is the *information function* defined such that $f(x, q) \in V_q$ for each attribute q and object $x \in U$. The set Q is often divided into a sub-set

$C \neq \emptyset$ of *condition attributes* and a sub-set $D \neq \emptyset$ of *decision attributes*, such that $C \cup D = Q$ and $C \cap D = \emptyset$. In this case, **S** is called a *decision table*.

In multicriteria decision making, the domains of the condition attributes are supposed to be ordered according to a decreasing or increasing preference. Such attributes are called *criteria*. The proponents of DRSA assume that the preference is increasing with $f(\cdot, q)$ for every $q \in C$. They also assume that the set of decision attributes $D = \{E\}$ is a singleton. The unique decision attribute E makes a partition of U into a finite number of preference-ordered decision classes $\mathbf{Cl} = \{Cl_t, t \in T\}$, $T = \{0, \cdots, n\}$, such that each $x \in U$ belongs to one and only one class.

The decision table used in our case study is given in Appendix A. As shown in this table, the learning set is composed of 98 spare parts described in terms of four criteria, namely Criticality, Annual Dollar Usage, Average Unit Cost, and Lead Time (a detailed description of these criteria is given in Section 4.3.1). These criteria have been identified by the decision maker based on his experience. However, it is worth mentioning that the proposed approach is generic enough and may be used with any number of criteria. The decision attribute E defines three classes: A, B and C. The preference order assumed $C \prec B \prec A$, where "$x \prec y$" means that y is preferred to x. The categorization of spare parts into the groups A, B and C will facilitate their management in the sense that a different stocking policy can be selected for each group. For instance, the spare parts in A, making up roughly 10% of the total inventory, should be controlled tightly, recorded accurately, and monitored closely due to their taking a large share of annual expenses; the spare parts in B, making up about 20% of the total inventory, are less tightly controlled or well recorded; and the spare parts in C, making up about 70% of the total inventory, are managed with the simplest controls and records.

3.1.2. Approximation

In DRSA the represented knowledge is a collection of *upward unions* $Cl_t^{\geq}$ and *downward unions* $Cl_t^{\leq}$ of classes defined as follows:

$$Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s, Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s.$$

The assertion "$x \in Cl_t^{\geq}$" means that "x belongs to at least class Cl_t" while assertion "$x \in Cl_t^{\leq}$" means that "x belongs to at most class Cl_t". The basic idea of DRSA is to replace the indiscernibility relation used in the conventional RST with a dominance relation. Let $P \subseteq C$ be a subset of condition criteria. The *dominance relation* Δ_P associated with P is defined for each pair of objects x and y as follows:

$$x \Delta_P y \Leftrightarrow f(x, q) \succeq f(y, q), \forall q \in P.$$

In the definition above, the symbol "$\succeq$" should be replaced with "$\preceq$" for criteria which are ordered according to decreasing preferences. To each object $x \in U$, we associate two sets: (i) the *P-dominating set* $\Delta_P^+(x) = \{y \in U : y \Delta_P x\}$ containing the objects that dominate x, and (ii) the *P-dominated set* $\Delta_P^-(x) = \{y \in U : x \Delta_P y\}$ containing the objects dominated by x.

Then, the P-lower and P-upper approximations of $Cl_t^{\geq}$ with respect to P are defined as follows:

- $\underline{P}(Cl_t^{\geq}) = \{x \in U : \Delta_P^+(x) \subseteq Cl_t^{\geq}\}$,
- $\bar{P}(Cl_t^{\geq}) = \{x \in U : \Delta_P^-(x) \cap Cl_t^{\geq} \neq \emptyset\}$.

Analogously, the P-lower and P-upper approximations of $Cl_t^{\leq}$ with respect to P are defined as follows:

- $\underline{P}(Cl_t^{\leq}) = \{x \in U : \Delta_P^-(x) \subseteq Cl_t^{\leq}\}$,
- $\bar{P}(Cl_t^{\leq}) = \{x \in U : \Delta_P^+(x) \cap Cl_t^{\leq} \neq \emptyset\}$.

The lower approximations group the objects which certainly belong to class unions $Cl_t^{\geq}$ (resp. $Cl_t^{\leq}$). The upper approximations group the objects which could belong to $Cl_t^{\geq}$ (resp. $Cl_t^{\leq}$).

The P-boundaries of $Cl_t^{\geq}$ and $Cl_t^{\leq}$ are defined as follows:

- $Bn_P(Cl_t^{\geq}) = \bar{P}(Cl_t^{\geq}) - \underline{P}(Cl_t^{\geq})$,
- $Bn_P(Cl_t^{\leq}) = \bar{P}(Cl_t^{\leq}) - \underline{P}(Cl_t^{\leq})$.

The boundaries group objects that can neither be ruled in nor out as members of class Cl_t.

The *quality of approximation* of a partition **Cl** by means of a set of criteria P is defined as the ratio of all P-correctly classified objects to all objects in the system. Mathematically,

$$\gamma(\mathbf{Cl}) = \frac{|U - ((\bigcup_{t\in T} Bn_P(Cl_t^{\geq}))\bigcup(\bigcup_{t\in T} Bn_P(Cl_t^{\leq})))|}{|U|}. \tag{1}$$

The accuracy of the rough-set representation of unions of classes is computed as the ratio between the number of objects in the lower approximation and the number of objects in the upper approximation. Mathematically,

$$\alpha(Cl_t^{\diamond}) = \frac{\underline{P}(Cl_t^{\diamond})}{\bar{P}(Cl_t^{\diamond})}, \tag{2}$$

where $\diamond \in \{\geq, \leq\}$. It is easy to see that $0 \leq \alpha(Cl_t^{\diamond}) \leq 1$, $\forall t$. This holds because, by definition, we have $\underline{P}(Cl_t^{\diamond}) \subseteq \bar{P}(Cl_t^{\diamond})$, $\forall t$. Clearly, when the upper and lower approximations are equal (i.e. the boundary region is empty), then $\alpha(Cl_t^{\diamond}) = 1$, and the approximation is perfect. At the other extreme, whenever the lower approximation is empty, the accuracy is $\alpha(Cl_t^{\diamond}) = 0$.

In addition to these measures, the authors in [13] introduce two additional measures to estimate the attainable predictive accuracy of a rough-set-based classifier. The first measure, called λ, estimates the attainable percentage of correctly classified objects of a classifier. With respect to the DRSA, the attainable percentage of correctly classified objects is defined for a subset $P \subseteq C$ of criteria as follows:

$$\lambda_P(\mathbf{Cl}) = \frac{|Cl_1 \cap POS_P(Cl_1^{\leq})|}{|U|} + \frac{\bigcup_{i=2}^{i=n-1} |Cl_i \cap (POS_P(Cl_i^{\geq}) \cup POS_P(Cl_i^{\leq}))|}{|U|} + \frac{|Cl_n \cap POS_P(Cl_n^{\geq})|}{|U|} \tag{3}$$

where $POS_P(Cl_i^{\geq})$ and $POS_P(Cl_i^{\leq})$ are the P-positive region of $Cl_i^{\geq}$ and $Cl_i^{\leq}$, respectively, defined as follows:

$$POS_P(Cl_i^{\geq}) = \bigcup_{y\in \underline{P}(Cl_t^{\geq})} \Delta_P^{+}(y), \tag{4}$$

and

$$POS_P(Cl_i^{\leq}) = \bigcup_{y\in \underline{P}(Cl_t^{\leq})} \Delta_P^{-}(y), \tag{5}$$

The second measure, called δ, estimates the attainable mean absolute error of a classifier. It is defined as the mean absolute difference between the index of the class to which an object is assigned by a classifier and the index of the class to which the object belongs. Obviously, δ can be employed only when the decision classes are ordered, i.e. in DRSA. The attainable mean absolute error is defined, for $i \in T$ and $y_j \in Cl_i^{\diamond}$ with $\diamond \in \{\geq, \leq\}$, as follows:

$$\delta_P(\mathbf{Cl}) = \frac{1}{|U|} \cdot \sum_{j=1}^{|U|} \min_{k: y_j \in POS_P(Cl_k^{\geq}) \vee y_j \in POS_P(Cl_k^{\leq})} |i-k| \tag{6}$$

The λ and δ measures are only useful if the quality of approximation is low or even equal to zero. Accordingly, these measures have not been considered in the case study given in Section 4 since the quality of approximation on the learning dataset is equal to 1.

The DRSA defines two concepts that may indicate some information about the importance of the criteria: the reduct and the core. A *reduct* is a minimal subset of criteria which can, by itself, fully characterize the knowledge in the decision table. The reduct of the decision table is not unique: there may be many subsets of criteria which preserve the equivalence classes. The set of attributes which is common to all reducts is called the *core*. Therefore, they are the criteria which cannot be removed from the decision table without causing the collapse of the equivalence classes. More information on these concepts is available in [41][42]. The results of the approximation of the decision table used as input in the considered case study are presented in Section 4.3.2 and summarized in Table 4.

3.1.3. The inference of the decision rules

The decision attribute induces a partition of U in a way that is independent of the criteria. Hence, a decision table may be seen as a set of 'if–then' decision rules. The condition part specifies the values assumed by one or more criteria, and the decision part specifies an assignment to one or more decision classes. Three types of decision rules may be considered: (i) certain rules generated from the lower approximations of unions of classes, (ii) possible rules generated from the upper approximations of unions of classes, and (iii) approximate rules generated from the boundary regions.

The general structures of certain decision rules are as follows:

IF *condition(s)*, THEN *At Most* Cl_t

IF *condition(s)*, THEN *At Least* Cl_t

The decision part of a certain rule takes the form of an assignment to at most class unions or at least class unions.

The general structures of possible decision rules are as follows:

IF *condition(s)*, THEN *Possibly At Most* Cl_t

IF *condition(s)*, THEN *Possibly At Least* Cl_t

In this case, the decision part specifies a possible assignment to at most class unions or at least class unions.

Finally, the general structure of approximate rules is as follows:

IF *condition(s)*, THEN *Belongs to* $Cl_s \cup Cl_{s+1} \cup \cdots \cup Cl_t$

Here, the decision part is defined as the union of several decision classes.

Decision rules are judged by their quality on the basis of the learning (or training) set, and by how they classify new unseen objects [71]. Several measures have been proposed to evaluate the performance of decision rules. An object *supports* a decision rule if the description of the object matches both the condition and the decision parts of this rule. The *support* of a rule is the number of objects supporting the rule. A decision rule *covers* an object if the description of the object matches at least the condition part of the rule. The *coverage* is the number of the objects covered by the rule. The *strength* of a rule is the number of positive examples covered by the rule. The *relative strength* is the number of positive examples covered by the rule divided by the number of all positive examples in the union of classes. The *confidence level* (some authors call it *consistency*, or the *certainty factor*, or the *precision*) is defined as the number of positive examples covered by the rule divided by the number of examples covered by the rule. For more information and the formal definitions of all these concepts, see [71][81]. We note that if the consequence is univocal (i.e. contains only one decision), the rule is *exact*, otherwise it is *approximate*.

At this level, we should note that a given decision object may be covered by one or more decision rules, or may not be covered by any rule, in other situations. This issue has been discussed in detail in [9], where the authors propose different solutions to classify an object using decision rules in one of three possible situations: it is covered by (i) no rule, (ii) exactly one rule, (iii) several rules. The authors in [9] showed how these issues are dealt with by the standard classification method and the new classification method that they introduce. In what follows, we summarize the solutions used by the standard classification method, as discussed in [9]:

1. *the decision object x is not covered by any decision rule*: in this case, the object x is assigned to all decision classes from **Cl**.
2. *the decision object x is covered by one decision rule*: in this case, the classification relies on the prudence principle. Two subclasses are distinguished here. First, if the decision rule is of at least type with a decision part of the form 'then $x \in Cl_t^{\geq}$', then the decision object x is assigned to the lowest class Cl_t of the union $Cl_t^{\geq}$ suggested in the decision part of the decision rule. Analogously, if the decision rule is of at most type with a decision part of the form 'then $x \in Cl_{t'}^{\leq}$', then the decision object x is assigned to the highest class $Cl_{t'}$ of the union $Cl_{t'}^{\leq}$ suggested in the decision part of the decision rule.
3. *the decision object x is covered by several decision rules*: in this case, the standard classification method proceeds in two steps. First, the decision object is assigned to an interval of the form $[Cl_t, Cl_s]$ where: (i) Cl_t is the lowest class in the intersection of suggested unions of all covering rules of type at least; and (ii) Cl_s is the highest class in the intersection of suggested unions of all covering rules of type at most. Then, if $Cl_t = Cl_s$, the assignment of x is univocal; otherwise, two cases are possible:
   (a) if $t < s$, then decision object x is assigned to classes $Cl_t, \cdots, Cl_s$, without the possibility of refinement, because of imprecise information;
   (b) if $t > s$, then decision object x is assigned to classes $Cl_s, \cdots, Cl_t$, without the possibility of discernment, because of contradictory information.

