Brane-world inflation: slow-roll corrections to the spectral index

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We quantify the slow-roll corrections to primordial density perturbations arising from inflation driven by a four-dimensional scalar field with a monomial potential in a five-dimensional non-compact bulk spacetime. Although the difference between the classical brane-world solutions and standard four-dimensional solutions is large at early times, the change to the amplitude at late times of perturbations generated from quantum fluctuations is first-order in slow-roll parameters, leading to second-order slow-roll corrections to the spectral index. This confirms that the leading-order effects are correctly given by previous work in the literature.

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I. INTRODUCTION

Recent proposals in theoretical physics have suggested that our four-dimensional Universe could lie on a brane embedded in a higher-dimensional space-time [1]. One of the most studied is the Randall–Sundrum (RS) model [2] where our four-dimensional universe is a brane embedded in five-dimensional anti-de Sitter space-time (AdS_5). Indeed it is possible in this framework to have only one brane in a non-compact space and this is the scenario we will consider in this letter.

This brane-world perspective may dramatically change our picture of the early universe. In this model, the Friedmann equation is modified [3] to

$$H^2 = \frac{\Lambda_4}{3} + \frac{\kappa_4^2}{3}\rho + \frac{\kappa_5^4\,\rho^2}{36}\,, \qquad (1)$$

so that the Hubble parameter $H \propto \rho$ at high energy-density. Bulk gravity is strongly coupled to the dynamics on the brane at high energy, which changes the evolution of cosmological perturbations [4]. Thus, it is important to establish whether the evolution of the perturbations generated during inflation could be significantly different from standard four-dimensional slow-roll inflation. In a recent paper [5] we calculated these effects for the simplest realization of slow-roll inflation on a brane. The purpose of this letter is to illustrate the effect of these brane-world corrections on the spectral index of density perturbations created during inflation in some specific models.

The simplest way to realize inflation in the RS model is to have a single scalar field, the inflaton, confined to the brane [6] and only gravity in the bulk. The amplitude of the resulting scalar curvature perturbations is given by

$$\langle \mathcal{R}_c^2 \rangle^{1/2} = \left(\frac{H}{\dot\phi}\right) \langle \delta\phi^2 \rangle^{1/2}, \qquad (2)$$

where $\dot\phi$ is the time derivative of the inflaton ϕ and $\delta\phi$ is the inflaton fluctuation on a spatially flat hypersurface. If one makes the assumption that back-reaction due to metric perturbations in the bulk can be neglected, the quantum expectation value of the inflaton fluctuations on super-horizon scales in the de Sitter space-time is $\langle \delta\phi^2 \rangle = H^2/(4\pi^2)$. It has been shown that the curvature perturbation $\mathcal{R}_c$ on comoving hypersurfaces is constant outside the horizon in this model [7] and, indeed, independently of the gravitational theory so long as energy-momentum is conserved [8]. One can then use Eq. (2) as initial conditions to calculate observables like Cosmic Microwave Background (CMB) temperature anisotropy. Although the formula (2) for the curvature perturbation is exactly the same as in the four-dimensional case, the relation between the Hubble parameter, H, and the scalar field potential, V, is different [6] due to the modified Friedmann equation (1). This changes the prediction of the spectrum of scalar perturbations for a given potential and observational predictions of brane-world inflation have been made using this approach [9, 10].

A crucial assumption is that back-reaction due to metric perturbations in the bulk can be neglected. In the extreme slow-roll limit this is necessarily correct because the coupling between inflaton fluctuations and metric perturbations vanishes; however, this is not necessarily the case when slow-roll corrections are included in the calculation. Previous work [11] has shown that such bulk effects can be subtle and interesting (see also [12] for other approaches). In particular, sub-horizon inflaton fluctuations on a brane excite an infinite ladder of Kaluza–Klein modes of the bulk metric perturbations at first order in slow-roll parameters, and a naive slow-roll expansion breaks down in the high-energy regime once one takes into account the back-reaction of the bulk metric perturbations, as confirmed by direct numerical simulations [13]. However, an order-one correction to the behaviour of inflaton fluctuations on sub-horizon scales does not necessarily imply that the amplitude of the inflaton perturbations receives corrections of order one on large scales; one must consistently quantise the coupled brane inflaton fluctuations and bulk metric perturbations. This requires a detailed analysis of the coupled brane-bulk system [5, 14].

