Cosmological perturbations in Horava-Lifshitz theory without detailed balance

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In the Horava-Lifshitz theory of quantum gravity, two conditions – detailed balance and projectability – are usually assumed. The breaking of projectability simplifies the theory, but it leads to serious problems with the theory. The breaking of detailed balance leads to a more complicated form of the theory, but it appears to resolve some of the problems. Sotiriou, Visser and Weinfurtner formulated the most general theory of Horava-Lifshitz type without detailed balance. We compute the linear scalar perturbations of the FRW model in this form of HL theory. We show that the higher-order curvature terms in the action lead to a gravitational effective anisotropic stress on small scales. Specializing to a Minkowski background, we study the spin-0 scalar mode of the graviton, using a gauge-invariant analysis, and find that it is stable in both the infrared and ultraviolet regimes for $0 \leq \xi \leq 2/3$. However, in this parameter range the scalar mode is a ghost.

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I. INTRODUCTION

Recently, Horava proposed a quantum gravity theory [1], motivated by the Lifshitz theory in solid state physics [2]. Horava-Lifshitz (HL) theory is non-relativistic and power-counting ultraviolet (UV)-renormalizable, and should recover general relativity in the infrared (IR) limit. The effective speed of light diverges in the UV regime, and this potentially resolves the horizon problem without invoking an inflationary scenario. In [3, 4], the general field equations were derived, and given explicitly for FRW cosmology, from which it can be seen that the spatial curvature is enhanced by higher-order curvature terms. This could open a new approach to the flatness problem and to a bouncing universe [5, 6]. It was also shown that almost scale-invariant super-horizon curvature perturbations can be produced without inflation [7].

Horava assumed two conditions – detailed balance and projectability. He also considered the case without detailed balance. So far most of the work [3, 4, 6, 8–10] on HL theory has abandoned projectability. However, breaking the projectability condition seems problematic [11, 12]. With detailed balance, it was shown that matter is not UV stable [5]. In addition, a non-zero negative cosmological constant is required, and it also breaks the parity in the purely gravitational sector [13].

Because Lorentz invariance is broken in the UV, HL theory contains a reduced set of diffeomorphisms, and as a result, a spin-0 mode of the graviton appears. This mode is potentially dangerous and may cause strong coupling problems that prevent the recovery of general relativity in the IR limit [14–16].

In order to avoid these problems, one possibility is to keep the projectability condition. With this condition, Mukohyama argued that the problems found in [14, 16] can be solved by the repulsive gravitational force due to the nonlinear higher curvature terms [11]. In addition, without the projectability condition, the theory seems to be inconsistent [17].

By abandoning detailed balance but still keeping the projectability condition, Sotiriou, Visser and Weinfurtner (SVW) showed that the most general such HL theory can be properly formulated with eight independent coupling constants, in addition to the Newton and cosmological ones [13]. Among these eight coupling constants, one is associated with the kinetic energy, which leads to the spin-0 scalar graviton, and the other seven are all related to the breaking of Lorentz invariance, which are highly suppressed by the Planck scale in the IR limit.

In this paper, we study linear scalar perturbations of FRW models in the SVW set-up. The paper is organized as follows: In Sec. II, we give a brief introduction to the generalized HL theory formulated by SVW [13]. In particular, we add matter fields (not considered in [13]) and generalize the dynamical equations and the Hamiltonian and super-momentum constraints. Calcagni constructed the action for a scalar field [5], while Kiritsis and Kofinas considered the same problem, and then generalized it to a vector field [4]. However, the general coupling of matter to this theory has not been worked out yet, since we no longer have the guide of Lorentz invariance. We shall not be concerned with this issue in the present paper, and simply assume that it can be done and represented by a general matter action, from which we can derive the conservation laws of the matter field. In Sec. III, we present the Friedmann-like field equations for FRW models with any curvature k. In Sec. IV, we first briefly discuss different gauge choices, and then study the linear scalar perturbations of FRW models, working in the quasi-longitudinal gauge. We obtain the perturbations of the dynamical equations and the Hamiltonian and super-momentum constraints. We also compute the perturbed matter conservation equations. In Sec. V, we study the spin-0 scalar mode of the graviton in a Minkowski background, by specializing the formulas developed in Sec. IV. We find that this scalar mode, which could potentially undermine the recovery of general relativity in the IR limit, is in fact stable in both the IR and UV regimes for $0 \leq \xi \leq 2/3$. However, it should be noted that in this range, the kinetic term of the scalar mode in the action has the wrong sign, as is evident in Horava's original results [1], so that the mode is a ghost [11, 16, 18]. In Sec. VI, we restrict to perturbations of the flat FRW model, and find that the corresponding field equations are considerably simplified. In Sec. VII, we present conclusions.

II. HORAVA-LIFSHITZ GRAVITY WITHOUT DETAILED BALANCE

We give a very brief introduction to HL gravity without detailed balance, but with the projectability condition. (For further details, see [13].) The dynamical variables are N, N^i and g_{ij} $(i,\ j=1,2,3)$, in terms of which the metric takes the ADM form,

$$ds^2 = -N^2dt^2 + g_{ij}\left(dx^i + N^idt\right)\left(dx^j + N^jdt\right). \quad (2.1)$$

The theory is invariant under the scalings

$$\begin{aligned} &t \to \ell^3 t, \quad x^i \to \ell x^i, \\ &N \to \ell^{-2}N,\ N^i \to \ell^{-2}N^i,\ g_{ij} \to g_{ij}. \end{aligned} \quad (2.2)$$

The projectability condition requires a homogeneous lapse function:

$$N = N(t), \quad N^i = N^i\left(t, x^k\right), \quad g_{ij} = g_{ij}\left(t, x^k\right). \quad (2.3)$$

This is invariant under the gauge transformations,

$$\tilde{t} = t + \chi^0(t), \tilde{x}^i = x^i + \chi^i\left(t, x^k\right). \quad (2.4)$$

