Induced gravity on intersecting brane-worlds

Part II: Cosmology

Olindo Corradini[a], **Kazuya Koyama**[b] and **Gianmassimo Tasinato**[c]

[a] *Dipartimento di Fisica, Università di Bologna and INFN Sezione di Bologna*
Via Irnerio, 46 - Bologna I-40126, Italy
E-mail: corradini@bo.infn.it

[b] *Institute of Cosmology and Gravitation, University of Portsmouth*
Portsmouth PO1 2EG, UK
E-mail: Kazuya.Koyama@port.ac.uk

[c] *Instituto de Fisica Teorica, UAM/CSIC*
Facultad de Ciencias C-XVI, C.U. Cantoblanco, E-28049-Madrid, Spain
E-mail: gianmassimo.tasinato@uam.es

Abstract

We explore cosmology of intersecting braneworlds with induced gravity on the branes. We find the cosmological equations that control the evolution of a moving codimension one brane and a codimension two brane that sits at the intersection. We study the Friedmann equation at the intersection, finding new contributions from the six dimensional bulk. These higher dimensional contributions allow us to find new examples of self-accelerating configurations for the codimension two brane at the intersection and we discuss their features.

Contents

1 Introduction

Brane-world models offer new perspectives for explaining the present day acceleration in purely geometrical terms, without the need to introduce dark energy [1, 2, 3] (for a review see [4]). A celebrated example is the Dvali-Gabadadze-Porrati (DGP) model in a 5d spacetime [1]. The brane action includes a quantum-induced Einstein-Hilbert action that recovers 4d gravity on small scales. This model realizes a so-called *self-accelerating* solution that features a 4d de Sitter phase even though the 3-brane is completely empty. However, so far, only codimension-one examples of such solutions have been proposed and these backgrounds are known to suffer from ghost instabilities [5]. An interesting possibility then is to look for other such solutions in higher codimensional set-ups, initially introduced to address the cosmological constant problem [6, 7, 9, 10, 11, 12]. This might lead to ghost free models [13] (see however [14]).

In this paper, as a continuation of [15], we consider a codimension two brane that lies at the intersection of two codimension one branes embedded in a six dimensional space. This system was studied in the past in the context of standard gravity [16] (cosmological properties were investigated in [17]), and Gauss-Bonnet gravity [18] elaborating an idea developed in [20]. The latter was generalized to higher-codimensional models in [19]. Models with a generic angle between two intersecting branes were first considered in [21] and then further generalised into the so-called Origami-world in [22]. More recently, in [15], we added brane induced gravity terms to this system to analyse the features of a configuration of static branes embedded in a time dependent, maximally symmetric background. We showed the existence of new self-accelerating solutions, and of configurations with potentially interesting self-tuning properties.

In the present paper, we continue the analysis of this system by studying cosmological models, obtained by the motion of one of the branes through the bulk, in a mirage approach [23]. The energy momentum tensor different from pure tension on the branes causes the brane to move and bend in the bulk, and induces cosmological evolutions from the point of view of observers sitting on the branes. We allow the branes to intersect at an arbitrary angle and to deform in the preferred shape.

The analysis of gravitational [24] and cosmological [25] aspects of codimension two brane-worlds is a subject that is receiving some attention. Cosmology is mainly studied in the context of a mirage approach. In higher codimensional brane-worlds, the mirage approach has usually some drawbacks (critically examined, for example, in the introduction of [26]), mainly due to fine-tuning relations that the brane energy momentum tensors must satisfy. These are usually associated with the fact that an analogue of Birkhoff theorem does not hold in this case, in contrast to the codimension one case. In codimension one case, this theorem ensures that a system composed by a homogeneous and isotropic brane, moving through a static higher dimensional space, fully catches all the relevant time dependence of the system [27]. In our case, this is not true: generically, a moving higher co-dimensional brane induces time-dependent effects in the bulk [18, 28]. In order to avoid the time dependence in the bulk, one must *impose* a static ansatz for the bulk geometry, and this is reflected on fine-tuning relations between matter on the brane and in the bulk. Nevertheless, it remains the most direct approach to study cosmological aspects of these models analytically.

The most interesting problem in this system is the isolation of the six dimensional effects in the induced Friedmann equation on the codimension two intersection. As we will see, the Friedmann equation at the intersection receives contributions due to induced gravity terms on it which ensure the recovery of normal 4d cosmology in the relevant regimes. Moreover, there are terms coming from induced gravity on the codimension one branes, of the typical DGP form [2]. Finally, and more interestingly in our framework, the Friedmann equation contains also contributions that come from the six dimensional bulk. They vanish in the limit in which the branes intersect at a right angle, but for generic brane configurations they play an important role for the cosmological evolutions. Indeed, they can provide the late time acceleration, regardless of the energy content of the codimension two brane, generalising the self-accelerating branch of the codimension one DGP model to higher codimensions [2]. This fact has been realized already in [15], but the present analysis is more general because we do not impose the maximal symmetry on the branes under consideration. By properly choosing the embedding for the codimension one branes, the six dimensional effects at the intersection can depend on the inverse of the induced Hubble parameter, and we will analyse the consequences of this in our discussion. Another peculiar feature of our construction is that six dimensional contributions to the Friedmann equation are also associated to the non-conservation of the energy density at the intersection. During the cosmological evolution, the energy density indeed flows from the codimension one to the codimension two branes, unless the codimension one branes intersect with a right angle.

This paper is organised as follow. In Section 2, we will present the general formalism that is necessary to study cosmological properties of the systems we are interested in. In Section 3, we apply this formalism to a particular embedding for the codimension one branes, and in Section 4 we study in some detail cosmological solutions derived from this embedding. Then, in Section 5, we study applications of these cosmological solutions to some interesting situations. We conclude in Section 6.

2 The general formalism

2.1 The model

We consider a system of two intersecting codimension one branes embedded in a six dimensional space-time. They intersect on a four dimensional codimension two brane, where observers like us can be localised. We take an Einstein-Hilbert action for gravity in the bulk and we allow for induced gravity terms on the codimension one branes, as well as on the intersection. Besides gravity, we allow for a cosmological constant term in the bulk, Λ_B, and for additional fields localised on the branes described by general Lagrangians L's. The general action takes the form

$$\begin{aligned} S &= \int_{\text{bulk}} d^6x\sqrt{-g}\left(\frac{M_6^4}{2}R-\Lambda_B\right)+\sum_{i=1}^{2}\int_{\Sigma_i} d^5x\sqrt{-g_{(i)}}\left(\frac{M_{5,\,i}^3}{2}R_{(i)}+L_{(i)}\right) \\ &+ \int_{\Sigma_\cap} d^4x\sqrt{-g_\cap}\left(\frac{M_4^2}{2}R_\cap+L_\cap\right), \end{aligned} \tag{1}$$

where $\Sigma_\cap \equiv \bigcap_i \Sigma_i$ denotes a three-brane at the intersection between all codimension-one branes Σ_i. We can have different fundamental scales in the different regions of the space, M_6, $M_{5,\,i}$, and M_4. The induced gravity terms could be generated, as it was proposed in the original model, by quantum corrections from matter loops on the brane. It is also interesting to note that induced curvature terms appear quite generically in junction conditions of higher codimension branes when considering natural generalisations of Einstein gravity [29, 30] as well as in string theory compactifications [31], orientifold models and intersecting D-brane models [32].

