Fractional Order Integral Sliding Mode Control for PWR Nuclear Power Plant

Piyush V. Surjagade[1], Jiamei Deng[1,*], Vineet Vajpayee[2], Victor M. Becerra[3], S. R. Shimjith[4] and A. John Arul[5]

Abstract— **This paper presents a robust control strategy for pressurized water type nuclear power plants by combining the optimal linear quadratic Gaussian control strategy with the fractional-order theory based integral sliding mode control strategy. The proposed control scheme follows the reference set-point effectively in spite of the presence of uncertainties in the system by spending minimal control efforts. The non-linear nuclear power plant model adopted in this study is characterized by 38 state variables. The non-linear model is first linearised around steady state operating point to obtain a linear model for which a proposed control strategy is designed. Stability of the closed-loop system is proved with the help of Lyapunov theory. Finally, efficacy of the proposed control scheme for different control loops of the nuclear power plant is demonstrated through simulations and compared with conventional techniques.**

I. Introduction

Operational safety and effective smooth operation of nuclear reactor core are of fundamental importance in the Nuclear Power Plants (NPPs). The operation and control of the NPPs represent a complex problem. The problems are further complicated as in nuclear reactor some system parameters vary with operating power level, fuel burn-up, ageing effect, and internal reactivity feedbacks. These variations in system parameters along with other system uncertainties, such as unmodeled dynamics and external disturbances, makes nuclear reactor control a very difficult task.

As such, active research is continuing to develop controllers for NPPs that can work successfully in presence of these uncertainties. In the last few decades, various control techniques such as optimal control [1], Sliding Mode Control (SMC) [2]–[7], predictive control [8], neural network and fuzzy control [9] have been developed and successfully applied to control NPPs. Among different robust control strategies, SMC has gained immense importance in the control community due to its inherent robustness towards matched uncertainties, simple structure and finite time convergence. SMC is characterized by a discontinuous control law that switches as the system crosses certain predefined manifold in the state space [10]. The early work on nuclear reactor control using SMC is reported by *Shtessel*, wherein the SMC technique is used to design reactor control system in order to provide the robust high accuracy thermal power tracking in a start up regime and a payload current tracking in an operation regime [2]. *Reddy et al.* [3] and *Munje et al.* [4] proposed SMC based spatial power control strategies for large heavy water reactors. *Qaiser et al.* [5] and *Ansarifar et al.* [6] proposed second order sliding mode control techniques based on super twisting algorithm for nuclear research reactor.

In recent years, Fractional-Order (FO) calculus has become more popular to model as well as to control various physical systems [11]. Fractional-order calculus, a branch of mathematics that generalizes the integer-order calculus, provides a more accurate realization than the integer-order one [12], [13]. Hence, fractional-order calculus becomes a strong controlling tool for linear as well as nonlinear systems. In literature, different fractional-order controllers have been designed and successfully tested on nuclear reactors [7], [14]–[16]. In [14]–[16], authors proposed robust FO Proportional Integral Derivative (PID) controllers for global power control of a pressurized heavy water reactor under step-back condition. *Nafiseh et al.* developed a non-linear reduced order FO-SMC for a non-linear FO model of a nuclear reactor system [7].

Compared to integer-order controllers, the FO controllers provide more flexibility to design the control system. For instance, for system modelling, in opposite to integer order systems, FO systems have memory effect and hereditary properties, thus FO system can provide more realistic and accurate behaviour of the system [11]. To date, FO-SMCs designed for nuclear reactor system focused on FO sliding surface to improve the closed-loop system performance [7], but they spent high control energy to achieve the desired objectives. Also, in NPP, not all the state variables are measurable. For example, delayed neutron precursors' concentration are not directly measurable. To overcome these problems, in this paper, a new optimal Fractional-Order Integral Sliding Mode Control (FO-ISMC) strategy is proposed based on optimal Linear Quadratic Gaussian (LQG) controller for Pressurized

*Corresponding author.

[1]Piyush V. Surjagade (p.surjagade@leedsbeckett.ac.uk) and Jiamei Deng (j.deng@leedsbeckett.ac.uk) are with the School of Computing, Creative Technologies & Engineering, Leeds Beckett University, Leeds, LS6 3QR, United Kingdom.

[2]Vineet Vajpayee (v.vajpayee@sheffield.ac.uk) is with the Department of Automatic Control & Systems Engineering, University of Sheffield, UK.

[3]Victor M. Becerra (victor.becerra@port.ac.uk) is with the School of Energy and Electronic Engineering, University of Portsmouth, Portsmouth, PO1 3DJ, United Kingdom.

[4]S. R. Shimjith (srshim@barc.gov.in) is with the Reactor Control Division, Bhabha Atomic Research Centre, Mumbai, 400 085, India and also with Homi Bhabha National Institute, Mumbai, 400 094, India.