The total action consists of kinetic, potential and matter parts,

$$S = \zeta^2 \int dt d^3x N\sqrt{g}\left(\mathcal{L}_K - \mathcal{L}_V + \zeta^{-2}\mathcal{L}_M\right), \quad (2.5)$$

where $g = \det g_{ij}$, and

$$\begin{aligned} \mathcal{L}_K &= K_{ij}K^{ij} - (1-\xi)K^2, \\ \mathcal{L}_V &= 2\Lambda - R + \frac{1}{\zeta^2}\left(g_2R^2 + g_3R_{ij}R^{ij}\right) \\ &\quad + \frac{1}{\zeta^4}\left(g_4R^3 + g_5R\ R_{ij}R^{ij} + g_6R^i_jR^j_kR^k_i\right) \\ &\quad + \frac{1}{\zeta^4}\left[g_7R\nabla^2R + g_8\left(\nabla_iR_{jk}\right)\left(\nabla^iR^{jk}\right)\right]. \end{aligned} \quad (2.6)$$

Here $\zeta^2 = 1/16\pi G$, the covariant derivatives and Ricci and Riemann terms all refer to the three-metric g_{ij}, and K_{ij} is the extrinsic curvature,

$$K_{ij} = \frac{1}{2N}\left(-\dot{g}_{ij} + \nabla_iN_j + \nabla_jN_i\right), \quad (2.7)$$

where $N_i = g_{ij}N^j$. The constants ξ, g_I $(I = 2, \ldots 8)$ are coupling constants, and Λ is the cosmological constant. It should be noted that Horava included a cross term $C_{ij}R^{ij}$, where C_{ij} is the Cotton tensor. This term scales as ℓ^5 and explicitly violates parity. To restore parity, SVW excluded this term [13].

In the IR limit, all the high order curvature terms (with coefficients g_I) drop out, and the total action reduces when $\xi = 0$ to the Einstein-Hilbert action.

Variation with respect to the lapse function $N(t)$ yields the Hamiltonian constraint,

$$\int d^3x\sqrt{g}\left(\mathcal{L}_K + \mathcal{L}_V\right) = 8\pi G\int d^3x\sqrt{g}\, J^t, \quad (2.8)$$

where

$$J^t = 2\left(N\frac{\delta\mathcal{L}_M}{\delta N} + \mathcal{L}_M\right). \quad (2.9)$$

Because of the projectability condition $N = N(t)$, the Hamiltonian constraint takes a nonlocal integral form. If one relaxes projectability and allows $N = N\left(t, x^i\right)$, then the corresponding variation with respect to N will yield a local super-Hamiltonian constraint $\mathcal{L}_K + \mathcal{L}_V = 8\pi GJ^t$. As argued in [17], this will result in an inconsistent theory. Results obtained by relaxing projectability should be treated with caution.

Variation with respect to the shift N^i yields the super-momentum constraint,

$$\nabla_j\pi^{ij} = 8\pi GJ^i, \quad (2.10)$$

where the super-momentum π^{ij} and matter current J^i are

$$\begin{aligned} \pi^{ij} &\equiv \frac{\delta\mathcal{L}_K}{\delta\dot{g}_{ij}} = -K^{ij} + (1-\xi)Kg^{ij}, \\ J^i &\equiv -N\frac{\delta\mathcal{L}_M}{\delta N_i}. \end{aligned} \quad (2.11)$$

Varying with respect to g_{ij}, on the other hand, leads to the dynamical equations,

$$\begin{aligned} \frac{1}{N\sqrt{g}}\left(\sqrt{g}\pi^{ij}\right)^{\cdot} &= -2\left(K^2\right)^{ij} + 2(1-\xi)KK^{ij} \\ &\quad + \frac{1}{N}\nabla_k\left[N^k\pi^{ij} - 2\pi^{k(i}N^{j)}\right] \\ &\quad + \frac{1}{2}\mathcal{L}_Kg^{ij} + F^{ij} + 8\pi G\tau^{ij}, \end{aligned} \quad (2.12)$$

where $\left(K^2\right)^{ij} \equiv K^{il}K_l^j,\ f_{(ij)} \equiv \left(f_{ij} + f_{ji}\right)/2$, and

$$F^{ij} \equiv \frac{1}{\sqrt{g}}\frac{\delta\left(-\sqrt{g}\mathcal{L}_V\right)}{\delta g_{ij}} = \sum_{s=0}^{8} g_s\zeta^{n_s}\left(F_s\right)^{ij}. \quad (2.13)$$

The constants are given by $g_0 = 2\Lambda\zeta^{-2}$, $g_1 = -1$, and $n_s = (2, 0, -2, -2, -4, -4, -4, -4, -4)$. The stress 3-tensor is defined as

$$\tau^{ij} = \frac{2}{\sqrt{g}}\frac{\delta\left(\sqrt{g}\mathcal{L}_M\right)}{\delta g_{ij}}, \quad (2.14)$$

and the geometric 3-tensors $(F_s)_{ij}$ are defined as follows:

$$
\begin{aligned}
(F_0)_{ij} &= -\frac{1}{2}g_{ij}, \\
(F_1)_{ij} &= R_{ij} - \frac{1}{2}Rg_{ij}, \\
(F_2)_{ij} &= 2\left(R_{ij} - \nabla_i\nabla_j\right)R - \frac{1}{2}g_{ij}\left(R - 4\nabla^2\right)R, \\
(F_3)_{ij} &= \nabla^2 R_{ij} - \left(\nabla_i\nabla_j - 3R_{ij}\right)R - 4\left(R^2\right)_{ij} \\
&\quad + \frac{1}{2}g_{ij}\left(3R_{kl}R^{kl} + \nabla^2 R - 2R^2\right), \\
(F_4)_{ij} &= 3\left(R_{ij} - \nabla_i\nabla_j\right)R^2 - \frac{1}{2}g_{ij}\left(R - 6\nabla^2\right)R^2, \\
(F_5)_{ij} &= \left(R_{ij} + \nabla_i\nabla_j\right)\left(R_{kl}R^{kl}\right) + 2R\left(R^2\right)_{ij} \\
&\quad + \nabla^2\left(RR_{ij}\right) - \nabla^k\left[\nabla_i\left(RR_{jk}\right) + \nabla_j\left(RR_{ik}\right)\right] \\
&\quad - \frac{1}{2}g_{ij}\left[\left(R - 2\nabla^2\right)\left(R_{kl}R^{kl}\right)\right. \\
&\quad \left. - 2\nabla_k\nabla_l\left(RR^{kl}\right)\right], \\
(F_6)_{ij} &= 3\left(R^3\right)_{ij} + \frac{3}{2}\left[\nabla^2\left(R^2\right)_{ij}\right. \\
&\quad \left. - \nabla^k\left(\nabla_i\left(R^2\right)_{jk} + \nabla_j\left(R^2\right)_{ik}\right)\right] \\
&\quad - \frac{1}{2}g_{ij}\left[R^k_l R^l_m R^m_k - 3\nabla_k\nabla_l\left(R^2\right)^{kl}\right], \\
(F_7)_{ij} &= 2\nabla_i\nabla_j\left(\nabla^2 R\right) - 2\left(\nabla^2 R\right)R_{ij} \\
&\quad + \left(\nabla_i R\right)\left(\nabla_j R\right) - \frac{1}{2}g_{ij}\left[\left(\nabla R\right)^2 + 4\nabla^4 R\right], \\
(F_8)_{ij} &= \nabla^4 R_{ij} - \nabla_k\left(\nabla_i\nabla^2 R^k_j + \nabla_j\nabla^2 R^k_i\right) \\
&\quad - \left(\nabla_i R^k_l\right)\left(\nabla_j R^l_k\right) - 2\left(\nabla^k R^l_i\right)\left(\nabla_k R_{jl}\right) \\
&\quad - \frac{1}{2}g_{ij}\left[\left(\nabla_k R_{lm}\right)^2 - 2\left(\nabla_k\nabla_l\nabla^2 R^{kl}\right)\right]. \qquad (2.15)
\end{aligned}
$$