The six dimensional bulk is characterised by a maximally symmetric geometry

$$\begin{aligned} ds^2 &= A^2(t,z^1,z^2)\left(\eta_{\mu\nu}dx^\mu dx^\nu+\delta_{kh}dz^k dz^h\right), \\ A(t,z^1,z^2) &= \frac{1}{1+\bar{H}t+k_i z^i}\,. \end{aligned} \tag{2}$$

The parameters $\bar{H}$ and k_i appearing in the warp factor A satisfy the following relation

$$\frac{\Lambda_B}{10} = \bar{H}^2-k_1^2-k_2^2, \tag{3}$$

in order to solve the Einstein equations in the bulk.

We embed a moving and a static codimension one branes (Σ_2 and Σ_1 respectively) on the background given by (2). The moving brane Σ_2 is characterised by an embedding

$$X^M_{(2)} = (t,\,\vec{x}_3,\,\mathcal{Z}_1(\omega_1),\,\mathcal{Z}_2(t,\omega_1))\,. \tag{4}$$

Here, w_1 is an embedding coordinate. In the following, for simplicity, we will demand that the intersection with the other brane lies at the position $w_1=0$, and that the function $\mathcal{Z}_1$ does not depend on time. The vectors V tangent to Σ_2 are given by (we introduce indices on the left of V: they indicate which brane we are talking about)

$${}^{(2)}V^M_{(a)} = \frac{\partial X^M}{\partial x^a}, \quad x^a=(t,x^i,w_1), \tag{5}$$

where ${}^{(2)}V^M_{(t)}$ is (proportional to) the velocity vector

$$ {}^{(2)}V^M_{(t)} = (1,\, 0^i,\, 0,\, \dot{\mathcal{Z}}_2) = \dot{X}^M \ . \tag{6} $$

The other four vectors are

$$ {}^{(2)}V^M_{(i)} \ = \ (0, \delta^M_i, 0, 0), \tag{7} $$

$$ {}^{(2)}V^M_{(w_1)} \ = \ (0,\, 0^i,\, \mathcal{Z}_1',\, \mathcal{Z}_2') = X'^M \ . \tag{8} $$

The normal vector to the brane is thus given by the conditions

$$ n_M V^M_{(a)} = 0, \quad \forall a \ . \tag{9} $$

Orthogonality with respect to the i vectors simply removes from n_M all its 3-dimensional space-like components. Imposing orthogonality w.r.t. ${}^{(2)}V^M_{(w_1)}$ we then find

$$ n^{(2)}_M \ = \ \frac{A}{\mathcal{N}} \left(-\dot{\mathcal{Z}}_2 \mathcal{Z}_1',\, \vec{0}_3,\, -\mathcal{Z}_2',\, \mathcal{Z}_1' \right), \tag{10} $$

with

$$ \mathcal{N} \ \equiv \ \sqrt{\mathcal{Z}_1'^2 + \mathcal{Z}_2'^2 - \dot{\mathcal{Z}}_2^2 \mathcal{Z}_1'^2} \ . \tag{11} $$

Doing exactly the same steps for the static brane Σ_1, with embedding

$$ X^M_{(1)} \ = \ (t,\, \vec{x}_3,\, 0,\, z_2) \ , \tag{12} $$

the vectors tangent to the brane, ${}^{(1)}V^M_{(a)}$, are immediate to find. And the normal is simply

$$ n^{(1)}_M \ = \ A \left(0,\, \vec{0}_3,\, 1,\, 0 \right) \ . \tag{13} $$

2.2 An useful change of coordinates

Proceeding identically as in the static case [15, 22], it is useful to change a frame in order to impose the Z_2 symmetry in the case of a general angle. We go to coordinates parallel to the branes $(z_1,\, z_2) \to (\tilde{z}_1,\, \tilde{z}_2)$, where

$$ d\tilde{z}^k \ \equiv \ \mathbf{n}^{(k)} \cdot d\mathbf{z} \, . $$

One obtains two two-vectors $\mathbf{l}_{(1)}$ and $\mathbf{l}_{(2)}$ parallel to the branes:

$$ \mathbf{l}_{(1)} \ = \ \frac{1}{\mathcal{Z}_1'} \left(\mathcal{Z}_1',\, \mathcal{Z}_2' \right), \quad \mathbf{l}_{(2)} \ = \ \frac{\mathcal{N}}{\mathcal{Z}_1'} \, (0,\, 1)\, , \tag{14} $$

and $d\mathbf{z} \ = \ \mathbf{l}_{(k)}\, d\tilde{z}^k$. Then the components of the vectors ${}^{(2)}V$ parallel to the moving brane become

$$ {}^{(2)}\tilde{V}^M_{(w_1)} \ = \ \frac{\partial \tilde{X}^M_{(2)}}{\partial w_1} \ = \ \left(0,\, \vec{0}_3,\, \mathcal{Z}_1',\, 0 \right) \ , \tag{15} $$

and

$$ {}^{(2)}\tilde{V}_{(0)} \,=\, \left(1,\, \vec{0}_3,\, 0,\, \frac{\mathcal{Z}_1' \dot{\mathcal{Z}}_2}{\mathcal{N}}\right) \,. \tag{16} $$

Notice that the consistency relation

$$ \frac{\partial^2 X^M_{(2)}}{\partial t\, \partial w_1} \,=\, \frac{\partial^2 X^M_{(2)}}{\partial w_1\, \partial t}, $$

in our case implies the condition

$$ \frac{\partial}{\partial w_1} \left(\frac{\mathcal{Z}_1' \dot{\mathcal{Z}}_2}{\mathcal{N}}\right) \,=\, 0\,. \tag{17} $$

The normal to the moving brane becomes in these coordinates

$$ \tilde{n}^{(2)}_M \,=\, A\,\mathcal{T}\,\left(-\frac{\dot{\mathcal{Z}}_2 \mathcal{Z}_1'}{\mathcal{N}},\, \vec{0}_3,\, 0,\, 1\right), \tag{18} $$

where $\mathcal{T}$ is a normalization factor that we will fix once we define the six dimensional metric.

In order to proceed, we must take into account that the branes are fixed points of Z_2 symmetries. We focus our analysis on the moving brane Σ_2 in order to compute Israel junction conditions at its position: the analysis for the static brane Σ_1 can be performed along similar lines. The Z_2 symmetry acting on the static brane Σ_1 implies the invariance of the 6d metric under $\tilde{z}^1 \to -\tilde{z}^1$, that can be obtained replacing $\tilde{z}^1 \to |\tilde{z}^1|$.