[5]A. John Arul (arul@igcar.gov.in) is with the Probabilistic Safety, Reactor Shielding and Nuclear Data Section, Indira Gandhi Centre for Atomic Research, Kalpakkam, 603 102, India.

Water Reactor (PWR)-type NPP. The proposed controller is designed in two steps: first the LQG controller is designed to obtain the optimal performance and to estimate the system states and then the FO-ISMC is designed to increase the robustness of the closed-loop system in the presence of uncertainties. The LQG controller design involves two steps: first is the Kalman filter design to estimate the system states and second is the Linear Quadratic Regulator (LQR) design based on the estimated states. The proposed control strategy is then applied for control of different PWR-type NPP subsystems, which are reactor core power control loop, temperature control loop, steam generator pressure control loop, pressurizer pressure and level control loop, and turbine speed control loop.

The rest of the paper is organized as follows: preliminaries of fractional calculus are discussed in Section II. Section III formulates the control problem. Section IV presents the proposed control design approach. Application of the proposed control scheme to PWR-type nuclear reactor is presented in Section V. Finally, conclusions are drawn in Section VI indicating main contributions.

II. Preliminaries of Fractional Calculus

Fractional-order calculus is the generalization of the integer-order calculus. Fractional calculus represents the fractional-order integration and fractional-order differentiation. The theorems and rules in fractional-order calculus are applicable to their integer-order counterparts in a more generalized representation but not always in a straightforward manner [11], [17]. The definition of fractional-order calculus mainly includes Grunwald-Letnikovs (GL) definition, Riemann-Liouvilles (RL) definition and Caputo definition [11]. However, the RL definition and the Caputo definition are the two most commonly used definitions, which are inspired by the definition of Cauchy generalized $n \in \mathbb{N}$–fold integral of function by replacing the factorial function by the more generalized Gamma function.

Definition 1: [11] The α^{th}-order fractional integration of the function $f : (0,\infty) \to \mathbb{R}$ with respect to $t > 0$ and terminal value $t_0 > 0$ is given by

$$ {}_{t_0}I_t^{\alpha} f(t) := \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{(1-\alpha)}}\ d\tau, \quad (1)$$

where $0 < \alpha < 1$ and $\Gamma : (0,\infty) \to$ is the Euler's Gamma function defined as:

$$\Gamma(\alpha) := \int_{0}^{\infty} x^{(\alpha-1)} e^{-x}\ dx \quad (2)$$

Definition 2: [11] The R-L definition of the α^{th}-order fractional derivative is given by

$$ {}_{t_0}^{RL}D_t^{\alpha} f(t) := \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{(\alpha-m-1)}}\ d\tau, \quad (3)$$

where $m \in \mathbb{N}$ such that $m \geq \lceil \alpha \rceil$, where $\lceil \alpha \rceil$ is the smallest integer greater than or equal to α.

Definition 3: [11] The Caputo definition of the α^{th}-order fractional derivative of the m times continuously differentiable function $f : (0,\infty) \to \mathbb{R}$ or $f \in C^m((0,\infty),\mathbb{R})$ is given by

$$ {}_{t_0}^{C}D_t^{\alpha} f(t) := \frac{1}{\Gamma(m-\alpha)} \int_{t_0}^{t} \frac{f^{(m)}(\tau)}{(t-\tau)^{(\alpha-m-1)}}\ d\tau. \quad (4)$$

In this work, the Caputo definition is employed to design a FO-ISMC.

III. Problem Formulation

Let us consider an uncertain linear time invariant single-input single-output (SISO) system, represented as

$$\dot{x}(t) = Ax(t) + B\big(u(t) + \xi(t)\big) + \omega(t) \quad (5a)$$
$$y(t) = Cx(t) + \nu(t), \quad (5b)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ is the control input, and $y(t) \in \mathbb{R}$ is the system output. $A \in \mathbb{R}^{n\times n}$ is the system matrix, $B \in \mathbb{R}^n$ is the input vector, and $C \in \mathbb{R}^{1\times n}$ is the output vector. Furthermore, the continuous function $\xi(t) \in \mathbb{R}$ represents the uncertainty, which includes uncertainty due to parameter variations and unmodeled dynamics, non-linear functions, and external disturbances. $\omega(t) \in \mathbb{R}^n$ and $\nu(t) \in \mathbb{R}$ are process noise and measurement noise with zero mean and covariance matrices $E(\omega(t)\omega^\top(t)) = \Xi$ and $E(\nu(t)\nu^\top(t)) = \Theta$, respectively, where $\Xi \geq 0 \in \mathbb{R}^{n\times n}$ and $\Theta > 0 \in \mathbb{R}$. For system (5) following assumptions are made

1) The system is fully controllable under the control input $u(t)$.
2) The unknown uncertainty $\xi(t)$ and its fractional order derivative $D^\alpha \xi(t)$ are bounded and they satisfy the inequalities

$$\|\xi(t)\| \leq \phi_\xi,\ \phi_\xi > 0 \text{ and } \|D^\alpha \xi(t)\| \leq \phi_\xi^\alpha,\ \phi_\xi^\alpha > 0. \quad (6)$$

Objective of the proposed control method is to design a robust fractional order controller for the linear uncertain system (5), such that the system output asymptotically tracks the desired trajectory.