One possible way to refine the assignment interval $[Cl_t, Cl_s]$ in case (a) above is to use some simple rules (such as the min, max, mean, floor, and ceiling operators) to reduce the assignment interval into a single decision class. This solution has been used in [20][21] for the reduction of the assignment intervals in the context of group decision making.

The new classification method proposed in [9] adopts the same strategy as the standard classification method for handling situation (1). With respect to situation (2), the new classification method computes, for each decision object x and rule ρ covering it, a score in $[0, 1]$ (which can be interpreted as the degree of certainty of the assignment of x to Cl_t in the decision part of rule ρ) and then assigns x to that Cl_t for which the score is the greatest. For situation (3), the authors in [9] use a combined score that considers the rules that are concordant with the assignment of the decision object x to decision class Cl_t and those which are discordant with this assignment. The combined score can be interpreted as a net balance of arguments in favor and argument against the assignment of the decision object to considered decision class. Then, decision object x is assigned to the class Cl_t for which the combined score is the greatest.

In our case study, we identified 11 certain rules that are discussed in Section 4.3.3. We should mention that, generally, only certain decision rules are used in practice. The other types of rules are mainly useful for sensitivity analysis.

3.2. Phase 2—Validation

The objective of the second phase is to check and validate the generated decision rules. In this paper, we propose three validation techniques:

- **Decision rules analysis**. The first and simplest validation technique is based on a direct analysis of the decision rules by the decision maker. The idea is based on asking the decision maker to scan all the decision rules and indicate his/her agreement level on a five-level Likert scale. A limited number of disagreements can be managed either by modifying some decision rules or by removing the decision rules with a high level of disagreements. Both options should, however, be authorized only for well-experienced decision makers.

- **Re-classification analysis**. The second validation technique consists of using the generated decision rules in order to re-classify the original spare parts. Ideally, the assignments obtained by re-classification should match completely with the original assignments. This is not always possible in practice, and generally a limited number of misclassifications may be accepted. When there are many misclassifications, the decision maker is called to revise his/her initial assignments in order to improve the quality of the decision rules.

- **Cross-validation analysis**. This validation strategy is used to evaluate the prediction accuracy of a machine learning technique. In essence, it starts with the partitioning of the available data into training and testing subsets. The training subset is used to train the model, and then the testing subset is used to measure the prediction accuracy. The key difference from re-classification is that the model is assessed by means of testing data that was completely unseen by the model. Multiple rounds of cross-validation are usually performed on different partitions, and the validation results are averaged over the rounds.

These complementary validation techniques can be used separately or jointly. They can help the decision maker better appreciate and refine the learning set and the obtained decision rules, which will naturally enhance the effectiveness of the decision making process and the successful implementation of the solution at the end. The use of these validation strategies is illustrated in Section 4.4 using real-world data in a case study.

3.3. Phase 3—Generalization

This phase aims to exploit the decision rules to classify spare parts other than those used initially for learning. For more advanced applications, decision rules can also be used to develop a rule-based decision support system by incorporating these rules into the knowledge base, but such an action is clearly beyond the scope of this paper.

The proposed approach is an iterative decision making process and, as can be seen in Figure 1, the process can be repeated whenever required. For example, at the end of the second phase, the agreement of the decision maker after an advanced analysis of decision rules is required to go through the generalization phase. Furthermore, the iterative structure of the decision making process enables the system to “learn” from past experiences. Indeed, and as shown in Figure 1, the final classifications can be used as input for inducing a set of more refined decision rules. This can enhance the system over time.

In our case study, detailed in the next section, we used a new set of data during the generalization phase. We then provided the results to the decision maker for appreciation. More information on this issue is given in Section 4.5.

4. Case study

The objective of this section is to illustrate the proposed approach through a case study in China. We first briefly introduce the company, under the pseudonym of the ‘Industrial Manufacturing Company’ (IMC), considered in this case study (Section 4.1). Then, we enumerate the problems faced by the IMC in their current spare parts management policies (Section 4.2). In the remaining sections, we provide a step-by-step application of the proposed approach to the IMC (Sections 4.3–4.6).

4.1. The company

The business of the IMC includes operations, manufacturing, and service activities. In recent years, its service business has made rapid progress. Hence, the activity of the IMC has been gradually turned towards maintenance, repair and overhaul (MRO) services for a variety of equipment, including vehicles, locomotives, engines, and electronic devices. Several MRO Service Centres have been created in order to satisfy the needs for different MRO services. The IMC also produces and supplies spare parts to customers. In order to support the customers’ needs, the IMC created a network of distribution warehouses. The spare parts related business of the IMC represents a very important part of the company’s profits, leading to a substantial increase in the annual business volume to more than 14 million dollars in the last three years.

4.2. The problem

Although the IMC's business made rapid progress in the preceding few years, the company, due to an increasingly competitive market, has faced several problems in its spare parts management. Firstly, the production costs, including the procurement of raw materials, employee salaries, and so on, have been growing and hence now make up a significant part in the company's expenses. Secondly, the spare parts management strategy used by the IMC is inappropriate. Indeed, the IMC uses the ABC classification technique to manage its spare parts. However, a high number of skilful and well experienced employees have retired in recent years, and the new young and inexperienced employees are unable to correctly classify the spare parts by themselves. Thirdly, the company manages more than 20,000 types of spare parts, which complicates the classification task, especially for the inexperienced employees.

The approach proposed in Section 3 permits handling all the above-cited problems by (i) reducing the production costs by correctly identifying the most critical spare parts (those assigned to group A) that should be controlled tightly, recorded accurately, and monitored closely, due to their important part in the annual expenses; (ii) extracting valuable knowledge (in terms of if–then rules) about spare parts management from the past experiences and historical data of the company; and (iii) automating the classification task through a learning-set based approach that uses a reduced set of spare parts as input and generates a collection of if–then decision rules that can be used to classify all the spare parts of the IMC.

4.3. Phase 1—Learning

Following Section 3.1, the learning phase is organized into three steps: (i) data structuring (Section 4.3.1), (ii) approximation (Section 4.3.2) and inference of decision rules (Section 4.3.3).

4.3.1. Data structuring

As introduced in Section 3.1.1, the input for the DRSA is a decision table containing a subset of typical spare parts described in terms of a collection of evaluation criteria.

Identification of the criteria. Initially, the IMC's practice of spare parts management relied on a single criterion, namely Annual Dollar Usage, to classify the spare parts. As strongly advocated by the spare parts manager, the use of Annual Dollar Usage alone is inefficient. For the purpose of this case study, and after a discussion with the spare parts manager, we decided to maintain four criteria: Criticality (Criticality), Annual Dollar Usage (AnnDollarUsage), Average Unit Cost (AvgUnitCost), and Lead Time (LeadTime). The description of these criteria is given in Table 3. The criteria Annual Dollar Usage and Average Unit Cost are continuous while the criteria Criticality and Lead Time are ordinal. The Criticality criterion can take one of the four values 1, 2, 3 and 4, where 1 corresponds to the lowest criticality and 4 corresponds to the highest criticality. The possible values for Lead Time are 1, 2 and 3, where 1 means a low lead time and 3 a high lead time. Finally, we note that all criteria are of type *gain*, i.e. the preference is increasing with the criteria values.

Table 3: Characteristics of used spare part management criteria

Code	Name	Description	Preference	Data type
Criticality	Criticality	It represents the influence of spare parts running out on the availability of equipment.	Gain	Ordinal
AnnDollarUsage	Annual Dollar Usage	It is calculated by spare part cost multiply demand volume.	Gain	Continuous
AvgUnitCost	Average Unit Cost	It refers to spare part cost.	Gain	Continuous
LeadTime	Lead Time	It refers to the time between the placement of an order and delivery of a new spare part from a IMC's supplier.	Gain	Ordinal

Generally, the assessment of ordinal criteria is not an obvious exercise. Two ordinal criteria have been considered in this paper: Criticality and Lead Time. For the purpose of this case study, the criteria Criticality and Lead Time have been assessed by the spare parts manager, based on his long experience within the IMC. Let us also mention that the authors in [27], for instance, emphasize the role of the lead time criterion in analysing the competitiveness of companies. In terms of criticality, one can also argue that it is a function of the criticality of the spare parts in the machine as well as the criticality of the machine within the whole operational system [33][38][57]. According to [82], the integration of production and maintenance is important and complex. For example, the criticality of a machine and hence its related spare parts can be based on different criteria, such as the capital cost of the machine, its rarity (i.e. absence of redundancy), the degree of deterioration (measured by assessing its conditions [85]), the difficulty of repair in case of downtime (measured by the mean time to repair [74]), its availability (measured by the mean time between failures [6]), the throughput of the machine (whether it is a bottleneck), whether the outputs of the machine are intended for important customers, or whether the machine has already produced its intended schedule of production (i.e. the current required demand).

Identification and assignment of learning examples. The definition of the assignment examples is a crucial step in our approach. It involves two operations: (i) the selection of a representative subset of spare parts, and (ii) the assignment—by the decision maker—of the selected spare parts on the three-level scale defined earlier. In the application considered in this paper, a subset of 98 spare parts (denoted 1 to 98) was selected. The evaluations of the selected spare parts in terms of the considered criteria, i.e. Criticality, AnnDollarUsage, AvgUnitCost and LeadTime, are summarized in the decision table in Appendix A. The values in the last column in the decision table correspond to the assignments, as expressed by the decision maker, of the spare parts to the decision classes C, B and A.

The selection of these spare parts from about 20 thousand spare parts managed by the IMC was a very difficult task. The inputs of the highly experienced spare parts manager have been crucial in this exercise. At this level, it is important to emphasize that there are no formal rules that can be used to coherently identify the learning set. In this respect, the authors in [59] identified some general guidelines that can be followed to obtain the 'best' set of assignment examples: (i) the spare parts should be as representative as possible by including different specifications and characteristics; (ii) the spare parts should be non-redundant (in terms of their evaluation with respect to different criteria); (iii) the spare parts should cover all the decision classes; and (iv) the spare parts should ideally be well known to the decision maker/expert. The authors in [59] also observe that there was no ideal theoretical number of examples. A limited number of examples might lead to a few and very generic decision rules and too great number of examples may lead to a high number of very specific and redundant decision rules.

4.3.2. Approximation

The DRSA has been designed to be used with any subset P of criteria from the set of criteria $C = \{$Criticality, AnnDollarUsage, AvgUnitCost, LeadTime$\}$. In our case study, we assumed that all the criteria are used, i.e. $P = C$. In addition, the domain of the decision attribute E is equal to $\{C, B, A\}$. These values correspond to the labels of the categories C, B and A introduced in Section 3.1.1. The decision attribute E divides the set U of spare parts into three preference-ordered classes: $Cl_1 = \{C\}$, $Cl_2 = \{B\}$, and $Cl_3 = \{A\}$. Thus, the class unions that should be approximated are:

- $Cl_1^{\leq}$, i.e. the objects belonging to (at most) class C,
- $Cl_2^{\leq}$, i.e. the objects belonging to at most class B,
- $Cl_2^{\geq}$, i.e. the objects belonging to at least class B,
- $Cl_3^{\geq}$, i.e. the objects belonging to (at least) class A.