After the change of frame, imposing the Z_2 symmetry, the six dimensional metric becomes

$$ \tilde{\gamma}_{mn} \quad=\quad \frac{1}{\mathcal{Z}_1'^2} \begin{pmatrix} \mathcal{Z}_1'^2 + \mathcal{Z}_2'^2 & \mathcal{N}\, \mathcal{Z}_2'\, sign(\tilde{z}_1) \\ \mathcal{N}\, \mathcal{Z}_2'\, sign(\tilde{z}_1) & \mathcal{N}^2 \end{pmatrix}\,, \tag{19} $$

with inverse

$$ \tilde{\gamma}^{mn} \quad=\quad \frac{1}{\mathcal{N}^2\, \mathcal{C}^2} \begin{pmatrix} \mathcal{N}^2 & -\mathcal{N}\, \mathcal{Z}_2'\, sign(\tilde{z}_1) \\ -\mathcal{N}\, \mathcal{Z}_2'\;\; sign(\tilde{z}_1) & \mathcal{Z}_1'^2 + \mathcal{Z}_2'^2 \end{pmatrix}\,, \tag{20} $$

where

$$ \mathcal{C}^2 \,=\, 1 + \frac{\mathcal{Z}_2'^2}{\mathcal{Z}_1'^2}\,\left(1 - sign^2(\tilde{z}_1)\right). \tag{21} $$

With this information, we determine the normalization factor $\mathcal{T}$ by requiring that the normal $\tilde{n}^{(2)}$ has a unit length. Then $\mathcal{T}$ is determined as

$$ \mathcal{T} \,=\, \frac{\mathcal{C}\,\mathcal{N}}{\mathcal{Q}}, \tag{22} $$

with $\mathcal{N}$ defined in formula (11), $\mathcal{C}$ in (21), and

$$ \mathcal{Q}^2 \,=\, \mathcal{Z}_1'^2 + \mathcal{Z}_2'^2 \,-\, \mathcal{C}^2\, \mathcal{Z}_1'^2\, \dot{\mathcal{Z}}_2{}^2. \tag{23} $$

On the other hand the induced metric is invariant under bulk reparametrisation, and thus reads

$$ds^2_{5,\Sigma_2} = A^2\,(t,w_1)\left[\; - \; \left(1-\dot{\mathcal{Z}}_2^2\right)dt^2 + d\vec{x}_3^2 + \left(\mathcal{Z}_1'^2+\mathcal{Z}_2'^2\right)dw_1^2 \right.$$
$$\left. + \;\; 2\,\dot{\mathcal{Z}}_2\mathcal{Z}_2'\,sign(\tilde{z}_1)\,dt\,dw_1\right]\;. \tag{24}$$

The inverse induced metric is given by

$$h^{ab} = A^{-2}\left(\begin{array}{cc}\delta^{ij} & 0\\ 0 & H^{\alpha\beta}\end{array}\right), \tag{25}$$

with

$$H^{\alpha\beta} = \mathcal{Q}^{-2}\left(\begin{array}{cc}-(\mathcal{Z}'^2_1+\mathcal{Z}'^2_2) & \dot{\mathcal{Z}}_2\mathcal{Z}'_2\,sign(\tilde{z}_1)\\ \dot{\mathcal{Z}}_2\mathcal{Z}'_2\,sign(\tilde{z}_1) & 1-\dot{\mathcal{Z}}_2^2\end{array}\right), \tag{26}$$

and $\mathcal{Q}$ defined in eq. (23). At the intersection, the four dimensional metric is given by

$$ds_4 = A^2(t)[-(1-\dot{\mathcal{Z}}_2^2)dt^2 + d\mathbf{x}^2]. \tag{27}$$

2.3 Extrinsic curvature

Given all this information, one can compute the components of the extrinsic curvature at the position of the brane Σ_2, using the general formula

$$K_{mn} = \tilde{V}^M_{(m)}\tilde{V}^N_{(n)}\tilde{\nabla}_M\tilde{n}_N\;. \tag{28}$$

Since the expression for K_{mn} is invariant under bulk reparametrisation, to evaluate the right hand side of the previous expression one can use the six dimensional metric in the original frame, or in the frame parallel to the brane.

The calculation of the regular part of K_{mn} is easier to work out in the original frame. The non vanishing components are the following

$$K_{00} = -\frac{A}{\mathcal{N}}\ddot{\mathcal{Z}}_2\mathcal{Z}_1' - \frac{A^2}{\mathcal{N}}\left(1-\dot{\mathcal{Z}_2}^2\right)\mathcal{K}(w,t)\,, \tag{29}$$

$$K_{w_1w_1} = \frac{A}{\mathcal{N}}\left(\mathcal{Z}_1''\mathcal{Z}_2' - \mathcal{Z}_2''\mathcal{Z}_1'\right) + \frac{A^2}{\mathcal{N}}\left({\mathcal{Z}_1}'^2+{\mathcal{Z}_2}'^2\right)\mathcal{K}(w,t)\,, \tag{30}$$

$$K_{0w_1} = -sign(\tilde{z}_1)\frac{A}{\mathcal{N}}\,\mathcal{Z}_1'\dot{\mathcal{Z}}_2' + sign(\tilde{z}_1)\,\frac{A^2}{\mathcal{N}}\mathcal{K}(w,t)\,\dot{\mathcal{Z}}_2\mathcal{Z}_2'\,, \tag{31}$$

$$K_{ij} = \frac{A^2\delta_{ij}\;\mathcal{K}(w,t)}{\mathcal{N}}\,. \tag{32}$$

where

$$\mathcal{K}(w,t) = k_1\mathcal{Z}_2' - k_2\mathcal{Z}_1' - \bar{H}\dot{\mathcal{Z}}_2\mathcal{Z}_1' \tag{33}$$

In addition, the component $K_{w_1w_1}$ of the extrinsic curvature may contain terms localised at the intersection due to the presence of the *sign* functions in the six dimensional metric. Let us then look for the singular pieces of the extrinsic curvature

$$K_{ab}|_{sing} = \tilde{V}^M_{(a)}\tilde{V}^N_{(b)}\nabla_M n_N|_{sing}\;. \tag{34}$$

This quantity is much easier to calculate in the tilted reference frame. There are, a priori, two classes of contributions to the singular pieces. The first one comes from partial derivatives acting on n_N, due to the *sign* function included in $\mathcal{T}$. However such a contribution is proportional to n_N itself as the *sign* function only appears in the prefactor of n_M, and thus vanishes due to the orthogonality between n_M and $V^M_{(a)}$.

Then

$$K_{ab}|_{sing} = -V^M_{(a)} V^N_{(b)} \; n_R \; \Gamma^R_{MN}|_{sing}, \tag{35}$$

and since Γ^0_{MN} has no singular part, one is left with $\Gamma^{\tilde{z}_2}_{MN}$. Its only singular component is $\Gamma^{\tilde{z}_2}_{\tilde{z}_1\tilde{z}_1} = g^{22}\partial_1 g_{12}$ since they involve derivatives of the *sign* function. Then, in the end, we find that

$$K_{w_1w_1}|_{sing} = -A\,\mathcal{K}\,\frac{\mathcal{Z}_1'^2 + \mathcal{Z}_2'^2}{\mathcal{C}^2\,\mathcal{N}}\,\mathcal{Z}_2'\left(\frac{\partial}{\partial\tilde{z}_1}\,sign(\tilde{z}_1)\right), \tag{36}$$

is the only singular component of the extrinsic curvature. Notice that the previous expression contains products of distributions, since the $\mathcal{C}$ contains squares of *sign* functions. In order to dispel any doubts about how to define such singular expression, it is convenient *not* to set $\frac{\partial\, sign(\tilde{z}_1)}{\partial\tilde{z}_1} = 2\frac{\delta(w_1)}{\mathcal{Z}_1'}$, but instead maintain the derivative of the *sign* function. Later, in the specific examples we will discuss, in order to extract the singular terms localized at the intersection we will perform an explicit integration on a small interval around the singularity. The result of this integration will provide the value of the various quantities localized at the intersection.

