

Non-comparative and comparative advertising in oligopolistic markets *

Maria Alipranti Evangelos Mitrokostas Emmanuel Petrakis[†]

Abstract

We study firms' advertising strategies in an oligopolistic market in which both non-comparative and comparative advertising are present. We show that in equilibrium firms mix over the two types of advertising, with the intensity of comparative advertising exceeding that of non-comparative advertising; moreover, that the intensity of comparative increases relatively to non-comparative advertising as market competition intensifies. Interestingly, the use of comparative advertising may lead to higher consumers' surplus and welfare in a mixed advertising market than in the absence of advertising or when either comparative or non-comparative advertising is not present.

JEL Classification: L13, M37.

Keywords: Comparative Advertising, Non-comparative advertising, Oligopoly, Product Differentiation.

*Acknowledgements: We thank two anonymous referees and the editor for their useful comments and suggestions. We also thank Simon Anderson and all the participants at 8th Conference on Research on Economic Theory and Econometrics, Tinos 2009, ASSET Meetings 2009 at Istanbul, XXV Jornadas de Economia Industrial, Madrid 2010 and III Conference on the Economics of Advertising and Marketing, Barcelona 2010, for their helpful comments and suggestions. Full responsibility for all shortcomings is ours.

[†]Maria Alipranti: Düsseldorf Institute for Competition Economics (DICE), Heinrich-Heine University of Düsseldorf, e-mail: alipranti@dice.hhu.de; Evangelos Mitrokostas: Economics and Finance Subject Group, Portsmouth Business School, University of Portsmouth, e-mail: evangelos.mitrokostas@port.ac.uk; Emmanuel Petrakis (corresponding author): Department of Economics, University of Crete, Univ. Campus at Gallos, Rethymnon 74100, Greece, tel: +302831077409, e-mail: petrakis@uoc.gr.

1 Introduction

Comparative advertising, “the advertising that compares alternative brands on objectively measurable attributes or price, and identifies the alternative brand by name, illustration or other distinctive information”,¹ is a widespread marketing practise met across various industries.² According to empirical observations in the U.S. market comparative advertising rates among 40% to 60% of total advertising (see e.g., Muehling et al., 1990; Pechmann and Stewart, 1990).³ Recent empirical evidence suggests that firms use both non-comparative and comparative advertising to approach consumers (see e.g., Anderson et al., 2013, 2015; Liaukonyte, 2012). For instance, Liaukonyte (2012) shows that in the U.S. over-the-counter analgetics market, Aleve devoted up to 90% of its total advertising in comparative ads and the rest in non-comparative ads, while the proportions for its competitors, Advil and Tylenol, were 70% and 26%, respectively. Consequently, one important question that a firm faces when it designs its advertising strategy is whether it should launch both non-comparative and comparative advertising campaigns and if so, what should be the optimal advertising mix.

The above questions have not been thoroughly addressed by the existing literature which even though it has studied comparative advertising it has done so by focusing on its informative attributes and its signalling role (Anderson and Renault, 2009; Barigozzi et al., 2009; Emons and Fluet, 2012). This paper contributes to the existing literature by studying the firms’ advertising strategies in an imperfectly competitive market in which firms can launch both non-comparative and comparative advertising campaigns. In particular, we address the following questions: Do firms have incentives to spend on both non-comparative and comparative advertising and if so, which is the optimal advertising mix? How does the intensity of market competition affect the firms’ expenditures on each type of advertising and their optimal

¹Statement of policy regarding comparative advertising, Federal Trade Commission, Washington, D.C., August 13, 1979.

²Typical examples of comparative advertising are, among others, the advertising campaigns of Subway that point out the higher nutritional value of its products in comparison to the Mc Donald’s ones, the “Get a Mac” commercials of Apple that promote the capabilities, the security and the attributes of a Mac in comparison to a PC, and the advertising battles of Pepsi and Coca Cola.

³Muehling et al. (1990) suggest that in the U.S. market almost 40% of all advertisements are comparative in content. Pechmann and Stewart (1990) show that in the U.S. market 60% of all the advertising campaigns contains indirect comparative claims, 20% contains direct comparative claims, and only the remaining 20% contains no comparative claims.

advertising mix? How does the presence of both types of advertising in a market affect market outcomes and welfare in comparison to markets in which either one or both types are absent?

We consider a horizontally differentiated duopolistic market in which firms can use non-comparative and comparative advertising to affect the consumers' perception of the products' qualities. Non-comparative advertising promotes the quality of own firm's product. Therefore, by increasing the consumers' perceived quality, it shifts the firm's demand outwards. Comparative advertising instead has a *push-me-pull-you* dual effect (Anderson et al, 2015): Not only it promotes the quality of the sponsoring firm's product, but also, by presenting it as superior to that of the rival's, it decreases the consumers' perceived quality of the targeted product. Comparative advertising thus increases the firm's own demand and decreases the demand of the rival. A two stage game is analyzed in which firms decide first over the type(s) and the intensity of their advertising campaigns and then they compete in quantities or prices in the market.

We show that in equilibrium firms launch both non-comparative and comparative advertising campaigns.⁴ Our analysis reveals that *within* each firm non-comparative and comparative advertising are strategic complements. Therefore, a firm optimally spends on both types of advertising due to the marginal profitability of one type being increasing in the level of expenditures in the other type of advertising. Further, we show that firms always spend more on comparative than on non-comparative advertising. This is due to the nature of comparative advertising. Evidently, comparative advertising is more appealing than non-comparative advertising due to its' *push-me-pull-you* dual effect. More importantly, as the competitive pressure increases in the market, firms spend relatively more on comparative than on non-comparative advertising. This finding indicates that in a more competitive market, firms adopt more aggressive advertising strategies, since there is more pressure for each firm to improve its own position and harm its rival's. Further, this is in line with the empirical evidence that comparative advertising is often met in highly competitive markets characterized by close substitutable goods, such as the soft drinks industry and the over the counter analgetics market in U.S.

Interestingly, equilibrium non-comparative and comparative advertising intensities are U-shaped in the degree of products' substitutability. In addition, the comparative advertising

⁴This result is in line with recent empirical evidence that suggest that oligopolistic firms use both non-comparative and comparative advertising in order to promote their products (see, e.g., Anderson et al., 2013, 2015; Liaukonyte, 2012).

intensity starts increasing for lower values of the degree of product substitutability than the non-comparative one. Intuitively, two opposing effects are in action: The *demand effect* and the *strategic effect*. The first effect arises because as the products become closer substitutes, each firm's demand decreases and thus its incentives to spend on advertising become weaker. The second effect, the strategic effect, captures the fact that closer products' substitutability translates into fiercer market competition, that reinforces the firm's incentives to spend on advertising so as to retain its market share. Clearly, when the products are poor substitutes, an increase in the degree of product substitutability decreases the advertising intensities, since the strategic effect is relatively weak and it is dominated by the demand effect. Exactly the opposite holds when the products are closer substitutes. Further, the comparative advertising intensity starts increasing in lower values of product substitutability, because, as already mentioned above, firms spend relatively more on comparative relative to non-comparative advertising as the competitive pressure in the market increases.

From a welfare perspective, our analysis indicates that the presence of both non-comparative and comparative advertising in a market can be welfare-enhancing in comparison to a market in which one or both types of advertising are absent. In fact, we show that consumers are always better-off when firms launch both non-comparative and comparative advertising campaigns. Although in the latter case firms' profits are lower than in a market in which either comparative or both types of advertising are banned,⁵ the higher consumers' surplus quite often offsets the lower profits, leading thus to higher welfare. In particular, a market with no restrictions in advertising typically leads to higher welfare than a market in which advertising is altogether banned (except if products are close substitutes and consumers' "quality consciousness" is rather low).⁶ It also leads to higher welfare than a market in which comparative advertising is banned whenever consumers are sufficiently quality conscious and products are differentiated enough. Therefore, from a policy perspective our findings suggest that authorities should carefully consider the specific features of a market before deciding whether to ban or not the use of comparative advertising.⁷

⁵Clearly, a ban on comparative advertising campaign is beneficial for firms, because in a symmetric equilibrium each firm's comparative advertising campaign is nullified by its rival's one. Therefore, firms' comparative advertising campaigns constitute a clear loss, as firms bear the cost of advertising without enjoying any benefit (i.e., comparative advertising expenses are wasteful).

⁶Emons and Fluet (2012), introduced the term quality consciousness.

⁷Stylized facts demonstrate that the discussion over the welfare effects of comparative advertising is still

Our main results do not depend on whether firms compete in quantities or prices in the market. It is worth noting that as the competitive pressure increases, as measured now by the mode of competition, firms switch to more aggressive advertising strategies, i.e., they spend relatively more on comparative than non-comparative advertising campaigns. Therefore, in a more competitive market environment, measured either by the degree of product substitutability or the mode of the market competition, the firms' optimal advertising mix goes in favor of comparative advertising. Nevertheless, the equilibrium advertising intensities are lower under price than under quantity competition, since the marginal profitability of advertising is lower under the fiercer price competition.