These class unions have been approximated using our decision table in Appendix A and the equations given in Section 3.1.2. The result of the approximation is summarized in Table 4. As shown in this table, all the boundaries are empty sets, which indicates that the approximation is perfect (see Section 3.1.2). The quality of the approximation and accuracy of the rough-set representation of the classes of our input data are summarized in Table 5. In our example, the quality of approximation of the partition $Cl = \{Cl_1, Cl_2, Cl_3\}$, the percentage of correctly classified objects, and the accuracy of the rough-set representation, are all equal to 1 and the attainable mean absolute error is equal to 0. This ensures the high quality of the learning set used as input. Additionally, the analysis with the DRSA shows that the set {LeadTime, AnnualDollarUsage, AverageUnitCost} constitutes the unique reduct and also the core of the data used as input.

Table 4: Result of approximation

Class union	Lower approximation	Upper approximation	Boundary
$Cl_1^{\leq}$ (At Most C)	1, 2, 3, 4, 5, 7, 8, 9, 16, 18, 19, 20, 21, 22, 24, 28, 29, 30, 31, 33, 34, 36, 37, 38, 41, 42, 43, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 61, 62, 63, 64, 66, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 86, 88, 89, 90, 91, 92, 93, 94, 97	1, 2, 3, 4, 5, 7, 8, 9, 16, 18, 19, 20, 21, 22, 24, 28, 29, 30, 31, 33, 34, 36, 37, 38, 41, 42, 43, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 61, 62, 63, 64, 66, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 86, 88, 89, 90, 91, 92, 93, 94, 97	$\emptyset$
$Cl_2^{\leq}$ (At Most B)	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97	$\emptyset$
$Cl_2^{\geq}$ (At Least B)	6, 10, 11, 12, 13, 14, 15, 17, 23, 25, 26, 27, 32, 35, 39, 40, 44, 45, 53, 54, 55, 59, 60, 65, 67, 71, 72, 73, 81, 82, 83, 84, 85, 87, 95, 96, 98	6, 10, 11, 12, 13, 14, 15, 17, 23, 25, 26, 27, 32, 35, 39, 40, 44, 45, 53, 54, 55, 59, 60, 65, 67, 71, 72, 73, 81, 82, 83, 84, 85, 87, 95, 96, 98	$\emptyset$
$Cl_3^{\geq}$ (At Least A)	23, 27, 40, 53, 55, 71, 73, 83, 84, 85, 98	23, 27, 40, 53, 55, 71, 73, 83, 84, 85, 98	$\emptyset$

Table 5: Quality of the approximation, accuracy of prediction and accuracy of rough-set representation

Quality of approximation	Percentage of correctly classified objects (λ)	Attainable mean absolute error (δ)	Accuracy			
			$Cl_1^{\leq}$ (At Most C)	$Cl_2^{\leq}$ (At Most B)	$Cl_2^{\geq}$ (At Least B)	$Cl_3^{\geq}$ (At Least A)
1	1	0	1	1	1	1

4.3.3. Inference of decision rules

The application of the inference algorithm DOMLEM [44] of the DRSA to the results of the approximation in Table 4 leads to a minimal set of 11 certain and exact decision rules, which are given in Table 6. A detailed description of these rules is given in Appendix B. By minimal set we mean a set of non-redundant rules that cover all the spare parts in the learning set. Table 6 indicates, in addition to the descriptions of the decision rules, the number of supporting objects, the relative strength, and the confidence of each decision rule. The description of these rules is straightforward. Rule #8, for instance, indicates that a spare part is classified as high priority (i.e. assigned to category A) once (i) the Annual Dollar Usage is greater than or equal to 3150 and (ii) it is of very high criticality. This decision rule is supported by 9 spare parts, has a relative strength of 81.82%, and a confidence level of 81.82%.

Table 6: Decision rules

#	Rule	Support	Relative strength (%)	Confidence level (%)
1	IF (AnnDollarUsage$\leq$1260) THEN (Class at most C)	53	86.89	100
2	IF (AvgUnitCost$\leq$27.3) & (Criticality$\leq$2) THEN (Class at most C)	16	26.23	100
3	IF (AvgUnitCost$\leq$24.07) & (LeadTime$\leq$2) & (AnnDollarUsage$\leq$1754) THEN (Class at most C)	47	77.05	100
4	IF (AnnDollarUsage$\leq$3071.25) THEN (Class at most B)	83	95.40	100
5	IF (LeadTime$\leq$1) THEN (Class at most B)	39	44.83	100
6	IF (AvgUnitCost$\leq$36.75) & (LeadTime$\leq$2) THEN (Class at most B)	64	73.56	100
7	IF (AnnDollarUsage$\geq$11025) THEN (Class at least A)	8	72.73	100
8	IF (AnnDollarUsage$\geq$3150) & (Criticality$\geq$4) THEN (Class at least A)	9	81.82	81.82
9	IF (AnnDollarUsage$\geq$1786.4) THEN (Class at least B)	32	86.49	100
10	IF (AvgUnitCost$\geq$71.66) THEN (Class at least B)	20	54.05	100
11	IF (AnnDollarUsage$\geq$1470) & (LeadTime$\geq$2) & (AvgUnitCost$\geq$29.4) THEN (Class at least B)	16	43.24	100

As discussed at the end of Section 3.1.3, a decision object can be covered by (i) no rule, (ii) exactly one rule, (iii) several rules. In the considered case study and as shown in Appendix B, all spare parts in the initial learning set are covered by at least one decision rule.

4.4. Phase 2—Validation

In the rest of this section, we apply the three validation strategies introduced in Section 3.2 to our case study.

4.4.1. Analysis of the decision rules

As mentioned earlier in Section 3.2, this validation strategy consists of asking the decision maker to scan all the decision rules and indicate his/her agreement level on a five-level Likert scale: Strongly Disagree, Disagree, Neutral, Agree, and Strongly Agree. The result of the analysis of the decision rules for our case study is given in Table 7. According to this table, the decision maker agrees with five decision rules, is neutral about five other decision rules, and disagrees with one decision rule.

Table 7: Decision rules analysis

#	Rule	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree	Comments
1	IF (AnnDollarUsage$\leq$1260) THEN (Class at most C)			✓			
2	IF (AvgUnitCost$\leq$27.3) & (Criticality$\leq$2) THEN (Class at most C)			✓			
3	IF (AvgUnitCost$\leq$24.07) & (LeadTime$\leq$2) & (AnnDollarUsage$\leq$1754) THEN (Class at most C)				✓		
4	IF (AnnDollarUsage$\leq$3071.25) THEN (Class at most B)			✓			
5	IF (LeadTime$\leq$1) THEN (Class at most B)				✓		
6	IF (AvgUnitCost$\leq$36.75) & (LeadTime$\leq$2) THEN (Class at most B)				✓		
7	IF (AnnDollarUsage$\geq$11025.) THEN (Class at least A)					✓	
8	IF (AnnDollarUsage$\geq$3150) & (Criticality$\geq$4) THEN (Class at least A)			✓			
9	IF (AnnDollarUsage$\geq$1786.4) THEN (Class at least B)		✓				The right-hand member of the condition is very low; consequently, the decision part should be C.
10	IF (AvgUnitCost$\geq$71.66) THEN (Class at least B)				✓		
11	IF (AnnDollarUsage$\geq$1470) & (LeadTime$\geq$2) & (AvgUnitCost$\geq$29.4) THEN (Class at least B)			✓			

The last column in Table 7 indicates the comments of the decision maker on the decision rules. In the present case study, the decision maker justifies his disagreement with decision rule #9 by the fact that the right-hand member of the unique condition (which is relative to the criterion Annal Dollar Usage) of this rule is very low; consequently, the decision part should be C. However, the decision provided by the decision maker is not consistent with the condition part of decision rule #9. To avoid any confusion, we contacted the decision maker again and proposed three solutions: (i) maintain decision rule #9 as it is and add 'by hand' to the list of inferred decision rules the following rule: 'IF (AnnDollarUsage<1786.39), THEN (Class at most C)'; (ii) maintain rule #9 as it is without adding any new rule; and (iii) remove decision rule #9. He finally opted for the second solution and he changed his agreement level from 'Disagree' to 'Neutral'.

4.4.2. Re-classification analysis

The second validation technique consists of using the generated decision rules to re-classify the spare parts. In the case study considered in this paper, the re-classification analysis shows that the original assignments (proposed by the decision maker) match with those proposed by the system for about 98% of the spare parts, and there are about 2% ambiguous assignments (for spare parts numbers 25 and 26). The result of the re-classification can be summarized through an $n \times n$ confusion matrix, where n is the number of decision classes. The intersection of a row and column indicates the number of original and possible assignments for the decision classes corresponding to the considered row and column. The confusion matrix for our case study is given in Table 8. As indicated by this table, all the spare parts originally assigned to decision classes C and A have been assigned to the same classes by the system. This table indicates also that 24 spare parts that had been initially assigned to class B were re-assigned to the same class by the system and that two spare parts (namely #25 and #26) that had been assigned to class B by the decision maker could be assigned to B or A.

It is important to mention that since there were no inconsistent assignments, normally there should be a perfect reclassification with 100% correct assignments and no ambiguous or wrong assignments. However, as shown in Table 8, there are two ambiguous assignments. Indeed, the value 2 in the confusion matrix can be read as there are two decision objects that had been assigned to class A instead of class B as initially proposed by the decision maker. This is due to decision objects #25 and #26, which are covered by conflicting rules.[1] Indeed, a careful examination of Appendix B shows that decision objects #25 and #26 are covered by one decision rule (namely rule #6) of type at most and three decision rules (namely rules #8, #9 and #11) of type at least. The assignment of decision objects #25 and #26 to both B and A holds since the DRSA software 4eMka2 [1][43] used in this paper implements the standard classification method mentioned in Section 3.1.3.

Table 8: Confusion matrix

	/	Possible C	B	A
Original	C	61	0	0
	B	0	26	2
	A	0	0	11

We provided the result of the re-classification analysis to the decision maker and asked him to revise his original assignments for spare parts #25 and #26 causing the confusion problem. The result of this exercise is summarized in Table 9. As shown in this table, the decision maker refused to revise his assignments and maintained the original assignments for spare parts #25 and #26.

Table 9: Result of revision

#	Criticality	Annual Dollar Usage	Average Unit Cost	Lead Time	Original assignment (by the decision maker)	Possible assignment (by the system)	Revision by the decision maker
25	4	3150.00	31.50	2	B	B or A	B
26	4	3675.00	36.75	2	B	B or A	B

4.4.3. Cross-validation analysis

The basic idea here is to randomly partition the data into parts or 'folds', and select one fold to be used for testing and the remaining to train the classification algorithm. Cross-validation is most commonly applied with k=10, where each fold should contain at least 30 items. For this purpose, we randomly created 10 mutually exclusive pairs of training and testing datasets. Then, we used the training sets to generate the decision rules, which were then applied to the testing sets. Each training and testing dataset was composed of 49 items (i.e. 49 for training and 49 for testing). Furthermore, we used stratified cross-validation to have proportional representation of each class in each fold.

Table 10 summarizes the recall and accuracy analysis for the decision classes C, B and A. In this table, we observe that the performance of the DRSA remains consistent for the three decision classes. It is important to emphasize here that decision class A has the fewest samples in the dataset, while decision class C has the highest number of samples. This indicates that the DRSA performs equally well regardless of the number of samples available in the dataset.

