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Limits on Isocurvature Perturbations from Non-Gaussianity in WMAP Temperature Anisotropy

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ABSTRACT

We study the effect of primordial isocurvature perturbations on non-Gaussian properties of CMB temperature anisotropies. We consider generic forms of the non-linearity of isocurvature perturbations which can be applied to a wide range of theoretical models. We derive analytical expressions for the bispectrum and the Minkowski Functionals for CMB temperature fluctuations to describe the non-Gaussianity from isocurvature perturbations. We find that the isocurvature non-Gaussianity in the quadratic isocurvature model, where the isocurvature perturbation S is written as a quadratic function of the Gaussian variable σ, $S=\sigma^2-\langle\sigma^2\rangle$, can give the same signal-to-noise as $f_{\rm NL}=30$ even if we impose the current observational limit on the fraction of isocurvature perturbations contained in the primordial power spectrum α. We give constraints on isocurvature non-Gaussianity from Minkowski Functionals using the WMAP 5-year data. We do not find a significant signal of isocurvature non-Gaussianity. For the quadratic isocurvature model, we obtain a stringent upper limit on the isocurvature fraction $\alpha < 0.070$ (95% CL) for a scale invariant spectrum which is comparable to the limit obtained from the power spectrum.

Key words: Cosmology: early Universe – cosmic microwave background – methods: statistical – analytical

1 INTRODUCTION

Recent observational progress in cosmology represented by surveys such as WMAP and SDSS has enabled a detailed analysis of cosmic density fields to investigate the physics of the early universe. In particular, (non-)Gaussianity of primordial density fields has been received much attention recently as a key observational probe to differentiate models of the early universe. In the simplest single field inflationary scenario, quantum fluctuations of the inflaton during inflation are assumed to be the origin of cosmic density fluctuations and such a model predicts adiabatic and almost Gaussian primordial fluctuations. However, other generation mechanisms of density fluctuations such as the curvaton scenario (Mollerach 1990; Linde & Mukhanov 1997; Moroi & Takahashi 2001; Enqvist & Sloth 2002; Lyth & Wands 2002), modulated reheating (Kofman 2003; Dvali, Gruzinov & Zaldarriaga 2004) and so on have been proposed. In these mechanism, the nature of primordial density fluctuations can be very different from that of a simple inflation model, in particular, in terms of Gaussianity of the fluctuations. In fact, it has been shown that large non-Gaussianity can be generated in curvaton models (Lyth, Ungarelli & Wands 2003; Bartolo et al. 2004; Enqvist & Nurmi 2005; Malik & Lyth 2006; Sasaki, Valiviita & Wands 2006; Assadullahi et al. 2007; Huang 2008; Ichikawa et al. 2008a; Enqvist & Takahashi 2008), modulated reheating scenarios (Zaldarriaga 2004; Suyama & Yamaguchi 2008; Ichikawa et al. 2008b), Dirac-Born-Infeld inflation models (Alishahiha et al. 2004; Chen et al. 2007b; Langlois et al. 2008a; Arroja, Mizuno & Koyama 2008), Ghost inflation (Arkami-Hamed et al. 2004), ekpyrotic models (Koyama et al. 2007; Buchbinder et. al. 2008;

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Lehners & Steinhardt 2008), single field inflation with a feature in its potential (Chen, Easther & Lim 2007a), a Gaussian-squared component in curvature or entropy perturbations (Linde & Mukhanov 1997; Boubekeur & Lyth 2006; Suyama & Takahashi 2008) and multi-brid inflation (Sasaki 2008; Naruko & Sasaki 2009).

The primordial non-Gaussianity has been quantitatively measured by higher-order statistics such as bispectra and Minkowski Functionals from CMB temperature maps obtained by WMAP (Komatsu et al. 2008; Creminelli et al. 2007; Yadav & Wandelt 2008; Hikage et al. 2008) and also from large-scale structure (Slosar et al. 2008). So far the observational results are consistent with the Gaussian hypothesis. There is, however, a hint of primordial non-Gaussianity at 2-3 σ level (Yadav & Wandelt 2008; Komatsu et al. 2008). Since the non-Gaussianity predicted in the simplest inflation model is too small to be detected by current observations, if a non-Gaussian signal is observed and it originates from primordial fluctuations, other mechanisms beyond the simplest model would be required in the dynamics of early universe.

As another probe of the early universe, the adiabaticity of primordial density fields has also been the subject of intense study. In fact, current cosmological observations of TT and TE spectra of CMB with some other distance measurements such as type Ia supernovae (SNe) and baryon acoustic oscillation (BAO) have already constrained the fraction of isocurvature perturbations to be less than 10% (e.g., Bean et al. 2006; Kawasaki & Sekiguchi 2007; Komatsu et al. 2008). Examples of isocurvature perturbations along with non-Gaussianity have been discussed in the context of non-Gaussian field potentials (Linde & Mukhanov 1997; Peebles 1999; Boubekeur & Lyth 2006; Suyama & Takahashi 2008), the curvaton scenario (Lyth, Ungarelli & Wands 2003; Bartolo et al. 2004; Beltran 2008; Moroi & Takahashi 2008), modulated reheating (Boubekeur & Creminelli 2006), baryon asymmetry (Kawasaki, Nakayama & Takahashi 2009) and the axion (Kawasaki et al. 2008, 2009). In particular, non-Gaussianity generated from isocurvature perturbations has been systematically investigated recently (Kawasaki et al. 2008; Langlois, Vernizzi & Wands 2008b; Kawasaki et al. 2009).

In this paper we discuss non-Gaussianity generated from the non-linearity of isocurvature perturbations and study a constraint on non-Gaussianity from isocurvature perturbations using Minkowski Functionals. For this purpose, we derive theoretical expressions for bispectra and Minkowski Functionals to characterize non-Gaussianity in CMB temperature maps generated from primordial mixed perturbations of adiabatic and isocurvature components. We characterize the non-linearity of isocurvature perturbations in two different forms which are theoretically motivated; one is a Gaussian variable plus its quadratic correction (Linear Model). The other form is given as a quadratic of Gaussian variables without a liner term (Quadratic Model). These two generic forms are applicable to a wide range of isocurvature models listed above. Then we give actual limits on the isocurvature non-Gaussianity using the WMAP 5-year data. As far as we know, this is the first attempt to put a limit on isocurvature perturbations from the non-Gaussianity of CMB anisotropies.

In this paper, we focus on CDM isocurvature perturbations. However, it is straightforward to apply our method to other types of isocurvature perturbations including baryon and neutrino ones. We adopt a set of cosmological parameters at the maximum likelihood values for a power-law ΛCDM model obtained from the WMAP 5-year data only fit (Dunkley et al. 2009); $\Omega_{\rm b} = 0.0432$, $\Omega_{\rm cdm} = 0.206$, $\Omega_\Lambda = 0.7508$, $H_0 = 72.4$ km s^{-1} Mpc^{-1}, $\tau = 0.089$, and $n_\phi = 0.961$. The total amplitude of primordial power spectra is set to be $\Delta_{\rm tot}(k = 0.002{\rm Mpc}^{-1}) = 2.41 \times 10^{-9}$.

This paper is organized as follows; in §2 we give two different forms of the non-linear isocurvature perturbations called the "Linear Model" and the "Quadratic Model". In §3 we derive analytical expressions for bispectra to describe isocurvature non-Gaussianity in CMB temperature anisotropies in the Linear Model. The isocurvature non-Gaussianity in the Quadratic Model is described in §4. In §5, we present generic perturbative formulae of Minkowski Functionals that can be applied to CMB temperature maps with adiabatic and isocurvature non-Gaussianity. In §6, we give limits on isocurvature non-Gaussianity from the WMAP 5-year temperature maps using Minkowski Functionals. In §7, we discuss implications of our results for an axion isocurvature model. §8 is devoted to a summary and our conclusions.

2 NON-LINEAR ADIABATIC AND ISOCURVATURE PERTURBATIONS

We consider the admixture of an adiabatic perturbation ζ with a CDM isocurvature perturbation $\mathcal{S}$. The curvature perturbation is written up to second order in a local form as

$$\zeta = \phi + \frac{3}{5} f_{\rm NL}(\phi^2 - \langle\phi^2\rangle), \tag{1}$$

where ϕ is the linear term of ζ that obeys a Gaussian statistics. The non-linear parameter $f_{\rm NL}$ represents the quadratic amplitude of the curvature perturbation Φ during the matter era (Komatsu & Spergel 2001), which is related to ζ by $\Phi = (3/5)\zeta$.

An isocurvature perturbation $\mathcal{S}$ between matter and radiation is defined as

$$\mathcal{S} \equiv \frac{\delta\rho_m}{\rho_m} - \frac{3\delta\rho_\gamma}{4\rho_\gamma}, \tag{2}$$

where ρ_m is the matter energy density and ρ_γ is radiation energy density. In this paper we focus on a CDM isocurvature perturbation.

We consider the non-linearity of the isocurvature perturbation in two different forms. One is a local form similar to the equation (1) which has a linear (Gaussian) term with a quadratic correction

$$\text{I. Linear Model :} \quad \mathcal{S} = \eta + f_{\rm NL}^{\rm (ISO)}(\eta^2 - \langle\eta^2\rangle), \tag{3}$$

where η is a Gaussian variable and its non-linearity is characterized by $f_{\rm NL}^{\rm (ISO)}$. The isocurvature non-Gaussianity in this form was studied in the context of the axion (Kawasaki et al. 2008) and the curvaton scenario (Langlois, Vernizzi & Wands 2008b).

The other is the case where the linear term is negligible compared with the quadratic term (e.g., Linde & Mukhanov 1997; Peebles 1999; Boubekeur & Lyth 2006; Kawasaki et al. 2008);

$$\text{II. Quadratic Model}: \quad \mathcal{S} = \sigma^2 - \langle\sigma^2\rangle, \tag{4}$$

where σ obeys Gaussian statistics. Linde & Mukhanov (1997) proposed a scenario to generate the quadratic form of isocurvature perturbations by introducing a massive free scalar field oscillating around the vacuum state which has a zero value. In this scenario, the isocurvature fluctuation has a blue spectrum and thus we here consider a wide range for the spectral index ranging from 1 to 3 in the Quadratic Model. Boubekeur & Lyth (2006) showed that a Gaussian-squared component of the primordial curvature perturbation would be bounded at 10% level by the WMAP bound on the bispectrum.

The auto and cross-correlation power spectra for fluctuations of I and J are defined as

$$\langle I_{\mathbf{k}} J_{\mathbf{k}'}\rangle = (2\pi)^3 \delta_D^{(3)}(\mathbf{k}+\mathbf{k}') P_{IJ}(k), \tag{5}$$

and then its dimensionless power is given by

$$\Delta_{IJ}(k) = \frac{P_{IJ}(k)k^3}{2\pi^2}, \tag{6}$$

where I and J denote ζ or $\mathcal{S}$.