The matter quantities $(J^t,\ J^i,\ \tau^{ij})$ satisfy the conservation laws [14],

$$
\int d^3x\sqrt{g}\left[\dot{g}_{kl}\tau^{kl} - \frac{1}{\sqrt{g}}\left(\sqrt{g}J^t\right)^{\cdot} + \frac{2N_k}{N\sqrt{g}}\left(\sqrt{g}J^k\right)^{\cdot}\right] = 0, \qquad (2.16)
$$

$$
\nabla^k\tau_{ik} - \frac{1}{N\sqrt{g}}\left(\sqrt{g}J_i\right)^{\cdot} - \frac{N_i}{N}\nabla_k J^k - \frac{J^k}{N}\left(\nabla_k N_i - \nabla_i N_k\right) = 0. \qquad (2.17)
$$

III. COSMOLOGICAL BACKGROUND

The homogeneous and isotropic universe is described by the FRW metric, $ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j$ where $\gamma_{ij} = \left(1 + \frac{1}{4}kr^2\right)^{-2}\delta_{ij}$, with $k = 0, \pm 1$. For this metric, $\bar{K}_{ij} = -a^2 H\gamma_{ij}$ and $\bar{R}_{ij} = 2k\gamma_{ij}$, where $H = \dot{a}/a$ and an overbar denotes a background quantity. Then we find that

$$
\begin{aligned}
\bar{\mathcal{L}}_K &= 3\left(3\xi - 2\right)H^2, \\
\bar{\mathcal{L}}_V &= 2\Lambda - \frac{6k}{a^2} + \frac{12\beta_1 k^2}{a^4} + \frac{24\beta_2 k^3}{a^6}, \qquad (3.1)
\end{aligned}
$$

where $\beta_1 = \zeta^{-2}\left(3g_2 + g_3\right)$ and $\beta_2 = \zeta^{-4}\left(9g_4 + 3g_5 + g_6\right)$.

Because of the spatial homogeneity, both $\bar{\mathcal{L}}_K$ and $\bar{\mathcal{L}}_V$ are independent of the spatial coordinates, and the matter quantities are

$$
\bar{J}^t = -2\bar{\rho},\quad \bar{J}^i = 0,\quad \bar{\tau}_{ij} = \bar{p}\,\bar{g}_{ij}, \qquad (3.2)
$$

where $\bar{\rho}$ and $\bar{p}$ are the total density and pressure. Then the Hamiltonian constraint (2.8) reduces to the super-Hamiltonian constraint, $\bar{\mathcal{L}}_K(t) + \bar{\mathcal{L}}_V(t) = 8\pi G\bar{J}^t(t)$, which leads to the modified Friedmann equation,

$$
\left(1 - \frac{3}{2}\xi\right)H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\bar{\rho} + \frac{\Lambda}{3} + \frac{2\beta_1 k^2}{a^4} + \frac{4\beta_2 k^3}{a^6}. \qquad (3.3)
$$

From Eqs. (2.11) and (2.13) we find that

$$
\begin{aligned}
\bar{F}^{ij} &= \left(-\Lambda + \frac{k}{a^2} + \frac{2\beta_1 k^2}{a^4} + \frac{12\beta_2 k^3}{a^6}\right)\bar{g}^{ij}, \\
\bar{\pi}^{ij} &= \left(3\xi - 2\right)H\,\bar{g}^{ij}. \qquad (3.4)
\end{aligned}
$$

Then the dynamical equation (2.12) reduces to [13]

$$
\left(2 - 3\xi\right)\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left(\bar{\rho} + 3\bar{p}\right) + \frac{2}{3}\Lambda - \frac{4\beta_1 k^2}{a^4} - \frac{16\beta_2 k^3}{a^6}. \qquad (3.5)
$$

Similarly to general relativity, the super-momentum constraint (2.10) is then satisfied identically, since $\bar{J}^i = 0$ and, from Eq. (3.4), $\vec{\nabla}_j\pi^{ij} \equiv \bar{\pi}^{ij}{}_{|j} = 0$, where $\vec{\nabla}_i$ denotes the covariant derivative with respect to γ_{ij}.

Using Eqs. (3.3) and (3.5), it follows that in the background the matter satisfies the same conservation law as in general relativity,

$$
\dot{\bar{\rho}} + 3H\left(\bar{\rho} + \bar{p}\right) = 0. \qquad (3.6)
$$

This can be also obtained from Eq. (2.16), while Eq. (2.17) is satisfied identically.

In deriving Eq. (3.3) we followed the usual assumption that the whole FRW universe is homogeneous and isotropic. In [11], it was argued that such an assumption might be too strong. If one relaxes the assumption and requires that only the observed patch of our universe is homogeneous and isotropic, one can introduce the notion of "dark matter as an integration constant" of the Hamiltonian constraint (2.8): $\bar{\rho}(t)$ in Eqs. (3.3) and (3.6) can be replaced by $\bar{\rho}(t) + \mathcal{E}(t)$ in the observable patch, where $\mathcal{E}(t) = \text{const}/a^3$ in the IR limit [11, 19]. Beyond the observable patch, $\mathcal{E}$ is necessarily inhomogeneous. In order to analyze perturbations on an FRW background, one needs to restrict the perturbations to the observable patch, which then raises issues about matching across the boundary of the observable patch. In our approach, the background is a homogeneous FRW spacetime, so that $\mathcal{E} = 0$ in the background.