Our work contributes to the literature that studies comparative advertising in competitive markets. Although there exists a large body of marketing literature that examines comparative advertising (see e.g., Grewal et al., 1997, for a survey), the respective economic literature is still scarce (e.g., Aluf and Shy, 2001; Anderson and Renault, 2009; Barigozzi et al., 2009; Chakrabarti and Haller, 2011; Emons and Fluet, 2012).⁸ This literature has focused mainly on the analysis of the informative and the signalling role of comparative advertising. Barigozzi et al. (2009) consider a market in which an entrant, whose quality is unknown, decides whether to use generic advertising (i.e., a standard money burning to signal quality) or comparative advertising (i.e., a comparison to the incumbent's quality which is known) to signal its quality. They show that comparative advertising can signal quality in instances where generic advertising fails, provided that the use of comparative advertising enables the incumbent to sue the entrant for manipulative advertising. Emons and Fluet (2012) examine the signalling role of comparative advertising in a duopolistic market in which non-comparative advertising discloses own firm's quality, while comparative advertising discloses the quality differential of the firms' products. They show that in the presence of comparative advertising in the market, firms never advertise together which may be the case when only non-comparative advertising is present. In a somewhat related context, Piccolo et al. (2015) consider a vertically differentiated duopoly in which the low quality firm can engage in deceptive advertising, potentially fooling a consumer into thinking that the product is better than it actually is. They show that

active. More specifically, while comparative advertising was allowed in the U.S. in 1979, the E.U. allowed comparative advertising only in 1997, with all EU member states harmonizing their policies by 2000 (Barigozzi et al., 2009).

⁸This can be viewed as a branch of a wider literature considering quality disclosure in competitive markets. See among others, Milgrom and Roberts (1986), Cheong and Kim (2004) and Hotz and Xiao (2013).

the consumer may benefit of such deceptive advertising and that stricter protection against deceptive practices does not necessarily improve consumer welfare.

Anderson and Renault (2009) consider comparative advertising as information disclosure for the horizontal match characteristics of the products. They show that if products are of similar quality, comparative advertising plays no role, since firms provide full information for their products. If, instead, products are of sufficiently different quality, the low quality firm engages in comparative advertising and discloses the horizontal characteristics of both products to improve its consumers base and survive in the market. The main differences to our setting is that we consider that advertising is costly and that it influences the consumers' perception of the quality of the products.⁹ Lastly, Chakrabarti and Haller (2011) extends the literature on comparative advertising by considering the n -firm oligopoly case in which firms decide not only their investment levels in comparative advertising but also the target of their advertising. They show that under perfect symmetry, investments in comparative advertising constitute a net loss for both the firms' performance and the welfare. The existing literature has mainly dealt with the analysis of the firms' decisions to use either non-comparative or comparative advertising in a market. Our paper extends this literature by considering, instead, a setting in which firms can launch both non-comparative and comparative advertising campaigns. This allows us to provide a detailed analysis on how firms mix over alternative advertising strategies and how the latter affects market outcomes and social welfare.

In Section 2, we present our basic model. Section 3 includes the equilibrium analysis and a comparison of our main results to those of a non-advertising, a mere non-comparative advertising and a mere comparative advertising market. In Section 4, we discuss extensions of our main model. Finally, Section 5 offers some concluding remarks. All proofs are relegated to the Appendix.

⁹Another strand of the literature considers the use of advertising to promote the horizontal characteristics of products. Sun (2011), Koessler and Renault (2012), Jansseny and Teteryatnikova (2013) and Celik (2014) analyze the incentives of firms to disclose their product characteristics focusing on horizontal differentiation. We rather focus on the use of both non-comparative and comparative advertising to influence the consumers' perception of the quality of the products.

2 The Model

We consider a market that consists of two firms, each producing one brand of a horizontally differentiated good. Each firm i , $i = 1, 2$, can launch both non-comparative and comparative advertising campaigns to influence the consumers' perception of the products' qualities. A non-comparative advertisement sends a positive message to consumers that promotes the quality of firm i 's product. A comparative advertisement, in line with Anderson et al. (2015), conveys a *push-me-pull-you* dual message to consumers presenting the sponsoring firm i 's product as of superior quality to that of the rival firm j 's product. It thus increases a consumer's perception of the sponsoring firm's product quality and decreases her perception of the rival's product quality.

On the demand side, there is a unit mass of consumers. The utility of a consumer depends on her perception of the two products' qualities, (τ_i, τ_j) , and is given by,

$$U(\tau_i, \tau_j) = (a + \tau_i s)q_i + (a + \tau_j s)q_j - (q_i^2 + q_j^2 + 2\gamma q_i q_j)/2 + z, \quad i, j = 1, 2, i \neq j. \quad (1)$$

where q_i, q_j , and z are respectively the quantities of goods i, j and the "composite" good that the consumer buys. The parameter $s > 0$ measures the consumer's valuation per unit of (perceived) quality. The parameter $\gamma \in [0, 1]$ denotes the degree of product substitutability, with $\gamma \rightarrow 0$ corresponding to the case of almost independent goods and $\gamma = 1$ to the case of perfect substitutes.¹⁰ Alternatively, γ can be interpreted as a measure of the intensity of market competition, i.e., the higher γ , the fiercer the market competition.

A consumer's perception of the quality level of good i can take values $\tau_i \in \{-2, -1, 0, 1, 2\}$; in other words, good i can be perceived as of very low, low, standard, high and very high quality, respectively. Prior to any firm's advertising campaigns, all consumers are identical and perceive the two firms' products as of standard quality, i.e., $\tau_i = \tau_j = 0$.¹¹ Each firm can influence a consumer's perception by sending her a non-comparative ad (message m_i) and/or a comparative ad (message c_i). Clearly, a consumer that receives no message by either firm continues to believe that both products are of standard quality.

Consider first that only firm i sends ads. If a consumer receives only a message m_i , she perceives firm i 's product as of high quality ($\tau_i = 1$). If a consumer receives only a message c_i , she perceives firm i 's product as of high quality and firm j 's product as of low quality ($\tau_i = 1$

¹⁰In Section 5, we briefly discuss the case of complement goods ($-1 \leq \gamma < 0$).

¹¹This could be so e.g., because she assigns equal probabilities to all possible quality levels for each good.

and $\tau_j = -1$). If she receives both messages m_i and c_i , she perceives firm i 's product as of very high quality and firm j 's product as of low quality ($\tau_i = 2$ and $\tau_j = -1$).¹² Consider next that both firms send ads. If a consumer receives messages c_i and c_j , the comparative ad messages nullify each other, and thus $\tau_i = \tau_j = 0$. In fact, due to c_i , the consumer perceives firm i 's product as of high quality and firm j 's product as of low quality, which are however offset by the exact opposite message that c_j conveys. This leaves the consumer perceiving both products to be of standard quality, $\tau_i = \tau_j = 0$. Further, if a consumer receives messages m_i , c_i and c_j , then, as the comparative ad messages nullify each other, the consumer ends up with $\tau_i = 1$ and $\tau_j = 0$. Finally, if she receives all four messages, m_i , c_i , m_j and c_j , then $\tau_i = \tau_j = 1$.

Each firm i launches non-comparative and comparative advertising campaigns with intensities μ_i and κ_i , $0 \leq \mu_i, \kappa_i \leq 1$, respectively. The intensity of a campaign represents the probability with which each consumer receives a respective ad. For instance, the probability of a consumer not receiving any message from either firm is: $(1 - \mu_i)(1 - \kappa_i)(1 - \mu_j)(1 - \kappa_j)$. To compute firm i 's inverse demand function, we distinguish sixteen groups of consumers based on the messages that each receives from the two advertising firms. Then the expected inverse demand function of firm i is the weighted (by their respective probabilities) sum of the inverse demand functions of the sixteen groups of consumers and is given by¹³

$$p_i(\cdot) = a + (\mu_i + \kappa_i - \kappa_j)s - q_i - \gamma q_j. \quad (2)$$

Observe that firm i 's demand increases in the intensity with which it launches comparative and non-comparative advertising campaigns, κ_i and μ_i , and decreases in the intensity with which its rival launches comparative advertising, κ_j .