For the static brane Σ_1, the extrinsic curvature is simply given by

$$K^a_b = -k_1\delta^a_b. \tag{37}$$

2.4 Junction conditions

The previous expressions for the extrinsic curvature are important in order to obtain the equations that govern the induced cosmology on the brane. They are dictated by the Israel junction conditions

$$2\left[\hat{K}_{ab}\right] \equiv 2\left[K_{ab} - K h_{ab}\right] = -\frac{1}{M_6^4}(S_{ab} + S^{loc}_{ab}), \tag{38}$$

where $[X] \equiv (X(\Sigma_{2,+}) - X(\Sigma_{2,-}))/2$, while the induced codimension one brane metric is h_{ab}. The extrinsic curvature tensor evaluated on Σ_2 is given by $K_{ab} = h^M_a h^N_b \nabla_M n_N$ with $K = K^a{}_a$, and energy momentum tensors relative to matter localised on Σ_2 , appearing on the right hand side of (38), are calculated in the usual way:

$$S_{ab} = -\frac{2}{\sqrt{-h_{(2)}}}\frac{\delta\left(\sqrt{-h_{(2)}}\mathcal{L}_{(2)}\right)}{\delta h^{ab}_{(2)}}, \tag{39}$$

$$\begin{aligned} S^{loc}_{ab} &= -\delta(\Sigma_1)\delta^\mu_a\delta^\nu_b\frac{2}{\sqrt{-h_{(2)}}}\frac{\delta\left(\sqrt{-h}\mathcal{L}_\cap\right)}{\delta h^{\mu\nu}_\cap} \\ &\equiv \delta(\Sigma_1)\sqrt{\frac{-h_\cap}{-h_{(2)}}}\delta^\mu_a\delta^\nu_b S_{\mu\nu}\;. \end{aligned} \tag{40}$$

In our model the localised energy-momentum tensor also includes contributions from the induced gravity terms. The last quantity S^{loc}_{ab} denotes energy momentum tensor that is localised on the intersection $\Sigma_\cap$

between the branes. Notice the presence of the factor $\sqrt{h_\cap/h_{(2)}}$ that renders the expression covariant with respect to the metric at the intersection.

In the previous discussion, we learned that the only singular term of the extrinsic curvature for the brane Σ_2, that is localised at the intersection, is contained in $K_{w_1 w_1}$. This implies that the six dimensional contributions to the energy momentum tensor must be proportional to the induced metric, $S^{loc}_{\mu\nu} = f(x^\mu)\, h_{\cap\,\mu\nu}$, for some function f. We still do not know whether this function f is a constant (in which case, it corresponds to a pure tension) or not, since we do not know whether the conservation of the energy holds at the intersection or not. The Codazzi equation holds in this case [1]

$$\nabla_a \hat{K}^a_{\ b} = 0 \qquad \Rightarrow \qquad \nabla_a S^a_{\ b} = 0, \tag{41}$$

which means that there is no exchange of energy between the bulk and the codimension one branes. But the previous relations may contain singular terms, associated with an exchange of energy between the codimension one and the codimension two branes. This is indeed what generically happens, and we will encounter an example of this phenomenon in Section 3. Singular terms in the first of the previous formulae can appear if $\hat{K}^a_b$ has singular pieces, or if some of its components become singular when covariant derivatives act on them. This possibility occurs when the angle between the brane is not right: then the component ${K_0}^{w_1}$ is non-vanishing at the intersection, and, being proportional to the *sign* function (see eq. (31)), it normally generates an additional singular term.

3 Applications

We then consider a system with the static brane Σ_1, and the moving brane Σ_2. Here, Σ_2 is free to move and bend arbitrarily. We assume that the induced gravity term on the moving codimension one brane vanishes: $M_{5,\,2} = 0$ for simplicity. For this brane, we take an embedding

$$X^M = (t,\, \vec{x}_3,\, \mathcal{Z}_1(w_1),\, \mathcal{Z}_2(t, w_1))\,, \tag{42}$$

with

$$\begin{aligned} \mathcal{Z}_1 &= w_1 \cos\alpha(w_1), &(43)\\ \mathcal{Z}_2 &= z_2(t, w_1) + w_1 \sin\alpha(w_1), &(44) \end{aligned}$$

where we wrote the two functions $\mathcal{Z}_i$ in terms of the auxiliary functions z_2 and α. We demand that these functions are continuous with respect to the variable w_1, at the position of the intersection $w_1 = 0$, and to avoid subtleties related with the reflection symmetry at the intersection we require smoothness conditions $z_2'(t,0) = z_2''(t,0) = \alpha'(0) = 0$ [2]. From these definitions, we have

$$\begin{aligned} &\dot{\mathcal{Z}}_1 = 0, \quad \mathcal{Z}_1' = \cos\alpha - w_1 \alpha' \sin\alpha,\\ &\dot{\mathcal{Z}}_2 = \dot{z}_2, \quad \mathcal{Z}_2' = z_2' + \sin\alpha + w_1 \alpha' \cos\alpha. \end{aligned} \tag{45}$$

Notice that the previous embedding satisfies the constraint (17); for a time-dependent angle (17) would

[1] The RHS of this formula vanishes because both the bulk and the static brane have maximal symmetry [33].

[2] Asking that only the first derivative vanish at the intersection may be enough to ensure sufficient smoothness to render the system well behaved. In fact later we will briefly mention a situation where a non-vanishing second derivative $z_2''(t,0) \neq 0$ can turn out to be useful.

instead imply $\dot{\alpha}(t,\, w_1 = 0) = 0$.

The junction condition on the static co-dimension one brane gives the tension λ_1 as

$$\lambda_1 = 6M_{5,1}^2(\bar{H}^2 - k_2^2) + 8M_6^4 k_1. \tag{46}$$

The induced metric on the brane Σ_2 is obtained by plugging the previous expressions in (24). The complete calculation of the cosmological behaviour for the moving codimension one brane is complicated as the brane is inhomogeneous, but it can be obtained straightforwardly from the general formulae (29)–(32). In the next subsection we discuss some of its properties that are useful when comparing them with cosmology on the codimension two brane.