II. THE BULK-FIELD EQUATION OF MOTION

The RS model is a brane-world model where the background spacetime is AdS_5 and matter fields are confined to a 3-brane. We will be considering the single-brane model first proposed in Ref. [2]. The brane-world inflation model under consideration has the action

$$S = \frac{1}{2\kappa_5^2}\int_{\mathcal{M}} d^5x\sqrt{-{}^{(5)}g}\left\{R-2\Lambda\right\} + \int_{\partial\mathcal{M}} d^4x\sqrt{-{}^{(4)}g}\left\{-\lambda+\frac{1}{\kappa_5^2}K-\frac{1}{2}(\partial\phi)^2-V(\phi)\right\}, \quad (3)$$

where K is the trace of the extrinsic curvature of the boundary $\partial\mathcal{M}$, the AdS curvature scale is defined by $\Lambda = -6\mu^2$ and the tension $\lambda = 6\mu/\kappa_5^2$ is tuned to make the brane flat if it has no other energy density.

The brane is our four-dimensional universe and, because we are interested in studying early-universe inflation, we will make the slow-roll approximation, where the induced metric is assumed to be de Sitter but with a slowly changing Hubble parameter. We will describe this in coordinates where the background line element has the form

$$\mathrm{d}s^2 = N(z)^2\Big[\mathrm{d}z^2 + \frac{1}{H^2\tau^2}(\mathrm{d}\vec{x}^2-\mathrm{d}\tau^2)\Big], \quad (4)$$

with the function $N(z)$ given by $N(z) = H/\mu\sinh(Hz)$ [3]. Here, H is the Hubble parameter, which is constant in a pure de Sitter universe, and the coordinate τ is conformal time on the brane. The brane is located at $z_\mathrm{b} = H^{-1}\,\mathrm{arcsinh}(H/\mu)$, which would be exactly constant for a pure de Sitter universe. During inflation the universe is approximately de Sitter in that the Hubble parameter H changes slowly as the field rolls slowly down the potential. This slow-rolling behaviour is usually expressed in terms of slow-roll parameters defined as

$$\epsilon = -\frac{\dot{H}}{H^2}\,, \qquad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}}\,, \quad (5)$$

where the overdot denotes derivatives with respect to proper time on the brane, not the conformal time τ. We will use the convention that τ is positive, so that early times corresponds to $\tau\to\infty$ and late times to $\tau\to 0$.

The equations of motion for the scalar metric perturbations are most conveniently expressed in terms of a master variable, a technique first used in the study of brane-world perturbations in Ref. [15, 16]. We define a master variable χ, whose equation of motion is [5]

$$\chi_{,\tau\tau} + k^2\chi - \frac{2}{\tau^2}\chi + \frac{1}{H^2\tau^2}\big(-\chi'' + U(z)\chi\big) = 0\,, \quad (6)$$

where

$$U(z) = \frac{H^2}{4}\left(9-\frac{1}{\sinh^2(Hz)}\right). \quad (7)$$

Note that Eq. (6) is separable. The other equations are simplified by defining a new variable, ξ, describing the scalar field and metric perturbations on the brane, and given by

$$\xi = -\left(\frac{d}{d\tau}+\frac{1}{\tau}\right)\left(\frac{\phi_{,\tau}}{H}\mathcal{R}_c\right) - \frac{\dot{\phi}}{3H\tau}\chi_\mathrm{b}\,, \quad (8)$$

in terms of the field χ evaluated on the brane and the curvature perturbation $\mathcal{R}_c$ on comoving hypersurfaces. This new variable obeys the following equation of motion [5]

$$\xi_{,\tau\tau} + k^2\xi = -\frac{k^2\dot{\phi}}{3H\tau}\chi_\mathrm{b}\,, \quad (9)$$

and the junction condition at the brane is

$$\left[\chi' + \left(\frac{1}{2}\frac{N'}{N}+\frac{\kappa_5^2\dot{\phi}^2}{3}\right)\chi\right]_\mathrm{b} = \kappa_5^2\dot{\phi}H\tau\xi\,. \quad (10)$$

Here we have neglected all terms suppressed by the slow-roll parameters except for the terms responsible for the coupling between ξ and χ, i.e., the terms that induce the mixing between the brane and bulk modes. It is useful to define the time-independent Wronskian of two states $f=(\xi,\chi)$ and $f^*=(\xi^*,\chi^*)$ by [5]

$$W(f^*,f) = \xi^*\xi_{,\tau} - \xi\xi^*_{,\tau} + \frac{1}{3\kappa_5^2}\int_{z_\mathrm{b}}^{\infty} dz\,\big(\chi^*\chi_{,\tau}-\chi\chi^*_{,\tau}\big)\,. \quad (11)$$

which is conserved.