We assume that the firms are endowed with identical constant returns to scale production technologies, with their marginal production cost given by c , $0 \leq c < a$. Moreover, we assume that the total cost of advertising is given by $b(\mu_i^2 + \kappa_i^2)$. It is separable across advertising campaigns and quadratic in each type of campaign, i.e., there are diminishing returns of advertising expenditures. The parameter b denotes the effectiveness of the advertising technology on shifting consumers' demand, with a higher b corresponding to a less effective advertising technology. As standard in the literature, the convexity assumption reflects that the cost of

¹²Note that our results remain qualitatively intact if we assume instead that a consumer that receives both m_i and c_i , perceives the product of firm i as of high quality ($\tau_i = 1$, instead of $\tau_i = 2$). However, this alternative assumption leads to unnecessary analytical complications.

¹³The derivation of firm i 's expected inverse demand function is presented in the Appendix A.1

advertising is increasing in the number of targeted consumers (see, e.g., Butters, 1977; Grossman and Sharipo, 1984; Tirole, 1988; Hernandez-Garcia, 1997; Bagwell, 2007 and Hamilton, 2009).

A crucial modeling assumption is the separability of advertising costs. This is well documented in a recent strand of the managerial literature stressing that, due to the vast advances in media technology, there is need for specialization in different advertising techniques applied by the respective agencies. According to Horsky (2006), firms would prefer to use different agencies to promote their products in different channels, based on their specialization. Arzaghi et al. (2008) mention that advertising agencies in the US have moved from "full service provider" of advertising campaigns to providers of specialized services. Therefore, agency compensation has moved from a proportional commission based on final number of targeted consumers to "fee for service" provided by each agency. The main reason is that the complexity and interaction among cotermporal media technologies have made it difficult to measure the final number of targeted consumers (Nichols, 2013). Therefore, in our case, given the different handling required for non-comparative and comparative ads, we treat the two types of advertising as separate projects with independent costs.

Firms play a two stage game with observable actions. In the first stage, firms independently and simultaneously decide their comparative and non-comparative advertising intensities. In the second stage, firms compete in the market by setting their quantities.¹⁴

To simplify the exposition, we adopt the following normalizations: $s_n = \frac{s}{a-c}$ and $b_n = \frac{b}{(a-c)^2}$. The parameter s_n is a measure of a consumer's valuation per unit of quality and per unit of market size (as captured by $a-c$). The parameter b_n measures the slope of the marginal advertising cost per unit of market size squared.

In the sequel, we make the following assumption.

Assumption 1. $b_n \geq 1/2$ and $s_n \leq \hat{s}_n(\gamma, b_n)$, where $\frac{\partial \hat{s}_n}{\partial \gamma} < 0$, $\frac{\partial \hat{s}_n}{\partial b_n} > 0$, $\hat{s}_n(1, b_n) = 0$ and $\lim_{b_n \rightarrow \infty} \hat{s}_n(\gamma, b_n) = \frac{1-\gamma^2}{(2+\gamma)(1+2\gamma)}$.¹⁵

Assumption 1 is sufficient for the second-order and stability conditions to hold under all cases. Moreover, it guarantees that the intensity of advertising does not exceed one, and

¹⁴In Section 4 we extend our analysis by examining price instead of quantity market competition.

¹⁵This is a sufficient condition in order to avoid corner solutions. In particular, $\hat{s}_n(\gamma, b_n)$ is the (real) solution of the equation $2s_n^2[1 - \gamma - (1 + 2\gamma)s_n] = b_n(4 - \gamma^2)[1 - \gamma^2 - (2 + \gamma)(1 + 2\gamma)s_n]$. If this condition fails to hold, then a consumer receiving both non-comparative and comparative ads from firm i , and no ads from firm j , will not buy a non-negative quantity of the firm j 's product.

that all types of consumers buy non-negative amounts of both goods under all circumstances. Moreover, it requires that the advertising technology is not too effective, i.e., marginal advertising costs are sufficiently steep,¹⁶ and that advertising does not alter too much a consumer's valuation per unit of quality.

3 Equilibrium Analysis

In the last stage, each firm i chooses its output to maximize profits

$$\max_{q_i} \Pi_i = [a + (\mu_i + \kappa_i - \kappa_j)s - q_i - \gamma q_j - c]q_i - b(\mu_i^2 + \kappa_i^2). \quad (3)$$

From the first order conditions, the reaction function of firm i is

$$R_i(q_j) = \frac{a - c - \gamma q_j}{2} + \frac{(\mu_i + \kappa_i - \kappa_j)s}{2}. \quad (4)$$

Observe that an increase in firm i 's advertising expenditures shifts its reaction function outwards, and therefore, tends to increase firm i 's output and (gross) profits. By contrast, an increase in firm j 's expenditures on comparative advertising shifts firm i 's reaction function inwards, tending to reduce its output and profits.

Solving the system of (4), the equilibrium quantities and profits are

$$q_i(\cdot) = \frac{(a - c)(2 - \gamma) + 2(\mu_i + \kappa_i - \kappa_j)s - \gamma(\mu_j + \kappa_j - \kappa_i)s}{4 - \gamma^2}; \quad (5)$$

$$\Pi_i(\cdot) = [q_i(\cdot)]^2 - b(\mu_i^2 + \kappa_i^2). \quad (6)$$

In the first stage, each firm i chooses its advertising intensities, μ_i and κ_i , to maximize profits $\Pi_i(\cdot)$, taking as given the rival's advertising intensities, μ_j and κ_j . The first order conditions give rise to the following reaction functions of non-comparative and comparative advertising (expressed in terms of s_n and b_n)

$$\mu_i(\cdot) = \frac{2s_n[2 - \gamma + (2 + \gamma)(\kappa_i - \kappa_j)s_n - \gamma\mu_j s_n]}{b_n(4 - \gamma^2)^2 - 4s_n^2}; \quad (7)$$

$$\kappa_i(\cdot) = \frac{s_n[2 - \gamma - (2 + \gamma)\kappa_j s_n + (2\mu_i - \gamma\mu_j)s_n]}{(2 + \gamma)[b_n(2 - \gamma)^2 - s_n^2]}. \quad (8)$$

¹⁶As standard in the relevant literature, non-existence of an equilibrium may arise because a sufficiently low advertising cost leads firms to savage advertising warfares that conclude to negative profits. Thus, advertising restrictions are required in order all the participants to be active in the market (see, e.g., Peters, 1984; Bester and Petrakis, 1995).

An immediate observation is that the firms' advertising intensities are strategic substitutes, i.e., $\frac{\partial \mu_i(\cdot)}{\partial \mu_j} < 0$, $\frac{\partial \mu_i(\cdot)}{\partial \kappa_j} < 0$, $\frac{\partial \kappa_i(\cdot)}{\partial \mu_j} < 0$, and $\frac{\partial \kappa_i(\cdot)}{\partial \kappa_j} < 0$. This implies that an increase in firm j 's advertising expenditures (either non-comparative or comparative) reduces firm i 's marginal revenue from either type of advertising and thus weakens its incentives to spend on advertising. More importantly, we observe that within each firm non-comparative and comparative advertising campaigns are strategic complements, i.e., $\frac{\partial \mu_i(\cdot)}{\partial \mu_i} > 0$ and $\frac{\partial \kappa_i(\cdot)}{\partial \mu_i} > 0$. That is, an increase in firm i 's expenditures on non-comparative advertising raises the marginal profitability of its' comparative advertising campaign (and vice versa). Intuitively, both non-comparative and comparative advertising campaigns have a positive direct effect on firm i 's demand. In particular, an increase in firm i 's non-comparative advertising intensity, by expanding firm i 's demand, raises the marginal profitability of its comparative advertising campaign and thus reinforces firm i 's incentives to spend on comparative advertising (and vice versa).

Solving the system of (7) and (8), the resulting equilibrium intensities in each type of advertising are

$$\mu^M = \frac{2s_n}{b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2}; \quad (9)$$

$$\kappa^M = \frac{(2+\gamma)s_n}{b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2}. \quad (10)$$

Further, the equilibrium advertising ratio of non-comparative to comparative advertising, namely the optimal advertising mix, is given by

$$M(\gamma) = \frac{\mu^M}{\kappa^M} = \frac{2}{2+\gamma}. \quad (11)$$

The following Proposition summarizes our findings.

Proposition 1 *i) In equilibrium firms launch both non-comparative and comparative advertising campaigns, i.e., $\kappa^M > 0$ and $\mu^M > 0$.*

ii) The optimal advertising mix $M(\gamma) < 1$ for all $\gamma > 0$, with $\frac{\partial M}{\partial \gamma} < 0$.

iii) The equilibrium intensities of non-comparative and comparative advertising are U shaped in γ , decreasing in b_n , and increasing in s_n .

Proposition 1 indicates that firms spend on both non-comparative and comparative advertising. Intuitively, firms launch both types of advertising campaigns to exploit the different effects that each type of advertising has on demand. That is, to increase their own demand by raising the consumers' quality perception of their products due to the self promoting attributes

of both non-comparative and comparative advertising messages, and to decrease their rival's demand due to the denigrating effect of comparative advertising. Note however that this is not the only reason for which firms spend on both non-comparative and comparative ads. As the two types of advertising are strategic complements within each firm, a firm by spending on one type of advertising raises the marginal profitability of the other type, and thus it has incentives to launch both non-comparative and comparative advertising campaigns.