IV. Design of Fractional-Order Integral Sliding Mode Controller

In a nuclear power plant, not all the system states are directly measurable. Therefore, in this work the Kalman filter is employed to estimate the unmeasurable states and then based on the estimated states the FO-ISMC strategy is designed.

To find the estimated state vector $\hat{x}(t)$ using Kalman filter estimation problem, the error covariance is chosen as

$$J_k = \lim_{t\to\infty} E\big\{\big(x(t) - \hat{x}(t)\big)\big(x(t) - \hat{x}(t)\big)^\top\big\}. \quad (7)$$

Minimizing (7) using Kalman filtering problem the Kalman gain K_k is obtained as

$$K_k = P_k C^\top \Theta^{-1}, \quad (8)$$

where $P_k \geq 0$ is symmetric matrix computed using algebraic Riccati equation as

$$AP_k + P_k A^\top + \Gamma_k \Xi \Gamma_k^\top - P_k C^\top \Theta^{-1} C P_k = 0, \quad (9)$$

where $\Gamma_k \in \mathbb{R}^n$ is disturbance input matrix. Thus, the estimated state vector $\hat{x}(t)$ for nominal system is obtained as

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K_k\big(y(t) - C\hat{x}(t)\big). \tag{10}$$

Defining estimation error $\tilde{x}(t)$ as

$$\tilde{x}(t) = x(t) - \hat{x}(t), \tag{11}$$

(10) can be written as

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K_kC\tilde{x}(t). \tag{12}$$

Let us assume that the state estimation error $\tilde{x}(t)$ and its fractional order derivatives $D^\alpha\tilde{x}(t)$ are bounded and they satisfy the inequalities

$$\|\tilde{x}(t)\| \leq \varphi_x,\ \varphi_x > 0 \text{ and } \|D^\alpha\tilde{x}(t)\| \leq \varphi_x^\alpha,\ \varphi_x^\alpha > 0. \tag{13}$$

Now, based on the estimated information given by (10) the fractional order integral sliding surface is designed as [18]

$$\sigma(t) = G\big[D^\alpha\big(\hat{x}(t) - \hat{x}(0)\big) - D^{\alpha-1}\big(A\hat{x}(t) + Bu_c(t)\big)\big], \tag{14}$$

where $G \in \mathbb{R}^{1\times n}$ is the projection vector and $u_c(t)$ is the nominal controller designed for nominal system. Here, G is selected as left pseudo-inverse of input distribution vector *i.e.,* $G = (B^\top B)^{-1}B^\top$ such that GB is invertible. Note that D^α represents the fractional derivative and $D^{-\alpha}$ represents the fractional integration. The nominal control $u_c(t)$ is designed as

$$u_c(t) = -K_x\hat{x}(t) - K_rr(t) \tag{15}$$

where K_x is the feedback control gain responsible for the performance of the nominal system and K_r is the feed-forward control gain which is introduced to track the reference signal $r(t)$.

In (15), the feedback control gain K_x can be designed by any state feedback control design method to achieve desired nominal performance. Here, K_x is designed satisfying the infinite horizon LQR cost function

$$J_c = \min_{u_c(t)} \int_0^\infty \big(\hat{x}^\top(\tau)Q\hat{x}(\tau) + u_c^\top(\tau)Ru_c(\tau)\big)d\tau \tag{16}$$

subject to

$$A\hat{x}(t) + Bu_c(t) = 0 \text{ and } C\hat{x}(t) = r(t) \tag{17}$$

where $Q \geq 0 \in \mathbb{R}^{n\times n}$ and $R > 0 \in \mathbb{R}$ are appropriate weighing matrices, to achieve optimal control input. With this, feedback control gain K_x and feed-forward control gain K_r are obtained as [19]

$$K_x = R^{-1}B^\top P_c, \text{ and } K_r = \big(C(A - BK_x)^{-1}B\big)^{-1}, \tag{18}$$

where $P_c > 0$ is the symmetric matrix which satisfies the algebraic Riccati equation

$$A^\top P_c + P_cA + Q - P_cBR^{-1}B^\top P_c = 0. \tag{19}$$

In sliding mode control, once the system states are on the sliding surface the closed-loop system is completely invariant towards the matched type of uncertainties. Thus, the control law which maintains the system states on the sliding surface (14) is designed based on the exponential reaching law as

$$u_d(t) = -(GB)^{-1}\Big\{D^{-\alpha}\Big(\mu_1\sigma(t) + \mu_2 \text{ sign}(\sigma(t))\Big)\Big\} \tag{20}$$

where $\mu_1 > 0$, $\mu_2 > 0$ and sign(.) is a standard signum function.