At late times, $k\tau\to 0$, a growing bound state solution is [5]

$$\begin{aligned}\xi_g &= A_g\,k\tau J_{-1/2}(k\tau)\,, \quad (12)\\ \chi_g &= C_g\,\tau\sqrt{\sinh(Hz)}\,\log\left(\frac{\cosh(Hz)-1}{\cosh(Hz)+1}\right),\end{aligned}$$

where A_g is the overall amplitude, while C_g can be determined through A_g. A decaying bound state is given by [5]

$$\begin{aligned}\xi_d &= A_d\,k\tau J_{1/2}(k\tau)\,, \quad (13)\\ \chi_d &= C_d\,\tau^2\sqrt{\sinh(Hz)}\Bigg[1 + \frac{\cosh(Hz)}{2}\log\left(\frac{\cosh(Hz)-1}{\cosh(Hz)+1}\right)\Bigg],\end{aligned}$$

where C_d is linearly related to A_d.

At early times (and high energies), $k\tau\to\infty$, the growing and decaying solutions both take the form [5]

$$\begin{aligned}\chi &\propto \tau^{1/3}\exp\Bigg\{i\tau - i\left(\frac{3\epsilon^2\tau}{2}\right)^{1/3} - \frac{z-z_\mathrm{b}}{2} - \left(\frac{2\epsilon\tau^2}{3}\right)^{1/3}(z-z_\mathrm{b}) + \frac{i}{3}\tau(z-z_\mathrm{b})^2\Bigg\} + \text{c.c.}\\ \xi &\propto \exp\left(i\tau - i\left(\frac{3\epsilon^2\tau}{2}\right)^{1/3}\right) + \text{c.c.}\,. \quad (14)\end{aligned}$$

The growing and decaying solutions have a different amplitude and phase, with the phase difference being approximately equal to $\pi/2$. We see that the amplitude of $\chi(z_b,\tau)$ increases as $\chi(z_b,\tau)\propto\tau^{1/3}$ while the amplitude of ξ tends to a constant at large τ, becoming increasingly dominated by higher modes and more closely bound to the brane.

The early-time solution (14) differs from the solution for the scalar field in 4D at leading order, where ξ satsifies the equation of motion

$$\xi_{,\tau\tau}+k^2\xi-\frac{2\epsilon-\eta}{\tau^2}\xi=0\,, \tag{15}$$

and has solutions at late times given by

$$\xi_g=\frac{2^\nu}{\Gamma(-\nu+1)}(k\tau)^{1/2-\nu}\,,\quad \xi_d=\frac{2^{-\nu}}{\Gamma(\nu+1)}(k\tau)^{1/2+\nu}\,, \tag{16}$$

where $\nu=1/2+2\epsilon-\eta$, and at early times by

$$\xi_g=N_d\cos(k\tau+\Delta_g)\,,\quad \xi_d=N_g\cos(k\tau+\Delta_d) \tag{17}$$

where $N_d=N_g$ and $|\Delta_g-\Delta_d|=\nu\pi$ for standard 4D inflation.

One might expect that the $\mathcal{O}(1)$ difference between the early-time solutions would lead to a significant difference to the amplitude of quantum perturbations generated during inflation. In fact this does not happen. We now illustrate how the quantum effects can be calculated and use the result to determine the slow-roll corrections to the spectral indices of scalar perturbations for some monomial potentials.

III. SUMMARY OF WRONSKIAN METHOD

In Ref. [5] we calculated the growing and decaying mode solutions; these were found to differ significantly from the solution in the standard 4D inflationary paradigm. However, the amplitude of large-scale perturbations from the quantum vacuum state in the early universe is only affected at the first order in the slow-roll parameters. We follow the method developed in Refs. [17] to relate the amplitude of the growing mode solution to the creation and annihilation operators, which is summarized here.