Table 10: Recall and accuracy analysis using the DRSA for all three classes

Class-wise recall			Class-wise accuracy			Total	Total
C	B	A	C	B	A	recall	accuracy
0.9033	0.8692	0.8167	0.9374	0.8019	0.8129	0.8631	0.8507

We have also used a series of well-known non-parametric statistics to compare the decision maker's assignments of the decision objects in the testing sets to those generated by the decision rules inferred from the training sets. The statistics considered in this paper are: Kendall's tau, Spearman's rho, and the Unweighted and Weighted Cohen's kappa. These statistics are often used to compare a set of rankings provided by two decision makers, experts, methods, etc. In addition, all of them accept ordinal data and can deal with ties. Kendall's tau lies in the range [-1,1]. If the agreement

[1]The results in Table 4 show empty boundaries, which should normally lead to a perfect re-classification and decision objects #25 and #26 should be assigned strictly to class B. After careful examination, it turns out that rule #8 should never been induced by the DOMLEM algorithm [44], as it has conclusion "at least A", and covers two decision objects (#25 and #26) from a worse class, B. It seems to be somewhat of an implementation error in the software 4eMka2 used to run DRSA. Despite this fact, rule #8 is maintained in the rest of the paper.

between the two rankings is perfect (i.e. the two rankings are the same) it is 1. If the disagreement between the two rankings is perfect (i.e. one ranking is the reverse of the other) it is -1. If two rankings are independent, then we would expect it to be approximately zero. Spearman's rho is in the range [-1,1]. A positive Spearman correlation coefficient indicates that both rankings vary in the same direction. A negative Spearman rho coefficient indicates a monotone decreasing relation between the two rankings. A Spearman rho coefficient of zero indicates that there is no tendency between the two rankings. There are two ways of calculating Cohen's kappa: unweighted and weighted. The weighted kappa is more appropriate for variables having more than two categories. In both cases, the value of Cohen's kappa lies in [0,1]. Conventionally, a kappa of <0.2 is considered poor agreement, 0.21–0.4 fair, 0.41–0.6 moderate, 0.61–0.8 strong, and more than 0.8 a near complete agreement.

The result of this statistical comparison is given in Table 11. According to this table, Kendall's tau and Spearman's rho show a very high agreement between the two assignment sets while the Cohen's kappa measures show a slightly less strong correlation between the two assignment sets. At this level, we note that despite the fact that the accuracy of fold number 7 is higher than the accuracy of fold number 2, the statistical analysis shows a higher correlation level for fold number 2. This can be explained by the fact that in fold number 2, there are three similar disagreements (assignment to B instead of A) while in fold number 7 there are two different disagreements (assignment to C instead of B and assignment to B instead of A). Indeed, the definitions of the statistics used take into account these aspects.

Table 11: Results of the cross-validation for DRSA—statistical analysis

Statistics	Fold 1	2	3	4	5	6	7	8	9	10	Min	Max	Average	Standard deviation
Kendall's tau	0.8254	0.9624	0.7831	0.8811	0.9117	0.8211	0.9538	0.9062	0.8195	0.8477	0.7831	0.9624	0.8712	0.0612
Spearman's rho	0.8482	0.9844	0.8116	0.9150	0.9290	0.8301	0.9632	0.9228	0.8405	0.8751	0.8116	0.9844	0.89199	0.0593
Unweighted Cohen's kappa	0.7224	0.8847	0.6554	0.7789	0.8464	0.7845	0.9230	0.8471	0.7454	0.7498	0.6554	0.923	0.7938	0.0812
Weighted Cohen's kappa	0.7817	0.9049	0.7232	0.8248	0.8852	0.8271	0.9391	0.8761	0.7969	0.8025	0.7232	0.9391	0.8361	0.0649
Mean Absolute Error	0.1633	0.0612	0.2041	0.1224	0.0816	0.1224	0.0408	0.0816	0.1429	0.1429	0.0408	0.2041	0.1163	0.0500

We also compared the ranking resulting from assignments given by the decision maker and the rankings resulting from application of decision rules induced on training sets by calculating the Mean Absolute Error (MAE) for all the training sets. The MAE is computed as the mean absolute difference between index of the class to which an object is assigned by the decision maker and index of the class to which it is assigned by rules. The results of this additional comparison are summarized in the last row of Table 11. These results indicate relatively high agreement levels (between the initial and predicted assignments) for all learning datasets.

Finally, we can conclude that the result of the cross-validation shows a high level of accuracy and agreement. This confirms the result of the previous validation techniques.

In the next phase of the decision making process, we should apply the validated decision rules to a new dataset of spare parts. At this level, we should mention that if the levels of the accuracy and agreement are not sufficient, the decision making process can be started by considering new input data by: (i) modifying the assignments of the spare parts in the learning set; (ii) selecting a new set of spare parts as a learning set, and/or (ii) adding (or removing) some evaluation criteria.

4.5. Phase 3—Generalization

The dataset used for the generalization phase consists of 123 spare parts that had not been used during the learning phase. The description of this new dataset is given in Appendix C. We used the decision rules given in Table 6 to classify these new items and then provided the results to the decision maker for comment. More specifically, we asked the decision maker to check the classification of the 123 new spare parts and indicate his agreement level on a five-level Likert scale. The result of this exercise is given in Appendix D and summarized in Table 12. As shown in this table, the decision maker agrees with 56.10% of the assignments, is neutral about 31.71%, and disagrees with 12.20%. This means that the decision maker is satisfied with 87.80% of the assignments.

Table 12: Summary of the decision maker agreement analysis

Agreement level	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree	Total
Number	2	13	39	16	53	123
Percent (%)	1.63	10.57	31.71	13.01	43.09	100

Following the assignment of unseen spare parts into the classes A, B and C, the decision maker should check and agree on the result of classification. If the decision maker accepts the assignments of unseen spare parts, then the decision process stops. Otherwise the decision maker should modify the input data and restarts the analysis approach from the beginning. In this particular case study, the acceptance rate has been judged relatively high. After discussing with the decision maker, we decided to update the initial learning set and to restart the analysis approach. More insights on this additional analysis are given in the following subsection.

4.6. Modification of the learning dataset

The assignment of the unseen spare parts by the obtained decision rules lead to an acceptance rate of 87.80%. As mentioned earlier, we decided, after discussing with the decision maker, to update the initial learning set used in the first phase by adding a subset of the spare parts used during the generalization phase. The spare parts to be added to the learning set are given in Table 13. These additional spare parts consist of 15 items from Appendix C whose assignments had been judged unacceptable by the decision maker, as indicated in Appendix D. The additional spare parts were assigned based on the information provided by the decision maker in Appendix D (last column).

Table 13: Additional spare parts to be added to the learning set

#	Criticality	Annual Dollar Usage	Average Unit Cost	Lead Time	Class
100	4	3805	38.05	1	B
114	3	10148.7	338.29	2	A
117	3	4420	11.05	1	C
119	3	1462.5	29.25	1	C
123	3	10176	101.76	2	A
137	3	3044	7.61	2	C
138	3	2013	6.71	2	C
141	3	9280.2	309.34	2	A
183	3	1138.95	227.79	2	B
195	3	1254.15	250.83	2	B
204	3	2144	5.36	1	C
205	2	4008	5.01	1	C
207	4	3150	31.5	2	B
212	3	10173.6	339.12	2	A
220	3	10482.2	524.11	2	A

We applied the DRSA to the new learning set in order to approximate the three decision classes C, B and A. It is fruitful to note that the quality of approximation, percentage of correctly classified objects, and accuracy of rough-set representation are all equal to 1, and the attainable mean absolute error is equal to 0. In addition, in this case there is a single reduct composed of all the criteria {Criticality, AnnualDollarUsage, AverageUnitCost, LeadTime}. This reduct constitutes also the core of the new learning set. We remark that the criterion Criticality was absent from the reduct and core during the approximation of the initial learning set (see Section 4.3.2).

The application of the inference algorithm to the result of the approximation of the revised learning set leads to a new set of decision rules that are given in Table 14. A detailed description of these rules is given in Appendix E. This table contains 11 certain and exact decision rules. By analysing the initial set of decision rules presented in Table 6 and the ones given in Table 14, we remark that there are two rules that are identical in both sets. There are also several rules with relaxed or stricter elementary conditions and/or decision. Finally, there are several different decision rules.

We followed the same steps given in Section 3.2 and Section 4.4 to validate the new decision rules. The direct analysis of the decision rules by the decision maker using a form similar to Table 7 showed that he agrees with all the rules. We also used the re-classification validation strategy to compare the assignments obtained using the decision

Table 14: Decision rules obtained from the updated learning set

#	Rule	Support	Relative strength (%)	Confidence level (%)
1	IF (AnnDollarUsage$\leq$1117.98) THEN (Class at most C);	52	77.61	100
2	IF (AvgUnitCost$\leq$29.25) & (LeadTime$\leq$2) THEN (Class at most C);	55	82.09	96
3	IF (AnnDollarUsage$\leq$3071.25) THEN (Class at most B);	89	91.75	100
4	IF (LeadTime$\leq$1) THEN (Class at most B);	44	45.36	100
5	IF (AvgUnitCost$\leq$36.75) & (LeadTime$\leq$2) THEN (Class at most B);	71	73.20	100
6	IF (AvgUnitCost$\geq$257.25) THEN (Class at least A);	12	75.00	100
7	IF (AnnDollarUsage$\geq$3150) & (Criticality$\geq$4) & (LeadTime$\geq$2) THEN (Class at least A);	9	56.25	75
8	IF (AnnDollarUsage$\geq$10176) & (LeadTime$\geq$2) THEN (Class at least A);	10	62.50	100
9	IF (AvgUnitCost$\geq$65.1) THEN (Class at least B);	28	60.87	100
10	IF (AnnDollarUsage$\geq$2063.4) & (Criticality$\geq$4) THEN (Class at least B);	19	41.30	100
11	IF (AnnDollarUsage$\geq$1470) & (AvgUnitCost$\geq$29.36) THEN (Class at least B);	37	80.43	100

rules given in Table 14 with those given by the decision maker. The result of this comparison is given in the confusion matrix in Table 15 and summarized in Table 16. The latter shows an accuracy of 96% and an error of 4%.

Table 15: Confusion matrix for the new learning set

		Possible		
	/	C	B	A
Original	C	67	0	0
	B	2	30	3
	A	0	0	16

Table 16: Summary of confusion matrix for the new learning set

Parameter	Correct assignment	Incorrect assignment	Ambiguous assignment	Accuracy (%)	Error (%)
Value	108	0	5	96	4

We also compared the assignments obtained using the decision rules given in Table 14 and those given by the decision maker using the non-parametric statistics Kendall's tau, Spearman rho, and the Unweighted and Weighted Cohen's kappa. The result is given in Table 17 where we distinguished two cases concerning the five ambiguous assignments: (i) case of best choice in which the five assignment intervals have been reduced into a single assignment equal to the one provided by the decision maker, and (ii) worst choice in which the five assignment intervals have been reduced into a single assignment different from the one provided by the decision maker. Concerning the statistical analysis using the best choices, all the statistics indicate a full agreement between the assignments obtained using the decision rules given in Table 14 and those given by the decision maker. For the statistics analysis using the wrong choices, the non-parametric statistics indicate a very high level of agreement.

Table 17: Statistics analysis for the new learning set

	Best choice				Wrong choice			
Statistics	Kendall's tau	Spearman's rho	Unweighted Cohen's kappa	Weighted Cohen's kappa	Kendall's tau	Spearman's rho	Unweighted Cohen's kappa	Weighted Cohen's kappa
Value	0.9999	1	1	1	0.95577	0.96730	0.9203	0.9403

Based on these results, and after discussion with the decision maker, we judged that there is no need to conduct a second cross-validation analysis since the two first validation strategies indicated a very high level of accuracy and agreement, and acceptance by the decision maker.

Finally, we used the new decision rules to classify 108 unseen spare parts (composed of the 123 spare parts used earlier and given in Appendix C minus those included in the new learning set). Then, we provided the output of this operation to the decision maker to indicate his agreement level. The results of this exercise are given in Appendix F. In this appendix, we also indicate the assignment given by the first set of decision rules and those corresponding to the new set of decision rules. We note that in Appendix F some rows (corresponding to objects moved from the testing set to the extended learning set) do not appear. Appendix F also shows that the decision maker globally agrees with all the new assignments.

5. Comparative study

We compared the DRSA to other widely-used classification methods, including fuzzy classification rule (FR), nearest neighbours (KNN), support vector machine (SVM), decision trees (DT), multi-layer perceptron network (MLPNN), and the Naive Bayes (NB) approach. We conducted two types of comparison: cross-validation and statistical analysis using the same data considered in Section 4.4.3. The cross-validation for the methods FR, KNN, SVM, DT, MLPNN and NB was conducted using the software KNIME (see [7] and `www.knime.org`) and for DRSA, we used the software 4eMka2 [1][43]. All the methods (except DRSA) were applied and tested using the software KNIME with their default parameter settings. The same learning and testing datasets have been used in each fold for all the compared methods including DRSA. We note that the scores of the criteria Criticality and LeadTime have been standardized since the methods FR, KNN, SVM, DT, MLPNN and NB require continuous data. The standardization operation was not required for the DRSA since it accepts ordinal data. Finally, it is also important to emphasize that the criteria Criticality and LeadTime are ordinal ones, with number-coded ordered categories (1,2,3,4 for Criticality and 1,2,3 for LeadTime). Thus the standardization of these criteria is mathematically wrong and it has been conducted only for comparison purposes.