3.1 Cosmology on the moving four brane

The cosmological evolution on the moving brane Σ_2 is complicated by the fact that its induced scale factor and energy momentum tensor must be inhomogeneous, in order to satisfy Israel junction conditions (29)–(32): such conditions require some off-diagonal components of the energy momentum tensor $S^a{}_b$ to be non vanishing. In what follows we will only need the explicit form of the codimension-one equations evaluated at the intersection. We thus concentrate on such a limit where the form of the needed energy momentum tensor is the following:

$$S^a{}_b = \begin{pmatrix} -\rho_2 & 0 & 0 & 0 & 0 \\ 0 & p_2 & 0 & 0 & 0 \\ 0 & 0 & p_2 & 0 & 0 \\ 0 & 0 & 0 & p_2 & 0 \\ \chi & 0 & 0 & 0 & p_2 \end{pmatrix}, \quad \text{at } w_1 = 0\ . \tag{47}$$

Then the junction conditions impose the following relations

$$\begin{aligned} \rho_2 &= -8M_6^4 \mathcal{N}^{-1}\mathcal{K}, && (48)\\ p_2 &= 8M_6^4(\mathcal{N}^{-1}\mathcal{K} + \frac{1}{4}\mathcal{N}^{-3}A^{-1}\ddot{z}_2\cos\alpha), && (49)\\ \chi &= 2M_6^4 A^{-1}\mathcal{N}^{-3}\dot{z}_2\ddot{z}_2\sin\alpha\cos\alpha, && (50) \end{aligned}$$

where we define $\mathcal{N} = \sqrt{1-\cos^2\alpha\dot{z}_2^2}$ and $\mathcal{K} = k_1\sin\alpha - (k_2 + \bar{H}\dot{z}_2)\cos\alpha$.

3.2 Cosmology at the intersection

We start from discussing the contributions from the brane Σ_2 to the codimension two brane. At the intersection, characterised by $w_1 = 0$, the induced metric is straightforwardly extracted from the five dimensional one and is simply given by

$$ds_4^2 = A^2(0,t)\Big\{ - \big[1-\dot{z}_2^2\big]\,dt^2 + d\vec{x}_3^2\Big\} = -d\tau^2 + a(\tau)^2 d\vec{x}_3^2, \quad d\tau = A(0,t)\sqrt{1-\dot{z}_2^2}dt. \tag{51}$$

Then the induced Hubble parameter is

$$H = \frac{1}{a(\tau)}\frac{da(\tau)}{d\tau} = \frac{\bar{H} + k_2\dot{z}_2}{\sqrt{1-\dot{z}_2^2}}. \tag{52}$$

In order to find the Friedmann equation at the intersection, we have to extract the singular part of the Israel junction conditions for the codimension one branes. This singular part receives contributions from the energy momentum tensor localised on the codimension two brane (containing also the induced gravity terms at the intersection), from the induced gravity terms on the codimension one branes, and from singular contributions of the extrinsic curvature terms. The final contribution represents the most interesting feature of our model since it corresponds to six dimensional contributions to four dimensional physics. We start our discussion with their evaluation.

Recall that only singular part on the extrinsic curvature for brane Σ_2 is contained in the (w_1, w_1) component. From its expression one straightforwardly obtains

$$\hat{K}_{\mu}^{\ \nu\,(sing)} = \frac{\mathcal{T}\,\sin\alpha}{A\,\mathcal{N}\,\mathcal{Q}^2\,\mathcal{C}^2}\,\left(1-\dot{z}_2^2\right)\left(\frac{\partial}{\partial\tilde{z}_1}sign(\tilde{z}_1)\right)\delta_{\mu}^{\nu}, \tag{53}$$

as the only singular piece, and it is easy to see that its contribution to the energy momentum tensor at the intersection results proportional to the induced metric (recall the definitions of $\mathcal{T}$, $\mathcal{N}$, $\mathcal{C}$ and $\mathcal{Q}$ respectively in eqs. (22), (11), (21) and (23)). Let us, for example, calculate its contribution to the energy density at the intersection. Plugging the previous expressions inside the Israel equations, one finds the relation

$$\frac{\delta\rho}{4M_6^4}\left(\frac{\partial}{\partial\tilde{z}_1}sign(\tilde{z}_1)\right) = \frac{\mathcal{T}\,\tan\alpha}{\mathcal{C}^2\mathcal{N}\,\mathcal{Q}}\sqrt{1-\dot{z}^2}\left(\frac{\partial}{\partial\tilde{z}_1}sign(\tilde{z}_1)\right), \tag{54}$$

where $\delta\rho$ indicates the contribution to the brane energy density, due to purely six dimensional effects.

The previous equation contains $sign^2(\tilde{z}_1)$ functions inside the expressions for $\mathcal{N}$, $\mathcal{Q}$, and $\mathcal{C}$. The safest way to handle them is to integrate both sides of (54) along an infinitesimally small interval centered at the origin, furnishing the value of the contribution to energy density localized at that point. Performing the integration we get the expression

$$\frac{\delta\rho}{2M_6^4} = \tan\alpha\sqrt{1-\dot{z}_2^2}\int_{-1}^{+1}\frac{d\,sign(\tilde{z}_1)}{\mathcal{C}\,\mathcal{Q}^2}. \tag{55}$$

After a few calculations, the previous expression can be written as

$$\frac{\delta\rho}{2M_6^4} = \frac{\sin\alpha}{\sqrt{1-\dot{z}_2^2}}\int_{-1}^{+1}d\,x\,\frac{1}{\sqrt{1-\sin^2\alpha\,x^2}}\,\frac{1}{1+\frac{\dot{z}_2^2\sin^2\alpha}{1-\dot{z}_2^2}x^2}. \tag{56}$$

By performing the integration, we get

$$\delta\rho = 4M_6^4\,\arctan\left[\frac{\tan\alpha}{\sqrt{1-\dot{z}_2^2}}\right]. \tag{57}$$

Notice that, in the limit of static brane $\dot{z}_2 = 0$, this contribution is proportional to the angle α, exactly as happens in the case of codimension two conical singularities.

Proceeding with our calculation, we can determine the contributions to the intersection from the induced gravity terms on Σ_1 (recall that we have chosen $M_{5,\,2} = 0$ so there are no induced gravity terms on the moving brane Σ_2). We find

$$\left(G_0^0\right)^{sing} = -\frac{6(k_2+\bar{H}\dot{z}_2)}{\sqrt{1-\dot{z}_2^2}}. \tag{58}$$

Putting all this information together, we find the following equation relating energy density to geometrical quantities (here ρ indicates the total energy density at the intersection)

$$\rho \;=\; 3M_4^2 H^2 + 6M_5^3\,\frac{k_2+\bar{H}\dot{z}_2}{\sqrt{1-\dot{z}_2^2}} + 4M_6^4\,\arctan\left(\frac{\tan\alpha}{\sqrt{1-\dot{z}_2^2}}\right), \tag{59}$$

that can be interpreted as the Friedmann equation for an observer localized at the intersection. The induced gravity terms on the codimension one branes do not induce a violation of the continuity equation at the intersection because the intersection can be seen as codimension one object from the point of view of the four branes and then the properties of the Israel formalism for the codimension one brane ensure the conservation of energy (see eq. (41)). On the other hand, the last, six dimensional term in Eq. (59) is explicitly time dependent, while we know that it appears as a tension term in the effective energy momentum tensor at the intersection. This is because it is proportional to the induced metric. This indicates that this term is likely to be associated with a violation of the continuity equation at the intersection.