IV. COSMOLOGICAL PERTURBATIONS

Linear perturbations of the metric give

$$\begin{aligned} \delta g_{ij} &= a^2(\eta) h_{ij}\left(\eta, x^k\right), \\ \delta N^i &= n^i\left(\eta, x^k\right), \ \delta N = a(\eta) n(\eta), \end{aligned} \tag{4.1}$$

where η is the conformal time. We decompose into scalar, vector and tensor modes [20],

$$\begin{aligned} n &= \phi, \quad n_i = B_{|i} - S_i, \\ h_{ij} &= -2\psi\gamma_{ij} + 2E_{|ij} + F_{i|j} + F_{j|i} + H_{ij}, \end{aligned} \tag{4.2}$$

Note that ϕ is a function of η only, while $B,\ S_i,\ \psi,\ E,\ F$ and H_{ij} are in general functions of both η and x^k, with the constraints,

$$S_i{}^{|i} = 0, \ \ F_i{}^{|i} = 0, \ \ H^i_i = 0 = H_{ij}{}^{|j}. \tag{4.3}$$

The perturbed energy quantity Eq. (2.9) is written as

$$\delta J^t = -2\delta\mu. \tag{4.4}$$

In general relativity, $\delta\mu$ reduces to the density perturbation $\delta\rho$.

The perturbed matter current in Eq. (2.11), on the other hand, decomposes as

$$\delta J^i = \frac{1}{a^2}\left(q^{|i} + q^i\right), \quad q^i{}_{|i} = 0, \tag{4.5}$$

and the perturbed stress tensor Eq. (2.14) decomposes as

$$\begin{aligned} &\delta\tau^{ij} = \frac{1}{a^2}\Big[\left(\delta\mathcal{P} + 2\bar{p}\psi\right)\gamma^{ij} + \Pi^{|\langle ij\rangle} + 2\Pi^{(i|j)} + \Pi^{ij}\Big], \\ &\Pi^i{}_{|i} = 0, \ \Pi^i_i = 0, \ \Pi^{ij}{}_{|j} = 0. \end{aligned} \tag{4.6}$$

The angled brackets on indices define the trace-free part:

$$f_{|\langle ij\rangle} \equiv f_{|ij} - \frac{1}{3}\gamma_{ij} f_{|k}{}^{|k}. \tag{4.7}$$

In general relativity, $q^{|i}$ and q^i reduce to the scalar and vector modes of the momentum perturbation $-a(\bar{\rho} + \bar{p})(v^{|i} + B^{|i} + v^i - S^i)$, while $\delta\mathcal{P}$ reduces to the pressure perturbation δp, and Π, Π^i and Π^{ij} reduce to the scalar, vector and tensor modes of the anisotropic pressure.

A. Gauge Transformations

Consider a gauge transformation as in Eq. (2.4), with

$$\chi^0 = \xi^0, \quad \chi^i = \xi^{|i} + \xi^i, \ \ \xi^i{}_{|i} = 0, \tag{4.8}$$

where $\xi^0 = \xi^0(\eta),\ \xi^i = \xi^i\left(\eta, x^k\right),\ \xi = \xi\left(\eta, x^k\right)$. Then the metric perturbations in Eq. (4.2) transform as

$$\begin{aligned} &\tilde{\phi} = \phi - \mathcal{H}\xi^0 - \xi^{0\prime}, \ \ \tilde{\psi} = \psi + \mathcal{H}\xi^0, \\ &\tilde{B} = B + \xi^0 - \xi', \ \ \tilde{E} = E - \xi, \\ &\tilde{S}_i = S_i + \xi_i', \ \tilde{F}_i = F_i - \xi_i, \ \tilde{H}_{ij} = H_{ij}, \end{aligned} \tag{4.9}$$

where $\mathcal{H} = a'/a$ and a prime denotes $\partial/\partial\eta$. Note that these gauge transformations are precisely the standard forms given in GR. The only difference is that in the HL case, ϕ and ξ^0 are homogeneous. We can omit ξ^0 from $\tilde{B}$, since only the gradient of $\tilde{B}$ occurs in the metric. However, we are free to maintain the ξ^0 term – and we do this in order that we can use the standard form of the gauge-invariant Bardeen potentials Φ, Ψ – see Eq. (4.14) below. Using the gauge freedom, we can restrict some of the quantities defined in Eq. (4.2).

Synchronous Gauge

This gauge is defined by

$$\tilde{\phi} = 0, \quad \tilde{B} = 0, \quad \tilde{S}_i = 0, \tag{4.10}$$

and from Eqs. (4.9) we find that

$$\begin{aligned} &\xi^0 = \frac{1}{a}\int a\phi d\eta + \frac{C_0}{a}, \ \xi_i = \int S_i d\eta + C_i(x), \\ &\xi = \int B d\eta + \int \frac{d\eta}{a}\left(\int a\phi d\eta\right) + C(x), \end{aligned} \tag{4.11}$$

where $C(x)$ and $C_i(x)$ are arbitrary functions of x^k with $C_i{}^{|i} = 0$, and C_0 is an arbitrary constant. Therefore, as in general relativity, this gauge does not completely fix all the gauge degrees of freedom. This gauge was used to study the scalar graviton mode in [13].

Quasi-longitudinal Gauge

In general relativity, the longitudinal gauge is defined by $\tilde{B} = \tilde{E} = \tilde{F}_i = 0$ [20]. However, due to the projectability condition, we see from Eq. (4.9) that we cannot set all 3 quantities to zero, although we are still free to set $\tilde{E} = 0$ and $\tilde{F}_i = 0$. In addition, using the remaining degree of freedom, we can further set $\tilde{\phi} = 0$. Thus we can set

$$\tilde{\phi} = 0, \ \ \tilde{E} = 0, \ \ \tilde{F}_i = 0, \tag{4.12}$$

with

$$\xi^0 = \frac{1}{a}\int a\phi d\eta + \frac{C_0}{a}, \;\; \xi = E, \; \xi_i = F_i, \qquad (4.13)$$

which are unique up to a constant C_0. We call this the quasi-longitudinal gauge (it has been used by [8, 15] in the case where projectability is abandoned, and $k = 0$).

