

Induced gravity on intersecting brane-worlds

Part I: Maximally symmetric solutions

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Abstract

We explore models of intersecting brane-worlds with induced gravity terms on codimension one branes and on their intersection. Maximally symmetric solutions for the branes and the intersection are found. We find new self-accelerating solutions. In a 6d spacetime, the solutions realize the see-saw modification of gravity where the UV scale of the modification to 4d gravity is determined by 6d Planck scale given by $M_6 \sim 10^{-3}\text{eV}$ and the IR scale of the modification is determined by $M_6^2/M_4 \sim H_0 \sim 10^{-42}\text{ GeV}$ where H_0 is present-day Hubble scale. We find that it is increasingly difficult to construct phenomenologically viable models in higher-dimensional spacetime due to the necessity to have the lower value for the fundamental Planck scale to realize the late time acceleration. It is found that the system also admits self-tuning solutions where the tension at the intersection does not change the geometry of the intersection. The induced gravity terms can avoid the necessity to compactify the extra dimensions. Finally, we discuss the possibility to have ordinary matter at the intersection, without introducing any regularisation, using the induced gravity terms.

Contents

1	Introduction	2
2	Maximally symmetric configurations of branes	3
3	Self-accelerating configurations	7
3.1	New self-accelerating solutions in 6d spacetime	8
3.2	Scales of gravity	9
3.3	More self-accelerating solutions in higher codimensions	9
4	Selftuning configurations	12
5	Conclusions and open issues	13

1 Introduction

Brane-world models with large distance modification of Einstein gravity are invoked in scenarios that aim to geometrically describe present day acceleration, without introducing dark energy [1, 2, 3] (for a review see [4]). A celebrated example is the DGP model in a 5d spacetime [1]. The brane action includes a quantum-induced Einstein-Hilbert that recovers 4d gravity on small scales. This model realizes a so-called *self-accelerating* solution that features a 4d de Sitter phase even though the 3-brane is completely empty. However, so far, only codimension-one examples of such solutions have been proposed and these backgrounds are known to suffer from ghost instabilities [5]. An interesting possibility would thus be to look for other such solutions in higher codimension set-ups, that might lead to ghost free models.

On the other hand, it is known that the presence of more than one extra dimensions might offer better possibilities for the solution of the (particle physics) cosmological constant problem. In fact, codimension-two brane-worlds with finite [6, 7, 8, 9] and infinite [11, 12] volume extra space have been extensively studied in the past few years towards a solution of the cosmological constant problem.¹ A common feature of all these models is that the energy momentum tensor on a codimension two brane only controls the *global* properties of the background geometry without affecting the local structure of the brane itself which remains flat. A realization of this idea is called the *selftuning* solution of the cosmological constant problem. Unfortunately, in the finite volume case, consistency conditions associated to the quantisation condition of a background flux which is introduced to compactify the extra-dimensional space are shown to be problematic and the selftuning mechanism in its simplest form does not work [13]. More in general, another delicate feature of codimension two brane models (both with finite- and infinite-volume) is related to the fact that only energy momentum tensor of the form of *pure tension* can be accommodated on a thin radially-symmetric codimension two brane, to ensure the regularity of the background metric at the brane position [14]. This last problem can be ameliorated by including higher order Gauss-Bonnet terms for gravity in the bulk [15]; however other difficulties arise when one tries to embed an isotropic and homogeneous fluid on such a radially-symmetric purely-conical codimension two brane [16]. This suggests the necessity of regularising the brane and allowing it to acquire some structure. For the finite

¹See [10] for attempts in this direction in the codimension one case.

volume scenario, this approach was started by [17] that shows how normal gravity can be recovered in this context, and it is further developed by various other works [18]². The regularisation of the co-dimension two brane with induced gravity has been also studied intensively [20, 21].

In this paper, we take a different route to tackle this regularisation problem. We consider a system in which the brane is a pure codimension two object, without an internal structure, but with induced gravity terms on it. This system, as we already mentioned at the beginning, is by itself interesting to investigate, for finding new self-accelerating configurations. The induced gravity terms can avoid the necessity to compactify the higher-dimensional space, by providing a mechanism to obtain four dimensional gravity in the relevant regimes with infinite volume extra-dimensions. Consequently the bulk gauge field is no more needed, and this suggests to re-consider the self-tuning mechanism in this context.

The model that we consider is a codimension two brane, that lies at the intersection of two codimension one branes embedded in a six dimensional space. This system was studied in the context of standard gravity [22] (cosmological properties were studied in [23]), and Gauss-Bonnet gravity [24] elaborating an idea developed in [25].³ The latter was generalised to higher codimensional models in [27]. Models with a generic angle between two intersecting branes were first considered in [28] and then further generalised into the so-called Origami-world in [29]. In our case, we consider a situation in which induced gravity terms are allowed on the codimension one branes and also at the intersection. We study various maximally symmetric configurations of branes and explore configurations that exhibit self-tuning or self-accelerating properties. Let us also point out that the presence of induced Einstein-Hilbert terms on intersecting codimension-one branes might allow for generic localised matter on the intersection and thus allow for generic FRW cosmology. We discuss this possibility in conclusions, briefly summarising the results of a companion paper [30].