The different confusion matrices of the cross-validation are given in Appendix G. A detailed analysis of the confusion matrices is given in Appendix H and summarized in Table 18. Based on the analysis of Table 18, we can conclude that the DRSA has the best average accuracy and the best average MAE. Table 19 summarizes the recall and accuracy analysis for all three classes using different classification methods. It can be seen that DRSA clearly outperforms all other methods on overall recall and accuracy, with the FR and MPLNN approaches performing very close to DRSA. However, on careful observation, we can see that the performance of DRSA remains consistent for all the three classes, of types A, B and C. It is important to emphasize here that class A has the fewest samples in the dataset, while class C has the highest number of samples, so clearly DRSA performs equally well regardless of the number of samples available in the dataset.

Table 18: Results of the cross-validation for the comparative study

	Correct Assign.				Wrong Assign.				Missing Assign.				Accuracy (%)				MAE (%)			
Method	Min	Max	Avg	Stdev	Min	Max	Avg	Stdev	Min	Max	Avg	Stdev	Min	Max	Avg	Stdev	Min	Max	Avg	Stdev
FR	39	45	42.6	2.0111	3	8	5.1	1.2867	0	5	1.3	1.8288	79.59	91.84	86.93	4.0981	8.16	20.41	13.06	4.1043
KNN	37	46	40.3	2.8304	3	12	9	2.8304	0	0	0	0	75.51	93.88	82.25	5.7764	6.12	24.49	17.76	5.7764
SVM	33	35	34.4	0.6992	14	16	15	0.6992	0	0	0	0	67.35	71.43	70.20	1.4271	28.57	32.65	29.80	1.4271
DT	33	43	39.4	3.0258	6	16	9.6	3.0258	0	0	0	0	67.35	87.76	80.41	6.1751	12.25	32.65	19.59	6.1751
MLPNN	40	46	43	1.8257	3	9	6	1.8257	0	0	0	0	81.63	93.88	87.76	3.7262	6.12	18.37	12.24	3.7262
NB	32	42	37	3.8006	7	17	12	3.8006	0	0	0	0	65.31	85.71	75.51	7.7563	14.29	34.69	24.49	7.7563
DRSA	39	47	43.3	2.4518	2	10	5.7	2.4518	0	0	0	0	79.59	95.92	88.37	5.0036	4.08	20.41	11.63	5.0036

Table 19: Recall and accuracy analysis for all three classes

	Class-wise Recall			Class-wise Accuracy			Total	Total
Method	A	B	C	A	B	C	Recall	Accuracy
FR	0.7667	0.7934	0.9581	0.8166	0.7876	0.9474	0.8394	0.8505
KNN	0.6833	0.5846	0.9533	0.7961	0.6410	0.8854	0.7404	0.7742
SVM	0.7333	0.0000	1.0000	0.8217	0.0000	0.8090	0.5778	0.5436
DT	0.9000	0.6000	0.8733	0.7382	0.6164	0.8999	0.7911	0.7515
MLPNN	0.7833	0.8385	0.9133	0.8157	0.7841	0.9325	0.8450	0.8441
NB	0.1500	0.5154	0.9800	0.2365	0.5924	0.8438	0.5485	0.5575
DRSA	0.8167	0.8692	0.9033	0.8129	0.8019	0.9374	0.8631	0.8507

The details of the statistical analysis are given in Appendix I and summarized in Table 20. The analysis of Table 20 indicates that the DRSA outranks all the other methods in terms of all the statistics. Although the MLPNN and FR results are quite high, they cannot be offered as an interactive tool for decision makers to suggest or amend any changes in an understandable manner. Only experts of MLPNN/FR can vary their parameters, while in DRSA, the decision makers do not need any expert knowledge to modify the extracted/suggested rules and/or provide feedback on these rules.

Table 20: Results of the statistical analysis for the comparative study

Method	Statistics			
	Kendall's tau	Spearman's rho	Unweighted Cohen's kappa	Weighted Cohen's kappa
FR	0.8236	0.8534	0.7616	0.7777
KNN	0.7146	0.7320	0.6421	0.6975
SVM	0.5393	0.5614	0.3143	0.4538
DT	0.7982	0.8329	0.6534	0.7375
MLPNN	0.8534	0.8734	0.7778	0.8230
NB	0.4522	0.4666	0.4437	0.3962
DRSA	0.8712	0.8920	0.7938	0.8361

6. Discussion

In this section, we first discuss the characteristics and main contributions of the proposed approach (Section 6.1). Then, we provide a straightforward guideline for using the proposed approach in practice (Section 6.2).

6.1. Characteristics of the proposed approach and main contributions

The proposed approach has several important characteristics that distinguish it from existing ones. These characteristics, which are discussed in more detail in the rest of this section, are an attempt to respond to the shortcomings of the existing literature discussed in Section 2.3.

6.1.1. Learning-set based analysis

The proposed approach uses a learning set as an input, representing a subset of the spare parts, to extract the preference of the decision maker. The idea of using a subset of data for inferring the preference of the decision maker is inspired by case-based reasoning (see, e.g. [39][76]), which is a powerful knowledge extraction technique that was initially developed in the field of Artificial Intelligence. This idea has been adopted in multicriteria analysis, where several multicriteria learning-set based methods have been proposed (e.g. [17][30][31][42][70]) and successfully applied to different real-world decision problems (see, e.g. [21][46][59][62]). A learning-set based analysis is particularity useful in spare parts management for large firms where tens of thousands of spare parts need to be managed (see, e.g. [32]). In such situations, it is not practical to identify the appropriate stock control strategy for each spare part. The use of a learning-set based approach will naturally minimize the cognitive effort required from the decision maker. Although all machine learning methods are learning-set based approaches, they fail to take into account the multicriteria aspects of the spare parts management problem.

6.1.2. Use of a powerful multicriteria classification method

The learning phase relies on the DRSA, which has several powerful and attractive characteristics [20] as it: (i) does not need any preference parameters, which reduces the cognitive effort required from the decision maker; (ii) produces if–then decision rules, which are easily understood by the decision maker [10]; (iii) is able to deal with incomplete/missing attribute values (see [14][77]); and (iv) is able to detect and deal with inconsistency problems (see [29][79]). At this stage, it is important to mention that the authors in [25] have also used the DRSA for ABC classification. However, the model proposed in [25] lacks effective validation strategies and a real-world application of the model.

6.1.3. Comprehensive collection of validation strategies

The proposed approach is enhanced with three validation strategies (namely, direct analysis of decision rules by the decision maker, re-classification analysis and cross-validation analysis) enabling the decision maker to analyse the validity of the results before using the obtained if–then decision rules in practice. These different strategies are very useful in practice in that they enable the decision maker to better appreciate and refine the learning set. From a practical point of view, the validation strategies will help the decision maker to check the quality of the generated decision rules. If the decision maker agrees with the extracted decision rules, they can then be used in practice to classify the spare parts. Otherwise, the decision maker may prefer to restart the decision making process by considering new input data. This will substantially improve the effectiveness of the decision making process and the successful implementation of the resulting solution.

6.1.4. Real-world case study and active involvement of the decision maker

The proposed approach has been applied to a real-world case study of a manufacturing company in China while most of the existing studies conducted example analysis or data analysis using data extracted from other papers. During this case study, the decision maker was involved in all the steps of the decision making process: (i) in the learning phase, the decision maker was involved in identifying the evaluation criteria and also in defining the learning set; (ii) in the validation phase, the decision maker was involved in the checking, analysing and revising of the obtained decision rules; and (iii) in the generalization phase, the decision maker was involved in the identification and assessment of a new set of spare parts and then in the analysis and revision of the new collection of decision rules. In all these activities, the participation of the decision maker was crucial and his expertise and feedback played an important role in refining the decision rules.

6.1.5. Comparative analysis

The proposed approach has been compared to several well-known classification techniques, namely FR, KNN, SVM, DT, MLPNN and NB, using the data of the case study. The results show that the proposed approach outranks all the other approaches in terms of accuracy of classification. In addition, the statistical analysis shows that the use of the DRSA leads to a high agreement between the assignments proposed by the decision maker and those computed by the use of the proposed approach. An important characteristic of the DRSA compared to the above cited and well-known classification techniques is its flexibility in the sense that it accepts almost any kind of data (binary, symbolic, nominal, ordinal, discrete, and continuous) while the other approaches require the use of continuous data. Furthermore, as with some other well-known methods like SVM and NB, DRSA is also able to deal with incomplete/missing values with some adaptation (see [14][77]).

6.2. Practical guidelines for using the proposed approach

The last point to discuss is related to the practical use of the proposed approach. We show in Table 21 some practical guidelines for using the proposed approach. For each step of the decision making process, this table indicates the input data, the operation and computing, the output data, and the guideline for use according to different analysis types and results. The description of Table 21 is straightforward. At this stage, we will only briefly comment on the last row in this table. Indeed, in the medium to long term and after the use of the proposed approach in practice, the decision maker can judge efficiently the decision rules. If he/she judges that the spare parts management system is still efficient, he/she can continue the use of the system and no action is required. However, when some insufficiencies are detected, the decision maker can use the progressively updated learning set to restart the process.

7. Conclusion and future research

We presented a learning-set based approach to implement an advanced multiple criteria ABC classification of spare parts. The proposed approach contains three phases. The first phase uses the dominance-based rough set approach (DRSA) to infer a set of if–then decision rules that summarize the preferences of the decision maker. The second phase uses different techniques (the direct analysis of the decision rules by the decision maker, a re-classification analysis, and a cross-validation analysis) in order to analyse and validate the generated decision rules. The third phase exploits the generated and validated decision rules in order to classify new spare parts. An important aspect of the proposed approach is the simplicity and the easily understandable if–then decision rules provided as output. Another interesting aspect of the proposed approach is the inclusion of several validation strategies permitting the decision maker to analyse the validity of the results before using the obtained if–then rules in practice. The proposed approach has been successfully applied to a manufacturing company in China. We also compared the proposed approach to several well-known classifications methods. The results show that the proposed approach outranks all the other approaches in terms of accuracy of classification.

Based on our findings, the following spare parts management policies are suggested for the company of the case study: (i) for those spare parts classified in group A, the Economic Order Quantity (EOQ) and reorder point will be determined, and a few of them should be held in inventory and ordered frequently; (ii) for those spare parts classified in group B, the EOQ and reorder point will also be determined, but the management of this group of spare parts needs less attention than those in group A—only periodic review is needed here; and (iii) those spare parts classified in group C should be kept in stock and ordered when required.