Finally, the total control law is designed as a combination of (15) and (20) as

$$u(t) = u_c(t) + u_d(t). \tag{21}$$

In the following, Lyapunov stability of the proposed controller (21) with the sliding surface (14) is analysed. Consider the Lyapunov function,

$$V(t) = \frac{1}{2}\sigma^2(t) \tag{22}$$

Taking the time derivative of $V(t)$ and using (5), (14) and (12), we get

$$\begin{aligned}
\dot{V}(t) &= \sigma(t)\dot{\sigma}(t) = \sigma(t)\big\{GD^\alpha\big[\dot{\hat{x}}(t) - A\hat{x}(t) - Bu_c(t)\big]\big\}\\
&= \sigma(t)\big\{GD^\alpha\big[A\hat{x}(t) + Bu_c(t) + Bu_d(t) + K_kC\tilde{x}(t)\\
&\qquad + B\xi(t) - A\hat{x}(t) - Bu_c(t)\big]\big\}\\
&= \sigma(t)\big\{GD^\alpha\big[Bu_d(t) + K_kC\tilde{x}(t) + B\xi(t)\big]\big\}\\
&= \sigma(t)\big\{GD^\alpha\big[-B(GB)^{-1}D^{-\alpha}\big(\mu_1\sigma(t)\\
&\qquad + \mu_2 \text{ sign}(\sigma(t))\big) + K_kC\tilde{x}(t) + B\xi(t)\big]\big\}\\
&= -\mu_1\sigma^2(t) - \mu_2\|\sigma(t)\| + \sigma(t)GK_kCD^\alpha\tilde{x}(t)\\
&\quad + \sigma(t)GBD^\alpha\xi(t)\\
&\leq -\mu_2\|\sigma(t)\| + \sigma(t)GK_kCD^\alpha\tilde{x}(t) + \sigma(t)GBD^\alpha\xi(t)\\
&\leq -\mu_2\|\sigma(t)\| + \|\sigma(t)\|\ \|GK_kC\|\ \|D^\alpha\tilde{x}(t)\|\\
&\quad + \|\sigma(t)\|\ \|GB\|\ \|D^\alpha\xi(t)\|\\
&\leq \|\sigma(t)\|\big(-\mu_2 + \varphi_x^\alpha\|GK_kC\| + \phi_\xi^\alpha\|GB\|\big)
\end{aligned} \tag{23}$$

Thus, for any choice of $\mu_2 \geq \varphi_x^\alpha\|GK_kC\| + \phi_\xi^\alpha\|GB\| + \eta$, (23) becomes

$$\dot{V}(t) = \sigma(t)\dot{\sigma}(t) \leq -\eta\|\sigma(t)\| \leq 0, \tag{24}$$

where η is a small positive constant. Hence, from (24) it is proved that the system trajectories remain on the sliding surface $\sigma(t)$ once they start from it at $t = t_0$ and then, asymptotically converge to equilibrium point.

V. Application to PWR Nuclear Power Plant

In this work, the non-linear dynamic model of PWR type nuclear reactor and associated subsystems given in Ref. [20] is adopted for the study. The model considers the dynamics of the reactor core, thermal hydraulics, piping and plenum, pressurizer, steam generator, condenser, and turbine-governor system, in addition to various actuators and sensors. For system equations, definitions of variables and values of parameters used in this work, the readers are referred to [20].

The proposed control strategy is applied to the different control loops (reactor core power control loop, temperature control loop, steam generator pressure control loop, pressurizer pressure and level control loop, and turbine speed

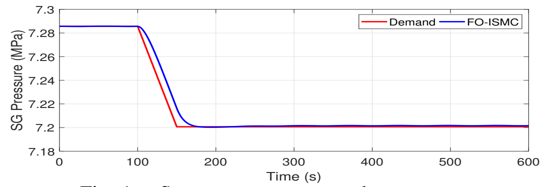

Fig. 1. Steam generator secondary pressure.

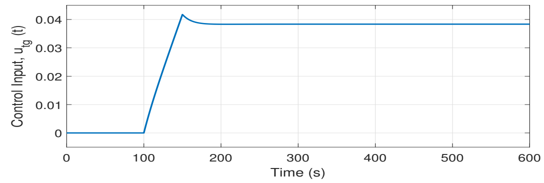

Fig. 2. Control signal to turbine governor valve.