While we were preparing this work, a relevant paper [31] appeared that studied a nested 4d brane in a 5d codimension brane in a 6d spacetime by generalizing [32].

2 Maximally symmetric configurations of branes

We consider a system of two intersecting codimension one branes embedded in a six dimensional space-time. They intersect on a four dimensional codimension two brane where observers like us can be localised. We take an Einstein-Hilbert action for gravity in the bulk and we allow for induced gravity terms on the codimension one branes, as well as on the intersection. Besides gravity, we allow for a cosmological constant term in the bulk, Λ_B , and for additional fields localised on the branes described by general Lagrangians L 's. The general action assumes the form

$$\begin{aligned}
 S &= \int_{\text{bulk}} d^6x \sqrt{-g} \left(\frac{M_6^4}{2} R - \Lambda_B \right) + \sum_{i=1}^2 \int_{\Sigma_i} d^5x \sqrt{-g^{(i)}} \left(\frac{M_{5,i}^3}{2} R_{(i)} + L_{(i)} \right) \\
 &+ \int_{\Sigma_\cap} d^4x \sqrt{-g_\cap} \left(\frac{M_4^2}{2} R_\cap + L_\cap \right), \tag{1}
 \end{aligned}$$

where $\Sigma_\cap \equiv \bigcap_i \Sigma_i$ denotes a three-brane at the intersection between all codimension-one branes four-branes Σ_i . We can have different fundamental scales in the different regions of the space, M_6 , $M_{5,i}$, and

²See also [19] for articles discussing gravity in higher codimensions.

³ Intersecting brane models are very important in string theory, for the possibility to build Standard-like models with chiral matter [26].

M_4 . The induced gravity terms could be generated, as it was proposed in the original model, by quantum corrections from matter loops on the brane. It is also interesting to note that induced curvature terms appear quite generically in junction conditions of higher codimension branes when considering natural generalisations of Einstein gravity [15, 19] as well as in string theory compactifications [33], orientifold models and intersecting D-brane models [34].

In this paper, we focus on a configuration with *static* codimension one branes. They are characterised by tensions Λ_1 and Λ_2 , while the intersection has tension λ . The branes intersect with a generic angle, along the lines of Origami-world [29]. The six dimensional bulk is characterised by a maximally symmetric geometry

$$\begin{aligned} ds^2 &= A^2(t, z^1, z^2) \left(\eta_{\mu\nu} dx^\mu dx^\nu + \delta_{kh} dz^k dz^h \right), \\ A(t, z^1, z^2) &= \frac{1}{Ht + k_i z^i}. \end{aligned} \quad (2)$$

Also the codimension one branes and their intersection are characterised by maximally symmetric geometries. We are going to relate the Hubble parameters on the branes with the geometrical parameters that control our system.

The parameters H and k_i appearing in the warp factor A satisfy the following relation

$$\frac{\Lambda_B}{10} = H^2 - k_1^2 - k_2^2, \quad (3)$$

in order to solve the Einstein equations in the bulk. The branes form a generic angle and are characterised by normal vectors

$$\mathbf{n}^{(1)} = (\sin \alpha_1, -\cos \alpha_1), \quad (4)$$

$$\mathbf{n}^{(2)} = (\sin \alpha_2, \cos \alpha_2), \quad (5)$$

in the two transverse space directions z^i , so that the angle between the two branes is $\alpha \equiv (\alpha_1 + \alpha_2)$. We refer to the brane Σ_i as orthogonal to the vector $\mathbf{n}^{(i)}$. It is useful to define new coordinates

$$\tilde{z}^k = \mathbf{n}^{(k)} \cdot \mathbf{z} \quad \text{with} \quad \mathbf{z} = (z^1, z^2), \quad (6)$$

that allow us to define the brane positions as $\tilde{z}^i = 0$. Following [29], we define new vectors $l_{(k)}$ as

$$\mathbf{l}_{(k)} \cdot \mathbf{n}^{(h)} = \delta_k^h \quad \rightarrow \quad \begin{cases} \mathbf{l}_{(1)} = \frac{1}{\sin \alpha} (\cos \alpha_2, -\sin \alpha_2), \\ \mathbf{l}_{(2)} = \frac{1}{\sin \alpha} (\cos \alpha_1, \sin \alpha_1), \end{cases} \quad (7)$$

that allow us to write

$$\mathbf{z} = \tilde{z}^k \mathbf{l}_{(k)}, \quad (8)$$

so that

$$\frac{\partial z^h}{\partial \tilde{z}^k} = l_{(k)}^h, \quad (9)$$

is the projection operator to the brane Σ_k . The two branes Σ_1 and Σ_2 are fixed points of Z_2 symmetries acting on the six dimensional space. We wish to investigate the conditions that their tensions must satisfy

in order to solve the Einstein equations. We will use the junction conditions obtained from the Israel formalism. We focus on the brane Σ_1 and find the extrinsic curvature at its position. Recall that the second brane Σ_2 is also a fixed point of Z_2 symmetry. To implement this information, we require the metric to be symmetric under $\tilde{z}^2 \rightarrow -\tilde{z}^2$. The transverse space line element in the new coordinate frame changes from