This issue can be understood by re-considering the Codazzi equation:

$$\nabla_M\,\hat{K}^M_N \;=\; \left(\nabla_M\,\hat{K}^M_N\right)^{(\mathrm{reg})} + \left(\nabla_M\,\hat{K}^M_N\right)^{(\mathrm{sing})} \;=\; 0, \tag{60}$$

where ∇ is the covariant derivative with respect to the five dimensional metric on the codimension one brane. From the previous formula, we learn that both the regular and singular parts must vanish simultaneously. However, it can happen that the covariant derivative induces singular contributions when it is applied to certain components of $\hat{K}^M_N$ by taking derivatives of *sign* functions. This is indeed what happens in our case. Consider the case $N=0$. The singular part of the previous formula tells us that

$$\frac{\partial}{\partial t}\,\left(K^{w_1}_{w_1}\right)^{sing} \;=\; \left(\nabla_M\,K^M_0\right)^{sing}, \tag{61}$$

where the left hand side contains the singular term associated with $K_{w_1 w_1}|_{sing}$, while in the right hand side the singular terms come from the covariant derivatives. But the piece in the left hand side corresponds precisely to the term associated with the six dimensional contribution at the intersection. Thus the six dimensional contribution to the Friedmann equation on the intersection does not satisfy the energy conservation and there is an exchange of energy from codimension two brane to the higher dimensional space.

In the light of this fact, one expects that the conservation of energy at the intersection does not hold. Instead, one finds the continuity equation

$$\dot{\rho} + 3H\,(\rho+p) \;=\; 4M_6^4\,\frac{\partial}{\partial\,\tau}\,\arctan\left(\frac{\tan\alpha}{\sqrt{1-\dot{z}_2^2}}\right). \tag{62}$$

Hence, the conservation of energy is ensured only when α vanishes, or when $\dot{z}_2$ is constant.

We close this section by summarising the equations that govern cosmology of the codimension one branes and the intersection;

$$\begin{aligned}
\frac{\Lambda_B}{10} &= \bar{H}^2 - k_1^2 - k_2^2, && (63)\\
\lambda_1 &= 6M_5^2(\bar{H}^2 - k_2^2) + 8M_6^4 k_1, && (64)\\
\rho_2 &= -8M_6^4 \frac{1}{\sqrt{1-\cos^2\alpha \dot{z}_2^2}} \left(k_1 \sin\alpha - (k_2 + \bar{H}\dot{z}_2)\cos\alpha\right), && (65)\\
\chi &= 2M_6^4 \frac{\dot{z}_2 \ddot{z}_2 \sin\alpha\cos\alpha}{A(0,t)(1-\cos\alpha \dot{z}_2^2)^{3/2}}, && (66)\\
\rho &= 3M_4^2 H^2 + 6M_{5,1}^3 \frac{k_2 + \bar{H}\dot{z}_2}{\sqrt{1-\dot{z}_2^2}} + 4M_6^4 \arctan\left(\frac{\tan\alpha}{\sqrt{1-\dot{z}_2^2}}\right), && (67)
\end{aligned}$$

where $A(0,t) = 1/(1+\bar{H}t + k_2 z_2(t))$, Λ_B is the bulk cosmological constant, ρ_2 is the energy density on the moving codimension one brane at $w_1 = 0$, χ is (w_1, t)-component of energy momentum tensor on the moving codimension one brane at $w_1 = 0$ and ρ is the energy density at the intersection. The Hubble parameter at the intersection is given by Eq. (52). The energy conservation at the intersection is given by Eq. (62). In the following we put $M_{1,5} = M_5$.

4 Cosmological solutions

In this section, we discuss the property of the cosmological solutions by focusing on the Friedmann equations on the moving four brane and at the intersection.

4.1 The branes at a right angle

We first consider the simplest case in which the branes are at a right angle $\alpha = 0$ and $\bar{H} = 0$. In this case, the energy momentum tensor on the moving four brane becomes the perfect fluid and there is no energy flow $\chi = 0$. Using the cosmic time τ, the 5D metric is given by

$$ds_5^2 = -d\tau^2 + A^2(w_1,\tau)\left(d\vec{x}_3^2 + dw_1^2\right), \quad A(w_1,\tau) = \frac{1}{1+k_1 w_1 + k_2 z_2(\tau)}. \tag{68}$$

Although the scale factor depends on w_1, the Hubble parameter in terms of the cosmic time is independent of w_1 and given by Eq. (52). By expressing $\dot{z}_2$ in terms of the Hubble parameter H, we get

$$\rho_2 = 8M_6^4\sqrt{H^2 + k_2^2}. \tag{69}$$

Since there is no energy flow, the energy density is conserved

$$\partial_\tau \rho_2 + 4H(\rho_2 + p_2) = 0. \tag{70}$$

At the intersection, the Friedmann equation is given by

$$\rho = 3M_4^2 H^2 + 6M_5^3 k_2 \sqrt{1 + \frac{H^2}{k_2^2}}, \tag{71}$$

and the standard continuity equation holds since $\tan\alpha = 0$. Notice that the static brane gives a contribution of the 5D DGP form.

Since the Hubble parameters are equal in both the equations (69) and (71), by expressing H as a function of the energies in the two cases and equalling the results, they will imply a fine tuning relation between the two homogeneous energy densities ρ_2 and ρ. Geometrically, this is because when the codimension one brane Σ_2 moves through the static bulk, it completely controls the dynamics of the brane $\Sigma_\cap$ that sits at the intersection with Σ_1. Then, $\Sigma_\cap$ can only follow the motion of Σ_2, without an independent dynamics on its own. The problem becomes clearer by the fact that the energy density and the Hubble parameter on the moving codimension brane do not depend on the coordinate w_1. Then, the energy density at the intersection actually fixes *all* the properties of the energy density on the moving brane Σ_2, including its equation of state.

There is a simple way out of a part of this problem. The fine-tuning we found is so strong because we demand that the moving brane Σ_2 keeps a straight shape – that is, it cannot deform along the z_1 direction. Suppose however that we allow the moving codimension one brane to be free to deform and bend, forming a non-trivial angle α with Σ_1 which explicitly depends both on z_1 and t. Then, the energy density on Σ_2 will explicitly depend on z_1. This implies that, although the energy density at the intersection must equal the energy density on Σ_2 calculated at $z_1 = 0$, nevertheless this fine-tuning is ameliorated with respect to the previous case. Indeed, it involves only the quantities calculated at the intersection.