Interestingly, Proposition 1 informs us that the optimal advertising mix always favors comparative instead of non-comparative advertising as long as the goods are horizontally differentiated. This is due to the dual *push-me-pull-you* effect of comparative advertising. In fact, a firm prefers to spend relatively more on comparative than on non-comparative advertising, since the former not only increases its own demand, but it also decreases the demand of the rival. More importantly, the optimal advertising mix decreases with the intensity of the market competition, i.e., $\frac{\partial M}{\partial \gamma} < 0$. Intuitively, fiercer market competition (larger γ) creates pressure to firms to adopt more aggressive advertising strategies. Clearly, as the market becomes more competitive, a firm spends relatively more on comparative advertising in order to reduce the demand of the rival (increasing at the same time its own demand).

Proposition 1 also indicates how firms adjust their advertising intensities as the market competition becomes fiercer. In particular, both non-comparative and comparative advertising intensities are U-shaped with γ .¹⁷ Note, however, that the comparative advertising intensity starts increasing with γ for much lower values of γ than the non-comparative advertising intensity. In more details, when the goods are poor substitutes, an increase in the competitive pressure (higher γ) leads firms to decrease their advertising intensities, whereas the opposite is true for goods that are closer substitutes. This is because there are two opposing effects in action: the negative *demand effect* and the positive *strategic effect*. The demand effect captures the fact that individual demands decrease with γ and as a consequence, firms' incentives to spend on advertising become weaker. On the other hand, the strategic effect captures the fact that market competition becomes fiercer as γ increases, reinforcing thus the firms' incentives to spend on advertising in order to retain their market shares. Clearly, when the goods are poor substitutes, the strategic effect is relatively weak and is dominated by the demand effect. As a consequence, firms' intensities in both types of advertising decrease with γ . The opposite

¹⁷It can be checked that $\frac{\partial \mu^M}{\partial \gamma} > 0$ if and only if $\gamma > \frac{2}{3}$; and $\frac{\partial \kappa^M}{\partial \gamma} > 0$ if and only if $0 < \gamma_c(b_n, s_n) < \gamma < 1$, where $\gamma_c(b_n, s_n)$ is the solution to $\gamma(2 + \gamma)^2 = \frac{s_n^2}{b_n}$. Moreover, Assumption 1 implies that $\gamma_c(b_n, s_n) \ll \frac{2}{3}$.

is true when the goods are close substitutes, in which case the strategic effect dominates. Moreover, since comparative ads become relatively more important as competition intensifies ($\frac{\partial M}{\partial \gamma} < 0$), it is clear that the strategic effect is stronger for comparative advertising and overturns the demand effect for lower values of γ . Finally, the equilibrium intensities of both types of advertising decrease with b_n and increase with s_n . As expected, as the advertising technology becomes more effective, firms advertising intensities increase. The same is true when the consumers valuation per unit of quality is higher, which is translated to higher demands for the firms' products.

Substituting (9) and (10) into (5) and (6), the equilibrium output and profits are

$$q^M = \frac{(a-c)b_n(4-\gamma^2)}{b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2}; \Pi^M = \frac{b_n(a-c)^2[b_n(4-\gamma^2)^2 - (\gamma^2 + 4\gamma + 8)s_n^2]}{[b_n(2-\gamma)(\gamma+2)^2 - 2s_n^2]^2}. \quad (12)$$

Proposition 2 *i) Equilibrium output is decreasing in γ and b_n , whereas it is increasing in s_n .
ii) Equilibrium profits are decreasing in γ and s_n , whereas they are increasing in b_n .*

Proposition 2 informs us that equilibrium output decreases as the products become closer substitutes and the advertising technology becomes less effective, whereas it increases as the consumers' valuation per unit of quality increases. Intuitively, a less effective advertising technology leads firms to spend less on both types of advertising, shifting inwards their reaction functions, which results to lower equilibrium output. In addition, equilibrium output decreases with γ , because the negative demand effect offsets the positive strategic effect. By contrast, when consumers care more about the products' quality, firms' expenditures on both types of advertising increase, resulting in fiercer market competition and higher equilibrium outputs.

Proposition 2 also indicates that equilibrium profits decrease as the products become closer substitutes and the consumers' valuation per unit of quality increases, whereas they increase as the advertising technology becomes less effective. Clearly, as b_n increases, competition in both, output and advertising, is relaxed and profits increase. The opposite is true when the consumers' valuation per unit of quality increases. In fact, an increase in s_n exacerbates the advertising warfare, as measured by advertising intensities, with a negative backlash on profits.

3.1 The Role of Mixed Advertising Strategies

We turn now to examine how the presence of both non-comparative and comparative advertising in a market affects market outcomes and social welfare. To do so, we consider three

alternative market settings. First, a standard Cournot market without any advertising activities: *non-advertising* market setting. Second, a market in which only non-comparative advertising is present: *mere non-comparative advertising* market setting. This is a market in which firms play the same game as in Section 2, with the only difference that $\kappa_i = 0$.¹⁸ Third, a market in which only comparative advertising is present: *mere comparative advertising* market setting. This is a market in which firms play a game as the one described in Section 2, with the only difference that $\mu_i = 0$.¹⁹ For notational reasons, we use superscripts N , I and C to denote the equilibrium values under the Cournot, the mere non-comparative and the mere comparative market settings, respectively. Comparing the equilibrium advertising intensities, output and profits in a mixed advertising market with the three alternative ones, we obtain the following result:

- Proposition 3** *i) The equilibrium advertising intensities satisfy: $\mu^M = \mu^I$ and $\kappa^M > \kappa^C$.*
ii) The equilibrium outputs satisfy: $q^M = q^I > q^N = q^C$.
iii) The equilibrium profits satisfy: $\Pi^I > \Pi^N > \Pi^M > \Pi^C$.

According to Proposition 3(i), the equilibrium comparative advertising intensity in a mixed advertising market always exceeds that of a mere comparative advertising market. This is mainly a consequence of the fact that in a mixed advertising market comparative and non-comparative advertising campaigns are strategic complements within each firm. As firms spend positively on non-comparative advertising in a mixed advertising market ($\mu^M > 0$), their marginal profitability from comparative ads is higher than in a mere comparative market. By contrast, the equilibrium non-comparative advertising intensity in a mixed advertising market is equal to that in a mere non-comparative market. Strategic complementarity between the two types of advertising within a firm in a mixed advertising market points towards higher non-comparative intensity in the latter than in a mere non-comparative market. Yet, strategic substitutability between the two types of advertising across firms in a mixed advertising

¹⁸This market setting corresponds to the case in which comparative advertising is prohibited by the law. It also corresponds to the case where even if the country's legislation allows for comparative advertising, comparative advertising campaigns are banned due to accusations of being misleading and manipulative to consumers (see for details, Barigozzi and Peitz, 2006; Barigozzi et al., 2009) and to the case where consumers perceive comparative ads as manipulative, and thus as non trustworthy messages (see for details, Wilkie and Farris, 1975; Barone and Miniard, 1999).

¹⁹Due to space limitations we provide the analysis of the three alternative market settings in Appendix A.3.

market works in the opposite direction. The two forces exactly offset each other and the non-comparative advertising intensities turn out to be equal in the mere non-comparative and the mixed advertising markets.

Proposition 3(ii) indicates that equilibrium output is the same in a mixed and in a mere non-comparative advertising market, and higher than that of a non-advertising and a mere comparative advertising market. This is because in equilibrium, the firms' comparative advertising intensities are equal and thus neutralize each other. In addition, as we have seen above, the equilibrium non-comparative advertising intensities are positive and equal in the mixed and the mere non-comparative market ($\mu^M = \mu^I > 0$), which shifts the firms' demands outwards and results to higher equilibrium output than in the mere comparative and the non-advertising markets.

Proposition 3(iii) informs us that firms obtain the highest profits in a mere non-comparative advertising market and the lowest in a mere-comparative advertising market. Moreover, firms' profits are higher in a non-advertising market than in a mixed advertising market. This result is driven by two effects on a firm's profits. The positive effect of advertising on a firm's demand and gross profits, and the negative effect of the advertising costs. It is straightforward that a mere comparative advertising market yields the lowest firms' profits, since in a symmetric equilibrium any potential benefit from a firm's spending on comparative advertising is nullified by its rival's one. Thus, firms enjoy no benefit and only bear the cost of advertising (i.e., comparative advertising expenses are wasteful).²⁰ It is also clear that a mere non-comparative advertising market yields the highest profits for the firms, as the shift in a firm's demand due to the self-promoting advertising more than compensates the cost of advertising. This, in turn, implies that the firms' profits in a non-advertising market, in which they are unable to promote their products, are lower than in a mere non-comparative advertising market. Lastly, a mixed advertising market yields lower profits for firms than a non-advertising market. This is due to the fact that the increase in profits from their non-comparative advertising campaigns does not compensate for the firms' wasteful advertising expenditures in comparative advertising.