$$\delta_{ij} dz^i dz^j = d\mathbf{z} \cdot d\mathbf{z}, \quad (10)$$

to

$$\mathbf{l}_{(1)} \cdot \mathbf{l}_{(1)} d\tilde{z}^1 d\tilde{z}^1 + 2\mathbf{l}_{(1)} \cdot \mathbf{l}_{(2)} \operatorname{sgn}(\tilde{z}^2) d\tilde{z}^1 d\tilde{z}^2 + \mathbf{l}_{(2)} \cdot \mathbf{l}_{(2)} d\tilde{z}^2 d\tilde{z}^2 = \tilde{\gamma}_{mn} d\tilde{z}^m d\tilde{z}^n, \quad (11)$$

with

$$\tilde{\gamma}_{mn} = \frac{1}{\sin^2 \alpha} \begin{pmatrix} 1 & \cos \alpha \operatorname{sgn}(\tilde{z}^2) \\ \cos \alpha \operatorname{sgn}(\tilde{z}^2) & 1 \end{pmatrix}. \quad (12)$$

The bulk line element reads ⁴

$$ds^2 = A^2(t, \tilde{z}^1, |\tilde{z}^2|) \left(\eta_{\mu\nu} dx^\mu dx^\nu + \tilde{\gamma}_{mn} d\tilde{z}^m d\tilde{z}^n \right), \quad (13)$$

$$A(t, \tilde{z}^1, |\tilde{z}^2|) = \frac{1}{Ht + \mathcal{C}_1 \tilde{z}^1 + \mathcal{C}_2 |\tilde{z}^2|}, \quad (14)$$

$$\mathcal{C}_1 = \frac{k_1 \cos \alpha_2 - k_2 \sin \alpha_2}{\sin \alpha}, \quad \mathcal{C}_2 = \frac{k_1 \cos \alpha_1 + k_2 \sin \alpha_1}{\sin \alpha}. \quad (15)$$

The inverse metric is given by

$$g^{MN} = A^{-2} \left(\begin{array}{c|c} \eta^{\mu\nu} & 0 \\ \hline 0 & \tilde{\gamma}^{mn} \end{array} \right), \quad (16)$$

where

$$\tilde{\gamma}^{mn} = \frac{\sin^2 \alpha}{1 - \cos^2 \alpha \operatorname{sgn}^2(\tilde{z}^2)} \begin{pmatrix} 1 & -\cos \alpha \operatorname{sgn}(\tilde{z}^2) \\ -\cos \alpha \operatorname{sgn}(\tilde{z}^2) & 1 \end{pmatrix}. \quad (17)$$

In the new coordinate frame the normal vector to the brane Σ_1 is

$$\tilde{n}_k^{(1)} = \frac{\partial z^h}{\partial \tilde{z}^k} n_h^{(1)} = \mathbf{l}_{(k)} \cdot \mathbf{n}^{(1)} = \delta_k^1 = (1, 0). \quad (18)$$

The normal vector that has a unit-length with respect to the bulk metric is

$$\tilde{n}_M^{(1)} = A \mathcal{K} (0_\mu, 1, 0), \quad (19)$$

⁴ We emphasize that we are imposing a Z_2 symmetry at the point $\tilde{z}_2 = 0$, while at this level we are not imposing a similar reflection symmetry at $\tilde{z}_1 = 0$. This last symmetry will be applied later at the end of this section, when using Israel formalism to find the junction conditions.

with

$$\mathcal{K} = \frac{\sqrt{1 - \cos^2 \alpha \operatorname{sgn}^2(\tilde{z}^2)}}{\sin \alpha}. \quad (20)$$

The projection operator (9) allows us to define the induced metric on the brane Σ_1 , whose $(\tilde{z}_2, \tilde{z}_2)$ component reads ⁵

$$g_{\tilde{z}_2 \tilde{z}_2}^{(5)} = l_{(\tilde{z}_2)}^k l_{(\tilde{z}_2)}^h A^2 \delta_{kh} = A^2 (\mathbf{1}_{(2)})^2 = \frac{A^2}{\sin^2 \alpha}. \quad (21)$$

Hence the line element on the brane Σ_1 is

$$ds_{\Sigma_1}^2 = A^2(t, 0, |\tilde{z}^2|) \left(\eta_{\mu\nu} dx^\mu dx^\nu + \frac{d\tilde{z}_2 d\tilde{z}_2}{\sin^2 \alpha} \right). \quad (22)$$