We assume the following power-law form of auto and cross power spectra for the Gaussian variables ϕ (eq.[1]), η (eq.[3]), and σ (eq.[4]);

$$\Delta_{XX}(k) = A_X \left(\frac{k}{k_0}\right)^{n_X-1}, \tag{7}$$

$$\Delta_{XY}(k) = (A_X A_Y)^{1/2} \cos\theta_{XY} \left(\frac{k}{k_0}\right)^{(n_X+n_Y)/2-1}, \tag{8}$$

where X and Y denote ϕ, η or σ. As $f_{\rm NL}A_\phi^{1/2}$ is observationally limited to be much smaller than unity, the power spectrum of the primordial adiabatic perturbation is given by

$$\Delta_{\zeta\zeta}(k) \simeq \Delta_{\phi\phi}(k). \tag{9}$$

We define the fraction of isocurvature perturbation as

$$\alpha \equiv \frac{P_{\mathcal{SS}}(k_0)}{P_{\zeta\zeta}(k_0) + P_{\mathcal{SS}}(k_0)}, \tag{10}$$

which is the same definition as Bean et al. (2006) for example. We here set $k_0 = 0.002\mathrm{Mpc}^{-1}$. The parameter α is related to another common parameter of adiabaticity $\delta_{\rm adi}^{(c,\gamma)}$ (e.g., eq. (41) of Komatsu et al. 2008) as $\delta_{\rm adi}^{(c,\gamma)} = [\alpha/(1-\alpha)]^{1/2}/3$. The upper limits of α from WMAP, BAO and SN combined are given by 0.067 (95% CL) for axion-type ($\cos\theta_{\zeta\mathcal{S}} = 0$) and 0.0037 (95% CL) for curvaton-type ($\cos\theta_{\zeta\mathcal{S}} = -1$) isocurvature perturbations (Komatsu et al. 2008).

3 NON-GAUSSIANITY OF ISOCURVATURE PERTURBATIONS I: LINEAR MODEL

3.1 Initial Perturbation

In the Linear Model (eq.[3]), power spectra for the isocurvature perturbation and its cross term with the adiabatic perturbation become

$$\Delta_{\mathcal{SS}}(k) \simeq \Delta_{\eta\eta}(k), \tag{11}$$

$$\Delta_{\zeta\mathcal{S}}(k) \simeq \Delta_{\phi\eta}(k). \tag{12}$$

Here, we have used the fact that $f_{\rm NL}^{\rm (ISO)} \lesssim 1/\sqrt{\Delta_{\mathcal{SS}}} \simeq 1/\sqrt{\alpha\Delta_{\phi\phi}} \sim 10^6$. In the case that $f_{\rm NL}^{\rm (ISO)}$ is larger than this upper limit, the linear term of η in Eq. (3) is negligible so that the model should be described by the quadratic model.

The ratio of the amplitude of the power spectra between ϕ and η is written in terms of α (eq.[10]) as

$$\frac{A_\eta}{A_\phi} = \frac{\alpha}{1-\alpha}. \tag{13}$$

We define the bispectra of ζ, $\mathcal{S}$ and their mixed contribution as

$$\langle I_{\mathbf{k}_1} J_{\mathbf{k}_2} K_{\mathbf{k}_3}\rangle \equiv (2\pi)^3 \delta_D^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \times B_{IJK}(k_1,k_2,k_3), \tag{14}$$

where I, J, and K denote ζ or $\mathcal{S}$.

When ϕ and η are initially uncorrelated, the adiabatic and isocurvature modes evolve independently under the linear approximation. There exist bispectra only from each mode given as

$$\begin{aligned} B_{\zeta\zeta\zeta}(k_1,k_2,k_3) &= \frac{6}{5} f_{NL}[P_{\phi\phi}(k_1)P_{\phi\phi}(k_2) \\ &+ P_{\phi\phi}(k_2)P_{\phi\phi}(k_3) + P_{\phi\phi}(k_3)P_{\phi\phi}(k_1)], \end{aligned} \tag{15}$$

$$\begin{aligned} B_{\mathcal{SSS}}(k_1,k_2,k_3) &= 2 f_{\rm NL}^{\rm (ISO)}[P_{\eta\eta}(k_1)P_{\eta\eta}(k_2) \\ &+ P_{\eta\eta}(k_2)P_{\eta\eta}(k_3) + P_{\eta\eta}(k_3)P_{\eta\eta}(k_1)]. \end{aligned} \tag{16}$$

When ϕ and η are initially correlated, the following cross-correlation terms may become important

$$\begin{aligned} B_{(\zeta\zeta\mathcal{S})}(k_1,k_2,k_3) &\equiv B_{\zeta\zeta\mathcal{S}}(k_1,k_2,k_3) + B_{\zeta\mathcal{S}\zeta}(k_1,k_2,k_3) \\ &+ B_{\mathcal{S}\zeta\zeta}(k_1,k_2,k_3) \\ &= 2 f_{\rm NL}^{\rm (ISO)}[P_{\phi\eta}(k_1)P_{\phi\eta}(k_2) \\ &+ P_{\phi\eta}(k_2)P_{\phi\eta}(k_3) + P_{\phi\eta}(k_3)P_{\phi\eta}(k_1)] \\ &+ \frac{6}{5} f_{\rm NL}[P_{\phi\phi}(k_1)\{P_{\phi\eta}(k_2) + P_{\phi\eta}(k_3)\} \\ &+ P_{\phi\phi}(k_2)\{P_{\phi\eta}(k_3) + P_{\phi\eta}(k_1)\} \\ &+ P_{\phi\phi}(k_3)\{P_{\phi\eta}(k_1) + P_{\phi\eta}(k_2)\}], \end{aligned} \tag{17}$$

$$\begin{aligned} B_{(\zeta\mathcal{SS})}(k_1,k_2,k_3) &\equiv B_{\zeta\mathcal{SS}}(k_1,k_2,k_3) + B_{\mathcal{S}\zeta\mathcal{S}}(k_1,k_2,k_3) \\ &+ B_{\mathcal{SS}\zeta}(k_1,k_2,k_3) \\ &= \frac{6}{5} f_{\rm NL}[P_{\phi\eta}(k_1)P_{\phi\eta}(k_2) \\ &+ P_{\phi\eta}(k_2)P_{\phi\eta}(k_3) + P_{\phi\eta}(k_3)P_{\phi\eta}(k_1)] \\ &+ 2 f_{\rm NL}^{\rm (ISO)}[P_{\phi\eta}(k_1)\{P_{\eta\eta}(k_2) + P_{\eta\eta}(k_3)\} \\ &+ P_{\phi\eta}(k_2)\{P_{\eta\eta}(k_3) + P_{\eta\eta}(k_1)\} \\ &+ P_{\phi\eta}(k_3)\{P_{\eta\eta}(k_1) + P_{\eta\eta}(k_2)\}]. \end{aligned} \tag{18}$$

3.2 Bispectra of CMB Temperature Anisotropy

Harmonic coefficients a_{lm} of CMB temperature anisotropies $\Delta T/T$ are defined as

$$a_{lm} \equiv \int d\hat{\mathbf{n}} \frac{\Delta T}{T}(\hat{\mathbf{n}}) Y_{lm}^*(\hat{\mathbf{n}}). \tag{19}$$

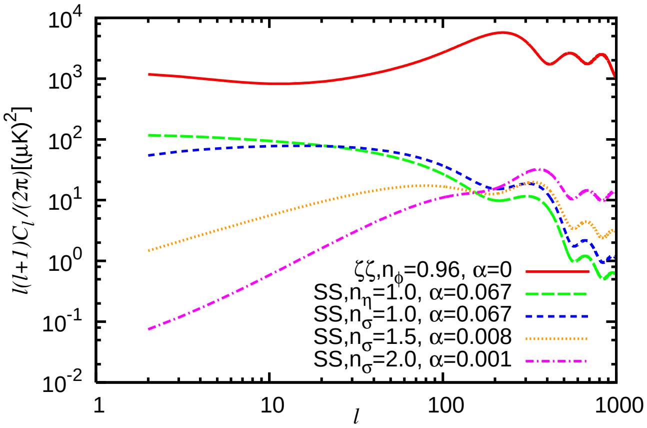

Figure 1. Angular power spectra C_l for adiabatic $\zeta\zeta$ and isocurvature perturbations $\mathcal{SS}$. The plotted adiabatic perturbation has the spectral index $n_\phi = 0.96$ (solid). For isocurvature perturbations, the spectral index of a Gaussian variable n_η is 1 (long-dashed) in the Linear Model and n_σ is 1 (short-dashed), 1.5 (dotted) and 2 (dot-dashed) in the Quadratic Model (see §4). The isocurvature fraction α is set to be 0.067 ($n_\eta = 1$ and $n_\sigma = 1$), 0.008 ($n_\sigma = 1.5$), and 0.001 ($n_\sigma = 2$) defined at $k_0 = 0.002\mathrm{Mpc}^{-1}$.

Introducing the radiative transfer function g^ζ_{Tl} (or $g^{\mathcal{S}}_{Tl}$), they are related to ζ (or $\mathcal{S}$) as

$$a_{lm} = a^\zeta_{lm} + a^{\mathcal{S}}_{lm}, \quad (20)$$

$$a^\zeta_{lm} = 4\pi(-i)^l \int \frac{d^3\mathbf{k}}{(2\pi)^3}\zeta_{\mathbf{k}} g^\zeta_{Tl}(k) Y^*_{lm}(\hat{\mathbf{k}}), \quad (21)$$

$$a^{\mathcal{S}}_{lm} = 4\pi(-i)^l \int \frac{d^3\mathbf{k}}{(2\pi)^3}\mathcal{S}_{\mathbf{k}} g^{\mathcal{S}}_{Tl}(k) Y^*_{lm}(\hat{\mathbf{k}}). \quad (22)$$

The angular power spectra of adiabatic and isocurvature components are

$$C^{\zeta\zeta}_l = \frac{2}{\pi}\int k^2 dk P_{\zeta\zeta}(k) g^\zeta_{Tl}(k)^2, \quad (23)$$

$$C^{\mathcal{SS}}_l = \frac{2}{\pi}\int k^2 dk P_{\mathcal{SS}}(k) g^{\mathcal{S}}_{Tl}(k)^2. \quad (24)$$

Fig. 1 shows the angular power spectrum from adiabatic and isocurvature perturbations. The radiation transfer functions for adiabatic and isocurvature perturbations are computed using the publicly-available CMBFAST code (Seljak & Zaldarriaga 1996). The spectral index of isocurvature perturbations in the Linear Model is $n_\eta = 1$. We set $\alpha = 0.067$ for $n_\eta = 1$ at $k_0 = 0.002\mathrm{Mpc}^{-1}$, which is a 2σ upper limit from the WMAP 5-year paper (Komatsu et al. 2008).