²⁰The term "wasteful advertising" was first introduced by Pigou (1924), in order to describe the prisoners' dilemma which arises when competing firms in a market invest equal efforts in advertising in order to attract the favor of the public from the others. As Pigou first showed, this concludes in a prisoners' dilemma where none of the firms gains anything at all.

Turning our attention to the welfare implications and comparing consumers' surplus and total welfare in the aforementioned markets, we obtain the following result:

Proposition 4 *i) $CS^M > CS^C > CS^N$ and $CS^M > CS^I > CS^N$.*

ii) $SW^M > SW^C$; $SW^M > SW^N$ except if γ is large enough and s_n very small; $SW^M > SW^I$ only if γ is small enough and s_n is large enough.

According to Proposition 4, consumers are better-off when both types of advertising are present in the market, whereas they are worse-off in the absence of advertising. It is clear that in the presence of both non-comparative and comparative advertising in a market, a larger fraction of consumers is exposed to the firms' advertising messages and thus their perception of the products' quality increases.

Moreover, total welfare in a mixed advertising market always exceeds that of a mere comparative advertising market. This is because both firms' profits and consumers' surplus are higher in the mixed than in the mere comparative advertising market. Interestingly, the welfare is (typically) higher in a mixed advertising market than in a non-advertising one. There is a small region of parameters, i.e., when products are close substitutes and consumers' valuation per unit of product's quality is too small, in which the opposite holds. Consumers' surplus is higher, whereas firms' profits are lower, in a mixed advertising than in a non-advertising market. Then the higher consumers' surplus dominates over the lower profits, except if advertising hardly alters consumers' perception of quality and market competition is fierce.

More importantly, total welfare in a mixed advertising market exceeds that of a mere non-comparative market when the goods are rather poor substitutes and consumers are highly quality conscious (for high s_n). Here too, consumers' surplus is higher, whereas firms' profits are lower, in the mixed advertising than in the mere non-comparative advertising market. When consumers are sufficiently quality conscious (high s_n) and market competition is rather soft (low γ), the higher consumers' surplus in the mixed advertising market dominates the higher industry profits in the mere non-comparative advertising market. This is because when the competitive pressure is weak, the difference in profits across the two market settings is small. In addition, as s_n increases, firms' advertising intensities increase in both market settings. As the fraction of consumers that are exposed to advertising messages in the mixed advertising compared to the non-comparative advertising market increases with s_n , so does the difference in consumers' surplus across the two market settings. Then for high enough s_n and low enough

γ , the profit differential is small and is dominated by the consumers' surplus differential. This is an interesting finding that adds to the discussion of the welfare effects of comparative advertising. More precisely, it demonstrates that, whereas comparative advertising campaigns can be detrimental to the firms' profitability, they can improve total welfare as long as they are launched together with non-comparative advertising campaigns (provided that consumers' are sufficiently quality conscious).

4 Extensions-Discussion

Next we extend our basic model to examine the robustness of our main results and explore the role of our assumptions.²¹

4.1 Bertrand Competition

In our basic model we have assumed that firms compete in quantities. We examine now what happens if firms compete in prices. Under price competition each firm i faces the following expected demand function,

$$q_i(\cdot) = \frac{(1 - \gamma)a + (\mu_i - \gamma\mu_j)s + (1 + \gamma)(\kappa_i - \kappa_j)s + \gamma p_j - p_i}{1 - \gamma^2}$$

To guarantee well-behaved interior solutions under all circumstances, we make the following assumption:

Assumption 1B. $b_n \geq \frac{1}{2}$, $\gamma \in [0, 0.76]$ and $s_n \leq \bar{s}_n(\gamma, b_n)$, with $\frac{\partial \bar{s}_n}{\partial \gamma} < 0$, $\frac{\partial \bar{s}_n}{\partial b_n} > 0$, $\bar{s}_n(1, b_n) = 0$ and $\lim_{b_n \rightarrow \infty} \bar{s}_n(\gamma, b_n) = \frac{1 - \gamma}{2 + 3\gamma - 2\gamma^2}$.²²

Note that stricter assumptions are required when firms compete in prices instead of quantities. This is in line with Singh and Vives (1984) and is due to the fact that price competition is fiercer than quantity competition.

We confirm that under price competition too, in equilibrium firms launch both non-comparative and comparative advertising campaigns. The respective equilibrium advertising

²¹The detailed analysis of the extensions presented below is available from the authors upon request.

²²Similarly to Cournot competition, $\bar{s}_n(\gamma, b_n)$ solves: $(2 - \gamma^2)s_n^2(1 - \gamma - s_n - 2\gamma s_n) = b_n(4 - \gamma^2)(1 + \gamma)[1 - \gamma - (2 - \gamma)(1 + 2\gamma)s_n]$, Then $s \leq \bar{s}_n(\gamma, b_n)$ guarantees that consumers buy non-negative quantities of both goods under all circumstances.

intensities and optimal advertising mix are

$$\begin{aligned}\mu_B^M &= \frac{(2 - \gamma^2)s_n}{b_n(2 - \gamma)^2(1 + \gamma)(2 + \gamma) - (2 - \gamma^2)s_n^2} \\ \kappa_B^M &= \frac{(2 - \gamma)(1 + \gamma)s_n}{b_n(2 - \gamma)^2(1 + \gamma)(2 + \gamma) - (2 - \gamma^2)s_n^2} \\ M_B(\gamma) &= \frac{\mu_B^M}{\kappa_B^M} = \frac{2 - \gamma^2}{(2 - \gamma)(1 + \gamma)}\end{aligned}$$

Interestingly, the optimal advertising mix is lower under price than under quantity competition, i.e., $M_B(\gamma) < M(\gamma)$ for $\gamma > 0$. That is, firms' spending in comparative relatively to non-comparative advertising are higher when market competition takes places in prices instead of quantities. This finding reveals that the more competitive the market environment, the more appealing the comparative advertising campaigns. Noting also that $\frac{\partial M_B}{\partial \gamma} < 0$, we conclude that an increase in competitive pressure, measured either by the degree of product substitutability or the mode of the market competition, leads firms to a more aggressive advertising warfare. In particular, firms choose a more aggressive mix of advertising strategies, i.e., higher comparative relatively to the non-comparative advertising intensities. Note however that the advertising intensities are lower under price than under quantity competition. This is because the rentability of sending messages are lower under the fiercer price competition.

We confirm that our main results hold also when firms compete in prices.²³ The only exception is that the equilibrium intensity in non-comparative advertising is decreasing (instead of U-shaped) in γ . This is because market competition is now fiercer and firms substitute away the less aggressive non-comparative advertising campaigns with the more aggressive comparative ones.

4.2 Complementary Goods

Throughout our analysis we have assumed that firms produce substitute goods. We discuss now what would happen in case of complementary goods, i.e., $\gamma \in [-1, 0)$ where $\gamma = -1$ captures perfect complementarity. Note that the analysis is the same as in Section 3, with γ now taking negative (instead of positive) values.

Surprisingly, in this case too firms launch both comparative and non-comparative advertising campaigns. However, the optimal advertising mix in this case favors non-comparative

²³For more details see Alipranti et al. (2016).

advertising, i.e., $M(\gamma) > 1$ for $-1 \leq \gamma < 0$ (see (11)). In particular, when goods are complements firms spend less on comparative than in non-comparative advertising campaigns. This is because in the case of complementary goods, the *push-me-pull-you* effect of comparative advertising has a different nature. In particular, the *"pull you"* effect of comparative advertising has adverse implications for the advertising firm. This is because a decrease in the consumers' perceived quality of the rival's product, and therefore a decrease in the rival's demand, decreases also the demand of the advertising firm. This makes comparative advertising less attractive in case of complementary goods, and thus firms spend relatively more on non-comparative advertising than under substitute goods.

In light of this, it is not surprising that, contrary to the case of substitute goods, when goods are complements firms' profits in a mix advertising market are typically higher than in a non-advertising market, i.e., $\Pi^M > \Pi^N$ except if the goods are weak complements (γ close to 0). This is because the optimal advertising mix favors non-comparative instead of comparative advertising, and therefore the positive effect of advertising on firm's demand and gross profits more than compensates the negative effect of the advertising costs. Accordingly, we find that $SW^M > SW^N$ always holds in this case, as both the consumers' surplus and the firms' profits are higher in a mixed advertising than in a non-advertising market. The rest of our findings are qualitatively the same as in the case of substitute goods.