Each induced brane metric corresponds to a maximally symmetric space with a constant Hubble parameter. For the brane Σ_1 , the Hubble parameter is given by

$$H_1^2 = H^2 - \sin^2 \alpha \mathcal{C}_2^2,$$

from which

$$\sin \alpha \mathcal{C}_2 = -\epsilon_1 \sqrt{H^2 - H_1^2}, \quad (23)$$

where ϵ_1 is equal to plus or minus one and distinguishes two different branches of solutions. An analogous relation connects H_2 to \mathcal{C}_1 . It is now straightforward to compute the components of the extrinsic curvature. Then, we can apply the formalism of the Israel junction conditions to extract information about the energy momentum tensor on the brane. Particularly interesting is its $(\tilde{z}_2, \tilde{z}_2)$ component that reads

$$K_{\tilde{z}_2 \tilde{z}_2} = \tilde{\nabla}_{\tilde{z}_2} \tilde{n}_{\tilde{z}_2} = -g_{\tilde{z}_2 \tilde{z}_2} \left(\mathcal{C}_1 - \cos \alpha \mathcal{C}_2 \right) - A \frac{\cotan \alpha}{\sqrt{1 - \cos^2 \alpha \operatorname{sgn}^2(\tilde{z}^2)}} \frac{d}{d\tilde{z}^2} \operatorname{sgn}(\tilde{z}^2). \quad (24)$$

Defining $w_2 \equiv \tilde{z}_2 / \sin(\alpha_1 + \alpha_2)$ in order to remove the $\sin(\alpha_1 + \alpha_2)$ factor from the metric (22), one obtains

$$K_{w_2 w_2} = - \left(\mathcal{C}_1 - \cos \alpha \mathcal{C}_2 \right) - A \frac{\cos \alpha}{\sqrt{1 - \cos^2 \alpha \operatorname{sgn}^2(w^2)}} \frac{d}{dw^2} \operatorname{sgn}(w^2), \quad (25)$$

where we take into account the Jacobean in the delta function $\delta(\tilde{z}_2) = \delta(w_2) / \sin \alpha$.

It is important to realise that *only* this component of the extrinsic curvature contains a singular term $\delta(w_2)$, so that $K_a^b - \delta_a^b K$ has singular contribution only along the four dimensional coordinates that characterise the intersection. It must be compensated by the energy momentum tensor of matter localised at the intersection itself. The maximally symmetric metric at the intersection reads

$$ds_4^{2 \text{ int}} = A^2(t, 0, 0) \eta_{\mu\nu} dx^\mu dx^\nu, \quad (26)$$

with Hubble parameter H .

⁵The induced metric is invariant under bulk diffeomorphisms. Using the bulk frame $(\tilde{z}_1, \tilde{z}_2)$ one simply gets $g_{\tilde{z}_2 \tilde{z}_2}^{(5)} = \delta_{\tilde{z}_2}^m \delta_{\tilde{z}_2}^n A^2 \gamma_{mn} = A^2 \gamma_{\tilde{z}_2 \tilde{z}_2}$ that obviously agrees with the result obtained in the original (z_1, z_2) frame.

The Israel junction conditions relate the components of the extrinsic curvature to the energy momentum tensor on the branes. A straightforward calculation using the relation between the \mathcal{C}_i functions and the induced Hubble parameters leads to the following conditions

$$\Lambda_1 = 6M_{5,1}^3 H_1^2 - \frac{8M_6^4}{\sin \alpha} \left(\epsilon_2 \sqrt{H^2 - H_2^2} - \epsilon_1 \cos \alpha \sqrt{H^2 - H_1^2} \right), \quad (27)$$

$$\Lambda_2 = 6M_{5,2}^3 H_2^2 - \frac{8M_6^4}{\sin \alpha} \left(\epsilon_1 \sqrt{H^2 - H_1^2} - \epsilon_2 \cos \alpha \sqrt{H^2 - H_2^2} \right), \quad (28)$$

for the Friedmann equations on codimension one branes. In the intersection we need to carefully determine the contribution from the singular part of the 6d extrinsic curvature namely

$$2M_6^2 \left(K_\mu^\nu - \delta_\mu^\nu K \right) \Big|_{sing} = -2M_6^2 K_{w^2} \Big|_{sing} \delta_\mu^\nu = \frac{\delta \lambda}{2A} \frac{d}{dw^2} \text{sgn}(w^2) \delta_\mu^\nu \quad (29)$$

so that making use of expression (25) and integrating both sides on an infinitesimal interval one gets

$$\delta \lambda = 4M_6^4 \left(\frac{\pi}{2} - \alpha \right) \quad (30)$$

and finally

$$\lambda = 3M_4^2 H^2 - 6 \left(\epsilon_1 M_{5,2}^3 \sqrt{H^2 - H_1^2} + \epsilon_2 M_{5,1}^3 \sqrt{H^2 - H_2^2} \right) + 4M_6^4 \left(\frac{\pi}{2} - \alpha \right). \quad (31)$$

It is important to recognise that the induced gravity terms on the codimension one branes provide contributions that are similar to the ones that appear in the DGP brane-worlds in five dimensions. At the intersection, we find that localised gravity terms on the codimension one branes induce contributions proportional to $M_{5,i}$ which are again similar to the ones in the DGP models in five dimensions. Note that by allowing the different 5d Newton constant on different codimension one branes, the parts of the Friedmann equation proportional to $M_{5,i}$ reproduce the asymmetric 5d model with induced gravity considered in Ref. [35]. In addition, we find a term inherited from the six dimensional bulk that vanishes in the limit in which the branes form a right angle [29].