The total angular bispectrum of CMB is written as the sum of bispectra with different combinations of adiabatic and isocurvature components:

$$b_{l_1l_2l_3} = \sum_{IJK} b^{IJK}_{l_1l_2l_3}, \quad (25)$$

where each component is defined as

$$\langle a^I_{l_1m_1} a^J_{l_2m_2} a^K_{l_3m_3}\rangle \equiv \mathcal{G}^{m_1m_2m_3}_{l_1l_2l_3} b^{IJK}_{l_1l_2l_3}, \quad (26)$$

$$\mathcal{G}^{m_1m_2m_3}_{l_1l_2l_3} \equiv \int d\hat{\mathbf{n}}\, Y_{l_1m_1}(\hat{\mathbf{n}}) Y_{l_2m_2}(\hat{\mathbf{n}}) Y_{l_3m_3}(\hat{\mathbf{n}}) \quad (27)$$

$$= \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix}, \quad (28)$$

where I and J denote ζ or $\mathcal{S}$. Then, $b^{IJK}_{l_1l_2l_3}$ can be written as

$$b^{IJK}_{l_1l_2l_3} = \frac{8}{\pi^3}\int r^2 dr \int k_1^2 dk_1 \int k_2^2 dk_2 \int k_3^2 dk_3\, g^I_{Tl_1}(k_1) j_{l_1}(k_1 r) \times g^J_{Tl_2}(k_2) j_{l_2}(k_2 r) g^K_{Tl_3}(k_3) j_{l_3}(k_3 r) B_{IJK}(k_1,k_2,k_3). \quad (29)$$

The explicit form of adiabatic, isocurvature bispectra and their cross-correlations are given in Appendix A.

Fig. 2 shows each component of the CMB bispectrum for equilateral triangles ($l_1 = l_2 = l_3 = l$). The purely adiabatic component $b^{\zeta\zeta\zeta}_{lll}$ with $f_{\rm NL} = 50$ is plotted with a solid line. A long-dashed line represents a mixed component $b^{(\zeta\zeta\mathcal{S})}_{lll}$ with $|\cos\theta_{\phi\eta}| = 1$ and the curvaton-type upper limit $\alpha = 0.0037$. The other mixed component $b^{(\zeta\mathcal{SS})}_{lll}$ with $|\cos\theta_{\phi\eta}| = 0.1$ and the axion-type limit $\alpha = 0.067$ is plotted with a short-dashed line. The pure isocurvature component $b^{\mathcal{SSS}}_{lll}$ with the axion-type limit $\alpha = 0.067$ is plotted with dotted lines. The non-linearity of isocurvature perturbation $f^{(\rm ISO)}_{\rm NL}$ is set to be 10^4. The spectral index of isocurvature perturbation is 1.

We numerically estimate how large $f^{(\rm ISO)}_{\rm NL}$, the non-Gaussianity from isocurvature perturbations, should be to obtain the same values of the signal-to-noise ratio of the CMB bispectrum as the one derived from the non-Gaussianity from purely adiabatic perturbations characterized by $f_{\rm NL}$. The signal to noise ratio is defined as

$$\left(\frac{S}{N}\right)^2 = \sum_{2\le l_1\le l_2\le l_3} I^2_{l_1l_2l_3}\frac{(b^{IJK}_{l_1l_2l_3})^2}{\mathcal{C}_{l_1}\mathcal{C}_{l_2}\mathcal{C}_{l_3}\Delta_{l_1l_2l_3}}, \quad (30)$$

where $\mathcal{C}_l$ is C_l of purely adiabatic perturbations ($\alpha = 0$) including noise and $I_{l_1l_2l_3}$ is defined as

$$I_{l_1l_2l_3} \equiv \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix}. \quad (31)$$

The factor $\Delta_{l_1l_2l_3}$ is equal to 6 ($l_1 = l_2 = l_3$), 2 ($l_1 = l_2$ or $l_2 = l_3$), and 1 ($l_1 \neq l_2 \neq l_3$). WMAP beam window functions are included both in the bispectrum and in the power spectra. The homogeneous noise distribution for the WMAP 5-year data is used to estimate the noise.

The pure isocurvature term $b^{\mathcal{SSS}}$ with $n_\eta = 1$ has non-Gaussianity corresponding to

$$f_{\rm NL} \simeq 15\left(\frac{\alpha}{0.067}\right)^2\left(\frac{f^{(\rm ISO)}_{\rm NL}}{10^4}\right), \quad (32)$$

where the above equations are valid when $\alpha \ll 1$. It is found that isocurvature non-Gaussianity reaches $f_{\rm NL} \sim 10$ only if the non-linearity in isocurvature perturbations $f^{(\rm ISO)}_{\rm NL}$ is of order 10^4 because isocurvature non-Gaussianity in the Linear Model is suppressed by α^2.

When there is a weak correlation between ϕ and η ($|\cos\theta_{\phi\eta}| \ll 1$), $b^{(\zeta\mathcal{SS})}$ has non-Gaussianity corresponding to

$$f_{\rm NL} \simeq 9.7\left(\frac{\alpha}{0.067}\right)^{3/2}\left(\frac{|\cos\theta_{\phi\eta}|}{0.1}\right)\left(\frac{f^{(\rm ISO)}_{\rm NL}}{10^4}\right). \quad (33)$$

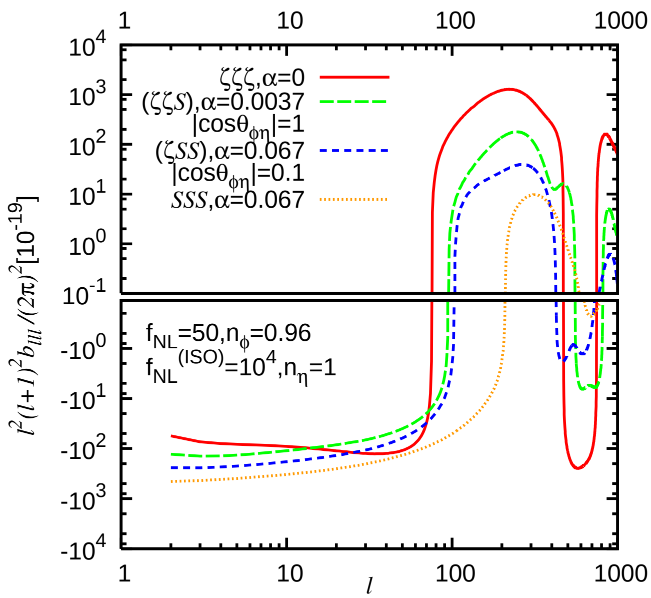

Figure 2. CMB angular bispectra of equilateral configurations $l^2(l+1)^2 b_{lll}/(2\pi)^2$ in the Linear Model; the adiabatic component $b_{lll}^{\zeta\zeta\zeta}$ (solid), the mixed components $b_{lll}^{(\zeta\zeta\mathcal{S})}$ (long-dashed) and $b_{lll}^{(\zeta\mathcal{S}\mathcal{S})}$ (short-dashed) and the isocurvature component $b_{lll}^{\mathcal{S}\mathcal{S}\mathcal{S}}$ (dotted). Upper (Lower) panel shows the positive (negative) side of bispectra plotted in logarithmic scale. The adiabatic perturbation has the power-law index $n_\phi = 0.96$ and its quadratic amplitude $f_{\rm NL} = 50$. The isocurvature perturbation has $n_\eta = 1$ and $f_{\rm NL}^{(ISO)} = 10^4$. The fraction of the isocurvature power spectrum α is set to be an axion-type upper limit 0.067 for $b_{lll}^{\mathcal{S}\mathcal{S}\mathcal{S}}$ and $b_{lll}^{(\zeta\mathcal{S}\mathcal{S})}$ with a weak correlation $|\cos\theta_{\phi\eta}| = 0.1$ and a curvaton-type upper limit 0.0037 for $b_{lll}^{(\zeta\zeta\mathcal{S})}$ with a strong correlation $|\cos\theta_{\phi\eta}| = 1$.

If the correlation is strong $|\cos\theta_{\phi\eta} \simeq 1|$ (e.g., curvaton-type isocurvature perturbation), the other correlated term $b^{(\zeta\zeta\mathcal{S})}$ becomes important with a more severe limit $\alpha < 0.0037$ for curvaton-type isocurvature perturbations;

$$f_{\rm NL} \simeq 7.2 \left(\frac{\alpha}{0.0037}\right) \cos^2\theta_{\phi\eta} \left(\frac{f_{\rm NL}^{\rm (ISO)}}{10^4}\right). \quad (34)$$

In both cases, however, isocurvature perturbations need to have a strong non-linearity $f_{\rm NL}^{(ISO)} \sim 10^4$ in order to generate non-Gaussianity corresponding to $f_{\rm NL} \sim 10$. Our result is consistent with previous theoretical estimations (Kawasaki et al. 2008; Langlois, Vernizzi & Wands 2008b).

4 NON-GAUSSIANITY OF ISOCURVATURE PERTURBATIONS II: QUADRATIC MODEL

4.1 Initial Perturbation

In the Quadratic Model (eq.[4]), the auto and cross power spectra of ζ and $\mathcal{S}$ become

$$P_{\mathcal{S}\mathcal{S}}(k) = 2\int_{L^{-1}} \frac{d^3\mathbf{p}}{(2\pi)^3} P_{\sigma\sigma}(p) P_{\sigma\sigma}(|\mathbf{k}+\mathbf{p}|), \quad (35)$$

$$P_{\zeta\mathcal{S}}(k) = \frac{6}{5} f_{\rm NL} \int_{L^{-1}} \frac{d^3\mathbf{p}}{(2\pi)^3} P_{\phi\sigma}(p) P_{\phi\sigma}(|\mathbf{k}+\mathbf{p}|), \quad (36)$$

where a finite box-size L gives an infrared cutoff. To avoid assumptions at scales far beyond the present horizon H_0^{-1}, L should be set not too much bigger than H_0^{-1} (Lyth 2007). Hereafter we set $L = 30$Gpc.