Table 21: Practical guidelines

Phase	Input	Operation	Output	Guidelines
Learning				
Selection of the learning set	Raw data	The decision maker, based on his/her experience, selects a subset of spare parts to be used as learning set.	Information table	The selected spare parts should be as representative as possible by including and covering different specifications and characteristics. In addition, they should be non-redundant (in terms of their evaluation on the different criteria). The spare parts should ideally be well known to the decision maker/expert. Note that there was no ideal theoretical number of examples. A limited number of examples might lead to a few and very generic decision rules and too great number of examples may lead to a high number of very specific and redundant decision rules.
Definition of the assignment examples	Information table	The decision maker, based on his/her experiences and knowledge, assigns the spare parts in the learning set into the classes A, B and C.	Decision table	The spare parts should cover all the decision classes; in other words, all decision classes should contain a sufficient number of decision objects.
Learning of decision rules	Decision table	Approximation by the DRSA and induction of decision rules.	Decision rules	If the quality of approximation is acceptable (say, for example, greater than or equal to 80%), then go to validation phase. Otherwise the decision maker should modify the input data.
Validation				
Decision rules analysis	Decision rules	Ask the decision maker to scan all the decision rules and indicate his/her agreement level on a five-level Likert scale (Strongly Disagree, Disagree, Neutral, Agree, and Strongly Agree).	Checked and validated decision rules.	If there is a limited number of disagreements, the decision rules can be used for the generalization phase. With a moderate number of disagreements, the decision maker can either remove the decision rules with high levels of disagreement or modify some of them. When there are many disagreements, the decision maker should revise his/her assignment examples and/or the criteria used.
Re-classification	Decision table and Decision rules	Use the decision rules in order to re-classify the initial spare parts.	Reclassification of the initial spare parts into the classes A, B and C.	If there are many misclassifications, the decision maker is called to revise his/her initial assignments in order to improve the quality of decision rules.
Cross-validation	k folds of training and testing sets	Use the training sets to generate the decision rules and apply them on the testing sets.	Accuracy of the assignment of the testing sets	If the accuracy is high (say, for example, higher than 90%), the decision rules can be used for the generalization phase. Otherwise, the decision maker should modify learning dataset and restart the process.
Generalization				
Short term	Unseen spare parts	Use the decision rules to classify any new spare part into classes the A, B and C.	Classification of the new spare parts into the classes A, B and C.	If the decision maker accepts the new classifications, then the decision process stops. Otherwise the decision maker should modify the input data and restart from the beginning.
Medium to long term	Spare parts that have been successfully managed using the decision rules.	Enrich and update the learning set.	Progressively enhanced learning set.	If the spare parts management system is still efficient, no action is required. Otherwise, the decision maker can use the new and updated learning set to restart the process.

The proposed approach does not provide an optimised inventory system parameters for each group of spare parts. However, the ABC classification permits the use of different stocking policies for different groups of spare parts. In future research, we intend to enhance the proposed approach by adding a new layer devoted to spare parts optimization. The idea consists of combining the qualitative approach proposed in this paper with a quantitative one, which leads to a bi-objective problem. Indeed, solving a bi-objective problem with one qualitative dimension and one quantitative dimension is computationally better than solving a pure multi-objective problem, as has been proven in [2]. Another variation, with respect to optimization, it is to use other advanced techniques such as genetic programming [38] or joint optimization such as in [87].

We also intend to investigate the use of some recent extensions of the DRSA in the literature, such as the Variable Consistency Dominance-based Rough Set Approach (VC-DRSA) [11][12][45][56], the Stochastic DRSA [28] or the Dominance-based Rough Set Approach for Group decisions (DRSAfG) [21][20]. The VC-DRSA is a variant of DRSA that enables the relaxation of the conditions for assignments of objects to the lower approximations by accepting a limited proportion of negative examples, which is particularly useful for large decision tables. The Stochastic DRSA allows inconsistencies to some degree. The DRSAfG, a method that extends the DRSA to group decisions, is appropriate to deal with spare parts management in the presence of multiple decision makers. We also intend to investigate the use of the aggregation/disaggregation approach [31] to address the spare parts management problem. The idea of this approach is to use a subset of data to infer the preference parameters and then the ELECTRE TRI method [34] is used to assign spare parts into different classes. In comparison to the DRSA, the aggregation/disaggregation approach allows the decision maker to specify an assignment interval for each spare part in the learning set, instead of a single assignment.

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AppendixA. Information table and assignment examples

#	Criticality	Annual Dollar Usage	Average Unit Cost	Lead Time	Class
1	2	1312.5	26.25	2	C
2	2	1365	27.3	2	C
3	1	347.76	19.32	1	C
4	1	74.8	7.48	1	C
5	1	1117.98	62.11	2	C
6	1	1289.88	71.66	2	B
7	3	193.55	38.71	1	C
8	4	1313	13.13	2	C
9	3	326	3.26	1	C
10	3	2268	126	1	B
11	3	4134.6	91.88	1	B
12	3	1587.6	88.2	1	B
13	3	2063.4	54.3	1	B
14	3	1786.4	44.66	1	B
15	3	10365.75	121.95	1	B
16	3	770.26	20.27	1	C
17	3	2646	52.92	1	B
18	3	113.4	5.67	1	C
19	1	650	65	1	C
20	1	418.88	14.96	1	C
21	1	948.3	31.61	1	C
22	3	410.7	13.69	1	C
23	3	26995.6	2699.56	2	A
24	4	746	7.46	2	C
25	4	3150	31.5	2	B
26	4	3675	36.75	2	B
27	3	27562.5	1837.5	2	A
28	4	840	8.4	3	C
29	4	1670	16.7	2	C
30	4	1754	17.54	2	C
31	4	437	4.37	2	C
32	4	2625	26.25	2	B
33	4	462	4.62	2	C
34	4	1260	12.6	2	C
35	4	2100	21	2	B
36	4	1050	10.5	2	C
37	4	1575	15.75	2	C
38	4	578	5.78	2	C
39	3	2936	29.36	1	B
40	4	19682.3	3936.46	2	A
41	4	1444.2	24.07	1	C
42	3	463.05	13.23	1	C
43	3	132.3	7.35	1	C
44	1	2734.2	97.65	1	B
45	1	3071.25	87.75	1	B
46	4	785.7	17.46	1	C
47	4	955.2	11.94	1	C
48	1	28.44	1.58	1	C
49	4	851	8.51	2	C

#	Criticality	Annual Dollar Usage	Average Unit Cost	Lead Time	Class
50	1	352.8	8.82	1	C
51	3	105.84	2.94	1	C
52	1	1304.48	21.04	1	C
53	4	3580.5	65.1	2	A
54	2	1325.52	73.64	1	B
55	4	18375	1837.5	3	A
56	2	236.7	2.63	1	C
57	2	862	8.62	1	C
58	3	735	7.35	1	C
59	1	2315.34	128.63	1	B
60	1	1984.5	110.25	1	B
61	1	157.5	15.75	1	C
62	1	340.2	18.9	2	C
63	1	642.6	35.7	2	C
64	4	346.5	34.65	2	C
65	3	1890	189	1	B
66	3	567	31.5	1	C
67	3	2126.25	47.25	1	B
68	3	623.7	34.65	2	C
69	3	420	4.2	2	C
70	4	840	8.4	2	C
71	4	3150	31.5	3	A
72	4	2625	26.25	3	B
73	4	13925.3	1392.53	3	A
74	3	199.5	5.25	2	C
75	3	472.5	9.45	2	C
76	3	336	16.8	2	C
77	1	57.8	5.78	2	C
78	1	161.84	5.78	2	C
79	3	840	8.4	2	C
80	4	840	8.4	3	C
81	4	2625	26.25	3	B
82	4	2100	21	3	B
83	4	25725	257.25	3	A
84	4	40056	400.56	3	A
85	4	3780	126	2	A
86	3	882	29.4	2	C
87	1	1470	36.75	2	B
88	3	126	12.6	2	C
89	4	1071	17.85	2	C
90	3	121.1	3.46	2	C
91	3	43.56	2.42	2	C
92	1	823.2	29.4	2	C
93	1	1029	5.78	2	C
94	4	1025.55	22.79	2	C
95	4	2688	33.6	2	B
96	3	1470	29.4	2	B
97	2	264.6	14.7	2	C
98	4	11025	5512.5	3	A

AppendixB. Detailed description of initial decision rules

#	Rule	Supporting objects	Relative strength (%)	Confidence level (%)
1	IF (AnnDollarUsage$\leq$1260) THEN (Class at most C)	3, 4, 5, 7, 9, 16, 18, 19, 20, 21, 22, 24, 28, 31, 33, 34, 36, 38, 42, 43, 46, 47, 48, 49, 50, 51, 56, 57, 58, 61, 62, 63, 64, 66, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 86, 88, 89, 90, 91, 92, 93, 94, 97	86.89	100
2	IF (AvgUnitCost$\leq$27.3) & (Criticality$\leq$2) THEN (Class at most C)	1, 2, 3, 4, 20, 48, 50, 52, 56, 57, 61, 62, 77, 78, 93, 97	26.23	100
3	IF (AvgUnitCost$\leq$24.07) & (LeadTime$\leq$2) & (AnnDollarUsage$\leq$1754) THEN (Class at most C)	3, 4, 8, 9, 16, 18, 20, 22, 24, 29, 30, 31, 33, 34, 36, 37, 38, 41, 42, 43, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 61, 62, 69, 70, 74, 75, 76, 77, 78, 79, 88, 89, 90, 91, 93, 94, 97	77.05	100
4	IF (AnnDollarUsage$\leq$3071.25) THEN (Class at most B)	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 24, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97	95.40	100
5	IF (LeadTime$\leq$1) THEN (Class at most B)	3, 4, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 39, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 54, 56, 57, 58, 59, 60, 61, 65, 66, 67	44.83	100
6	IF (AvgUnitCost$\leq$36.75) & (LeadTime$\leq$2) THEN (Class at most B)	1, 2, 3, 4, 8, 9, 16, 18, 20, 21, 22, 24, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 61, 62, 63, 64, 66, 68, 69, 70, 74, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97	73.56	100
7	IF (AnnDollarUsage$\geq$11025) THEN (Class at least A)	23, 27, 40, 55, 73, 83, 84, 98	72.73	100
8	IF (AnnDollarUsage$\geq$3150) & (Criticality$\geq$4) THEN (Class at least A)	25, 26, 40, 53, 55, 71, 73, 83, 84, 85, 98	81.82	81.82
9	IF (AnnDollarUsage$\geq$1786.4) THEN (Class at least B)	10, 11, 13, 14, 15, 17, 23, 25, 26, 27, 32, 35, 39, 40, 44, 45, 53, 55, 59, 60, 65, 67, 71, 72, 73, 81, 82, 83, 84, 85, 95, 98	86.49	100
10	IF (AvgUnitCost$\geq$71.66) THEN (Class at least B)	6, 10, 11, 12, 15, 23, 27, 40, 44, 45, 54, 55, 59, 60, 65, 73, 83, 84, 85, 98	54.05	100
11	IF (AnnDollarUsage$\geq$1470) & (LeadTime$\geq$2) & (AvgUnitCost$\geq$29.4) THEN (Class at least B)	23, 25, 26, 27, 40, 53, 55, 71, 73, 83, 84, 85, 87, 95, 96, 98	43.24	100