The angle between the branes can be determined by (3). In terms of the H 's and the α it becomes

$$\Lambda_B = -\frac{10}{\sin^2 \alpha} \left[H^2 (1 + \cos^2 \alpha) - H_1^2 - H_2^2 - 2 \epsilon_1 \epsilon_2 \cos \alpha \sqrt{H^2 - H_1^2} \sqrt{H^2 - H_2^2} \right], \quad (32)$$

that can be easily solved in terms of the angles:

$$\cos \alpha = \frac{\epsilon_1 \epsilon_2 \sqrt{(H^2 - H_1^2)(H^2 - H_2^2)} - \sqrt{\left(H_1^2 - \frac{\Lambda_B}{10}\right) \left(H_2^2 - \frac{\Lambda_B}{10}\right)}}{H^2 - \frac{\Lambda_B}{10}}. \quad (33)$$

3 Self-accelerating configurations

Potentially interesting configurations in our system are self-accelerating solutions. It is well known that induced gravity terms may provide accelerating cosmological solutions in which acceleration is induced

by gravity itself and is independent of the presence of matter or cosmological constant in the system. This very important observation [2, 3] has received much attention, since it can provide a model for dark energy without cosmological constant. However, it has also been realized that this scenario has a serious drawback. The most interesting branch of solutions in the standard DGP brane-world (the one that contains the self-accelerating configuration) is plagued by ghosts [5]. It would be interesting to study how generic this conclusion is in higher codimensional models. In order to investigate this issue, we start from seeking new self-accelerating solutions in higher codimensions.

3.1 New self-accelerating solutions in 6d spacetime

We consider now self-accelerating solutions for our system. Let us then choose $\Lambda_1 = \Lambda_2 = \lambda = 0$, while the induced gravity terms (parametrised by M_4 and $M_{5,i}$) do not vanish. For simplicity we assume $M_{5,1} = M_{5,2}$. It is then possible to solve our system of equations (27)-(32). Taking the difference between (27) and (28) one obtains

$$\left(\epsilon_1 \sqrt{H^2 - H_1^2} - \epsilon_2 \sqrt{H^2 - H_2^2} \right) \left[\frac{1 + \cos \alpha}{\sin \alpha} - \frac{6M_5^3}{8M_6^4} \left(\epsilon_1 \sqrt{H^2 - H_1^2} + \epsilon_2 \sqrt{H^2 - H_2^2} \right) \right] = 0, \quad (34)$$

so that we have two possible types of solutions that we now describe.

(i) *Symmetric solution:* $H_1 = H_2$ and $\epsilon_1 = \epsilon_2$. The flat-bulk condition (32) yields

$$H_1^2 = \frac{H^2}{2} (1 - \cos \alpha). \quad (35)$$

From the codimension one equation (27) one thus gets either the trivial solution $H = 0$, or

$$H = \frac{8\epsilon_1 M_6^4}{3 \sin \alpha M_5^3} \sqrt{\frac{1 + \cos \alpha}{2}}, \quad (36)$$

which, along with (35), can be plugged into the intersection equation (31) to give

$$\frac{1 + \cos \alpha}{\sin \alpha} - \frac{1}{4} \left(\frac{\pi}{2} - \alpha \right) = \frac{2 M_6^4 M_4^2 (1 + \cos \alpha)}{3 M_5^6 \sin^2 \alpha}, \quad (37)$$

that sets the value for the angle between the codimension-one branes. Notice that since the physical range for the angle is $0 < \alpha < \pi$, then in order to have a positive H , eq. (36) requires a positive ϵ_1 .

(ii) *Asymmetric solution:* $H_1 \neq H_2$. Defining

$$\mathcal{D} = \frac{6M_5^3 \sin \alpha}{8M_6^4} \left(\epsilon_1 \sqrt{H^2 - H_1^2} + \epsilon_2 \sqrt{H^2 - H_2^2} \right), \quad (38)$$

we get

$$\mathcal{D} = 1 + \cos \alpha, \quad (39)$$

from (34). Then the sum of (27) and (28), and (31) give

$$\sqrt{H_1^2 + H_2^2} = \frac{4M_6^4}{3M_5^3}, \quad (40)$$

$$H^2 = \frac{4M_6^4}{3M_4^2} \left[2 \frac{1 + \cos \alpha}{\sin \alpha} - \left(\frac{\pi}{2} - \alpha \right) \right], \quad (41)$$

respectively. Finally from the condition (32) one gets a relation that sets the value of the angle α and from the positivity condition on α we learn that in this case at least one of the ϵ 's must be positive as well.

3.2 Scales of gravity

A simple manipulation of the previous formulae shows that, in both cases, the resulting Hubble parameter is of the order

$$H \simeq \frac{M_5^3}{M_4^2}, \quad (42)$$

that is, the same result as in the five dimensional DGP models. In order to explain the late time acceleration $H \sim 10^{-42}$ GeV, we should require $M_5 = 10^{-2}$ GeV with $M_4 = 10^{18}$ GeV. For $\alpha \sim \pi/2$, the six dimensional Newton constant is roughly given by

$$M_6^4 \simeq \frac{M_5^6}{M_4^2} \simeq M_4^2 H^2. \quad (43)$$

Thus M_6 must be $M_6 = 10^{-3}$ eV to explain the late time acceleration. With these parameters we find that the cross-over scales are given by

$$r_{c,5} \sim r_{c,6} \sim H^{-1}, \quad r_{c,5} = \frac{M_4^2}{M_5^3}, \quad r_{c,6} = \frac{M_4}{M_6^2}. \quad (44)$$