Using the power-law form as in the equations (7), the isocurvature power spectra are written as

$$\Delta_{\mathcal{S}\mathcal{S}}(k) = A_\sigma^2 F(n_\sigma) \left(\frac{k}{k_0}\right)^{2(n_\sigma - 1)}, \quad (37)$$

where the factor F is approximately given by

$$F(n_\sigma) = \begin{cases} 4\log(kL) & (n_\sigma = 1), \\ 4(n_\sigma - 1)^{-1}(1-(kL)^{1-n_\sigma}) & (n_\sigma \neq 1), \end{cases} \quad (38)$$

As we take L to be much larger than the range of scales we are interested in, the k dependence in F is weak.

As $\mathcal{S}$ is the square of a Gaussian variable, the cross-correlation coefficient becomes nearly zero regardless of $\cos\theta_{\phi\sigma}$;

$$\frac{\Delta_{\zeta\mathcal{S}}}{(\Delta_{\zeta\zeta}\Delta_{\mathcal{S}\mathcal{S}})^{1/2}} \simeq f_{\rm NL}(A_\phi F)^{1/2} \cos^2\theta_{\phi\sigma} < f_{\rm NL}(A_\phi F)^{1/2} \leq \mathcal{O}(10^{-2}). \quad (39)$$

Following the definition of α in the equation (10), the ratio between A_ϕ and A_σ becomes

$$\frac{A_\sigma}{A_\phi} = \left(\frac{\alpha}{1-\alpha}\right)^{1/2} (A_\phi F)^{-1/2}. \quad (40)$$

The amplitude of σ has an additional factor $(A_\phi F)^{-1/2} \sim \mathcal{O}(10^4)$ relative to the amplitude of ϕ because the isocurvature perturbation is given as quadratic in σ (eq. [4]).

The bispectra from isocurvature perturbations are written as

$$B_{\mathcal{S}\mathcal{S}\mathcal{S}}(k_1,k_2,k_3) = \frac{8}{3}\int_{L^{-1}} \frac{d^3\mathbf{p}}{(2\pi)^3} P_{\sigma\sigma}(p) \times[P_{\sigma\sigma}(|\mathbf{k}_1+\mathbf{p}|)P_{\sigma\sigma}(|\mathbf{k}_2-\mathbf{p}|) + P_{\sigma\sigma}(|\mathbf{k}_2+\mathbf{p}|)P_{\sigma\sigma}(|\mathbf{k}_3-\mathbf{p}|) + P_{\sigma\sigma}(|\mathbf{k}_3+\mathbf{p}|)P_{\sigma\sigma}(|\mathbf{k}_1-\mathbf{p}|)]. \quad (41)$$

When we extract the dominant contributions around the poles, the equation (41) is approximately written as (Kawasaki et al. 2008)

$$B_{\mathcal{S}\mathcal{S}\mathcal{S}}(k_1,k_2,k_3) \simeq 2\Delta_{\sigma\sigma}(k_b)F(n_\sigma)[P_{\sigma\sigma}(k_1)P_{\sigma\sigma}(k_2) + P_{\sigma\sigma}(k_2)P_{\sigma\sigma}(k_3) + P_{\sigma\sigma}(k_3)P_{\sigma\sigma}(k_1)], \quad (42)$$

where $k_b = Min(k_1,k_2,k_3)$.

The cross-correlation terms are

$$B_{(\zeta\zeta\mathcal{S})}(k_1,k_2,k_3) = 2[P_{\phi\sigma}(k_1)P_{\phi\sigma}(k_2) + P_{\phi\sigma}(k_2)P_{\phi\sigma}(k_3) + P_{\phi\sigma}(k_3)P_{\phi\sigma}(k_1)], \quad (43)$$

$$B_{(\zeta\mathcal{S}\mathcal{S})}(k_1,k_2,k_3) = \frac{24}{5} f_{\rm NL} \int_{L^{-1}} \frac{d^3\mathbf{p}}{(2\pi)^3} P_{\sigma\sigma}(\mathbf{p}) \times[P_{\phi\sigma}(|\mathbf{k}_1+\mathbf{p}|)P_{\phi\sigma}(|\mathbf{k}_2-\mathbf{p}|) + P_{\phi\sigma}(|\mathbf{k}_2+\mathbf{p}|)P_{\phi\sigma}(|\mathbf{k}_3-\mathbf{p}|) + P_{\phi\sigma}(|\mathbf{k}_3+\mathbf{p}|)P_{\phi\sigma}(|\mathbf{k}_1-\mathbf{p}|)]. \quad (44)$$

The amplitude of each bispectrum component relative to the pure adiabatic one at k_0 is estimated as

$$\frac{B_{\mathcal{SSS}}}{B_{\zeta\zeta\zeta}} = \frac{5}{3} f_{\rm NL}^{-1}\left(\frac{\alpha}{1-\alpha}\right)^{3/2}(A_\phi F)^{-1/2}, \qquad (45)$$

$$\frac{B_{(\zeta\zeta\mathcal{S})}}{B_{\zeta\zeta\zeta}} = \frac{5}{3} f_{\rm NL}^{-1}\left(\frac{\alpha}{1-\alpha}\right)^{1/2}\cos^2\theta_{\phi\sigma}(A_\phi F)^{-1/2}, \qquad (46)$$

$$\frac{B_{(\zeta\mathcal{SS})}}{B_{\zeta\zeta\zeta}} = 3\left(\frac{\alpha}{1-\alpha}\right)\cos^2\theta_{\phi\sigma}. \qquad (47)$$

The bispectra $B_{(\zeta\zeta\mathcal{S})}$ and $B_{\mathcal{SSS}}$ has an additional factor $(A_\phi F)^{-1/2} \sim \mathcal{O}(10^4)$ relative to $B_{\zeta\zeta\zeta}$. This is because the isocurvature perturbation has no Gaussian part (eq.[4]) and thus its cubic term does not vanish unlike the adiabatic case. The isocurvature non-Gaussianity therefore can be substantial even if $\alpha \sim 0.067$. We neglect the last term $B_{(\zeta\mathcal{SS})}$ due to the current observational limit on $\alpha < 0.067$.

4.2 Bispectra for CMB Temperature Anisotropy

The angular power spectra for isocurvature perturbations in the Quadratic Model are plotted in Fig. 1 for different spectral indices n_σ=1, 1.5 and 2. We set $\alpha = 0.067$ for $n_\sigma = 1$ as in the Linear Model. The power spectrum in the Quadratic Model is slightly different from that in the Linear Model due to the weak scale dependence of F (eq.[38]). For other spectral indexes, we set α=0.008 ($n_\sigma = 1.5$) and 0.001 ($n_\sigma = 2$) so that the amplitude of C_l around l=200 becomes roughly the same.

The angular bispectrum from the isocurvature term $B_{\mathcal{SSS}}$ is approximately given by putting k_b as one of the wavenumbers of $P_{\sigma\sigma}$ in the equation (42);

$$\begin{aligned} b^{\mathcal{SSS}}_{l_1l_2l_3} = 2\int r^2dr[& b^{\mathcal{S},\mathcal{SS}}_{Ll_1}(r)b^{\mathcal{S},\sigma\sigma}_{Ll_2}(r)b^{\mathcal{S}}_{NLl_3}(r) \\ & +b^{\mathcal{S},\sigma\sigma}_{Ll_1}(r)b^{\mathcal{S}}_{NLl_2}(r)b^{\mathcal{S},\mathcal{SS}}_{Ll_3}(r) \\ & +b^{\mathcal{S}}_{NLl_1}(r)b^{\mathcal{S},\mathcal{SS}}_{Ll_2}(r)b^{\mathcal{S},\sigma\sigma}_{Ll_3}(r)], \end{aligned} \qquad (48)$$

where

$$b^{\mathcal{S},\mathcal{SS}}_{Ll}(r) \equiv \frac{2}{\pi}\int_{L^{-1}} k^2dkP_{\mathcal{SS}}(k)g^{\mathcal{S}}_{Tl}(k)j_l(kr). \qquad (49)$$

In order to test the validity of the above approximation, we compare the equation (48) with full calculation without the pole approximation which is used to derive the equation (42). The angular bispectrum $B_{\mathcal{SSS}}$ without the pole approximation is analytically given as (eq. (C.7) of Komatsu 2002)

$$\begin{aligned} b^{\mathcal{SSS}}_{l_1l_2l_3} = & \frac{8}{3}\int r^2dr\int_{L^{-1}}\frac{p^2dp}{(2\pi)^3}P_{\sigma\sigma}(p)\left[\sum_{l_1'l_2'l}\mathcal{F}^{l_1l_2l_3}_{l_2'l_1'l}\right. \\ & \times \frac{2}{\pi}\int_{L^{-1}} k_1^2dk_1\tilde{P}^{(+)}_{\sigma\sigma\ l}(k_1,p)g^{\mathcal{S}}_{Tl_1}(k_1)j_{l_1'}(k_1r)(-i)^{l_1-l_1'} \\ & \times \frac{2}{\pi}\int_{L^{-1}} k_2^2dk_2\tilde{P}^{(-)}_{\sigma\sigma\ l}(k_2,p)g^{\mathcal{S}}_{Tl_2}(k_2)j_{l_2'}(k_2r)(-i)^{l_2-l_2'} \\ & \times \frac{2}{\pi}\int_{L^{-1}} k_3^2dk_3g^{\mathcal{S}}_{Tl_3}(k_3)j_{l_3}(k_3r) + ({\rm cyc.})], \end{aligned} \qquad (50)$$

where

$$\begin{aligned} \mathcal{F}^{l_1l_2l_3}_{l_2'l_1'l} \equiv & \frac{(2l_1'+1)(2l_2'+1)(2l+1)}{4\pi}\begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix}^{-1} \\ & \times \begin{Bmatrix} l_1 & l_2 & l_3 \\ l_2' & l_1' & l \end{Bmatrix}\begin{pmatrix} l_1' & l_2' & l_3 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} l_1 & l_1' & l \\ 0 & 0 & 0 \end{pmatrix} \\ & \times \begin{pmatrix} l_2 & l_2' & l \\ 0 & 0 & 0 \end{pmatrix}(-1)^{l_1'+l_2'+l}, \end{aligned} \qquad (51)$$

and

$$P_{\sigma\sigma}(|\mathbf{k}\pm\mathbf{p}|) = \sum_{lm}\tilde{P}^{(\pm)}_{\sigma\sigma\ l}(k,p)Y_{lm}(\hat{\mathbf{k}})Y^*_{lm}(\hat{\mathbf{p}}). \qquad (52)$$

When the power spectrum is given in power-law form as in equation (7),

$$\frac{k^3\tilde{P}^{(-)}_{\sigma\sigma\ l}(k,p)}{2\pi^2} = \begin{cases} 4\pi A_\sigma\left(\dfrac{k}{p}\right)^3\left(\dfrac{p}{k_0}\right)^{n_\sigma-1}h_l\left(\dfrac{k}{p}\right), & \text{when } p > k+L^{-1}, \\ 4\pi A_\sigma\left(\dfrac{k}{k_0}\right)^{n_\sigma-1}h_l\left(\dfrac{p}{k}\right), & \text{when } p < k-L^{-1}, \end{cases} \qquad (53)$$

$$\tilde{P}^{(+)}_{\sigma\sigma\ l}(k,p) = (-1)^l\tilde{P}^{(-)}_{\sigma\sigma\ l}(k,p), \qquad (54)$$

where $h_l(x)$ is the expansion coefficient by Legendre function $P_l(z)$ as

$$h_l(y) = \int_{-1}^{1}dz(y^2-2yz+1)^{(n_\sigma-4)/2}P_l(z). \qquad (55)$$

When n_σ is equal to 1, $h_l(y)$ is analytically given as $2y^l/(1-y^2)$.