4.2 Arbitrary angle between the branes

Now let us consider the case in which $\alpha \neq 0$. In this case, the Hubble parameter at $w_1 = 0$ is given by Eq. (52). Again taking $\bar{H} = 0$ for simplicity, the Friedmann equation on the moving four brane at the position of the intersection is given by

$$\rho_2 = 8M_6^4(k_2\cos\alpha - k_1\sin\alpha)\sqrt{\frac{H^2+k_2^2}{H^2\sin^2\alpha + k_2^2}}. \tag{72}$$

On the other hand, the Friedmann equation at the intersection is given by

$$\rho \;=\; 3M_4^2H^2 + 6M_5^3\,k_2\,\sqrt{1+\frac{H^2}{k_2^2}} + 4M_6^4\,\arctan\left[\tan\alpha\sqrt{1+\frac{H^2}{k_2^2}}\right]\,. \tag{73}$$

When the angle α vanishes and k_2 remains finite, we recover the results of the previous subsection. Notice also that comparison between Eqs. (72) and (73) imposes a fine-tuning relation between ρ and the energy density ρ_2 of the codimension one brane, when evaluated at the intersection. Nevertheless, this fine-tuning is much milder than the one we met in the previous subsection. This fine-tuning is associated with the restrictive Ansatz we have chosen for the bulk metric.

The form of the previous Friedmann equation is quite complicated to study with full generality. While the second term on the right hand side of Eq. (73) corresponds to the well-known DGP-like term, the last term in the right hand side of Eq. (73) is less standard, and is associated with six dimensional contributions.

Notice that, in the limit of large H, this term approaches the constant value $4M_6^4\,sign(k_2)$, and then can help to drive acceleration in this regime. We will discuss in the last part of the paper more general situations, where six dimensional contributions can provide sources of acceleration for the induced cosmology at the intersection.

The continuity equation is given by

$$\dot{\rho} + 3H\,(\rho + p) \;=\; 4M_6^4\,\frac{\partial}{\partial\,\tau}\;\arctan\left[\tan\alpha\sqrt{1+\frac{H^2}{k_2^2}}\right], \tag{74}$$

so, under the assumption that k_2 is non zero, the conservation of energy is ensured only when α vanishes, or when H is constant. For $\alpha \neq 0$, it is also necessary to have the energy flow on the moving codimension one brane, χ, which also breaks the conservation of energy on the moving codimension one brane. Then we can understand that the energy at the intersection is transmitted to the moving codimension one brane.

5 Applications

In this section, we derive some interesting consequences of the cosmological equations we discussed in the previous section. In the first two subsections, we examine some of the cosmological properties of the solutions we discussed in [15] in the present context. In the last subsection, we will instead derive a new selfaccelerating configuration in which the codimension one branes are *not* maximally symmetric, a case that we did not discuss in our previous work.

5.1 Self-tuning solutions

Here we re-examine the selftuning solution presented in [15]. We take $k_1 = 0$, $k_2 = 0$, $M_{5,1} = 0$ and $\dot{z} = 0$. Then we have

$$\lambda_4 = 3M_4^2\bar{H}^2 + 4M_6^4\alpha, \quad \bar{H}^2 = \frac{\Lambda_B}{10}. \tag{75}$$

where $H = \bar{H}$. Then the expansion rate does not depend on λ_4. The self-tuning mechanism consists on the fact that if we change tension λ_4, α changes so that induced cosmology remains the same. Unfortunately, our embedding is not well suited to study the self-tuning property of the solution as $\alpha =$constant is actually imposed by hand. A possible way out would be to consider a situation in which $z''(t,0) \neq 0$ at the intersection. This would generalise (17) with new pieces that would not necessarily impose that $\dot{\alpha}(t,0) = 0$. It would be nice to study in more detail this kind of generalisation to understand whether it can be compatible with the reflection symmetries of our system or not. It is important to understand whether the eventual self-tuning property would be compatible with the recovery of small scale 4d general relativity on the intersection, in order not to contradict big bang nucleosyntesis and other cosmological tests. In order for this last tricky issue to be solved, it seems to be necessary that the dynamical angle reacts *only* to the vacuum energy density component of the localised matter on the intersection: a priori this is rather counterintuitive. However, a few observations are in order here: it is well known that 6d brane worlds with conical singularities treat tension-type of matter on a completely different footing with respect to a generic fluid ($\omega \neq -1$) [34]. In fact, also in the present setup the six-dimensional contribution to the 4d stress tensor has a tension-like structure; moreover as we showed, a generic fluid localised on the intersection does not seem to render the bulk geometry singular as opposed to what happens in the thin conical setups [34], but violates the conservation of energy. It is therefore not excluded that the self-tuning might be at work here.

5.2 Self-accelerating solutions with $\bar{H} \neq 0$

Here we re-consider the self-accelerating solution presented in [15], for maximally symmetric configurations. We assume there is no cosmological constant nor matter in the system $\Lambda_B = \rho = \rho_2 = \lambda_1 = 0$ with $\dot{z} = 0$. Then we get

$$
\begin{aligned}
0 &= \bar{H}^2 - k_1^2 - k_2^2, && (76)\\
0 &= 3M_4^2 \bar{H}^2 + 6M_5^3 k_2 + 4M_6^4 \alpha, && (77)\\
0 &= k_1 \sin\alpha - k_2 \cos\alpha, && (78)\\
0 &= 6M_5^3(\bar{H}^2 - k_2^2) + 8M_6^4 k_1. && (79)
\end{aligned}
$$

If $k_1 < 0$ and $k_2 < 0$, there are non-trivial solutions for k_1, k_2, α and $\bar{H}$. The solution is roughly given by

$$
\bar{H} \sim \frac{M_6^2}{M_4}, \quad M_5^3 \sim M_4 M_6^2. \tag{80}
$$

in accordance with what we found in the previous paper. Once α, k_2 and $\bar{H}$ are fixed, Eqs. (62) and (67) determine the cosmological dynamics with ρ without ambiguity. The resulting cosmology is complicated due to the non-conservation of energy. Instead of dealing with this complicated case, we will discuss a simpler, different situation in the next subsection.

5.3 Self-accelerating solution with $\bar{H} = 0$

We consider the case $\bar{H} = 0$, $k_1 = 0$, and $k_2 = -\epsilon_2 \beta\, |\sin\alpha|\, M_6$, for some positive constant β and $\epsilon_2 = \pm 1$. To make a more direct comparison to analog studies in the codimension one case, we focus on a regime in which the quantity H^2 satisfies the condition

$$
H^2 \gg \beta^2 \sin^2\alpha M_6^2, \tag{81}
$$

and we will later check in which cases this relation can be satisfied in our context. The continuity equation becomes

$$
\dot{\rho} + 3H\,(\rho + p) \;=\; -4\,\epsilon_1\; M_6^4\, \frac{\partial}{\partial\tau}\; \mathrm{arctan}\left[\frac{H}{\beta\, M_6\; \cos\alpha}\right], \tag{82}
$$

with $\epsilon_1 \equiv -\frac{\sin\alpha}{|\sin\alpha|} = \pm 1$. Notice that $\epsilon_1 = 1$ means that we are taking negative values for the angle α: in the conical case, it would correspond to take an excess angle for the conical singularity.