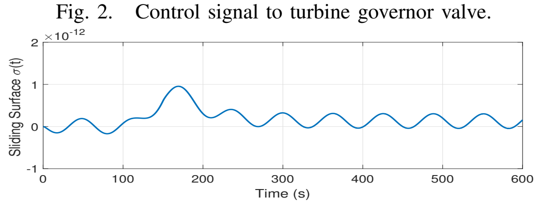

Fig. 3. Sliding surface.

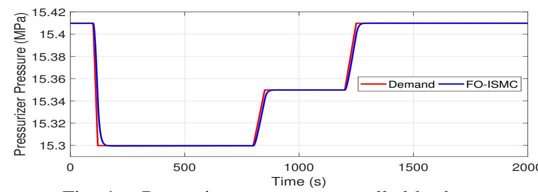

Fig. 4. Pressurizer pressure controlled by heater.

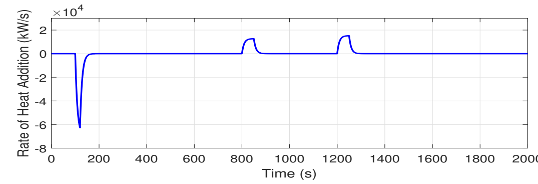

Fig. 5. Rate of heat addition.

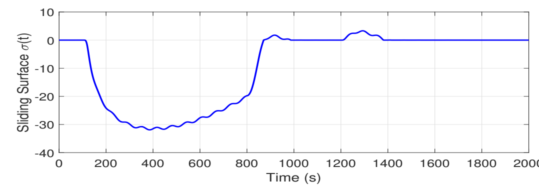

Fig. 6. Sliding surface.

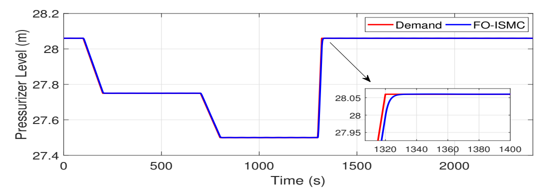

Fig. 7. Pressurizer level.

control loop) of PWR NPP and its performance is tested in the presence of external disturbance for load following operation. Here, in each control loop a sinusoidal external disturbance in the control input is considered throughout system response as

$$\xi(t) = \xi_0 \times \sin(0.1t), \tag{25}$$

where ξ_0 is the magnitude of the disturbance.

First, the non-linear model of PWR NPP is linearised around steady state operating point to obtain the linear model on which the effectiveness of the proposed controller has been tested. The definition of input and output signals for every SISO control loop and the value of controller parameters are given in Table I.

A. *Steam Generator Pressure Control Loop*

In this control loop, the steam generator pressure, P_s is controlled by adjusting input signal, u_{tg} to the turbine-governor valve. Here, the performance of the proposed controller is evaluated for a set-point change in steam generator pressure in the presence of external disturbance (25) where the value of ξ_0 is considered as 1×10^{-3}. Initially, it is assumed that secondary pressure is at 7.2857 MPa and then the set-point is decreased to 7.2 MPa during time $t = 100$ s to $t = 150$ s. During this transient, variation of output secondary pressure, control input, and sliding surface with the proposed controller are shown in Figs. 1, 2, and 3, respectively. It can be observed that, the set-point is reached without any overshoot and at the same time the proposed controller is able to mitigate the disturbance present in the system.

B. *Pressurizer Pressure Control Loop*

In this control loop our aim is to maintain the coolant pressure within permissible limit. Primary coolant pressure, P_p can be controlled by bank of heaters, spray flow rate, power operated relief valves, or safety valves. However, in this study, the coolant pressure control is studied only by actuating a bank of heaters, Q_{heat}. Performance of the proposed controller is tested for a set-point change in pressurizer pressure in the presence of external disturbance (25) where the value of ξ_0 is considered as 20. Initially, it is assumed that pressurizer pressure is at 15.4097 MPa and then the set-point is reduced to 15.3 MPa during time $t = 100$ s to $t = 120$ s and again it is increased to 15.4097 MPa in two steps. During this transient, variation of output secondary pressure, control input, sliding surface with proposed controller are shown in Figs. 4, 5, and 6, respectively. It can be observed that the proposed controller is able to follow the set-point with minimum tracking error in spite of the presence of uncertainty.