In the left panel of Fig. 3, we compare the bispectrum of the adiabatic component with those of the isocurvature components with different spectral indexes n_σ of 1, 1.5 and 2. We set $f_{\rm NL} = 50$ and $\alpha = 0.067$ for $n_\sigma = 1$, $\alpha = 0.008$ for $n_\sigma = 1.5$ and $\alpha = 0.001$ for $n_\sigma = 2$ (same as Fig. 1). The box-size L is set to be 30Gpc.

Computationally, it is very difficult to evaluate the full expression at high ℓ. Thus we check the validity of the pole approximations at low ℓ, less than 10. The thick lines represent the full calculations of bispectra given in equation (50). It is found that the isocurvature bispectra approximated as the equation (48) roughly agree with the full calculations within a factor 2 at least for $l \leq 10$. The full calculation has comparable or larger amplitude for all n_σ, and a slightly steeper slope than the pole approximation. This indicates that the isocurvature bispectrum at higher l would become larger than the pole approximation and then the proper signal may be larger than the pole approximation. The amplitude of the isocurvature bispectrum is proportional to $\alpha^{3/2}$ and therefore the effect on the estimation of α is suppressed at the power of two-third.

The isocurvature bispectrum in the Quadratic Model depends on the assumed box-size L, which is set to be 30Gpc in this analysis. The equation (45) indicates that the ratio of the isocurvature component in the primordial perturbation at $k = k_0 = 0.002{\rm Mpc}^{-1}$ is proportional to a box-size dependent factor $F^{-1/2}$ (eq. [38]). When L is set to be ten times larger (300Gpc), the amplitude decreases by 20% for $n_\sigma = 1$, 5% for $n_\sigma = 1.5$, and 1% for $n_\sigma = 2$. We find that the overall amplitude of the CMB bispectrum also decreases at the same level at $l > 10$, while additional large-scale power at more than 30Gpc slightly increases the amplitude of the bispectrum at smaller l . The isocurvature bispectrum depends on $\alpha^{3/2}$ and therefore the error in α increases $(1/0.8)^{2/3} \simeq 16\%$ for $n_\sigma = 1$.

Using the equation (30), we estimate the isocurvature non-Gaussianity in terms of $f_{\rm NL}$;

$$f_{\rm NL} = 30\left(\frac{\alpha}{0.067}\right)^{3/2}. \quad (56)$$

It is found that the isocurvature non-Gaussianity in the Quadratic Model can reach $f_{\rm NL} \sim 30$ given the current 2σ limit on α.

The angular bispectrum from the correlation term $B_{(\zeta\zeta\mathcal{S})}$ is given as

$$b^{(\zeta\zeta\mathcal{S})}_{l_1l_2l_3} = 2\int r^2dr[b^{\zeta,\phi\sigma}_{Ll_1}(r)b^{\zeta,\phi\sigma}_{Ll_2}(r)b^{\mathcal{S}}_{NLl_3}(r) + b^{\zeta,\phi\sigma}_{Ll_1}(r)b^{\mathcal{S}}_{NLl_2}(r)b^{\zeta,\phi\sigma}_{Ll_3}(r) + b^{\mathcal{S}}_{NLl_1}(r)b^{\zeta,\phi\sigma}_{Ll_2}(r)b^{\zeta,\phi\sigma}_{Ll_3}(r)]. \quad (57)$$

The correlated term $b^{(\zeta\zeta\mathcal{S})}_{l_1l_2l_3}$ is plotted in the right panel of Fig. 3 for different spectral indexes n_σ=1, 1.5 and 2. The correlated coefficient $\cos\theta_{\phi\sigma}$ is set to be 0.1. Using equation (30), the isocurvature non-Gaussianity with $n_\sigma = 1$ corresponds to

$$f_{\rm NL} = 240\left(\frac{\alpha}{0.067}\right)^{1/2}\cos^2\theta_{\phi\sigma}. \quad (58)$$

If ϕ and σ are strongly correlated initially, the correlation term can generate substantial non-Gaussianity, while the effect of the correlation on C_l is negligible as shown in equation (39).

5 PERTURBATIVE FORMULAE OF MINKOWSKI FUNCTIONALS

We adopt the perturbation formulae for Minkowski Functionals developed by Matsubara (2003) to describe the non-Gaussianity of the CMB temperature anisotropy (Hikage, Komatsu & Matsubara 2006; Hikage et al. 2008). We separate the analytical formulae of Minkowski Functionals into the amplitude and a function of ν, which is defined as $\Delta T/T$ divided by its standard deviation, as follows;

$$V_k(\nu) = A_k v_k(\nu). \quad (59)$$

The amplitude A_k is given using the angular power spectrum C_l as

$$A_k = \frac{1}{(2\pi)^{(k+1)/2}}\frac{\omega_2}{\omega_{2-k}\omega_k}\left(\frac{\sigma_1}{\sqrt{2}\sigma_0}\right)^k, \quad (60)$$

$$\sigma_j^2 \equiv \frac{1}{4\pi}\sum_l (2l+1)\left[l(l+1)\right]^j C_l W_l^2, \quad (61)$$

where $\omega_k \equiv \pi^{k/2}/\Gamma(k/2+1)$ gives $\omega_0 = 1$, $\omega_1 = 2$, $\omega_2 = \pi$ and W_l represents the smoothing kernel determined by the pixel and beam window functions and an additional smoothing. In our analysis, we add a Gaussian kernel $W_l = \exp[-l(l+1)\theta_s^2/2]$ where θ_s is a smoothing scale. For weakly non-Gaussian fields, the function $v_k(\nu)$ can be divided into the Gaussian term $v_k^{(G)}$ and the non-Gaussian term Δv_k;

$$v_k(\nu) = v_k^{(G)}(\nu) + \Delta v_k(\nu, f_{\rm NL}), \quad (62)$$

$$v_k^{(G)} = e^{-\nu^2/2}H_{k-1}(\nu), \quad (63)$$

$$\Delta v_k(\nu, f_{\rm NL}) = e^{-\nu^2/2}\left\{\left[\frac{1}{6}S^{(0)}H_{k+2}(\nu) + \frac{k}{3}S^{(1)}H_k(\nu) + \frac{k(k-1)}{6}S^{(2)}H_{k-2}(\nu)\right]\sigma_0 + \mathcal{O}(\sigma_0^2)\right\} \quad (64)$$

where $H_n(\nu)$ is the n-th Hermite polynomials. The non-Gaussian term Δv_k is characterized by three skewness parameters $S^{(k)}$ at lowest order in σ_0. The skewness parameters $S^{(k)}$ are given by the integral of the reduced bispectrum as

$$S^{(0)} = \frac{3}{2\pi\sigma_0^4}\sum_{2\le l_1\le l_2\le l_3} I^2_{l_1l_2l_3}b_{l_1l_2l_3}W_{l_1}W_{l_2}W_{l_3}, \quad (65)$$

$$S^{(1)} = \frac{3}{8\pi\sigma_0^2\sigma_1^2}\sum_{2\le l_1\le l_2\le l_3}[l_1(l_1+1)+l_2(l_2+1)+l_3(l_3+1)] \times I^2_{l_1l_2l_3}b_{l_1l_2l_3}W_{l_1}W_{l_2}W_{l_3}, \quad (66)$$

$$S^{(2)} = \frac{3}{4\pi\sigma_1^4}\sum_{2\le l_1\le l_2\le l_3}\{[l_1(l_1+1)+l_2(l_2+1)-l_3(l_3+1)] \times l_3(l_3+1) + ({\rm cyc.})\}\, I^2_{l_1l_2l_3}b_{l_1l_2l_3}W_{l_1}W_{l_2}W_{l_3}, \quad (67)$$

where $I_{l_1l_2l_3}$ was previously defined in Eq. (31).

In Fig. 4, we plot the three skewness parameters $S^{(k)}$ from each component of bispectrum in the Quadratic Model as a function of θ_s. Upper panels show the skewness of isocurvature components with different spectral indices n_σ =1, 1.5 and 2 in comparison with the adiabatic case. The skewness from a mixed component $b^{(\zeta\zeta\mathcal{S})}$ is plotted in the lower panels. The smaller scale (higher l) information in the bispectrum is reflected in skewness parameters with a higher number (that is $S^{(2)}$ rather than $S^{(0)}$) because it is weighted towards higher l. We can extract further detailed scale-dependent information by changing the smoothing scale θ_s.

Fig. 5 illustrates an example of non-Gaussian effects on each Minkowski Functional Δv_k (eq.[64]) at different smoothing scales θ_s=10, 20, 40, 70 and 100 arcmin. Here we consider isocurvature non-Gaussianity in the Quadratic Model with $n_\sigma = 1$ and $\alpha = 0.067$. It is found that the non-Gaussian effect on Minkowski Functionals is 1% or less.

6 LIMITS ON ISOCURVATURE NON-GAUSSIANITY FROM WMAP DATA

We use the WMAP 5-year data to constrain the non-Gaussianity associated with primordial isocurvature perturbations. We use the linearly co-added maps for Q, V and W frequency bands with $N_{\rm side} = 512$. The co-added maps are masked with the *Kq75* galaxy mask including the point-source mask provided by Gold et al. (2009), which leaves 71.8% of the sky available for the data analysis. The field is smoothed with a Gaussian filter at a scale of θ_s. We obtain the normalized Minkowski Functional (eq.[64]) for the WMAP data using the same procedure described in Hikage et al. (2008).