AppendixC. Data used for the generalization phase

#	Criticality	Annual Dollar Usage	Average Unit Cost	Lead Time
100	4	3805	38.05	1
101	3	66256	331.28	2
102	3	13396.5	267.93	2
103	1	16555	165.55	2
104	3	12645	126.45	2
105	2	868	17.36	2
106	3	2537.5	50.75	2
107	3	26955	53.91	2
108	1	1837	36.74	2
109	3	12122	121.22	2
110	3	4887	48.87	2
111	3	31136	622.72	3
112	3	8814.6	293.82	2
113	3	6519	130.38	2
114	3	10148.7	338.29	2
115	3	15389.1	512.97	2
116	3	594	11.88	1
117	3	4420	11.05	1
118	3	2125.5	42.51	1
119	3	1462.5	29.25	1
120	1	900	4.5	2
121	4	20126.25	4025.25	3
122	1	756	3.78	2
123	3	10176	101.76	2
124	2	786.5	15.73	1
125	2	3611	72.22	1
126	3	706800	883.5	3
127	3	4121	41.21	2
128	1	4121	41.21	2
129	3	6473	64.73	2
130	3	3469	69.38	2
131	3	6108	61.08	2
132	2	1629	16.29	2
133	3	976	9.76	2
134	2	3488	69.76	2
135	3	27225	544.5	2
136	3	892	4.46	2
137	3	3044	7.61	2
138	3	2013	6.71	2
139	3	13627.3	2725.46	3
140	4	210909.8	42181.96	3
141	3	9280.2	309.34	2
142	3	22105.2	736.84	2
143	3	3506.1	116.87	2
144	3	7529.1	250.97	2
145	3	3506.1	116.87	2
146	3	5898.6	589.86	2
147	3	14980.8	249.68	2
148	3	3875.7	129.19	2
149	3	856	17.12	2
150	3	1266.5	25.33	2
151	1	55	0.55	1
152	2	1618.5	32.37	2
153	3	3283.2	109.44	2
154	2	1312	3.28	1
155	3	3321.5	66.43	2
156	4	61249.35	12249.87	3
157	3	5898.6	589.86	2
158	2	1440	14.4	2
159	2	9720	97.2	2
160	3	21507	143.38	2
161	3	155400	51.8	2
162	3	8872.8	295.76	2
163	3	4455	148.5	2
164	3	18917.1	630.57	3
165	3	2358	23.58	2
166	3	1646	16.46	2
167	3	3150	31.5	2
168	3	1377.9	45.93	2
169	3	14496	144.96	2
170	3	12600	126	2
171	1	311	3.11	1
172	3	23	0.23	1
173	3	7072	35.36	2
174	3	462	7.7	1
175	3	868	17.36	2
176	3	288.5	5.77	1
177	3	279.6	4.66	2
178	3	3702.85	740.57	3
179	3	9474	94.74	2
180	3	594	11.88	2
181	3	5636	112.72	2
182	1	915.5	18.31	2
183	3	1138.95	227.79	2
184	2	1232	24.64	2
185	3	3317.25	663.45	3
186	3	3150	63	2
187	3	1800	18	2
188	3	8453.4	281.78	2
189	3	16306.5	326.13	2
190	3	14055.6	468.52	2
191	3	31136	622.72	3
192	3	3585	35.85	2
193	3	13768.5	275.37	2
194	3	1855.5	37.11	2
195	3	1254.15	250.83	2
196	3	12145	2429	3
197	4	16851.45	3370.29	3
198	3	8229.9	274.33	2
199	3	11743.5	391.45	2
200	3	1375.2	45.84	2
201	3	1301	26.02	2
202	3	1072	5.36	2
203	2	626	6.26	1
204	3	2144	5.36	1
205	2	4008	5.01	1
206	3	1428.8	8.93	2
207	4	3150	31.5	2
208	1	302	3.02	2
209	1	55	0.55	2
210	2	165	1.65	2
211	4	630	12.6	1
212	3	10173.6	339.12	2
213	1	5880	29.4	2
214	3	3091	61.82	2
215	3	18917.1	630.57	3
216	1	552	5.52	1
217	3	1396	27.92	2
218	1	504	16.8	2
219	1	420	4.2	2
220	3	10482.2	524.11	2
221	1	10908	545.4	3
222	3	27344.4	911.48	3

AppendixD. Result of the generalization phase using the initial set of decision rules

#	Decision by the set of rules	Agreement level					Desirable decision
		Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree	
100	B or A		✓				B
101	A					✓	
102	A					✓	
103	A				✓		
104	A					✓	
105	C					✓	
106	B				✓		
107	A			✓			
108	B			✓			
109	A					✓	
110	B				✓		
111	A					✓	
112	B			✓			
113	B			✓			
114	B	✓					A
115	A					✓	
116	C					✓	
117	B	✓					C
118	B			✓			
119	B		✓				C
120	C					✓	
121	A					✓	
122	C					✓	
123	B		✓				A
124	C					✓	
125	B			✓			
126	A					✓	
127	B					✓	
128	B			✓			
129	B				✓		
130	B				✓		
131	B				✓		
132	C			✓			
133	C				✓		
134	B			✓			
135	A					✓	
136	C					✓	
137	B		✓				C
138	B		✓				C
139	A					✓	
140	A					✓	
141	B		✓				A
142	A					✓	
143	B			✓			
144	B			✓			
145	B			✓			
146	B			✓			
147	A					✓	
148	B			✓			
149	C					✓	
150	B			✓			
151	C					✓	
152	B			✓			
153	B			✓			
154	C					✓	
155	B				✓		
156	A					✓	
157	B			✓			
158	C					✓	
159	B			✓			
160	A				✓		
161	A				✓		

#	Decision by the set of rules	Agreement level					Desirable decision
		Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree	
162	B			✓			
163	B			✓			
164	A					✓	
165	B				✓		
166	C			✓			
167	B			✓			
168	B			✓			
169	A					✓	
170	A					✓	
171	C					✓	
172	C					✓	
173	B				✓		
174	C					✓	
175	C					✓	
176	C					✓	
177	C					✓	
178	B			✓			
179	B			✓			
180	C					✓	
181	B				✓		
182	C				✓		
183	C or B		✓				B
184	C			✓			
185	B			✓			
186	B				✓		
187	B			✓			
188	B			✓			
189	A					✓	
190	A					✓	
191	A					✓	
192	B			✓			
193	A					✓	
194	B			✓			
195	C or B		✓				B
196	A					✓	
197	A					✓	
198	B			✓			
199	A					✓	
200	B			✓			
201	B			✓			
202	C					✓	
203	C					✓	
204	B		✓				C
205	C or B		✓				C
206	C				✓		
207	B or A		✓				B
208	C					✓	
209	C					✓	
210	C					✓	
211	C					✓	
212	B		✓				A
213	B			✓			
214	B			✓			
215	A					✓	
216	C					✓	
217	B			✓			
218	C					✓	
219	C					✓	
220	B		✓				A
221	B			✓			
222	A					✓	

AppendixE. Detailed description of revised decision rules

#	Rule	Supporting objects	Relative strength (%)	Confidence (%)
1	IF (AnnDollarUsage$\leq$1117.98) THEN (Class at most C);	3, 4, 5, 7, 9, 16, 18, 19, 20, 21, 22, 24, 28, 31, 33, 36, 38, 42, 43, 46, 47, 48, 49, 50, 51, 56, 57, 58, 61, 62, 63, 64, 66, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 86, 88, 89, 90, 91, 92, 93, 94, 97	77.61	100
2	IF (AvgUnitCost$\leq$29.25) & (LeadTime$\leq$2) THEN (Class at most C);	1, 2, 3, 4, 8, 9, 16, 18, 20, 22, 24, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42, 43, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 61, 62, 69, 70, 74, 75, 76, 77, 78, 79, 88, 89, 90, 91, 93, 94, 97, 117, 119, 137, 138, 204, 205	82.09	96
3	IF (AnnDollarUsage$\leq$3071.25) THEN (Class at most B);	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 24, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 119, 137, 138, 183, 195, 204	91.75	100
4	IF (LeadTime$\leq$1) THEN (Class at most B);	3, 4, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 39, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 54, 56, 57, 58, 59, 60, 61, 65, 66, 67, 100, 117, 119, 204, 205	45.36	100
5	IF (AvgUnitCost$\leq$36.75) & (LeadTime$\leq$2) THEN (Class at most B);	1, 2, 3, 4, 8, 9, 16, 18, 20, 21, 22, 24, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 61, 62, 63, 64, 66, 68, 69, 70, 74, 75, 76, 77, 78, 79, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 117, 119, 137, 138, 204, 205, 207	73.20	100
6	IF (AvgUnitCost$\geq$257.25) THEN (Class at least A);	23, 27, 40, 55, 73, 83, 84, 98, 114, 141, 212, 220	75.00	100
7	IF (AnnDollarUsage$\geq$3150) & (Criticality$\geq$4) & (LeadTime$\geq$2) THEN (Class at least A);	25, 26, 40, 53, 55, 71, 73, 83, 84, 85, 98, 207	56.25	75
8	IF (AnnDollarUsage$\geq$10176) & (LeadTime$\geq$2) THEN (Class at least A);	23, 27, 40, 55, 73, 83, 84, 98, 123, 220	62.50	100
9	IF (AvgUnitCost$\geq$65.1) THEN (Class at least B);	6, 10, 11, 12, 15, 23, 27, 40, 44, 45, 53, 54, 55, 59, 60, 65, 73, 83, 84, 85, 98, 114, 123, 141, 183, 195, 212, 220	60.87	100
10	IF (AnnDollarUsage$\geq$2063.4) & (Criticality$\geq$4) THEN (Class at least B);	25, 26, 32, 35, 40, 53, 55, 71, 72, 73, 81, 82, 83, 84, 85, 95, 98, 100, 207	41.30	100
11	IF (AnnDollarUsage$\geq$1470) & (AvgUnitCost$\geq$29.36) THEN (Class at least B);	10, 11, 12, 13, 14, 15, 17, 23, 25, 26, 27, 39, 40, 44, 45, 53, 55, 59, 60, 65, 67, 71, 73, 83, 84, 85, 87, 95, 96, 98, 100, 114, 123, 141	80.43	100

AppendixF. Result of the generalization using the new set of decision rules

	Assignment		Agreement level				
#	Initial decision rules	New decision rules	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
101	A	A					✓
102	A	A					✓
103	A	A				✓	
104	A	A					✓
105	C	C					✓
106	B	B				✓	
107	A	A			✓		
108	B	B			✓		
109	A	A					✓
110	B	B				✓	
111	A	A					✓
112	B	A			✓		
113	B	B			✓		
115	A	A					✓
116	C	C					✓
118	B	B			✓		
120	C	C					✓
121	A	A					✓
122	C	C					✓
124	C	C					✓
125	B	B			✓		
126	A	A					✓
127	B	B					✓
128	B	B			✓		
129	B	B				✓	
130	B	B				✓	
131	B	B				✓	
132	C	C			✓		
133	C	C				✓	
134	B	B			✓		
135	A	A					✓
136	C	C					✓
139	A	A					✓
140	A	A					✓
142	A	A					✓
143	B	B			✓		
144	B	B			✓		
145	B	B			✓		
146	B	A			✓		
147	A	A					✓
148	B	B			✓		
149	C	C					✓
150	B	C			✓		
151	C	C					✓
152	B	B			✓		
153	B	B			✓		
154	C	C					✓
155	B	B				✓	
156	A	A					✓
157	B	A			✓		
158	C	C					✓
159	B	B			✓		
160	A	A				✓	
161	A	A				✓	

	Assignment		Agreement level				
#	Initial decision rules	New decision rules	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
162	B	A			✓		
163	B	B			✓		
164	A	A					✓
165	B	C				✓	
166	C	C			✓		
167	B	B			✓		
168	B	B			✓		
169	A	A					✓
170	A	A					✓
171	C	C					✓
172	C	C					✓
173	B	B				✓	
174	C	C					✓
175	C	C					✓
176	C	C					✓
177	C	C					✓
178	B	A			✓		
179	B	B			✓		
180	C	C					✓
181	B	B				✓	
182	C	C				✓	
184	C	C			✓		
185	B	A			✓		
186	B	B				✓	
187	B	C			✓		
188	B	A			✓		
189	A	A					✓
190	A	A					✓
191	A	A					✓
192	B	B			✓		
193	A	A					✓
194	B	B			✓		
196	A	A					✓
197	A	A					✓
198	B	A			✓		
199	A	A					✓
200	B	B			✓		
201	B	C			✓		
202	C	C					✓
203	C	C					✓
206	C	C				✓	
208	C	C					✓
209	C	C					✓
210	C	C					✓
211	C	C					✓
213	B	B			✓		
214	B	B			✓		
215	A	A					✓
216	C	C					✓
217	B	C			✓		
218	C	C					✓
219	C	C					✓
221	B	A			✓		
222	A	A					✓