We should note that these scales are exactly the see saw scale of [36]. For large distance, $r > r_{c,5}, r_{c,6}$, gravity is five-dimensional or six-dimensional. The laws of four-dimensional gravity are valid all the way down to the distances of the order M_6^{-1} . Below this length scale, the effective theory of gravity breaks down. As a result, there is a lower bound on the scale M_6 which comes from accelerators, astroparticle and cosmological data, that is $M_6 > 10^{-3}$ eV [37]. This is exactly the scale we need to explain the present day acceleration of the Universe. Note that for a small angle $\alpha \ll 1$, M_6 is given by $M_6^4 = \alpha M_5^6 / M_4^2$. Hence the constraint becomes difficult to satisfy.

However, one realizes that, in order to have H positive, we must choose at least one of the ϵ_i with the positive sign. Then, we are in the same class of self-accelerating solutions of [2], that are notoriously plagued by ghosts. It is then likely that our codimension one branes contain ghost excitations. There is a suggestion that if there is no matter on the self-accelerating universe, it is possible to quantise the theory without having the ghost instability. In our model, we do not need matter on codimension one brane. It would be interesting to study at what level the ghost couples with matter on the brane at the intersection.

3.3 More self-accelerating solutions in higher codimensions

In this section we show that the methods previously described can be easily generalised to a system of N codimension one branes that intersect in a $(4 + N)$ -dimensional space-time, along the lines of [27]. Their common intersection - a four dimensional, codimension- N brane - should correspond to the space-time we observe. We consider an Einstein-Hilbert action in the bulk, with the addition of a bulk Lagrangian L , and we allow for induced gravity terms on all the branes, and at the intersections.

The action for the system is then the following

$$\begin{aligned} S = & \int_{\text{bulk}} d^{4+N} x \sqrt{-g} \left(\frac{M_{4+N}^{2+N}}{2} R + L \right) + \sum_{i=1}^N \int_{\Sigma_i} d^{3+N} x \sqrt{-g_{(i)}} \left(\frac{M_{3+N}^{1+N}}{2} R_{(i)} + L_{(i)} \right) \\ & + \sum_{i \neq j} \int_{\Sigma_i \cap \Sigma_j} d^{2+N} x \sqrt{-g_{(ij)}} \left(\frac{M_{2+N}^N}{2} R_{(ij)} + L_{(ij)} \right) + \dots + \int_{\cap} d^4 x \sqrt{-g_{\cap}} \left(\frac{M_4^2}{2} R_{\cap} + L_{\cap} \right), \end{aligned} \quad (45)$$

where $\cap \equiv \bigcap_i \Sigma_i$ denotes the three-brane at the intersection between all codimension-one branes. Here we concentrate on the case where the Lagrangians are simply tensions and the codimension-one branes intersect to form right angles. We also describe an empty “self-accelerated” scenario where all the branes undergo a de Sitter phase with no cosmological constant nor tensions in the system. The bulk geometry is again a maximally-symmetric one $ds^2 = A^2(t, z_1, \dots, z_N) \eta_{MN} dx^M dx^N$ with $A(t, z_1, \dots, z_N) = 1/Ht + \sum_{i=1}^N k_i |z_i|$ and each static codimension-one brane Σ_i is defined by the subspace $\{z_i = 0\}$. The bulk Einstein equation gives the relation

$$H^2 - \sum_{i=1}^N k_i^2 = \frac{2}{(2+N)(3+N)} \Lambda, \quad (46)$$

whereas the equation of motion for the codimension-one branes Σ_i can be computed by means of the Israel junction condition. Assuming Z_2 symmetry across the branes

$${}^{(i)}K^m_n = -\frac{1}{2M_{4+N}^{2+N}} \left({}^{(i)}T^m_n - \frac{1}{2+N} \delta_n^m {}^{(i)}T \right) \Big|_{\text{smooth}}, \quad (47)$$

where ${}^{(i)}T^m_n$ includes a regular “matter” part $-\Lambda_{(i)} \delta_n^m$ and a part from the smooth part of the induced EH term, namely $-M_{3+N}^{1+N} {}^{(i)}G^m_n = M_{3+N}^{1+N} \frac{(1+N)(2+N)}{2} (H^2 - \sum_{i'} k_{i'}^2) \delta_n^m$. Hence

$${}^{(i)}T^m_n \Big|_{\text{smooth}} = \left(-\Lambda_{(i)} + M_{3+N}^{1+N} \frac{(1+N)(2+N)}{2} \left(H^2 - \sum_{i'} k_{i'}^2 \right) \right) \delta_n^m. \quad (48)$$