C. *Pressurizer Level Control Loop*

The purpose of pressurizer level control loop is to maintain the water level for the reactor core coolant system. In this simulation study, the controller performance is analysed by varying set-point in the pressurizer level in the presence of external disturbance (25) where the value of ξ_0 is considered as 5×10^{-2}. It is assumed that initially the system is at steady state and pressurizer level is at 28.06 m. The set point is then reduced to 27.5 m in two steps and again it is increased to 28.06. Fig. 7 shows the variation of output pressurizer level with respect to demand. Variation of control input is shown in Fig. 8. Fig. 9 shows the plot for sliding surface.

TABLE I
CONTROLLER PARAMETERS

Loop	Signal		LQR		Kalman Filter		FOISMC		
	I/P	O/P	Q	R	Ξ	Θ	μ_1	μ_2	α
A	u_{tg}	P_s	$5 \times 10^{-3} I_n$	1×10^{2}	$5 \times 10^{-5} I_n$	1	1	1×10^{-1}	8×10^{-1}
B	Q_{heat}	P_p	$1 I_n$	1×10^{-8}	$1 \times 10^{-2} I_n$	1	50	25	8×10^{-1}
C	$\dot{m}_{sur}$	l_w	$1 \times 10^{3} I_n$	1×10^{-2}	$6 I_n$	1	1	1×10^{-1}	8×10^{-1}
D	u_{tg}	ω_{tur}	$2 \times 10^{3} I_n$	1×10^{-2}	$1 I_n$	1	1	1×10^{-1}	8×10^{-1}
E.1	v_{rod}	i_{rtd}	$1 \times 10^{-3} I_n$	1×10^{3}	$1 \times 10^{0} I_n$	1	5	1×10^{-1}	8×10^{-1}
E.2	v_{rod}	i_{lo}	$1 \times 10^{-3} I_n$	1×10^{5}	$5 \times 10^{0} I_n$	1	10	1	8×10^{-1}

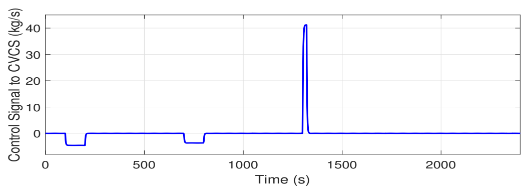

Fig. 8. Input signal to CVCS system.

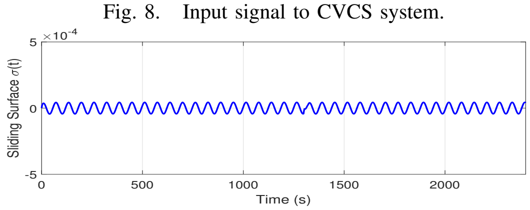

Fig. 9. Sliding surface.

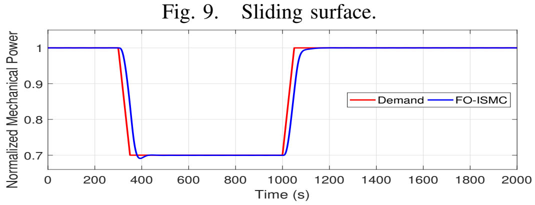

Fig. 10. Normalized mechanical power.

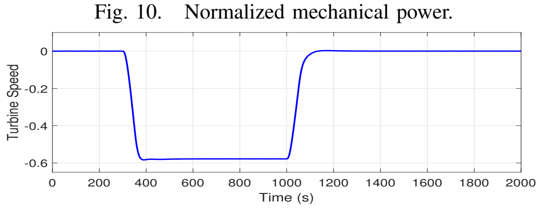

Fig. 11. Turbine speed.

D. Turbine Speed Control loop

This loop is responsible for controlling the shaft speed by varying the steam flow. In this simulation study the controller performance is tested by varying the demand power from the generator in the presence of external disturbance (25) where the value of ξ_0 is considered as 1×10^{-4}. During the transient, variation of output turbine speed, control input, and sliding surface are shown in Figs. 10, 11, and 12, respectively. Here, it can be observed that the output turbine speed follows the demand with acceptable undershoot. In case of nuclear reactor, minimum value of overshoot and undershoot is always preferable.

E. Load Following Mode of Operation (Power Control Loop)

In load following mode of operation the reactor power adjusts according to demand of electricity throughout the day. The reactor power can be controlled using neutronic power or through average coolant temperature. Also, in this

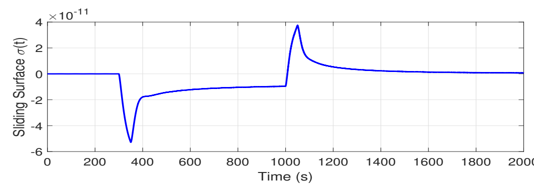

Fig. 12. Sliding surface.

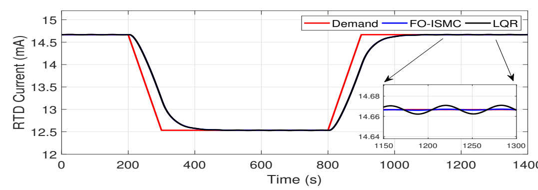

Fig. 13. RTD current.