We use Bayes's theorem to find a probability of α or other combined parameters, together denoted by x, from the observed set of $\Delta\nu^{(\rm obs)}$ as follows;

$$\mathcal{P}(x|\Delta\nu^{(\rm obs)}) \propto \mathcal{L}(\Delta v^{(\rm obs)}|x)\mathcal{P}(x), \quad (68)$$

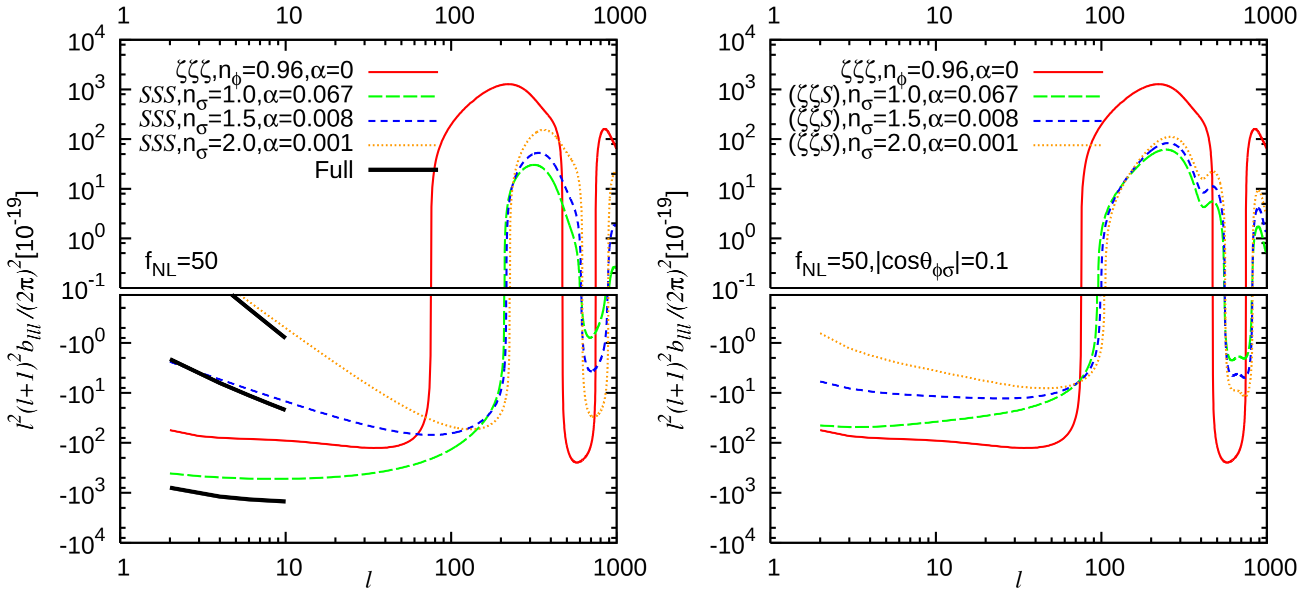

Figure 3. CMB angular bispectra of equilateral configurations $l^2(l+1)^2b_{lll}/(2\pi)^2$ in the Quadratic Model; the isocurvature components b_{lll}^{SSS} (*Left*) and the mixed components $b_{lll}^{(\zeta\zeta S)}$ (*Right*). The power-law index of isocurvature perturbation n_σ is set to be 1 (long-dashed), 1.5 (short-dashed) and 2 (dotted). For comparison, the adiabatic bispectrum $b_{lll}^{\zeta\zeta\zeta}$ with $f_{\rm NL}=50$ is plotted in both panels (thin solid lines). The fraction of isocurvature power spectrum α is 0.067 ($n_\sigma=1$), 0.008 ($n_\sigma=1.5$) and 0.001 ($n_\sigma=2$) defined at $k_0=0.002{\rm Mpc}^{-1}$. Thick solid lines in Left panels show the full calculations of isocurvature terms (eq.[50]) for each n_σ at $l\le 10$. The box-size $L_{\rm max}$ is set to be 30Gpc.

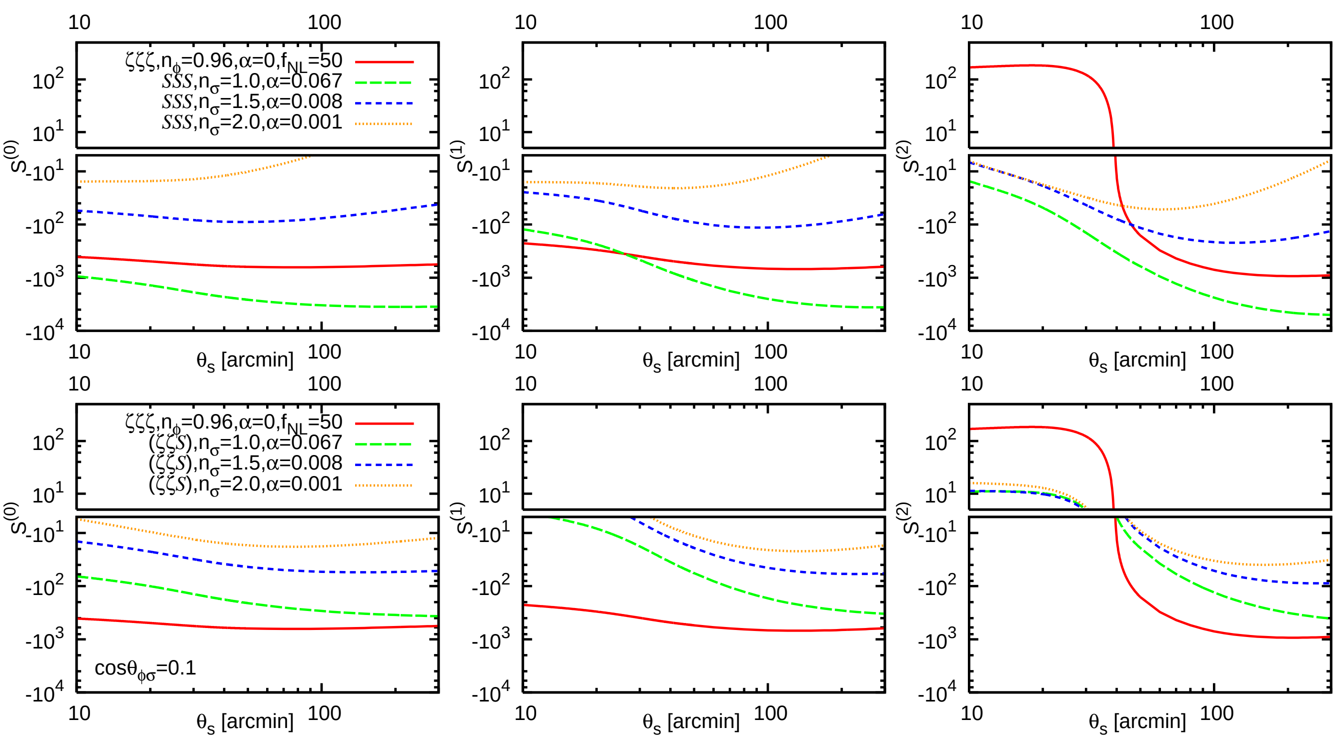

Figure 4. Skewness parameters $S^{(k)}$ (*left:* $k=0$, *center:* $k=1$, *right:* $k=2$) of each component in the Quadratic Model plotted as a function of θ_s. The skewness values for a pure isocurvature component are plotted in upper panels, while those for a mixed component are plotted in lower panels. The parameters of isocurvature perturbation are n_σ=1 with $\alpha=0.067$ (long-dashed), n_σ=1.5 with $\alpha=0.008$ (short-dashed), and n_σ=2 with $\alpha=0.001$ (dotted). For comparison, the adiabatic skewness with $n_\phi=0.96$ and $f_{\rm NL}=50$ are plotted with solid lines in all panels.

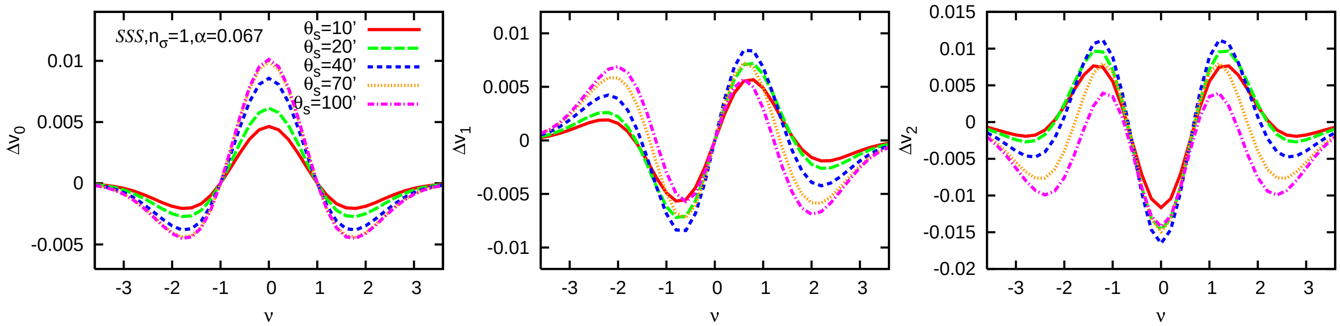

Figure 5. Non-Gaussian term of Minkowski Functionals Δv_k (eq.[64]) from isocurvature bispectrum in Quadratic Model (*left:* $k=0$, *center:* $k=1$, *right:* $k=2$). The isocurvature perturbation has a spectral index n_σ=1 and the fraction α is 0.067. The smoothing scales are shown by different lines: $\theta_s=10$ (solid), $\theta_s=20$ (long-dashed), $\theta_s=40$ (short-dashed) $\theta_s=70$ (dotted), and $\theta_s=100$ (dot-dashed) arcmin.

where $\mathcal{L}$ is the likelihood function of $\Delta v^{(\rm obs)}$ when a non-Gaussian parameter has a value x and the probability $\mathcal{P}(x)$ represents the prior for x. In general, we need to analyze all non-Gaussian components together, however we consider the cases in which a single non-Gaussian term dominates over the other terms for simplicity; in the Linear Model, the pure isocurvature term (eq.[A2]) dominates when there is no correlation between ϕ and η. If there exists a weak correlation, the correlation term of ($\zeta\mathcal{SS}$) (eq. [A8]) becomes important. The other correlation term of ($\zeta\zeta\mathcal{S}$) (eq. [A7]) dominates when the correlation is strong ($\cos\theta_{\phi\eta}\simeq 1$). In these three cases, we set the parameter x to be $\alpha^2 f_{\rm NL}^{(\rm ISO)}$, $\alpha^{3/2}\cos\theta_{\phi\eta}f_{\rm NL}^{(\rm ISO)}$ and $\alpha\cos^2\theta_{\phi\eta}f_{\rm NL}^{(\rm ISO)}$. In the Quadratic Model, the pure isocurvature term (eq.[48]) dominates at $\cos\theta_{\phi\sigma}\ll 1$. Then we set x to be α (not $\alpha^{3/2}$). When the correlation between ϕ and σ is strong, the correlation term (eq.[57]) is dominant and then x is $\alpha^{1/2}\cos^2\theta_{\phi\sigma}$. We assume a flat prior for all x. We furthermore impose a non-negative constraint on the parameters in the Quadratic Model, α and $\alpha^{1/2}\cos^2\theta_{\phi\sigma}$ by definition of α (eq.[10]).