4.3 Advertising Cost Asymmetries

We performed our analysis so far under the assumption that the costs of the non-comparative and comparative advertising campaigns are the same. However, in reality when a firm invests in comparative advertising, it runs the risk of being prosecuted to the courts by the rivals and to be accused for misleading advertising.²⁴ Motivated by the latter, we examine what happens when the cost of comparative advertising exceeds that of non-comparative advertising. Assuming that the cost of comparative advertising is $d\kappa_i^2$, where $d = tb$ with $t > 1$ and keeping all the other features of our model intact, we reconfirm that the firms' optimal mix of advertising favors comparative instead of non-comparative as long as t is sufficiently small ($t < \frac{2+\gamma}{2}$). We also confirm that our main results do not qualitatively change when comparative is more

²⁴For instance, in 2000 Papa John's was forced by the court to pay over 468.000\$ in damages to Pizza Hut due to the advertising campaign "Better ingredients. Better pizza" that has been judged as misleading, since such claims can not be proved (see for details, Barigozzi and Peitz, 2006; Barigozzi et al., 2009).

expensive than non-comparative advertising.

5 Concluding remarks

We analyzed firms' advertising strategies in a duopolistic market in which firms can launch both non-comparative and comparative advertising campaigns. We also studied the market and societal implications of the presence of both types of advertising in the market in comparison with markets in which one or both types of advertising are absent due, e.g., to legal restraints.

We found that in equilibrium, firms spend on both non-comparative and comparative advertising. A central contribution of our analysis is that firms' advertising warfare intensifies when firms are able to launch both non-comparative and comparative advertising campaigns. In particular, firms spend relatively more on comparative than on non-comparative advertising. Most importantly, the higher the competitive pressure (as measured by either the degree of product substitutability or the mode of market competition), the higher the share of comparative advertising in the chosen mix of both competitors. This finding highlights that a more competitive market environment makes the aggressive comparative advertising strategy more attractive than the traditional self-promoting non-comparative one.

Regarding the welfare implications, we find that a blend of advertising types always benefits consumers, i.e., consumers' surplus takes its highest value when firms launch both non-comparative and comparative advertising campaigns. In addition, a mixed advertising market often leads to higher welfare than markets in which one or both types of advertising are not present. More specifically, it leads to higher welfare than a market in which firms can launch only non-comparative advertising campaigns, i.e., in markets in which comparative advertising is either banned or mistrusted by consumers, as long as products are sufficiently differentiated and consumers are highly concerned over the products' quality. Although the use of comparative advertising is detrimental to the firms' profitability (i.e., firms' profits are lower in the presence than in the absence of comparative advertising), firms' spending on comparative advertising campaigns can improve not only the consumers' surplus but also the social welfare as long as they are launched together with non-comparative advertisements. An important policy implication of our analysis is that the regulator should not ban comparative advertising, especially when its objective is maximize consumers' surplus.

Our analysis leads to a number of testable implications. First, we should observe that

firms launch both non-comparative and comparative advertising campaigns in horizontally differentiated industries with few firms endowed with similar production technologies. Second, if the risk of being prosecuted to the courts by the rivals and to be accused for misleading advertising is rather small, we expect firms to spend relatively more on comparative than on non-comparative advertising. Finally, we should observe dissimilar reaction patterns of advertising expenses to an increase in the industry competitive pressure. In particular, in highly competitive markets, i.e., markets with a high degree of product substitutability, we should observe a positive relationship between competitive pressure and advertising expenses. Whereas the opposite is expected to occur in markets with low competitive pressure.

In contrast to common wisdom, we found that comparative advertising campaigns are used even when firms' products are complementary, although with relatively lower intensity compared to non-comparative advertising. This is due to the dual, pull-me-push-you, role of comparative advertising, i.e., it is used by each firm to promote, along with non-comparative advertising, its product quality to consumers. Of course, in this case a different type of advertising, e.g., an individual firm's advertising campaign over the bundle of the products, seems to be more appropriate. Whether firms still use comparative advertising in the presence of the latter type of advertising is left for future research.

Appendix

A.1 We present here how we derive firm i 's expected inverse demand function. First, as described in Section 2, we distinguish sixteen groups of consumers, $n = 1, 2, \dots, 16$, based on the messages that a consumer receives from the two advertising firms. The share of each group in the market, ρ^n , is given by the respective probability with which a consumer receives messages from the firms. Thus, the expected inverse demand function of firm i is the weighted (by their respective probabilities) sum of the inverse demand functions of these sixteen groups of consumers. In the following we present the share of each group of consumers in the market (stated in the column, ρ^n) and its respective inverse demand function (stated in the column, $p_i^n(\tau_i, \tau_j)$).

ρ^n	$p_i^n(\tau_i, \tau_j)$
$(1 - \mu_i)(1 - \mu_j)(1 - \kappa_i)(1 - \kappa_j)$	$a - q_i - \gamma q_j$
$(1 - \mu_i)(1 - \mu_j)(1 - \kappa_i)\kappa_j$	$a - s - q_i - \gamma q_j$
$(1 - \mu_i)\mu_j(1 - \kappa_i)(1 - \kappa_j)$	$a - q_i - \gamma q_j$
$(1 - \mu_i)\mu_j(1 - \kappa_i)\kappa_j$	$a - s - q_i - \gamma q_j$
$(1 - \mu_i)(1 - \mu_j)\kappa_i(1 - \kappa_j)$	$a + s - q_i - \gamma q_j$
$(1 - \mu_i)(1 - \mu_j)\kappa_i\kappa_j$	$a - q_i - \gamma q_j$
$(1 - \mu_i)\mu_j\kappa_i(1 - \kappa_j)$	$a + s - q_i - \gamma q_j$
$(1 - \mu_i)\mu_j\kappa_i\kappa_j$	$a - q_i - \gamma q_j$
$\mu_i(1 - \mu_j)(1 - \kappa_i)(1 - \kappa_j)$	$a + s - q_i - \gamma q_j$
$\mu_i(1 - \mu_j)(1 - \kappa_i)\kappa_j$	$a - q_i - \gamma q_j$
$\mu_i\mu_j(1 - \kappa_i)(1 - \kappa_j)$	$a + s - q_i - \gamma q_j$
$\mu_i\mu_j(1 - \kappa_i)\kappa_j$	$a - q_i - \gamma q_j$
$\mu_i(1 - \mu_j)\kappa_i(1 - \kappa_j)$	$a + 2s - q_i - \gamma q_j$
$\mu_i(1 - \mu_j)\kappa_i\kappa_j$	$a + s - q_i - \gamma q_j$
$\mu_i\mu_j\kappa_i(1 - \kappa_j)$	$a + 2s - q_i - \gamma q_j$
$\mu_i\mu_j\kappa_i\kappa_j$	$a + s - q_i - \gamma q_j$
$p_i = \sum_{n=1}^{16} \rho^n p_i^n(\tau_i, \tau_j) = a + (\mu_i + \kappa_i - \kappa_j)s - q_i - \gamma q_j$	

A.2 We present now how we derive consumers surplus and social welfare under mix advertising. In equilibrium, we have: $p_i = p_j = p^M$, $q_i = q_j = q^M$, $\mu_i = \mu_j = \mu^M$, and $\kappa_i = \kappa_j = \kappa^M$. Hence the inverse demand function (2) can be rewritten as: $p^M = a + \mu^M s - (1 + \gamma)q^M$, and thus $q^M = \frac{a - p^M}{1 + \gamma} + \frac{\mu^M s(1 - \gamma)}{1 - \gamma^2}$.

Let τ_i and τ_j be an individual consumer's perceived quality for firm i 's and firm j 's products, respectively. Her consumer surplus $cs(x_i, x_j, \tau_i, \tau_j)$ is given by

$$\begin{aligned}
& U(x_i, x_j, \tau_i, \tau_j) - p^M x_i - p^M x_j \\
&= [(a + \tau_i s)x_i + (a + \tau_j s)x_j - (x_i^2 + x_j^2 + 2\gamma x_i x_j)]/2 - p^M x_i - p^M x_j
\end{aligned}$$

where x_i and x_j denote the quantity of the product i and j that this consumer buys, respectively. As the firms' prices are equal in equilibrium, the first order conditions of the individual consumer's utility maximization can be written as: $a + \tau_i s - x_i - \gamma x_j = a + \tau_j s - x_j - \gamma x_i = p^M$.