AppendixG. Confusion Matrices

Method																					
FR	Fold	1				2				3				4				5			
			C	B	A		C	B	A		C	B	A		C	B	A		C	B	A
		C	26	3	0	C	29	1	0	C	30	0	0	C	28	2	0	C	30	0	0
		B	0	10	3	B	1	12	0	B	0	12	0	B	2	11	0	B	4	7	2
		A	0	2	4	A	0	3	3	A	0	3	3	A	0	1	5	A	0	0	6
	Fold	6				7				8				9				10			
			C	B	A		C	B	A		C	B	A		C	B	A		C	B	A
		C	23	3	0	C	30	0	0	C	29	1	0	C	30	0	0	C	28	2	0
		B	2	10	0	B	3	6	0	B	2	11	0	B	3	9	1	B	2	9	0
		A	0	0	6	A	0	1	5	A	0	2	4	A	0	1	5	A	0	1	5
KNN	Fold	1				2				3				4				5			
			C	B	A		C	B	A		C	B	A		C	B	A		C	B	A
		C	30	0	0	C	28	2	0	C	27	3	0	C	30	0	0	C	29	1	0
		B	3	10	0	B	4	9	0	B	5	8	0	B	2	11	0	B	7	6	0
		A	0	0	6	A	0	3	3	A	3	0	3	A	0	4	2	A	0	1	5
	Fold	6				7				8				9				10			
			C	B	A		C	B	A		C	B	A		C	B	A		C	B	A
		C	28	2	0	C	30	0	0	C	29	1	0	C	29	1	0	C	26	4	0
		B	9	4	0	B	9	4	0	B	7	6	0	B	3	10	0	B	5	8	0
		A	1	0	5	A	1	1	4	A	2	0	4	A	0	2	4	A	0	1	5
SVM	Fold	1				2				3				4				5			
			C	B	A		C	B	A		C	B	A		C	B	A		C	B	A
		C	30	0	0	C	30	0	0	C	30	0	0	C	30	0	0	C	30	0	0
		B	10	0	3	B	13	0	0	B	13	0	0	B	13	0	0	B	13	0	0
		A	0	1	5	A	3	0	3	A	1	0	5	A	2	0	4	A	1	0	5
	Fold	6				7				8				9				10			
			C	B	A		C	B	A		C	B	A		C	B	A		C	B	A
		C	30	0	0	C	30	0	0	C	30	0	0	C	30	0	0	C	30	0	0
		B	13	0	0	B	13	0	0	B	13	0	0	B	13	0	0	B	13	0	0
		A	1	0	5	A	2	0	4	A	2	0	4	A	2	0	4	A	1	0	5
DT	Fold	1				2				3				4				5			
			C	B	A		C	B	A		C	B	A		C	B	A		C	B	A
		C	24	6	0	C	28	2	0	C	21	9	0	C	27	3	0	C	28	2	0
		B	0	9	4	B	1	8	4	B	4	6	3	B	0	10	3	B	2	8	3
		A	0	0	6	A	0	2	4	A	0	0	6	A	0	1	5	A	0	0	6
	Fold	6				7				8				9				10			
			C	B	A		C	B	A		C	B	A		C	B	A		C	B	A
		C	24	6	0	C	30	0	0	C	30	0	0	C	24	6	0	C	26	4	0
		B	1	9	3	B	4	9	0	B	4	6	3	B	1	9	3	B	2	4	7
		A	0	0	6	A	0	2	4	A	0	1	5	A	0	0	6	A	0	0	6
MLPNN	Fold	1				2				3				4				5			
			C	B	A		C	B	A		C	B	A		C	B	A		C	B	A
		C	26	4	0	C	29	1	0	C	28	2	0	C	26	4	0	C	30	0	0
		B	0	10	3	B	0	13	0	B	1	11	1	B	0	12	1	B	3	8	2
		A	0	0	6	A	0	3	3	A	0	1	5	A	0	2	4	A	0	1	5
	Fold	6				7				8				9				10			
			C	B	A		C	B	A		C	B	A		C	B	A		C	B	A
		C	24	6	0	C	29	1	0	C	28	2	0	C	30	0	0	C	24	6	0
		B	2	11	0	B	4	9	0	B	1	12	0	B	2	11	0	B	0	12	1
		A	0	1	5	A	0	1	5	A	0	2	4	A	0	1	5	A	0	1	5
NB	Fold	1				2				3				4				5			
			C	B	A		C	B	A		C	B	A		C	B	A		C	B	A
		C	30	0	0	C	30	0	0	C	30	0	0	C	30	0	0	C	29	1	0
		B	6	5	2	B	11	2	0	B	10	3	0	B	10	3	0	B	9	4	0
		A	5	0	1	A	6	0	0	A	6	0	0	A	6	0	0	A	4	0	2
	Fold	6				7				8				9				10			
			C	B	A		C	B	A		C	B	A		C	B	A		C	B	A
		C	29	1	0	C	29	1	0	C	29	1	0	C	28	2	0	C	30	0	0
		B	1	12	0	B	3	10	0	B	4	9	0	B	0	13	0	B	7	6	0
		A	4	1	1	A	4	1	1	A	2	2	2	A	3	2	1	A	5	0	1
DRSA	Fold	1				2				3				4				5			
			C	B	A		C	B	A		C	B	A		C	B	A		C	B	A
		C	24	6	0	C	30	0	0	C	23	7	0	C	28	2	0	C	30	0	0
		B	0	11	2	B	0	13	0	B	0	11	2	B	0	11	2	B	2	9	2
		A	0	0	6	A	0	3	3	A	0	1	5	A	0	2	4	A	0	0	6
	Fold	6				7				8				9				10			
			C	B	A		C	B	A		C	B	A		C	B	A		C	B	A
		C	25	5	0	C	30	0	0	C	29	1	0	C	26	4	0	C	26	4	0
		B	1	12	0	B	1	12	0	B	1	12	0	B	1	11	1	B	0	11	2
		A	0	0	6	A	0	1	5	A	0	2	4	A	0	1	5	A	0	1	5

AppendixH. Analysis of confusion matrices

Method	Parameter	Fold 1	2	3	4	5	6	7	8	9	10
FR	Correct assignment	40	44	45	44	43	39	41	44	44	42
	Wrong assignment	8	5	3	5	6	5	4	5	5	5
	Missing assignment	1	0	1	0	0	5	4	0	0	2
	Accuracy (%)	81.63	89.76	91.84	89.76	87.76	79.59	83.67	89.8	89.8	85.71
	Mean Absolute Error (%)	18.37	10.2	8.16	10.2	12.25	20.41	16.33	10.2	10.2	14.29
KNN	Correct assignment	46	40	38	43	40	37	38	39	43	39
	Wrong assignment	3	9	11	6	9	12	11	10	6	10
	Missing assignment	0	0	0	0	0	0	0	0	0	0
	Accuracy (%)	93.88	81.63	77.55	87.76	81.63	75.51	77.55	79.59	87.76	79.59
	Mean Absolute Error (%)	6.12	18.37	22.45	12.25	18.37	24.49	22.45	20.41	12.25	20.41
SVM	Correct assignment	35	33	35	34	35	35	34	34	34	35
	Wrong assignment	14	16	14	15	14	14	15	15	15	14
	Missing assignment	0	0	0	0	0	0	0	0	0	0
	Accuracy (%)	71.43	67.35	71.43	69.39	71.43	71.43	69.39	69.39	69.39	71.43
	Mean Absolute Error (%)	28.57	32.65	28.57	30.61	28.57	28.57	30.61	30.61	30.61	28.57
DT	Correct assignment	39	40	33	42	42	39	43	41	39	36
	Wrong assignment	10	9	16	7	7	10	6	8	10	13
	Missing assignment	0	0	0	0	0	0	0	0	0	0
	Accuracy (%)	79.59	81.63	67.35	85.71	85.71	79.59	87.76	83.67	79.59	73.47
	Mean Absolute Error (%)	20.41	18.37	32.65	14.29	14.29	20.41	12.25	16.33	20.41	26.53
MLPNN	Correct assignment	42	45	44	42	43	40	43	44	46	41
	Wrong assignment	7	4	5	7	6	9	6	5	3	8
	Missing assignment	0	0	0	0	0	0	0	0	0	0
	Accuracy (%)	85.714	91.84	89.8	85.71	87.76	81.63	87.76	89.8	93.88	83.67
	Mean Absolute Error(%)	14.286	8.16	10.2	14.29	12.25	18.37	12.25	10.2	6.12	16.33
NB	Correct assignment	36	32	33	33	35	42	40	40	42	37
	Wrong assignment	13	17	16	16	14	7	9	9	7	12
	Missing assignment	0	0	0	0	0	0	0	0	0	0
	Accuracy (%)	73.47	65.31	67.35	67.35	71.43	85.71	81.63	81.63	85.71	75.51
	Mean Absolute Error (%)	26.53	34.69	32.65	32.65	28.57	14.29	18.37	18.37	14.29	24.49
DRSA	Correct assignment	41	46	39	43	45	43	47	45	42	42
	Wrong assignment	8	3	10	6	4	6	2	4	7	7
	Missing assignment	0	0	0	0	0	0	0	0	0	0
	Accuracy (%)	83.67	93.88	79.59	87.76	91.84	87.76	95.92	91.84	85.71	85.71
	Mean Absolute Error (%)	16.33	6.12	20.41	12.24	8.16	12.24	4.08	8.16	14.29	14.29

AppendixI. Statistical analysis

		Statistics			
Fold	Method	Kendall's tau	Spearman's rho	Unweighted Cohen's kappa	Weighted Cohen's kappa
1	FR	0.7732	0.8151	0.6814	0.7043
	KNN	0.904	0.9092	0.8819	0.9094
	SVM	0.6696	0.706	0.3982	0.5765
	DT	0.8176	0.853	0.6564	0.738
	MLPNN	0.8614	0.8887	0.7523	0.8106
	NB	0.4582	0.4781	0.4145	0.3806
2	FR	0.8944	0.9165	0.8078	0.8414
	KNN	0.7611	0.7842	0.6446	0.7104
	SVM	0.4314	0.4482	0.2183	0.3199
	DT	0.828	0.8767	0.6677	0.7467
	MLPNN	0.9274	0.949	0.8483	0.874
	NB	0.1945	0.2021	0.1368	0.0962
3	FR	0.9368	0.9745	0.8478	0.8498
	KNN	0.5102	0.52	0.5471	0.5396
	SVM	0.5694	0.5916	0.3473	0.4983
	DT	0.6086	0.6468	0.4408	0.5697
	MLPNN	0.8751	0.8948	0.8134	0.853
	NB	0.2408	0.2502	0.2016	0.1431
4	FR	0.8525	0.8652	0.8099	0.8487
	KNN	0.892	0.9197	0.7618	0.8022
	SVM	0.5036	0.5233	0.2843	0.4121
	DT	0.8634	0.8974	0.748	0.8054
	MLPNN	0.8387	0.864	0.7472	0.7937
	NB	0.2408	0.2502	0.2016	0.1431
5	FR	0.852	0.8716	0.7633	0.8256
	KNN	0.7285	0.7481	0.6285	0.716
	SVM	0.5694	0.5916	0.3473	0.4983
	DT	0.841	0.8681	0.7396	0.8059
	MLPNN	0.8661	0.8935	0.7652	0.8229
	NB	0.3898	0.4005	0.3553	0.359

		Statistics			
Fold	Method	Kendall's tau	Spearman's rho	Unweighted Cohen's kappa	Weighted Cohen's kappa
6	FR	0.5652	0.5803	0.6611	0.5652
	KNN	0.5624	0.5811	0.49	0.5839
	SVM	0.5694	0.5916	0.3473	0.4983
	DT	0.7825	0.8133	0.6505	0.7311
	MLPNN	0.7384	0.7538	0.675	0.7348
	NB	0.6315	0.6496	0.7125	0.6223
7	FR	0.801	0.869	0.6961	0.6953
	KNN	0.659	0.6789	0.5153	0.6005
	SVM	0.5036	0.5233	0.2843	0.4121
	DT	0.8512	0.8705	0.7578	0.8088
	MLPNN	0.8216	0.8364	0.7625	0.8146
	NB	0.5702	0.5869	0.6195	0.5466
8	FR	0.8735	0.8911	0.8063	0.844
	KNN	0.6045	0.617	0.5724	0.6064
	SVM	0.5036	0.5233	0.2843	0.4121
	DT	0.8273	0.8613	0.6844	0.7675
	MLPNN	0.8731	0.8898	0.8114	0.8462
	NB	0.674	0.6931	0.6272	0.629
9	FR	0.8781	0.8984	0.8032	0.849
	KNN	0.8415	0.8601	0.7644	0.8115
	SVM	0.5036	0.5233	0.2843	0.4121
	DT	0.7825	0.8133	0.6505	0.7311
	MLPNN	0.9223	0.9331	0.8829	0.908
	NB	0.6848	0.7055	0.7243	0.6644
10	FR	0.8095	0.8519	0.7392	0.7538
	KNN	0.6831	0.7019	0.6148	0.6953
	SVM	0.5694	0.5916	0.3473	0.4983
	DT	0.7795	0.8281	0.5381	0.6711
	MLPNN	0.8101	0.8307	0.7196	0.7724
	NB	0.4377	0.45	0.4437	0.3779