In principle, this method involves the construction of properly normalised positive- and negative-frequency solutions at early times and the evaluation of the inner product (Wronskian) between these solutions and the bound states. However, there is a short-cut: all we need are the values of the real coefficients of the positive and negative frequency parts. We first explain this short-cut by making use of the familiar four-dimensional example, and then proceed to the brane-world model.

Let us begin with the standard four-dimensional case. The Wronskian is defined as

$$W(\xi^*,\xi)=\frac{1}{k^2}(\xi^*\xi_{,\tau}-\xi\xi^*_{,\tau}). \tag{18}$$

We are interested in the quantum field that behaves at late times as $\hat\xi\to\hat Z\xi_g$, where $\hat Z$ is a time-independent quantum operator. This field is quantised on small scales where we can expand $\hat\xi$ in terms of negative- and positive-frequency modes,

$$\hat\xi=\hat a\varphi^{(-)}+\hat a^\dagger\varphi^{(+)}, \tag{19}$$

where $\hat a$ and $\hat a^\dagger$ are annihilation and creation operators, respectively, which define the vacuum $\hat a|0\rangle=0$. Note that because τ decreases as proper time increases, the negative- and positive-frequency functions are $\varphi^{(-)}\propto e^{ik\tau}$ and $\varphi^{(+)}\propto e^{-ik\tau}$. The mode functions should be normalised as $W(\varphi^{(-)},\varphi^{(+)})=-i$ to ensure the canonical commutational relation between $\hat\xi$ and its conjugate momentum.

We use the constancy of the Wronskian to express $\hat Z$ in terms of $\hat a$ and $\hat a^\dagger$. At late times, $\hat Z=W(\hat\xi,\xi_d)/W(\xi_g,\xi_d)$ where the numerator can be calculated using the early-time solutions and the denominator using the late-time solutions (16) as $W(\xi_g,\xi_d)=2/(k\pi)$ up to the first order in slow-roll parameters. We expand the growing mode and the decaying mode solutions at early times by

$$\xi_g=c_g\varphi^{(-)}+c_g^*\varphi^{(+)}\,,\quad \xi_d=c_d\varphi^{(-)}+c_d^*\varphi^{(+)}\,. \tag{20}$$

This allows us to evaluate the expectation value of the square of the operator $\hat Z$ as

$$\langle\hat Z^\dagger\hat Z\rangle=k^2\left(\frac{\pi}{2}\right)^2|c_d|^2. \tag{21}$$

which fully characterizes the Gaussian random field. Thus, the problem reduces to finding c_d, the expansion coefficient for the decaying mode solution.

The positive- and negative-frequency modes have the form $\varphi^{(\pm)}=|\varphi|e^{\mp i\delta}e^{\mp ik\tau}$, where the normalisation factor $|\varphi|$ is not needed here, and δ is some phase (which is irrelevant in the four-dimensional case). Comparing the two expressions (17) and (20), and using the Wronskian $W(\xi_g,\xi_d)$, $|c_d|^2$ is determined to be

$$|c_d|^2=\frac{1}{2}\frac{N_d}{N_g}\frac{1}{\sin|\Delta_g-\Delta_d|}\frac{1}{k}\left(\frac{2}{\pi}\right)\,. \tag{22}$$

Thus, the quantity $|c_d|^2$ entering (21) is expressed in terms of the ratio N_d/N_g of the amplitudes and difference $(\Delta_g-\Delta_d)$ of the phases of the decaying and growing modes evolved back in time. These are evaluated analytically as $N_d/N_g=1$ and $|\Delta_d-\Delta_g|=\pi/2$, giving

$$|c_d|^2=\frac{1}{k\pi}\,,\quad \langle\hat Z^\dagger\hat Z\rangle=\frac{k\pi}{4}. \tag{23}$$

The quantity that is related to observables is the comoving curvature perturbation $\mathcal{R}_c$, which is defined as

$$\xi=-\left[\frac{d}{d\tau}+(1+2\epsilon-\eta)\frac{1}{\tau}\right]\left(\frac{\phi_{,\tau}}{H}\mathcal{R}_c\right). \tag{24}$$

Thus, one finds the power spectrum of the curvature perturbation to be

$$\mathcal{P}_{\mathcal{R}_c}^{1/2}(k) = \left[1 + C_1\epsilon + C_2\eta\right] \frac{H^2}{2\pi\dot{\phi}}\bigg|_{k=aH}, \qquad (25)$$

where $C_1 = 3 - 2\ln 2 - 2\gamma$ and $C_2 = -2 + \ln 2 + \gamma$, using the fact that conformal time is $\tau = (1+\epsilon)/(aH)$ up to the first order in slow-roll parameters.