The likelihood function is computed by

$$-2\ln\mathcal{L}(\Delta v^{(\rm obs)}|x) \propto \sum_{ij}[\Delta v_i^{(\rm obs)}-\Delta v_i^{(\rm theory)}(x)]C_{ij}^{-1} \times [\Delta v_j^{(\rm obs)}-\Delta v_j^{(\rm theory)}(x)], \qquad (69)$$

where i and j denote the binning number of threshold values ν, different kinds of Minkowski Functional k, and smoothing scales parameterized with θ_s. We choose 18 threshold values at an equal spacing in the range of ν from -3.6 to 3.6. The full covariance matrix C_{ij} is estimated from 1000 Gaussian simulation maps from purely adiabatic perturbations. They include the pixel and beam window function, *Kq75* survey mask, and inhomogeneous noise for WMAP 5-year maps. Applying our procedure to limit the non-Gaussianity from curvature perturbations by putting x as $f_{\rm NL}$, we obtain $-63<f_{\rm NL}<76$ at 95% CL (Hikage et al. in preparation).

Table 1 lists the mean and one-sigma error of the isocurvature non-Gaussianity in the Linear Model characterized by $\alpha\cos^2\theta_{\phi\eta}f_{\rm NL}^{(\rm ISO)}$, $\alpha^{3/2}\cos\theta_{\phi\eta}f_{\rm NL}^{(\rm ISO)}$, and $\alpha^2 f_{\rm NL}^{(\rm ISO)}$. Significant isocurvature non-Gaussian signals are not found. If isocurvature perturbations exist and there is no correlation between ϕ and η, the non-linear parameter $f_{\rm NL}^{(\rm ISO)}$ is constrained from isocurvature non-Gaussianity for a fixed α to be

$$f_{\rm NL}^{(\rm ISO)}=(-3300\pm 13000)(\alpha/0.067)^{-2}. \qquad (70)$$

If there is a strong correlation between ϕ and η represented by curvaton-type isocurvature perturbations ($\cos\theta_{\phi\eta}=-1$), the correlated term ($\zeta\zeta\mathcal{S}$) becomes important and then its non-Gaussianity is limited as

$$f_{\rm NL}^{(\rm ISO)}=(4900\pm 43000)(\alpha/0.0037)^{-1}. \qquad (71)$$

Table 2 lists the maximum likelihood value $\alpha_{\rm ML}$, at which $\mathcal{P}(\alpha)$ has a maximum value, and the 95% confidence limit of α when the pure isocurvature term in the Quadratic Model dominates. We do not find a significant non-zero value of α. The upper limit of α is given by

$$\begin{aligned}\alpha &< 0.070 \quad (n_\sigma=1),\\ \alpha &< 0.042 \quad (n_\sigma=1.5),\\ \alpha &< 0.0064 \quad (n_\sigma=2),\end{aligned} \qquad (72)$$

at 95% CL. Our constraint on α for $n_\sigma=1$ is comparable to that from the joint limit from WMAP(TT and TE spectra)+BAO+SN, which is smaller than 0.067 (95% CL), obtained by Komatsu et al. (2008). The limits on the correlated term $\alpha^{1/2}\cos^2\theta_{\phi\sigma}$ are also listed in Table 3. Our result roughly agrees with Boubekeur & Lyth (2006) who showed the fraction of a Gaussian-squared component of primordial curvature perturbation is limited to be less than 0.18, which corresponds to $\alpha<0.031$, when $|f_{\rm NL}|<100$. The difference from their analysis is that we calculate a Gaussian-squared component of CMB isocurvature bispectra including its full radiative transfer function.

7 IMPLICATIONS FOR AXION ISOCURVATURE

Here we briefly discuss implications of our results in some explicit models. Although, for the linear model, the constraint from non-Gaussianity obtained here is not as strong as that from the power spectrum, the constraint for the quadratic model is severe as we showed in the previous section. Thus we consider the case of the axion as discussed in Kawasaki et al. (2008). Assuming that Pecci-Quinn symmetry has already been spontaneously broken during inflation, the mean value of the axion field can be written as

$$a = f_a \theta_a, \tag{73}$$

where f_a is the axion decay constant and θ_a is the phase of the axion. During inflation, the axion field has quantum fluctuations

$$\delta a = \frac{H_{\rm inf}}{2\pi}, \tag{74}$$

where $H_{\rm inf}$ is the Hubble parameter during inflation. When the average value of the axion is much less the mean-square inhomogeneity of a, i.e., when $f_a\theta_a \leq H_{\rm inf}/2\pi$, the density fluctuation of a is written as

$$\frac{\delta\rho_a}{\rho_a} = \left(\frac{\delta a}{a}\right)^2. \tag{75}$$

Thus power spectrum for isocurvature fluctuations can be written as

$$\frac{k^3 P_{SS}}{2\pi^2} = \frac{(\Omega_a h^2)^2}{(\Omega_{\rm cdm}h^2)^2}, \tag{76}$$

where Ω_a is the energy density of the axion at present. When $f_a\theta < H_{\rm inf}/2\pi$, the abundance of the axion is given by (Turner 1986)

$$\Omega_a h^2 = 0.2\left(\frac{f_a}{10^{12}\ {\rm GeV}}\right)^{-0.825}\left(\frac{H_{\rm inf}/2\pi}{10^{12}\ {\rm GeV}}\right)^2. \tag{77}$$

Since the total amplitude of primordial perturbations should be $\Delta_{\rm tot} \simeq \Delta_{\zeta\zeta} + \Delta_{SS} = k^3/2\pi^2(P_{\zeta\zeta} + P_{SS}) = 2.4\times10^{-9}$ which is required from WMAP5, by using the constraint on α presented in the previous section, $\alpha < 0.070$, we obtain a limit for the Hubble parameter during inflation

$$H_{\rm inf} < 1.7\times10^{10}\ \left(\frac{f_a}{10^{12}\ {\rm GeV}}\right)^{0.41}\ {\rm GeV}, \tag{78}$$

where we adopt $\Omega_{\rm cdm}h^2 = 0.108$ which is the mean value for a ΛCDM model from the WMAP5 analysis. The condition $f_a\theta_a < H_{\rm inf}/2\pi$ gives

$$\theta_a < 2.7\times10^{-3}\left(\frac{f_a}{10^{12}\ {\rm GeV}}\right)^{-0.59}. \tag{79}$$

Although our results obtained in this paper may also have implications for other models with isocurvature fluctuations, a detailed study of this issue will be given elsewhere.

8 SUMMARY AND CONCLUSIONS

We have explored the effect of non-Gaussianity from primordial isocurvature perturbations on CMB temperature anisotropies. Considering the linear and quadratic forms of isocurvature perturbations, which are applicable to a wide range of theoretical models, we derived theoretical expressions for bispectra and Minkowski Functionals of CMB temperature maps with isocurvature non-Gaussianity. We find that the amplitude of a quadratic correction $f_{\rm NL}^{(ISO)}$ in the Linear Model (a Gaussian variable plus its quadratic correction) needs to be of the order of 10^4 to generate CMB non-Gaussianity at a level of $f_{\rm NL} \sim 10$. The isocurvature non-Gaussianity in the Quadratic Model (quadratic in a Gaussian variable without linear terms) can reach $f_{\rm NL} = 30$ while respecting the current upper limit on the isocurvature contribution to the power spectrum $\alpha < 0.067$. Isocurvature perturbations provide a possible mechanism to explain primordial non-Gaussianity recently suggested from the observed bispectrum of the CMB (Yadav & Wandelt 2008; Komatsu et al. 2008).

We give limits on isocurvature non-Gaussianity from Minkowski Functionals for the WMAP 5-year data. In the Quadratic Model of isocurvature perturbations, we obtain a stringent limit $\alpha < 0.070$ (95% CL) from the non-Gaussianity, which is comparable to the current constraints from WMAP TT and TE spectra, BAO and SN combined $\alpha < 0.067$ (95% CL). We apply our results to a QCD axion isocurvature model and then obtain a limit for the Hubble parameter and the phase of the axion.

We estimate isocurvature non-Gaussianity in the Quadratic Model using the pole approximation (eq.[42]). The validity can be checked by comparing Minkowski Functionals with non-Gaussian simulations with a Gaussian-squared perturbation. We plan to perform this analysis in the near future.

We employ Minkowski Functionals to characterize non-Gaussianity in CMB maps. The application of our work to other higher-order statistics is important to utilize non-Gaussian information in a more complete manner. Different statistics are sensitive to different aspects of density fields and they are affected by possible observational systematics (e.g., foregrounds and point sources) in a different way. There is actually some friction between the limits on $f_{\rm NL}$ from Minkowski Functionals and the bispectrum (Hikage et al. 2008); Minkowski Functionals analysis indicate a maximum likelihood value around nearly 0, while bispectrum analysis favours a more positive $f_{\rm NL}$ around 50 or more. Complementary analyses with different statistical approaches will provide a more robust way to analyze primordial non-Gaussianity.