Solving the latter system of equations, and using the expression obtained above for q^M , we have

$$x_i(\tau_i, \tau_j) = \frac{a - p^M}{1 + \gamma} + s \frac{\tau_i - \gamma \tau_j}{1 - \gamma^2} = q^M + s \frac{\tau_i - \gamma \tau_j - \mu^M(1 - \gamma)}{1 - \gamma^2}, \quad i, j = 1, 2, i \neq j.$$

Further, using the first order conditions above, the individual consumer's surplus can be written as: $cs(\tau_i, \tau_j) = \frac{1}{2}[x_i^2(\tau_i, \tau_j) + x_j^2(\tau_i, \tau_j) + 2\gamma x_i(\tau_i, \tau_j)x_j(\tau_i, \tau_j)]$. Moreover, since $x_j(\tau_j, \tau_i) = x_i(\tau_i, \tau_j)$, then $cs(\tau_i, \tau_j) = cs(\tau_j, \tau_i)$. Hence, we can summarize the sixteen types of consumers into six groups with (τ_i, τ_j) being respectively, $(0, 0)$, $(1, 0)$, $(1, -1)$, $(2, -1)$, $(2, 0)$ and $(1, 1)$, where the first element corresponds to any of the two products that is perceived (weakly) better than the other. It follows that consumers' surplus is the sum of the surplus of these groups weighted by their respective probabilities of appearance in the market:

$$\begin{aligned} CS^M &= [(1 - \kappa^M)^2 + (\kappa^M)^2](1 - \mu^M)^2 cs(0, 0) + (\mu^M)^2 [(\kappa^M)^2 + (1 - \kappa^M)^2] cs(1, 1) \\ &\quad + 2(1 - \mu^M)^2 \kappa^M (1 - \kappa^M) cs(1, -1) + 2\kappa^M \mu^M (1 - \kappa^M) (1 - \mu^M) cs(2, -1) \\ &\quad + 2(\mu^M)^2 \kappa^M (1 - \kappa^M) cs(2, 0) + 2\mu^M (1 - \mu^M) [1 - \kappa^M + (\kappa^M)^2] cs(1, 0). \end{aligned}$$

After some manipulations, consumers surplus is given by

$$CS^M = \frac{(a - c)^2 [b_n^2 (1 - \gamma) (4 + 4\gamma - \gamma^2 - \gamma^3)^2 + 2b_n (2 + \gamma)^2 (6 + 3\gamma - \gamma^2 - \gamma^3) s_n^3 - 2s_n^4 \Phi(\cdot)]}{(1 - \gamma^2) [b_n (2 - \gamma) (2 + \gamma)^2 - 2s_n^2]^2}$$

where $\Phi(\cdot) = 6(1 + s_n) + 2\gamma(4 + 3s_n) + \gamma^2(5 + 2s_n) + \gamma^3$. Total welfare is then, $SW^M = CS^M + 2\Pi^M$.

A.3 Non-advertising. This is the standard Cournot market with horizontally differentiated goods. Solving each firm's maximization problem, given in (3) after setting $\mu_i = 0$ and $\kappa_i = 0$, the equilibrium output and profits are, $q^N = \frac{a-c}{2+\gamma}$ and $\Pi^N = \frac{(a-c)^2}{(2+\gamma)^2}$. Further, consumers surplus and total welfare are, $CS^N = (1 + \gamma) \frac{(a-c)^2}{(2+\gamma)^2}$ and $SW^N = (3 + \gamma) \frac{(a-c)^2}{(2+\gamma)^2}$.

Mere Non-Comparative Advertising. In this case firms can use only non-comparative advertising. Solving each firm's maximization problem, given in (3) after setting $\kappa_i = 0$, the equilibrium advertising intensity, output and profits are, $\mu^I = \frac{2s_n}{b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2}$, $q^I = \frac{(a-c)b_n(4-\gamma^2)}{b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2}$ and $\Pi^I = \frac{(a-c)^2 b_n [b_n(4-\gamma^2)^2 - 4s_n^2]}{[b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2]^2}$. Further, consumers surplus and total welfare are,²⁵

$$CS^I = \frac{(a - c)^2 [b_n^2 (1 - \gamma) (\gamma^3 + \gamma^2 - 4\gamma - 4)^2 + 2b_n (2 - \gamma) (2 + \gamma)^2 s_n^3 - 4s_n^4 (1 + s_n)]}{(1 - \gamma^2) [b_n (2 - \gamma) (2 + \gamma)^2 - 2s_n^2]^2},$$

²⁵ CS^I is obtained following the same steps as in the mixed advertising case. Here there are only three types of consumers characterized by (τ_i, τ_j) being $(0, 0)$, $(1, 0)$ and $(1, 1)$. Their respective probabilities of appearance are $(1 - \mu^I)^2$, $2\mu^I(1 - \mu^I)$ and $(\mu^I)^2$.

$$SW^I = CS^I + 2\Pi^I.$$

Mere Comparative Advertising. In this case firms can use only comparative advertising. Solving each firm's maximization problem, given in (3) after setting $\mu_i = 0$, the equilibrium advertising intensity, output and profits are, $\kappa^C = \frac{s_n}{b_n(4-\gamma^2)}$, $q^C = \frac{a-c}{2+\gamma}$ and $\Pi^C = \frac{(a-c)^2[b_n(2-\gamma)^2 - s_n^2]}{b_n(4-\gamma^2)^2}$. Further, consumers' surplus and total welfare are,²⁶

$$CS^C = \frac{(a-c)^2[b_n^2(2-\gamma)^2(1-\gamma^2) - 2s_n^4 + 2b_n(4-\gamma^2)]}{b_n^2(1-\gamma)(4-\gamma^2)^2},$$

$$SW^C = CS^C + 2\Pi^C.$$

B.1 Proof of Proposition 1: i)+ii) As $M(\gamma) = \frac{\mu^M}{\kappa^M} = \frac{2}{2+\gamma} \leq 1$, $\kappa^M > \mu^M$ for all $\gamma > 0$. Further, $\frac{\partial M(\gamma)}{\partial \gamma} = -\frac{2}{(2+\gamma)^2} < 0$.

iii) By differentiating μ^M and κ^M with respect to b_n , s_n and γ , we obtain:

- a) $\frac{\partial \mu^M}{\partial b_n} = -\frac{2(2-\gamma)(2+\gamma)^2 s_n}{[b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2]^2} < 0$ and $\frac{\partial \kappa^M}{\partial b_n} = -\frac{(2-\gamma)(2+\gamma)^3 s_n}{[b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2]^2} < 0$
- b) $\frac{\partial \mu^M}{\partial s_n} = \frac{2b_n(2-\gamma)(2+\gamma)^2 + 4s_n^2}{[b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2]^2} > 0$ and $\frac{\partial \kappa^M}{\partial s_n} = \frac{b_n(2-\gamma)(2+\gamma)^3 + 2(2+\gamma)s_n}{[b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2]^2} > 0$
- c) $\frac{\partial \mu^M}{\partial \gamma} = \frac{2b_n(2+\gamma)(2-3\gamma)s_n}{[b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2]^2} > 0$ if and only if $\gamma > \frac{2}{3}$; otherwise $\frac{\partial \mu^M}{\partial \gamma} < 0$
- d) $\frac{\partial \kappa^M}{\partial \gamma} = \frac{2s_n(b_n\gamma(2+\gamma)^2 - s_n^2)}{[b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2]^2} > 0$ if and only if $b_n > \frac{s_n^2}{\gamma(2+\gamma)^2}$; otherwise $\frac{\partial \kappa^M}{\partial \gamma} < 0$. ■

Proof of Proposition 2: i) By differentiating q^M with respect to b_n , s_n and γ , we obtain:
 $\frac{\partial q^M}{\partial b_n} = -\frac{b_n(a-c)(4-\gamma^2)s_n^2}{[b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2]^2} < 0$; $\frac{\partial q^M}{\partial s_n} = \frac{b_n(a-c)(4-\gamma^2)s_n}{[b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2]^2} > 0$ and $\frac{\partial q^M}{\partial \gamma} = -\frac{b_n(a-c)[b_n(4-\gamma^2)^2 - 4\gamma s_n^2]}{[b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2]^2} < 0$

ii) By differentiating Π^M with respect to b_n , s_n and γ , we obtain:

$$\begin{aligned} \frac{\partial \Pi^M}{\partial s_n} &= -\frac{2b_n(a-c)^2 s_n \Omega}{[b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2]^3} < 0, \text{ with } \Omega \equiv b_n\gamma(2-\gamma)(8+\gamma)(2+\gamma)^2 + 2(8+\gamma(4+\gamma))s_n^2 \\ \frac{\partial \Pi^M}{\partial \gamma} &= -\frac{2b_n(a-c)^2 s_n \Xi}{[b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2]^3} < 0, \text{ with } \Xi \equiv b_n^2(4-\gamma^2)^3 - 2b_n(2+\gamma)^2(2-\gamma^2-4\gamma)s_n^2 - 2(2+\gamma)s_n^4 \\ \frac{\partial \Pi^M}{\partial b_n} &= \frac{(a-c)^2 s_n^2 \Omega}{[b_n(2-\gamma)(2+\gamma)^2 - 2s_n^2]^3} > 0. \blacksquare \end{aligned}$$