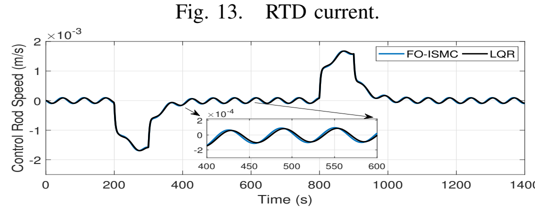

Fig. 14. Control rod speed movement.

subsection, superiority of the proposed controller over some conventional control strategies are shown.

1) Temperature Control Loop: In this case study the reactor power is controlled with the help of coolant temperature. Here, the performance of the proposed controller is compared with the state feedback LQG controller in the presence of external disturbance (25) where the value of ξ_0 is considered as 1×10^{-4}. The variation of measured Resistance Temperature Detector (RTD) current corresponding to the output power with proposed controller and the LQR controller is shown in Fig. 13. It can be observed that, the proposed controller is able to overcome the disturbance and tracks the demand perfectly. While, with the LQR controller the disturbance superimposed on the output signal and fails to maintain the desired demand. Variation of control input for both the controllers and variation of sliding surface for the proposed controller are shown in Figs. 14 and 15, respectively.

2) Core Neutronics Control Loop: In this control loop the reactor power is controlled directly with the help of ex-core detectors. Here, the performance of the proposed controller is compared with the state feedback LQG controller (setting $u_d(t) = 0$) and conventional Integral Sliding Mode Control (ISMC) (setting $\alpha = 0$) in the presence of external disturbance (25) where the value of ξ_0 is considered

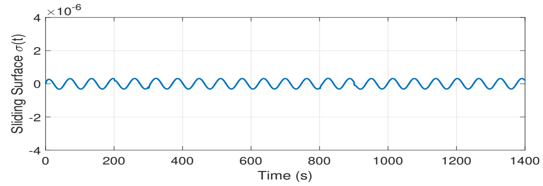

Fig. 15. Sliding surface.

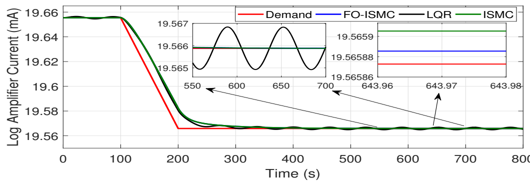

Fig. 16. Excore detector current during demand power manoeuvring.

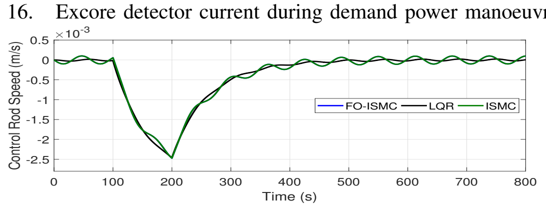

Fig. 17. Control rod speed movement.

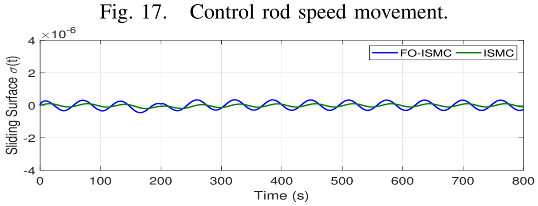

Fig. 18. Sliding surface.

as 1×10^{-4}. Initially, it is assumed that the NPP is operating at full power. Then the demand power is reduced from full power to 0.9 FFP during time $t = 100$ s to $t = 200$ s. During this transient, variation of excore detector logarithmic amplifier output current correspond to the reactor power with the proposed controller, LQG controller and ISMC is shown in Fig. 16. It can be observed that the proposed controller and ISMC are able to follow the change in demand in spite of presence of uncertainty in the system and the performance of the closed-loop system is improved with the proposed controller as compared to ISMC. Whereas, the LQG controller fails to maintain the demand. The control input for three controllers is shown in Fig. 17. The variation of sliding surface for proposed controller is shown in Fig. 18.

VI. Conclusions

This paper presents an optimal fractional-order integral sliding mode control scheme, which assures asymptotic tracking of reference set-point in the presence of uncertainties and external disturbances. To obtain the optimal performance linear quadratic Gaussian control is combined with fractional-order integral sliding mode control. The proposed control scheme offers robustness towards uncertainties and guarantees minimal use of control energy. Simulation results show that the proposed control scheme provides satisfactory tracking performance in the presence of parametric uncertainty and external disturbance for the different control loops of nuclear power plant.