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APPENDIX A: ADIABATIC, ISOCRUVATURE, & MIXED COMPONENTS OF CMB ANGULAR BISPECTRUM

Explicit forms of the angular bispectra from pure adiabatic and isocurvature mode are calculated as (Komatsu & Spergel 2001)

$$\begin{aligned} b^{\zeta\zeta\zeta}_{l_1l_2l_3} &= \frac{6}{5}f_{\rm NL}\int r^2dr[b^{\zeta,\phi\phi}_{Ll_1}(r)b^{\zeta,\phi\phi}_{Ll_2}(r)b^{\zeta}_{NLl_3}(r) \\ &\quad +b^{\zeta,\phi\phi}_{Ll_1}(r)b^{\zeta}_{NLl_2}(r)b^{\zeta,\phi\phi}_{Ll_3}(r) \\ &\quad +b^{\zeta}_{NLl_1}(r)b^{\zeta,\phi\phi}_{Ll_2}(r)b^{\zeta,\phi\phi}_{Ll_3}(r)], \qquad (A1) \end{aligned}$$

$$\begin{aligned} b^{\mathcal{SSS}}_{l_1l_2l_3} &= 2f^{(ISO)}_{\rm NL}\int r^2dr[b^{\mathcal{S},\eta\eta}_{Ll_1}(r)b^{\mathcal{S},\eta\eta}_{Ll_2}(r)b^{\mathcal{S}}_{NLl_3}(r) \\ &\quad +b^{\mathcal{S},\eta\eta}_{Ll_1}(r)b^{\mathcal{S}}_{NLl_2}(r)b^{\mathcal{S},\eta\eta}_{Ll_3}(r) \\ &\quad +b^{\mathcal{S}}_{NLl_1}(r)b^{\mathcal{S},\eta\eta}_{Ll_2}(r)b^{\mathcal{S},\eta\eta}_{Ll_3}(r)], \qquad (A2) \end{aligned}$$

where

$$b^{\zeta,\phi\phi}_{Ll}(r) \equiv \frac{2}{\pi}\int k^2dkP_{\phi\phi}(k)g^{\zeta}_{Tl}(k)j_l(kr), \qquad (A3)$$

$$b^{\mathcal{S},\eta\eta}_{Ll}(r) \equiv \frac{2}{\pi}\int k^2dkP_{\eta\eta}(k)g^{\mathcal{S}}_{Tl}(k)j_l(kr), \qquad (A4)$$

$$b^{\zeta}_{NLl}(r) \equiv \frac{2}{\pi}\int k^2dkg^{\zeta}_{Tl}(k)j_l(kr), \qquad (A5)$$

$$b^{\mathcal{S}}_{NLl}(r) \equiv \frac{2}{\pi}\int k^2dkg^{\mathcal{S}}_{Tl}(k)j_l(kr). \qquad (A6)$$

The angular bispectra from the correlation terms are given by

$$\begin{aligned} b^{(\zeta\zeta\mathcal{S})}_{l_1l_2l_3} &\equiv b^{\zeta\zeta\mathcal{S}}_{l_1l_2l_3}+b^{\zeta\mathcal{S}\zeta}_{l_1l_2l_3}+b^{\mathcal{S}\zeta\zeta}_{l_1l_2l_3} \\ &= 2f^{(ISO)}_{\rm NL}\int r^2dr\,\Big[b^{\zeta,\phi\eta}_{Ll_1}(r)b^{\zeta,\phi\eta}_{Ll_2}(r)b^{\mathcal{S}}_{NLl_3}(r) \\ &\quad +b^{\zeta,\phi\eta}_{Ll_1}(r)b^{\mathcal{S}}_{NLl_2}(r)b^{\zeta,\phi\eta}_{Ll_3}(r) \\ &\quad +b^{\mathcal{S}}_{NLl_1}(r)b^{\zeta,\phi\eta}_{Ll_2}(r)b^{\zeta,\phi\eta}_{Ll_3}(r)\Big] \end{aligned}$$

Table 1. Mean values and 1 σ uncertainties of each isocurvature non-Gaussianity parameter in the Linear Model; the pure isocurvature term $\alpha^2 f_{\rm NL}^{\rm (ISO)}$ (eq.[A2]) and the correlated terms with adiabatic perturbations $\alpha^{3/2}\cos\theta_{\phi\eta} f_{\rm NL}^{\rm (ISO)}$ (eq.[A8]) and $\alpha\cos^2\theta_{\phi\eta} f_{\rm NL}^{\rm (ISO)}$ (eq.[A7]). The limits are obtained from Minkowski Functionals for WMAP 5-year data at different smoothing scales θ_s and their combination.

θ_s [arcmin]	$\alpha^2 f_{\rm NL}^{\rm (ISO)}$	$\alpha^{3/2}\cos\theta_{\phi\eta} f_{\rm NL}^{\rm (ISO)}$	$\alpha\cos^2\theta_{\phi\eta} f_{\rm NL}^{\rm (ISO)}$
100	33 ± 84	26 ± 62	91 ± 220
70	36 ± 74	28 ± 56	104 ± 210
40	71 ± 78	56 ± 61	256 ± 230
20	0 ± 89	-3 ± 66	73 ± 240
10	1 ± 91	-30 ± 60	41 ± 230
10, 20, 40, 70, 100	-15 ± 60	-18 ± 43	18 ± 160

Table 2. Limits on α for the Quadratic Model from Minkowski Functionals for WMAP 5-year data at different smoothing scales θ_s. The listed values are maximum likelihood values $\alpha_{\rm ML}$ and 95% CL on α. We neglect non-Gaussianity from curvature perturbations and their cross-correlation. We impose a non-negative condition on α following from its definition in equation (10).

θ_s [arcmin]	$n_\sigma = 1$		$n_\sigma = 1.5$		$n_\sigma = 2$	
	$\alpha_{\rm ML}$	95% CL	$\alpha_{\rm ML}$	95% CL	$\alpha_{\rm ML}$	95% CL
100	0.037	< 0.115	0.022	< 0.100	0.005	< 0.047
70	0.036	< 0.105	0.029	< 0.079	0.013	< 0.030
40	0.051	< 0.112	0.010	< 0.057	0	< 0.014
20	0	< 0.101	0	< 0.048	0	< 0.0085
10	0	< 0.093	0	< 0.047	0	< 0.0071
10, 20, 40, 70, 100	0	< 0.070	0	< 0.042	0	< 0.0064

$$
\begin{aligned}
&+\frac{6}{5} f_{\rm NL}\int r^2 dr \times \\
&\left[b^{\zeta,\phi\phi}_{Ll_1}(r)\left\{b^{\mathcal{S},\phi\eta}_{Ll_2}(r) b^{\zeta}_{NLl_3}(r) + b^{\mathcal{S},\phi\eta}_{Ll_3}(r) b^{\zeta}_{NLl_2}(r)\right\}\right. \\
&+b^{\zeta,\phi\phi}_{Ll_2}(r)\left\{b^{\mathcal{S},\phi\eta}_{Ll_1}(r) b^{\zeta}_{NLl_3}(r) + b^{\mathcal{S},\phi\eta}_{Ll_3}(r) b^{\zeta}_{NLl_1}(r)\right\} \\
&\left.+b^{\zeta,\phi\phi}_{Ll_3}(r)\left\{b^{\mathcal{S},\phi\eta}_{Ll_1}(r) b^{\zeta}_{NLl_2}(r) + b^{\mathcal{S},\phi\eta}_{Ll_2}(r) b^{\zeta}_{NLl_1}(r)\right\}\right],
\end{aligned} \quad \text{(A7)}
$$

$$
\begin{aligned}
b^{(\zeta\mathcal{S}\mathcal{S})}_{l_1 l_2 l_3} &\equiv b^{\zeta\mathcal{S}\mathcal{S}}_{l_1 l_2 l_3} + b^{\mathcal{S}\zeta\mathcal{S}}_{l_1 l_2 l_3} + b^{\mathcal{S}\mathcal{S}\zeta}_{l_1 l_2 l_3} \\
&= \frac{6}{5} f_{\rm NL}\int r^2 dr\left[b^{\mathcal{S},\phi\eta}_{Ll_1}(r) b^{\mathcal{S},\phi\eta}_{Ll_2}(r) b^{\zeta}_{NLl_3}(r)\right. \\
&+b^{\mathcal{S},\phi\eta}_{Ll_1}(r) b^{\zeta}_{NLl_2}(r) b^{\mathcal{S},\phi\eta}_{Ll_3}(r) \\
&\left.+b^{\zeta}_{NLl_1}(r) b^{\mathcal{S},\phi\eta}_{Ll_2}(r) b^{\mathcal{S},\phi\eta}_{Ll_3}(r)\right] \\
&+2 f_{\rm NL}^{(ISO)}\int r^2 dr \times \\
&\left[b^{\zeta,\phi\eta}_{Ll_1}(r)\left\{b^{\mathcal{S},\eta\eta}_{Ll_2}(r) b^{\mathcal{S}}_{NLl_3}(r) + b^{\mathcal{S},\eta\eta}_{Ll_3}(r) b^{\mathcal{S}}_{NLl_2}(r)\right\}\right. \\
&+b^{\zeta,\phi\eta}_{Ll_2}(r)\left\{b^{\mathcal{S},\eta\eta}_{Ll_1}(r) b^{\mathcal{S}}_{NLl_3}(r) + b^{\mathcal{S},\eta\eta}_{Ll_3}(r) b^{\mathcal{S}}_{NLl_1}(r)\right\} \\
&\left.+b^{\zeta,\phi\eta}_{Ll_3}(r)\left\{b^{\mathcal{S},\eta\eta}_{Ll_1}(r) b^{\mathcal{S}}_{NLl_2}(r) + b^{\mathcal{S},\eta\eta}_{Ll_2}(r) b^{\mathcal{S}}_{NLl_1}(r)\right\}\right],
\end{aligned} \quad \text{(A8)}
$$

where

$$
b^{\zeta,\phi\eta}_{Ll}(r) \equiv \frac{2}{\pi}\int k^2 dk P_{\phi\eta}(k) g^{\zeta}_{Tl}(k) j_l(kr), \quad \text{(A9)}
$$

$$
b^{\mathcal{S},\phi\eta}_{Ll}(r) \equiv \frac{2}{\pi}\int k^2 dk P_{\phi\eta}(k) g^{\mathcal{S}}_{Tl}(k) j_l(kr). \quad \text{(A10)}
$$

The terms proportional to $f_{\rm NL}$ in $b^{(\zeta\zeta\mathcal{S})}_{l_1 l_2 l_3}$ and $b^{(\zeta\mathcal{S}\mathcal{S})}_{l_1 l_2 l_3}$ are very small relative to the adiabatic bispectrum under the current observational constraints and thus they are neglected in the following analysis.

Table 3. Same as Table 2 but for the parameter $\alpha^{1/2}\cos^2\theta_{\phi\sigma}$ where the correlation term $b^{(\zeta\zeta S)}_{l_1 l_2 l_3}$ in the Quadratic Model dominates. We also impose a non-negative condition on $\alpha^{1/2}\cos^2\theta_{\phi\sigma}$.

θ_s [arcmin]	$n_\sigma = 1$		$n_\sigma = 1.5$		$n_\sigma = 2$	
	$(\alpha^{1/2}\cos^2\theta_{\phi\sigma})_{\rm ML}$	95% CL	$(\alpha^{1/2}\cos^2\theta_{\phi\sigma})_{\rm ML}$	95% CL	$(\alpha^{1/2}\cos^2\theta_{\phi\sigma})_{\rm ML}$	95% CL
100	0.018	< 0.095	0.018	< 0.12	0.010	< 0.089
70	0.021	< 0.092	0.021	< 0.11	0.014	< 0.073
40	0.045	< 0.12	0.040	< 0.14	0.017	< 0.075
20	0.012	< 0.098	0.026	< 0.11	0.013	< 0.041
10	0	< 0.071	0.003	< 0.063	0.0026	< 0.019
10, 20, 40, 70, 100	0	< 0.049	0	< 0.039	0	< 0.012