Proof of Proposition 3: i) First, we observe that, $\mu^M = \mu^I$; second,

$$\kappa^M - \kappa^C = \frac{2s_n^3}{b_n[b_n(2-\gamma)^2(2+\gamma)^3 - 2(4-\gamma)s_n^2]} > 0; \text{ thus } \kappa^M > \kappa^C.$$

²⁶ CS^C is obtained following the same steps as in the mixed advertising case. Here there are only two types of consumers characterized by (τ_i, τ_j) being $(0, 0)$ and $(1, -1)$. Their respective probabilities of appearance are $(1 - \kappa^C)^2 + (\kappa^C)^2$ and $2\kappa^C(1 - \kappa^C)$.

ii) We observe that, $q^M = q^I$; $q^N = q^C$ and $q^M - q^C = \frac{2(a-c)s_n^2}{b_n(2-\gamma)(2+\gamma)^3 - 2(2+\gamma)s_n^2} > 0$; hence, $q^M = q^I > q^N = q^C$.

iii) $\Pi^I - \Pi^N = \frac{4(a-c)^2 s_n^2 [b_n(1-\gamma)(2+\gamma)^2 + s_n^2]}{[b_n(2-\gamma)(2+\gamma)^3 + 2(2+\gamma)s_n^2]^2} > 0$; $\Pi^N - \Pi^M = \frac{(a-c)^2 s_n^2 [b_n\gamma(8+\gamma)(2+\gamma)^2 + 4s_n^2]}{[b_n(2-\gamma)(2+\gamma)^3 + 2(2+\gamma)s_n^2]^2} > 0$ and $\Pi^M - \Pi^C = \frac{4(a-c)^2 s_n^2 [b_n^2(1-\gamma)(4-\gamma^2)^2(2+\gamma)^2 - b_n(12-\gamma^2-\gamma^3) + s_n^4]}{b_n[b_n(2-\gamma)^2(2+\gamma)^3 - 2(4-\gamma^2)s_n^2]^2} > 0$; hence, $\Pi^I > \Pi^N > \Pi^M > \Pi^C$. ■

Proof of Proposition 4: i) First, taking the following differences, $CS^M - CS^I$ and $CS^M - CS^C$, we find that they are always positive. Second, taking the differences $CS^I - CS^N$ and $CS^C - CS^N$ we observe that they are always positive. ii) First, taking the following differences, $SW^M - SW^C$, we find that it is always positive. Second, taking the difference $SW^M - SW^I$, we observe that it is positive if and only if γ is small enough and $s_n > \tilde{s}_n(\gamma, b_n)$.²⁷ Finally, taking the difference $SW^M - SW^N$, we observe that it is positive except if γ is large enough and s_n is sufficiently close to zero. ■

References

- [1] Alipranti, M., Mitrokostas, E., Petrakis, E., 2016. Non-comparative and Comparative Advertising in Oligopolistic Markets. DICE Discussion Paper No. 231.
- [2] Aluf, Y., Shy, O., 2001. Comparison advertising and competition. Mimeo, University of Haifa.
- [3] Anderson, S., Renault, R., 2009. Comparative advertising: disclosing horizontal match information. *Rand Journal of Economics* 40, 558-581.
- [4] Anderson, S., Ciliberto, F., Liaukonyte, J., Renault, R., 2015. Push-me pull-you: comparative advertising in the OTC analgesics industry. CESIFO working paper 5418.
- [5] Anderson, S., Ciliberto, F., Liaukonyte, 2013. Information content of advertising: Empirical evidence from the OTC analgesic industry. *International Journal of Industrial Organization* 31, 355-367.
- [6] Arzaghi, M., Berndt, E., Davis, J., Silk, A., 2008. Economic factors underlying the unbundling of advertising agency services. NBER working paper 14345.

²⁷In fact, $\tilde{s}_n(\gamma, b_n) \leq \hat{s}_n(\gamma, b_n)$ (see Assumption 1) but only if γ is small enough.

- [7] Bagwell, K., 2007. The economic analysis of advertising. In: Mark Armstrong and Rob Porter (eds.), *The Handbook of Industrial Organization* 3, 1701-1844. North-Holland.
- [8] Barigozzi, F., Garella, P., Peitz, M., 2009. With a little help from my enemy: comparative advertising as a signal of quality. *Journal of Economics and Management Strategy* 18, 1071-1094.
- [9] Barigozzi, F., Peitz M., 2006. Comparative advertising and competition policy. In: Choi, J.P.(ed.), *Recent Trends in Antitrust: Theory and Evidence*, 215-263, MIT Press. Cambridge.
- [10] Barone, M., Miniard, P., 1999. How and when factual ad claims mislead consumers: examining the deceptive consequences of copy X copy interactions for partial comparative advertisements. *Journal of Marketing Research* 36, 58-74.
- [11] Bester, H., Petrakis, E., 1995. Price competition and advertising in oligopoly. *European Economic Review* 39, 1075-1088.
- [12] Butters, G., 1977. Equilibrium distribution of sales and advertising prices. *Review of Economic Studies* 44, 465-492.
- [13] Celik, L., 2014. Information unraveling revisited: Disclosure of horizontal attributes, *Journal of Industrial Economics* 62(1), 113–136.
- [14] Chakrabarti, S., Haller, H., 2011. An analysis of advertising wars. *The Manchester School* 79, 100-124.
- [15] Cheong, I., Kim, J.-Y., 2004. Costly information disclosure in oligopoly. *Journal of Industrial Economics* 52, 121–132.
- [16] Emons, W., Fluett, C., 2012. Non-comparative versus comparative advertising as a quality signal. *International Journal of Industrial Organization* 30, 352-360.
- [17] Grewal, D., Kavanoor, S., Fern, E., Costley, C., Barnes, J., 1997. Comparative versus noncomparative advertising: a meta-analysis. *Journal of Marketing* 61, 1–15.
- [18] Grossman, G., Sharipo, C., 1984. Informative advertising with differentiated products. *Review of Economics Studies* 51, 63-81.

- [19] Hamilton, S., 2009. Informative advertising in differentiated oligopoly markets. *International Journal of Industrial Organization* 27, 60- 69.
- [20] Hernandez-Garcia, J., 1997. Informative advertising, imperfect targeting and welfare. *Economics Letters* 55, 131-137.
- [21] Horsky, S., 2006. The changing architecture of advertising agencies. *Marketing science* 25, 367-383.
- [22] Hotz, V. J., Xiao, M., 2013. Strategic information disclosure: The case of multi-attribute products with heterogeneous consumers. *Economic Inquiry* 51, 865–881.
- [23] Koessler, F., Renault R., 2012. When does a firm disclose product information? *RAND Journal of Economics* 43 (4), 630-649.
- [24] Jansseny, M., Teteryatnikova M., 2013. Disclosing horizontal attributes. *Vienna Economic Papers* 1205.
- [25] Liaukonyte, J., 2012. Is comparative advertising an active ingredient in the market for pain relief?. Mimeo. University of Virginia.
- [26] Muehling, D., Stoltman, J., Grossbart, S., 1990. The impact of comparative advertising on levels of message involvement. *Journal of Advertising* 19, 41-50.
- [27] Milgrom, P., Roberts, J. M., 1986. Relying on the information of interested parties. *Rand Journal of Economics* 17, 18–32.
- [28] Nichols, W., 2013. Advertising analytics 2.0. *Harvard Business Review* 91, 60-68.
- [29] Pechmann, C., Stewart, D., 1990. The effect of comparative advertising on attention, memory and purchase intention. *Journal of Consumer Research* 17, 180-191.
- [30] Peters, M., 1984. Restrictions on price advertising. *Journal of Political Economy* 92, 472-485.
- [31] Piccolo, S., Tedeschi, P., Ursino, G., 2015. How limiting deceptive practices harms consumers. *Rand Journal of Economics* 46, 611-624.
- [32] Pigou, A., 1924. *The economics of welfare*. London: Macmillan and Co., Limited. Second Edition.

- [33] Singh, N., Vives X., 1984. Price and quantity competition in a differentiated duopoly. *Rand Journal of Economics* 15, 546-554.
- [34] Sun, M. J., 2011. Disclosing multiple product attributes. *Journal of Economics and Management Strategy* 20, 195-224.
- [35] Tirole, J., 1988. *The Theory of Industrial Organization*. Cambridge, MA: M.I.T. Press.
- [36] Wilkie, W., Farris, P., 1975. Comparative advertising: problems and potential. *Journal of Marketing* 39, 7-15.