On the other hand the Friedmann equation acquires the form

$$
\rho \simeq 3\, M_4^2\, H^2 - 6M_5^3\, \epsilon_2\, H - 4\,\epsilon_1\; M_6^4\; \mathrm{arctan}\left[\frac{H}{\beta\, M_6\; \cos\alpha}\right], \tag{83}
$$

with $\epsilon_2 = \pm 1$ corresponding to the usual DGP choice of branches for the codimension one brane. The first term in the right hand side is dominant at large H, and ensures the correct four dimensional form for early time cosmology. The second term is the typical DGP contribution, while the third term, a six dimensional effect, is less standard as we discussed before. The previous expression can be easily rewritten as

$$
\frac{\rho}{3M_4^2} \;=\; \left(H - \epsilon_2\, \frac{M_5^3}{M_4^2}\right)^2 - \frac{M_5^6}{M_4^4} - \frac{4\,\epsilon_1\, M_6^4}{3M_4^2}\, \mathrm{arctan}\left[\frac{H}{\beta\, M_6\; \cos\alpha}\right], \tag{84}
$$

from which we obtain

$$H \;=\; \epsilon_2 \frac{M_5^3}{M_4^2} + \sqrt{\frac{\rho}{3M_4^2} + \frac{M_5^6}{M_4^4} + \epsilon_1 \frac{4\; M_6^4}{3M_4^2}\,\text{arctan}\left[\frac{H}{\beta\, M_6\; \cos\alpha}\right]}, \tag{85}$$

by imposing that the quantity inside the square root is positive. Now, the choice $\epsilon_1 = \epsilon_2 = 1$ corresponds to the standard DGP self-accelerating branch, and the quantity inside the square root is always positive. It implies that, even when ρ vanishes, the Hubble parameter satisfies the inequality

$$H \geq \frac{M_5^3}{M_4^2}\left(1 + \sqrt{1 + \frac{4\; M_6^4\, M_4^2}{3M_5^6}\,\text{arctan}\left[\frac{H}{\beta\, M_6\; \cos\alpha}\right]}\right), \tag{86}$$

and so we find a lower bound for H, as in the well-known self-accelerating branch of DGP model.

This case is very similar to the standard five dimensional case, since the acceleration is mainly driven by the effects of the codimension one brane. It is however also possible to study the case in which $M_5 = 0$, to understand whether six dimensional effects provide acceleration by themselves. We will focus on this case, choosing $\epsilon_1 = 1$, for the remaining discussion. Then, the continuity equation (82) can be formally integrated as

$$\rho \;=\; \rho_0 \left(\frac{a(t)}{a_0}\right)^{-3(1+\omega)} \exp\left[-\int_{X(\infty)}^{X(H)} \frac{d\tilde{x}}{\frac{3M_4^2}{4M_6^2}\beta^2 \cos^2\alpha\; \tan^2\tilde{x} - \tilde{x}}\right], \tag{87}$$

defining

$$X(H) \equiv \text{arctan}\left[\frac{H}{\beta \cos\alpha\, M_6}\right], \tag{88}$$

and introducing the constants ρ_0, a_0 (corresponding to the quantities evaluated at a fiducial time) and calling $w = p/\rho$. This solution shows that, in the limit $H \to \infty$, one recovers the usual relation between energy density and scale factor because in this limit the last factor in the previous equation becomes 1. Using the Friedmann equation, it is straightforward to get the following relation for the acceleration:

$$\frac{\ddot{a}}{a} \;=\; \dot{H} + H^2 \;=\; -\frac{1}{2M_4^2}\left[\frac{1}{3} + \omega - \frac{8M_6^4}{3}\,\frac{\text{arctan}\left[H/\left(\beta\cos\alpha\, M_6\right)\right]}{3M_4^2\, H^2 - 4\, M_6^4\; \text{arctan}\left[H/\left(\beta\cos\alpha\, M_6\right)\right]}\right]\rho, \tag{89}$$

so we have the acceleration when

$$\omega \;<\; -\frac{1}{3} + \frac{8M_6^4}{3}\,\frac{X(H)}{3M_4^2\, H^2 - 4\beta\, M_6^4\, X(H)}. \tag{90}$$

Then for small H we learn that the six dimensional contributions help to provide the acceleration and it can be achieved even when $\omega > -\frac{1}{3}$.

To conclude, we discuss the late time cosmological evolution for our system. In an expanding universe, at late times $\rho \to 0$; then H approaches a constant value, given by the solution of the equation

$$H^2 - \frac{4M_6^4}{3M_4^2}\,\text{arctan}\left[\frac{H}{\beta\cos\alpha\, M_6}\right] \;=\; 0. \tag{91}$$

Let us study two limiting cases, compatible with the condition (81), in which equation (91) admits simple solutions. The first is the case in which α is not too small, from which applying (81) in (91) we get the equation

$$H^2 \;\simeq\; \frac{4M_6^4}{3M_4^2}\,. \tag{92}$$

In this case, H can be rendered small by choosing M_4 sufficiently larger than M_6. On the other hand, condition (81) requires $M_6 \gg \beta\, M_4$, so the constant β must be chosen correspondingly small. Notice that relation (92) is similar to the one we already met in eq. (80), although in the present case we have $M_5 = 0$.

The second possibility is to consider α extremely small, and at the same time focus on a regime in which $H \ll \beta M_6$. The the solution of (91) provides a different relation between H and the mass scales:

$$H \simeq \frac{4M_6^3}{3\,\beta\, M_4^2}\,, \tag{93}$$

and also in this case we can get a sufficiently small value of H by choosing M_4 much larger than M_6.

Notice that the present examples of self-acceleration require a different analysis, with respect to the ones discussed in the previous subsection originally found in [15]. This is because, for the particular choice of our embedding, the codimension one branes do not need to be maximally symmetric, nor empty. It would be nice to understand whether in this case ghosts are present in the low energy spectrum, and if so how do they manifest themselves.

6 Conclusions and Open Issues

In this paper, we explored the cosmological features of a codimension two brane with induced gravity terms, sitting at the intersection between two codimension one branes in six dimensions. We found that the cosmological expansion at the intersection is controlled by contributions coming from the codimension one branes, and from the six dimensional bulk. We first showed that the effect of the codimension one branes on the Friedmann equation at the intersection have the well-known DGP form. Then, we learned that six dimensional contributions are much less standard. They can have an important role for late time cosmology providing a new source of the geometrical acceleration, controlled by the angle between the branes. At the same time, they are also associated with a violation of the energy conservation at the intersection, allowing a flow of energy between the codimension two brane and the higher dimensional space. We discussed consistency relations that matter on the codimension two brane must satisfy and the connection with the choice of energy momentum tensor localised on the codimension one branes.

The main aim of this work was to formulate a general and powerful formalism based on the approach of *mirage cosmology* that can be used to study cosmological solutions in this and similar models, and to apply it to a couple of representative examples. Due to the fact that the codimension one branes that intersect with general angle are not homogeneous and isotropic, a numerical analysis is likely to be needed in order to analyse in full details the cosmological evolution of this class of models. As a natural continuation of the present work, it would be interesting to study the low energy effective action for the light modes associated with our brane configurations. This analysis would be necessary in order to investigate whether ghosts are present in the spectrum of the low energy theory, and, if so, whether they can be eliminated with a mechanism similar to the one of [13]. We leave these issues to future work.

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