The reason for preferring this method for dealing with the brane-world case is that one does not need to know the properly-normalised positive- and negative-frequency modes at early times, merely the quantities N_d/N_g and $|\Delta_d - \Delta_g|$. For the brane-world a similar calculation presented in our previous work [5] allow us to include the effect of interactions between the brane and bulk. The Wronskian method, using the formula in Eq. (11), allows us to deal with complicated early-time behaviour. The expectation value of the square of the operator $\hat{Z}$ is obtained as

$$\langle \hat{Z}^\dagger \hat{Z} \rangle = \frac{k\pi}{4}\Big[1 + K(\epsilon; H/\mu)\Big], \qquad (26)$$

where $K(\epsilon; H/\mu)$ is determined by numerical solution. The left panel of Fig. 1 shows the dependence of K on the energy scale of the inflation H/μ. At low energies, $H/\mu \ll 1$, the correction K is negligible and we recover the standard vacuum (23). In the high-energy limit, $H/\mu \gg 1$, the correction can be approximated as a linear function of ϵ , which we write as $K(\epsilon; H/\mu) = K_1\epsilon$. The right panel of Fig. 1 shows $K(\epsilon; H/\mu)/\epsilon$ in the high energy limit. This modifies Eq. (25) for the power spectrum by changing the coefficient C_1 to

$$\tilde{C}_1 = 3 - 2\ln 2 - 2\gamma + (K_1/2), \qquad (27)$$

while $\tilde{C}_2 = C_2$.

IV. CORRECTIONS TO THE SPECTRAL INDEX

One of the strengths of the slow-roll inflation paradigm is that the perturbations generated by quantum fluctuations have a power spectrum that is almost scale-invariant. In 4D inflation the first-order correction to the tilt, $n - 1 \equiv \mathrm{d}\ln\mathcal{P}_{\mathcal{R}_c}/\mathrm{d}\ln k$, is well known to be $n - 1 = -4\epsilon + 2\eta$. The second-order correction was calculated by Stewart and Lyth [18]. This was extended to high energy inflation in the RS model in Ref. [19], including the modified effect of the Friedmann equation (1) in Eq. (25) but without including bulk gravity effects on the initial vacuum state. This gives

$$\begin{aligned} n - 1 \; + \; & 4\epsilon - 2\eta = 2(C_1 - 2)\epsilon^2 \\ & + (-4C_1 + 2C_2 + 2)\epsilon\eta - 2C_2\delta^2, \end{aligned} \qquad (28)$$

where $\delta^2 = \dddot{\phi}/(H^2\dot{\phi}) - \eta^2$. The actual values of ϵ and η for any given potential will change. Alternative slow-roll parameters ϵ_V and η_V are defined in terms of the potential by

	4D Inflation		High-Energy BW		
α	$\epsilon_{V,50}$	$\eta_{V,50}$	$\epsilon_{V,50}$	$\eta_{V,50}$	K_1
2	$\frac{1}{101}$	$\frac{1}{101}$	$\frac{1}{101}$	$\frac{1}{202}$	1.21
4	$\frac{1}{51}$	$\frac{3}{102}$	$\frac{1}{76}$	$\frac{3}{304}$	1.15
6	$\frac{3}{103}$	$\frac{5}{103}$	$\frac{3}{203}$	$\frac{5}{406}$	1.14

TABLE I: Showing the values of the slow-roll parameters ϵ_V and η_V for three monomial potentials $V \propto \phi^\alpha$ and 50 e-folds of inflation.