Taking the normal vector to the brane to be pointing into the bulk $n_M^+ = A \delta_M^{z_i}$ we get ${}^{(i)}K^m_n = g^{mr} \nabla_r n_n = \delta_n^m A^{-2} \partial_i A \Big|_{\Sigma_i} = -\delta_n^m k_i$. Then Eq. (47) gives

$$\Lambda_i = M_{3+N}^{1+N} \frac{(1+N)(2+N)}{2} \left(H^2 - \sum_{\ell \neq i} k_\ell^2 \right) + 2(2+N) M_{4+N}^{2+N} k_i, \quad (49)$$

which in the special case $N = 2$ simply reduces to (27) for a right angle between the branes. The process can be iterated. In fact each brane Σ_i is intersected by other $N - 1$ branes. From the point of view of an observer sitting on Σ_i each of these intersections separate the brane itself in two half parts whose boundaries are given by that intersection. Consider for definiteness the intersection $\Sigma_i \cap \Sigma_j$ we will then have a contribution to the singular part of the Einstein tensors associated to $R_{(i)}$ localised at $z_j = 0$ and a contribution to the singular part of the Einstein tensors associated to $R_{(j)}$ localised at $z_i = 0$. Such contributions can be evaluated by computing the extrinsic curvatures associated to the intersection $\Sigma_i \cap \Sigma_j$ as measured from Σ_i and Σ_j point of view. Namely,

$${}^{(ij)}K^m_n + {}^{(ji)}K^m_n = -\delta_n^m (k_i + k_j), \quad (50)$$

and (49) thus generalises as

$$\Lambda_{(ij)} = M_{2+N}^N \frac{N(1+N)}{2} \left(H^2 - \sum_{l \neq i, j} k_l^2 \right) + 2(1+N) M_{3+N}^{1+N} (k_i + k_j), \quad (51)$$

and so on. In particular for the 3-brane intersection of all the codimension-one branes we have N contributions from the induced EH terms localised on the $N - 1$ -codimension N intersections. Hence

$$\Lambda_\cap = 3M_4^2 H^2 + 6M_5^3 \sum_{i=1}^N k_i . \quad (52)$$

It is thus clear from the previous setup that the presence of induced EH terms on the branes allows for localised matter on the branes themselves, at least in the form of tension.

We then concentrate on a special case of the previous results that gives rise to a de Sitter solution in absence of any form of matter in the system, namely a ‘‘self-accelerated’’ solution. We set $k_i = -H_i$: the bulk equation of motion simply requires $H^2 = \sum_i H_i^2$, whereas from the junctions conditions (49) one obtains

$$M_{3+N}^{1+N} \frac{(1+N)}{4} \left(H^2 - \sum_{\ell \neq i} H_\ell^2 \right) = M_{4+N}^{2+N} H_i , \quad (53)$$

that, using the bulk equation of motion, reduces to

$$H_i = \frac{M_{3+N}^{1+N}}{M_{4+N}^{2+N}} \frac{1+N}{4} H_i^2 \quad (54)$$

which fixes

$$K \equiv H_i = \frac{4}{1+N} \frac{M_{4+N}^{2+N}}{M_{3+N}^{1+N}}, \quad \forall i . \quad (55)$$

The process can again be iterated all the way down to codimension- N . At the last level, we have N contributions to the extrinsic curvature on the 3-brane, from N possible intersections of $N - 1$ codimension-one branes. Using (52,55) and the bulk equation of motion one obtains the 4d Hubble scale and a relation between all the masses in the system

$$H = \sqrt{N} K = \sqrt{N} 2 \frac{M_5^3}{M_4^2}, \quad (56)$$

$$2 \frac{M_5^3}{M_4^2} = \frac{4}{3} \frac{M_6^4}{M_5^3} = \frac{M_7^5}{M_6^4} = \dots = \frac{4}{1+N} \frac{M_{4+N}^{2+N}}{M_{3+N}^{1+N}} . \quad (57)$$

Let us also stress that all the branes (codimension-one and intersections) undergo a de Sitter phase. As to the codimension-one branes Σ_i the rate is simply $H_i = K$, and similarly for $\Sigma_i \cap \Sigma_j$ one has $H_{ij} = \sqrt{H_i^2 + H_j^2} = \sqrt{2}K$: for a generic codimension- n intersection, one obtains $H_{(n)} = \sqrt{n}K$.

Note that if we require the de Sitter rate H to be equal to today’s Hubble rate $H_0 \sim 10^{-42} \text{ GeV}$, from (56,57), and using $M_4 \sim 10^{18} \text{ GeV}$, we get the following values for the mass scales involved in the system

$$M_5 \sim 10^{-2} \text{ GeV}, \quad M_6 \sim M_5 \left(\frac{M_5}{M_4} \right)^{1/2} \sim 10^{-12} \text{ GeV}, \quad M_7 \sim M_6 \left(\frac{M_6}{M_5} \right)^{3/5} \sim 10^{-18} \text{ GeV} \quad \dots , \quad (58)$$

Hence the higher is the codimension, the lower the fundamental scale must be. This requirement contradicts to the constraints on the fundamental scale $M_{4+N} < 10^{-3} \text{ eV}$ and it is impossible to realize the self-accelerated scenarios in higher codimensional spacetime with $N > 2$.