It should be noted that, as in general relativity, in each of these gauges only two scalars are left, and we can define the same gauge-invariant potentials as in general relativity [20]

$$\begin{aligned} \Phi &= \phi + \mathcal{H}\left(B - E'\right) + \left(B - E'\right)', \\ \Psi &= \psi - \mathcal{H}\left(B - E'\right). \end{aligned} \qquad (4.14)$$

(Note that in [15] a different set of gauge-invariant variables was used.)

B. Scalar Perturbations in Quasi-longitudinal Gauge

In the quasi-longitudinal gauge, the metric scalar perturbations are given by

$$ds^2 = a^2\left[-d\eta^2 + 2B_{|i}dx^i d\eta + (1-2\psi)\gamma_{ij}dx^i dx^j\right]. \qquad (4.15)$$

Then from Eqs. (2.6) and (2.7), we find that

$$\begin{aligned} K_{ij} &= \bar{K}_{ij} + a\left[B_{|ij} + (\psi' + 2\mathcal{H}\psi)\gamma_{ij}\right], \\ K &= \bar{K} + a^{-1}\left(\vec{\nabla}^2 B + 3\psi'\right), \\ K^{ij} &= \bar{K}^{ij} + a^{-3}\left[B^{|ij} + (\psi' - 2\mathcal{H}\psi)\gamma^{ij}\right], \\ \mathcal{L}_K &= \bar{\mathcal{L}}_K + 2\mathcal{H}a^{-2}(2-3\xi)\left(\vec{\nabla}^2 B + 3\psi'\right), \end{aligned} \qquad (4.16)$$

and

$$\begin{aligned} \mathcal{L}_V &= \bar{\mathcal{L}}_V - \frac{4}{a^2}\left(1 - \frac{4\beta_1 k}{a^2}\right)\left(\vec{\nabla}^2 + 3k\right)\psi \\ &\quad + \frac{48\beta_2 k^2}{a^6}\left(\vec{\nabla}^2 + 3k\right)\psi \\ &\quad + \frac{24 g_7 k}{\zeta^4 a^6}\vec{\nabla}^2\left(\vec{\nabla}^2 + 3k\right)\psi. \end{aligned} \qquad (4.17)$$

To first-order the Hamiltonian constraint (2.8) is

$$\int d^3x\sqrt{\gamma}\left(\delta\mathcal{L}_K + \delta\mathcal{L}_V\right) = -16\pi G\int d^3x\sqrt{\gamma}\,\delta\mu. \qquad (4.18)$$

Using Eqs. (4.16) and (4.17) we find that

$$\begin{aligned} \int \sqrt{\gamma}d^3x\Bigg[&\left(\vec{\nabla}^2 + 3k\right)\psi - \frac{(2-3\xi)\mathcal{H}}{2}\left(\vec{\nabla}^2 B + 3\psi'\right) \\ &- 2k\Big(\frac{2\beta_1}{a^2} + \frac{6\beta_2 k}{a^4} + \frac{3g_7}{\zeta^4 a^4}\vec{\nabla}^2\Big)\left(\vec{\nabla}^2 + 3k\right)\psi \\ &- 4\pi G a^2\delta\mu\Bigg] = 0. \end{aligned} \qquad (4.19)$$

The integrand is a generalization of the general relativity Poisson equation [20]. Note that the Laplacian terms can be dropped from this equation, using the identity,

$$\int d^3x\sqrt{\gamma}\,\vec{\nabla}^2 f = 0. \qquad (4.20)$$

At first-order the supermomentum constraint (2.10) is

$$\Big[(2-3\xi)\,\psi' - 2kB - \xi\vec{\nabla}^2 B\Big]_{|i} = 8\pi G a\, q_{|i}\,, \qquad (4.21)$$

which generalizes the general relativity $0i$ constraint [20]. Note that in the general relativity limit ($\xi = 0$) and in a Minkowski background ($q = 0 = k$), Eq. (4.21) implies

$$\psi = G(x), \qquad (4.22)$$

where we used the Hamiltonian constraint (4.19) to set a homogeneous function of integration to zero. This result is closely related to the fact that the spin-0 scalar mode of the graviton becomes stabilized in the limit $\xi = 0$, as we show in the next section.

The perturbed dynamical equations require the perturbed $(F_s)_{ij}$ of Eq. (2.15). The results are given by Eq. (A.1) in the Appendix. Using Eqs. (A.2) and (4.6) in Eq. (2.12), we can find the perturbed dynamical equations. The trace part gives

$$\begin{aligned} \psi'' &+ 2\mathcal{H}\psi' - \mathcal{F}\psi - \frac{1}{3(2-3\xi)}\gamma^{ij}\delta F_{ij} \\ &+ \frac{1}{3}\left(\vec{\nabla}^2 B' + 2\mathcal{H}\vec{\nabla}^2 B\right) = \frac{8\pi G a^2}{(2-3\xi)}\delta\mathcal{P}. \end{aligned} \qquad (4.23)$$

Here $\delta F_{ij} = \sum g_s\zeta^{n_s}\delta(F_s)_{ij}$, with $\delta(F_s)_{ij}$ given by Eq. (A.1), and $\mathcal{F}$ is defined as

$$\mathcal{F} = \frac{2a^2}{(2-3\xi)}\left(-\Lambda + \frac{k}{a^2} + \frac{2\beta_1 k^2}{a^4} + \frac{12\beta_2 k^3}{a^6}\right). \qquad (4.24)$$

The trace-free part is

$$B'_{|\langle ij\rangle} + 2\mathcal{H}B_{|\langle ij\rangle} + \delta F_{\langle ij\rangle} = -8\pi G a^2\Pi_{|\langle ij\rangle}. \qquad (4.25)$$

These two equations generalize the general relativity ij perturbed field equations [20].

The perturbed parts of the conservation laws (2.16) and (2.17) give

$$\int \sqrt{\gamma}d^3x\Big[\delta\mu' + 3\mathcal{H}\left(\delta\mathcal{P} + \delta\mu\right) - 3\left(\bar{\rho} + \bar{p}\right)\psi'\Big] = 0, \qquad (4.26)$$

$$\left[q' + 3\mathcal{H}q - a\delta\mathcal{P} - \frac{2a}{3}\left(\vec{\nabla}^2 + 3k\right)\Pi\right]_{|i} = 0. \qquad (4.27)$$

The energy conservation equation is an integrated generalization of the general relativity energy equation, and the momentum equation generalizes the general relativity momentum equation [20].