Acknowledgement

This research was funded by the Engineering and Physical Sciences Research Council (EPSRC) grant number EP/R021961/1.

References

[1] V. Vajpayee, V. Becerra, N. Bausch, J. Deng, S. Shimjith, and A. J. Arul, "LQGI/LTR based robust control technique for a pressurized water nuclear power plant," *Annals of Nuclear Energy*, vol. 154, p. 108105, 2021.

[2] Y. Shtessel, "Enhanced sliding mode control of the space nuclear reactor system," in *Proceedings of 1995 34th IEEE Conference on Decision and Control*, vol. 3, 1995, pp. 2468–2473.

[3] G. D. Reddy, B. Bandyopadhyay, and A. P. Tiwari, "Multirate output feedback based sliding mode spatial control for a large PHWR," *IEEE Transactions on Nuclear Science*, vol. 54, no. 6, pp. 2677–2686, 2007.

[4] R. K. Munje, B. M. Patre, S. R. Shimjith, and A. P. Tiwari, "Sliding mode control for spatial stabilization of advanced heavy water reactor," *IEEE Transactions on Nuclear Science*, vol. 60, no. 4, pp. 3040–3050, 2013.

[5] S. Qaiser, A. Bhatti, M. Iqbal, R. Samar, and J. Qadir, "Model validation and higher order sliding mode controller design for a research reactor," *Annals of Nuclear Energy*, vol. 36, no. 1, pp. 37–45, 2009.

[6] G. R. Ansarifar and M. Rafiei, "Second-order sliding-mode control for a pressurized water nuclear reactor considering the xenon concentration feedback," *Nuclear Engineering and Technology*, vol. 47, no. 1, pp. 94–101, 2015.

[7] N. Zare Davijani, G. Jahanfarnia, and A. Esmaeili Abharian, "Nonlinear fractional sliding mode controller based on reduced order FNPK model for output power control of nuclear research reactors," *IEEE Transactions on Nuclear Science*, vol. 64, no. 1, pp. 713–723, 2017.

[8] V. Vajpayee, S. Mukhopadhyay, and A. P. Tiwari, "Data-driven subspace predictive control of a nuclear reactor," *IEEE Transactions on Nuclear Science*, vol. 65, no. 2, pp. 666–679, 2018.

[9] M. Boroushaki, M. B. Ghofrani, C. Lucas, and M. J. Yazdanpanah, "An intelligent nuclear reactor core controller for load following operations, using recurrent neural networks and fuzzy systems," *Annals of Nuclear Energy*, vol. 30, no. 1, pp. 63–80, 2003.

[10] C. Edwards and S. K. Spurgeon, *Sliding Mode Control: Theory and Applications*. Taylor and Francis, 1998.

[11] B. Bandyopadhyay and S. Kamal, *Stabilization and Control of Fractional Order Systems: A Sliding Mode Approach*. Springer International Publishing, 2014.

[12] C. Li and W. Deng, "Remarks on fractional derivatives," *Applied Mathematics and Computation*, vol. 187, no. 2, pp. 777–784, 2007.

[13] J. T. Machado, V. Kiryakova, and F. Mainardi, "Recent history of fractional calculus," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 3, pp. 1140–1153, 2011.

[14] S. Saha, S. Das, R. Ghosh, B. Goswami, R. Balasubramanian, A. K. Chandra, S. Das, and A. Gupta, "Design of a fractional order phase shaper for iso-damped control of a PHWR under step-back condition," *IEEE Transactions on Nuclear Science*, vol. 57, no. 3, pp. 1602–1612, 2010.

[15] S. Bhase and B. Patre, "Robust FOPI controller design for power control of PHWR under step-back condition," *Nuclear Engineering and Design*, vol. 274, pp. 20–29, 2014.

[16] M. Bongulwar and B. Patre, "Design of $PI^{\lambda}D^{\mu}$ controller for global power control of pressurized heavy water reactor," *ISA Transactions*, vol. 69, pp. 234–241, 2017.

[17] I. Podlubny, *Fractional Differential Equations*. New York: Academic, 1999.

[18] F. Yang, X. Shao, S. M. Muyeen, D. Li, S. Lin, and C. Fang, "Disturbance observer based fractional-order integral sliding mode frequency control strategy for interconnected power system," *IEEE Transactions on Power Systems*, pp. 1–1, 2021.

[19] D. S. Naidu, *Optimal Control Systems*, R. C. Dorf, Ed. Boca Raton, FL, USA: CRC Press, Inc., 2002.

[20] V. Vajpayee, V. Becerra, N. Bausch, J. Deng, S. Shimjith, and A. J. Arul, "Dynamic modelling, simulation, and control design of a pressurized water-type nuclear power plant," *Nuclear Engineering and Design*, vol. 370, p. 110901, 2020.