4 Selftuning configurations

The system of equations (27)-(32) relates the bulk cosmological constant and brane tensions with the brane Hubble parameters and the angle between the branes. Since we have induced gravity terms on the branes, it is possible that they are enough to ensure that four dimensional gravity is obtained in the relevant regimes at the intersection, with no need to compactify the extra-dimensions. Then, the angle between the branes is a free parameter; it is natural to ask whether the *selftuning* solutions exist where, by changing the tension at the intersection λ , only the angle α between the branes changes and all the other quantities are fixed.

The following is an example of such solutions

$$\Lambda_1 = 6M_{5,1}^3 H_1^2, \quad (59)$$

$$\Lambda_2 = 6M_{5,2}^3 H_2^2, \quad (60)$$

$$H^2 = H_1^2 = H_2^2 = \frac{\Lambda_B}{10}, \quad (61)$$

$$\frac{\pi}{2} - \alpha = \frac{\lambda - 3M_4^2 H^2}{4M_6^4}. \quad (62)$$

The Hubble parameter at the intersection is *independent* of the tension λ . The tension only controls the angle between the branes. Then, if we localise standard model physics at the intersection, any contribution to the four dimensional cosmological constant, that is, the brane tension λ , does not curve the four dimensional spacetime. Instead, it only changes a parameter in the higher dimensional spacetime that is the angle between the branes, realising the desired *selftuning mechanism*.

It is important to point out the similarity and difference between this idea and the selftuning scenarios in six dimensions using braneworlds on conical singularities [7, 9] where two extra dimensions are compactified by a magnetic monopole. In these cases, the selftuning mechanism suffers from a quantisation condition for the magnetic monopole responsible for compactifying the extra space and we need a fine-tuning between the cosmological constant and other parameters of the models. Our case is different because in principle we do *not need* to compactify the extra dimensions. In our model, the induced gravity is supposed to recover 4D behaviours of gravity on scales smaller than the cross-over scales $r_{c,5}$ or $r_{c,6}$. Similarly to what happens in [7, 9] and in their infinite-volume counterparts [12, 15] (which are more alike to the present configuration), also in this case the vacuum energy density has a finite critical value that corresponds to the maximum of the deficit angle (minimum angle between the branes): in the limit $\lambda \gg 3M_4^2 H^2$ and using $\alpha_{crit} = 0$, one gets

$$\lambda_{crit} = 2\pi M_6^4. \quad (63)$$

Although there is a critical value for the brane tension, our solution is topologically different from a vortex due to the existence of codimension one branes. This would have an important implication for the existence of the ghost. It has been shown that there appears a ghost if we consider a 4d vortex with induced gravity in a 6d spacetime depending on the regularisations [20]. We should carefully re-examine this issue in our model, as a mechanism like the one found in [31] might be at work and help cancelling ghosts.

5 Conclusions and open issues

In this paper, we studied maximally symmetric solutions for intersecting brane-worlds with induced gravity terms. We found new classes of the self-accelerating solutions that realize de Sitter solution without any cosmological constant in the system. In a 6d spacetime, this realizes the see-saw modification of the gravity. The 6d Planck scale must be $M_6 = 10^{-3}\text{eV}$ to explain the late time acceleration $H = H_0 = 10^{-42}\text{ GeV}$. This scale determines the IR scale where 4d gravity is modified $M_6^2/M_4 \sim H_0$. On the other hand, M_6 defines the UV scale where 4d gravity is modified. It was shown that the lower bound for the fundamental Planck scale is roughly 10^{-3} eV from various constraints which is consistent in the 6d case. The small angle between the two branes makes it difficult to satisfy this constraint. We have shown that the construction of the self-accelerating universe with right angles between the branes can be naturally extended to N -dimensional spacetime. However, it was shown that, in order to realize the observed late time acceleration, the higher-dimensional Planck scale is increasingly lower and violates the lower bound for the fundamental scale. We should mention that the problem of the ghost instability would exist even in these new solutions. It would be important to study at what level the ghost couples with matter on the brane at the intersection.

It is also possible to find the self-tuning solution where the cosmological constant on the intersection only depends on the angle between the branes. Unlike other examples, in principle we do not need to compactify the extra dimensions, since the induced gravity terms may be sufficient to provide four dimensional gravity in the relevant regimes. Then, the angle between the branes is a free parameter that does not have to satisfy any constraint. In the vortex case, it is shown that a regularisation is needed to remove the divergence of 6d propagator and a ghost can appear depending on the regularisation. It is vital to re-examine these issues for our intersecting branes.

Finally, let us briefly discuss the problem of obtaining consistent four dimensional gravity at the intersection in this set-up. The first simple step toward the understanding of this issue is to generalise the maximally symmetric solutions to the case of FRW cosmological configurations on the branes. Then, by studying Israel junction conditions, one can derive the effective cosmological equations on the codimension one and codimension two brane-worlds. By obtaining the Hubble equation on the codimension two brane-world at the intersection, it is possible to determine on which scales standard four dimensional cosmological expansion can be obtained at the intersection. The resulting cosmological system presents many interesting features. It is possible to show that the preferred energy momentum tensor can be localised at the intersection, although in the case in which it is different from pure tension, there is normally an exchange of energy density between the codimension one branes and the intersection. The details of the induced cosmology, as well as their consequences, are going to be analysed in a companion paper [30].

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