V. SCALAR GRAVITON ON MINKOWSKI BACKGROUND

In the general relativity limit on a Minkowski background, the scalar graviton mode should be suppressed. Otherwise the recovery of general relativity would be obstructed. By contrast, when $\xi \neq 0$, we expect the scalar mode may play a significant role.

We set $a = 1$, $k = 0 = \Lambda$, $J^t = 0 = J^i$ and $\tau^{ij} = 0$. Then

$$\begin{aligned} \delta F_{ij} &= -\left(1 + \alpha_1 \vec{\nabla}^2 + \alpha_2 \vec{\nabla}^4\right)\left(\psi_{,ij} - \delta_{ij}\vec{\nabla}^2\psi\right), \\ \delta\mathcal{L}_K &= 0, \quad \delta\mathcal{L}_V = -4\vec{\nabla}^2\psi, \end{aligned} \tag{5.1}$$

where $\alpha_1 \equiv \zeta^{-2}(8g_2 + 3g_3)$, $\alpha_2 \equiv -\zeta^{-4}(8g_7 - 3g_8)$. Note that $\psi = \Psi$ since $\mathcal{H} = 0$, so that ψ is gauge invariant. The Hamiltonian constraint (4.18) is satisfied identically. (It is interesting to note that if the projectability condition is given up, the Hamiltonian constraint becomes the super-Hamiltonian constraint $\delta\mathcal{L}_K + \delta\mathcal{L}_V = 0$, which gives the strong condition $\vec{\nabla}^2\psi = 0$.)

The super-momentum constraint (4.21) gives

$$(2 - 3\xi)\,\dot{\psi} - \xi\vec{\nabla}^2 B = 0. \tag{5.2}$$

Substituting Eq. (5.1) into the dynamical equations (4.23) with $f = 0$, we find

$$(2 - 3\xi)\left(3\dot{\psi} + \vec{\nabla}^2 B\right)^{\cdot} = 2\left(1 + \alpha_1\vec{\nabla}^2 + \alpha_2\vec{\nabla}^4\right)\vec{\nabla}^2\psi. \tag{5.3}$$

We consider the cases $\xi \neq 0$ and $\xi = 0$ (general relativity limit) separately.

A. $\xi \neq 0$

From Eqs. (5.2) and (5.3) we obtain the wave equation

$$\ddot{\psi} - c_\psi^2\left(1 + \alpha_1\vec{\nabla}^2 + \alpha_2\vec{\nabla}^4\right)\vec{\nabla}^2\psi = 0, \tag{5.4}$$

where $c_\psi^2 \equiv \xi/(2 - 3\xi)$. In Fourier space,

$$\ddot{\psi}_n + \omega_n^2\psi_n = 0, \quad \omega_n^2 \equiv n^2 c_\psi^2\left(1 - \alpha_1 n^2 + \alpha_2 n^4\right), \tag{5.5}$$

where n is the wave-number. It is clear that the solution is stable in the IR limit, provided that $0 \leq \xi \leq 2/3$, which is equivalent to $1/3 \leq \lambda \leq 1$, where $\lambda = 1 - \xi$ is the parameter used in [1]. This is in contrast to the conclusions obtained in [21], in which it was found that ψ is not stable for any choice of ξ. The main reason is that in [21] the authors considered the case with detailed balance, or at most with 'soft' breaking (and not full breaking) of detailed balance. With the most general breaking of detailed balance [13], we find that to have stability in the IR, it is necessary that $0 \leq \xi \leq 2/3$ (or $1/3 \leq \lambda \leq 1$).

In addition, ψ is also stable in the UV regime for $\alpha_2 > 0$. It is stable in intermediate regimes provided that either (a) $\alpha_2 > 0$ and $\alpha_1 \leq 0$ or $\alpha_1^2 < 4\alpha_2$; or (b) $\alpha_2 = 0$ and $\alpha_1 \leq 0$.

B. $\xi = 0$

When $\xi = 0$, from Eq. (4.22) we find that $\psi(t, x) = G(x)$. Inserting this into Eq. (5.3) and integrating,

$$\vec{\nabla}^2 B = H(x) + t\left(1 + \alpha_1\vec{\nabla}^2 + \alpha_2\vec{\nabla}^4\right)\vec{\nabla}^2 G(x), \tag{5.6}$$

where $H(x)$ is an arbitrary integration function. Thus it appears that the scalar graviton has a growing mode $\propto t$. However, it is important to note that B is not gauge-invariant and therefore B does not directly determine the stability of the spin-0 scalar graviton mode. The gauge-invariant variables defined by Eq. (4.14) are in this case

$$\Phi = \dot{B} = I(x), \qquad \Psi = \psi = G(x). \tag{5.7}$$

Here $I(x)$ is determined by Eq. (5.6): in Fourier space, $I_n = (1 - \alpha_1 n^2 + \alpha_2 n^4)G_n$.

Clearly, neither of the gauge-invariant variables is growing with time. As a result, the spin-0 scalar graviton is indeed stable in the general relativity limit ($\xi = 0$) on a Minkowski background.

This conclusion appears to contradict the one obtained by SVW [13]. However, a closer analysis shows that in terms of gauge-invariant variables, the results are consistent. The synchronous gauge variables used in [13] are ψ and

$$h = -6\psi + \vec{\nabla}^2 E, \quad B = 0, \tag{5.8}$$

and they find that

$$E = L(x) + M(x)t + Q(x)t^2. \tag{5.9}$$

The scalar mode appears to be growing because h is. However, by Eq. (4.14), the gauge-invariant variables for the SVW solution are $\Phi = -\ddot{E} = -2Q(x)$ and $\Psi = \psi = G(x)$ – neither of which is growing.

Our conclusion is also consistent with the results obtained recently by Mukohyama [11].

It is interesting to note that the coupling of the spin-0 scalar graviton to a dust fluid on a Minkowski background does not alter this conclusion. In fact, one can show that ψ and B will satisfy the same equations as above in both cases, $\xi \neq 0$ and $\xi = 0$. The only difference is that now the Hamiltonian constraint (4.19) requires the matter energy quantity to satisfy the condition $\int d^3x\,\delta\mu = 0$.

VI. SCALAR PERTURBATIONS OF THE FLAT FRW MODEL

We return now to an FRW background. In the flat case, $k = 0$, we find that the perturbation equations simplify considerably.

The super-momentum constraint (4.21) reduces to

$$(2-3\xi)\psi' = \xi\vec{\nabla}^2 B + 8\pi G a q. \tag{6.1}$$

Integrating it over space and using the Hamiltonian constraint (4.19), we find

$$\int d^3x\,(3\mathcal{H}q + a\delta\mu) = 0. \tag{6.2}$$

When $k=0$, we also have

$$\begin{aligned}\delta F_{ij} &= 2\Lambda a^2\psi\gamma_{ij}\\ &\quad - \left(1+\frac{\alpha_1}{a^2}\vec{\nabla}^2+\frac{\alpha_2}{a^4}\vec{\nabla}^4\right)\left(\vec{\nabla}_i\vec{\nabla}_j-\delta_{ij}\vec{\nabla}^2\right)\psi. \end{aligned}\tag{6.3}$$

Then the trace-free dynamical equation (4.25) gives

$$\left(a^2B\right)' = \left(a^2+\alpha_1\vec{\nabla}^2+\frac{\alpha_2}{a^2}\vec{\nabla}^4\right)\psi - 8\pi G a^4\Pi, \tag{6.4}$$

while the trace equation (4.23) reduces to

$$\begin{aligned}\psi'' &+ 2\mathcal{H}\psi' - \frac{\xi}{2-3\xi}\left(1+\frac{\alpha_1}{a^2}\vec{\nabla}^2+\frac{\alpha_2}{a^4}\vec{\nabla}^4\right)\vec{\nabla}^2\psi\\ &= \frac{8\pi G a^2}{3(2-3\xi)}\Big[3\delta\mathcal{P}+(2-3\xi)\vec{\nabla}^2\Pi\Big]. \end{aligned}\tag{6.5}$$

The conservation laws Eqs. (4.26) and (4.26) reduce to

$$\int d^3x\Big[\delta\mu' + 3\mathcal{H}\left(\delta\mathcal{P}+\delta\mu\right) - 3\left(\bar\rho+\bar p\right)\psi'\Big] = 0, \tag{6.6}$$

$$q' + 3\mathcal{H}q = a\delta\mathcal{P} + \frac{2}{3}a\vec{\nabla}^2\Pi. \tag{6.7}$$

Note that not all the equations are independent. Equation (6.5) can be derived from Eqs. (6.1), (6.4) and (6.7). Therefore, we are left with three first-order evolution equations, (6.1), (6.4) and (6.7), and two integral constraints, Eqs. (6.2) and (6.6), for the six unknowns, $\psi, B, \delta\mu, \delta\mathcal{P}, q$ and Π.

In terms of the gauge-invariant variables defined in Eq. (4.14), we can rewrite Eq. (6.4) as

$$\Phi - \Psi = -8\pi G a^2\Pi + \frac{1}{a^2}\left(\alpha_1+\frac{\alpha_2}{a^2}\vec{\nabla}^2\right)\vec{\nabla}^2\psi. \tag{6.8}$$

The last term on the right acts as an effective anisotropic stress from HL gravity, i.e. from the higher-order curvature terms:

$$\Pi_{\rm grav} = -\frac{1}{8\pi G a^4}\left(\alpha_1+\frac{\alpha_2}{a^2}\vec{\nabla}^2\right)\vec{\nabla}^2\psi. \tag{6.9}$$

This stress is strongest on small scales, and is suppressed on large scales. It might provide a signal to distinguish the HL theory from general relativity.

VII. CONCLUSIONS

In this paper, we systematically studied the linear scalar perturbations of the FRW models in the SVW setup [13], which is the most general theory of HL type [1] when detailed balance is abandoned, but projectability is maintained (and so is parity).

We generalized [13], who considered only a vacuum Minkowski background. In addition to generalizing the geometrical terms, we included matter and derived the conservation laws. We have not specified the type of matter, as it is still an open question how to construct the matter Lagrangian $\mathcal{L}_M$, although scalar and vector fields have recently been studied [4, 5].

Working in the quasi-longitudinal gauge, we obtained explicitly the perturbed Hamiltonian constraint (4.19), the super-momentum constraint (4.21), and the dynamical equations (4.23) and (4.25). The perturbed conservation laws are given by Eqs. (4.26) and (4.26).

A crucial issue in the HL theory and its generalizations is the spin-0 scalar graviton mode. By specializing the FRW background to its Minkowski limit, we showed via a gauge-invariant treatment that this mode is stable in the IR limit for $0 \le \xi \le 2/3$. It is also stable in the UV regime, provided that the arbitrary coupling constants g_7 and g_8 are suitably chosen. The apparent contradiction with the results of [13] is resolved via a gauge-invariant reformulation of their results. This is consistent with the results of Mukohyama [11]. We also showed that this conclusion is true when coupling the scalar graviton to a dust fluid in Minkowski spacetime. This result is also different from the one obtained in [21], in which it was shown that the scalar mode is not stable for any given ξ. The main reason is that in [21] the authors considered the case with detailed balance, or at most 'soft' breaking of detailed balance.

The stability condition $0 \le \xi \le 2/3$ has the unwanted consequence that the scalar mode is a ghost [1, 11, 16, 18]. To tackle this problem, one may consider the theory in the range $\xi < 0$ and then try to remove the instability of the scalar mode via the Vainshtein mechanism [22].

Our general formulas for the FRW background provide the basis for further work to analyze cosmological tensor perturbations, inflationary perturbations and large-scale structure formation in the framework of the generalized HL theory. We showed that there is an effective gravitational contribution to the anisotropic stress on small scales, Eq. (6.8), so that in HL theory we have $\Phi \neq \Psi$ even in the absence of matter anisotropic stresses.

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Appendix A: Perturbed $(F_s)_{ij}$

To first-order, the $(F_s)_{ij}$ are given by

$$
\begin{aligned}
(F_0)_{ij} &= -\frac{1}{2}a^2\gamma_{ij} + a^2\psi\gamma_{ij}, \\
(F_1)_{ij} &= -k\gamma_{ij} + \left[\psi_{|ij} - \left(\vec{\nabla}^2\psi\right)\gamma_{ij}\right], \\
(F_2)_{ij} &= \frac{6k^2}{a^2}\gamma_{ij} + \frac{4k}{a^2}\left[3\psi_{|ij} + \gamma_{ij}\left(\vec{\nabla}^2 + 3k\right)\psi\right] \\
&\quad -\frac{8}{a^2}\left(\vec{\nabla}_i\vec{\nabla}_j - \gamma_{ij}\vec{\nabla}^2\right)\left(\vec{\nabla}^2 + 3k\right)\psi, \\
(F_3)_{ij} &= \frac{2k^2}{a^2}\gamma_{ij} + \frac{2k}{a^2}\gamma_{ij}\left(\vec{\nabla}^2 + 2k\right)\psi \\
&\quad -\frac{1}{a^2}\left(3\vec{\nabla}^2 + 10k\right)\left(\psi_{|ij} - \gamma_{ij}\vec{\nabla}^2\psi\right) \\
&\quad +\frac{4}{a^2}\left[\vec{\nabla}^2\left(\psi_{|ij}\right) - \left(\vec{\nabla}^2\psi\right)_{|ij}\right], \\
(F_4)_{ij} &= \frac{108k^3}{a^4}\gamma_{ij} + \frac{36k^2}{a^4}\left[3\psi_{|ij} + \gamma_{ij}\left(5\vec{\nabla}^2 + 12k\right)\psi\right] \\
&\quad -\frac{144k}{a^4}\left(\vec{\nabla}_i\vec{\nabla}_j - \gamma_{ij}\vec{\nabla}^2\right)\left(\vec{\nabla}^2 + 3k\right)\psi, \\
(F_5)_{ij} &= \frac{36k^3}{a^4}\gamma_{ij} + \frac{24k^2}{a^4}\left[\psi_{|ij} + 2\gamma_{ij}\left(\vec{\nabla}^2 + 3k\right)\psi\right] \\
&\quad -\frac{2k}{a^4}\left[3\vec{\nabla}^2\left(\vec{\nabla}_i\vec{\nabla}_j - 3\gamma_{ij}\vec{\nabla}^2\right)\psi\right. \\
&\qquad +6\left(\vec{\nabla}_i\vec{\nabla}_j - 3\gamma_{ij}\vec{\nabla}^2\right)\vec{\nabla}^2\psi \\
&\qquad \left. +2\gamma_{ij}\left(\vec{\nabla}^2 - 37k\right)\vec{\nabla}^2\psi\right], \\
(F_6)_{ij} &= \frac{12k^3}{a^4}\gamma_{ij} + \frac{12k^2}{a^4}\left(\vec{\nabla}^2 + 4k\right)\psi\gamma_{ij} \\
&\quad -\frac{6k}{a^4}\left[2\left(\vec{\nabla}^2\psi\right)_{|ij} + \vec{\nabla}^2\left(\psi_{|ij}\right)\right. \\
&\qquad \left. -\gamma_{ij}\vec{\nabla}^2\left(3\vec{\nabla}^2 + 8k\right)\psi\right], \\
(F_7)_{ij} &= \frac{8}{a^4}\left\{\left[\left(\vec{\nabla}^2 + 3k\right)\vec{\nabla}^2\psi\right]_{|ij}\right. \\
&\qquad \left. -\gamma_{ij}\left(\vec{\nabla}^2 + 3k\right)\left(\vec{\nabla}^2 + 2k\right)\vec{\nabla}^2\psi\right\}, \\
(F_8)_{ij} &= \frac{1}{a^4}\left\{\vec{\nabla}^4\left(\psi_{|ij}\right) - \vec{\nabla}^k\vec{\nabla}_i\vec{\nabla}^2\left(\psi_{|jk}\right)\right. \\
&\qquad -\vec{\nabla}^k\vec{\nabla}_j\vec{\nabla}^2\left(\psi_{|ik}\right) - 2\vec{\nabla}_i\vec{\nabla}_j\vec{\nabla}^2\left(\vec{\nabla}^2 + 4k\right)\psi \\
&\qquad +2\gamma_{ij}\vec{\nabla}^4\left(\vec{\nabla}^2 + 4k\right)\psi \\
&\qquad \left. +\gamma_{ij}\vec{\nabla}^k\vec{\nabla}^l\vec{\nabla}^2\left(\psi_{|kl}\right)\right\},
\end{aligned}
\tag{A.1}
$$

where $\vec{\nabla}^k\psi \equiv \psi^{|k}$. In addition, we also have

$$
\begin{aligned}
&\frac{1}{N\sqrt{g}}\left(\sqrt{g}\pi^{ij}\right)' = -\left(2-3\xi\right)\frac{\ddot{a}}{a^3}\gamma^{ij} \\
&\quad +\frac{1}{a^3}\left\{\left(2-3\xi\right)\gamma^{ij}\left(a\ddot{\psi} + 2\dot{a}\dot{\psi} - 2\ddot{a}\psi\right)\right. \\
&\quad \left. -\left[\dot{B}^{|ij} - \left(1-\xi\right)\gamma^{ij}\vec{\nabla}^2\dot{B}\right]\right\}, \\
&\left(K^2\right)^{ij} - \left(1-\xi\right)KK^{ij} = \left(3\xi-2\right)\frac{H^2}{a^2}\gamma^{ij} \\
&\quad +\frac{H}{a^3}\left\{\left(1-3\xi\right)B^{|ij} + \left(1-\xi\right)\gamma^{ij}\vec{\nabla}^2B\right. \\
&\quad \left. +2\left(2-3\xi\right)a\left(\dot{\psi} - H\psi\right)\gamma^{ij}\right\}, \\
&\mathcal{L}_K g^{ij} = -\frac{3(2-3\xi)H^2}{a^2}\gamma^{ij} \\
&\quad +\frac{2(2-3\xi)H}{a^3}\left[\vec{\nabla}^2B + 3a\left(\dot{\psi} - H\psi\right)\right]\gamma^{ij}, \\
&N^k\nabla_k\pi^{ij} + 2\pi^{k(i}\nabla_k N^{j)} + \pi^{ij}\nabla_k N^k \\
&\quad = \frac{(2-3\xi)H}{a^2}\left(2B^{|ij} - \gamma^{ij}\vec{\nabla}^2B\right).
\end{aligned}
\tag{A.2}
$$

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