	4D Inflation		High-Energy BW	
α	1st order	2nd order	1st order	2nd order
2	0.9604	0.9601	0.9505	0.9507
4	0.9412	0.9389	0.9408	0.9406
6	0.9223	0.9169	0.9360	0.9355

TABLE II: Showing the spectral index for scalar perturbations with 50 e-folds of inflation to first and second order in the slow-roll parameters.

$$\epsilon_V = \frac{2\lambda}{\kappa_4^2}\frac{V'^2}{V^3}, \qquad \eta_V = \frac{2\lambda}{\kappa_4^2}\frac{V''}{V^2}, \qquad (29)$$

for brane-world inflation in the high-energy limit and

$$\epsilon_V = \frac{1}{2\kappa_4^2}\frac{V'^2}{V^2}, \qquad \eta_V = \frac{1}{\kappa_4^2}\frac{V''}{V}, \qquad (30)$$

for standard 4D inflation. These are related to the slow-roll parameters given in Eq.(5) by $\epsilon \approx \epsilon_V$ and $\eta \approx \eta_V - \epsilon_V$. Note that the different Friedmann relation means that the field, ϕ will have a different value at horizon crossing.

The calculations presented in the preceding section show that we can determine the spectral index to second order for brane-worlds, including bulk gravity effects at high energies as well as the modified Friedmann equation, by using the formula given in Eq. (25), replacing C_1 with $\tilde{C}_1$ as given in Eq. (27).

V. RESULTS FOR MONOMIAL POTENTIALS

Tables I and II show the slow-roll parameters and spectral indices for some monomial potentials. The differences at first order arise from the modified evolution on the brane, not from the brane-bulk mixing. The second-order corrections in brane-worlds have a contribution from the brane-bulk mixing [5] in addition to the familiar Stewart–Lyth correction, and these are of the same order of magnitude. For both the standard and brane-world inflation models the second-order correction is less than 1 part in 10^3 so the first-order result should be sufficient for observational cosmology.

In the standard inflation paradigm slow-roll corrections change the dynamics on sub-horizon scales by introducing an effective mass that is negligible at early times

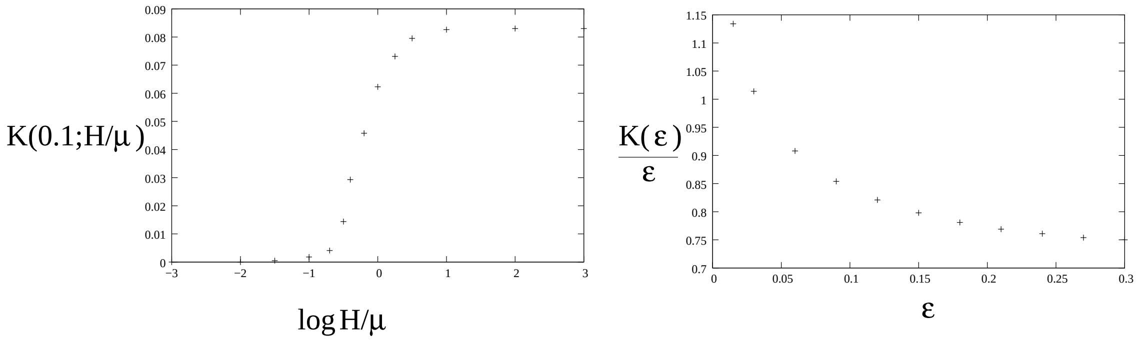

FIG. 1: *Left:* H/μ dependence of $K(\epsilon; H/\mu)$ for $\epsilon = 0.1$. *Right:* $K(\epsilon)/\epsilon$ in the high energy limits.

(though this is not negligible on super-horizon scales). The problem thus reduces to quantizing a massless scalar field in de Sitter space. By contrast, in the brane-world model it is essential to use the slow-roll approximation even to formulate the problem in a way where the bulk wave equation is separable. In the brane-world model we have shown that the coupling to bulk metric perturbations cannot be ignored in the equations of motion. Indeed, we have shown that there are order-unity differences between the classical solutions without coupling and with slow-roll induced coupling. However, the change in the amplitude of quantum-generated perturbations is at next-to-leading order [5] because there is still no mixing at leading order between positive and negative frequencies when scales observable today crossed the horizon, so the Bogoliubov coefficients receive no corrections at leading order. The amplitude of perturbations generated is also subject to the usual slow-roll corrections on super-horizon scales. Thus, the correction we calculate is in addition to the usual Stewart–Lyth correction [18] and we find it to be of the same order of magnitude.

Our results also show that the ratio of tensor-to-scalar perturbation amplitudes are not influenced by brane-bulk interactions at leading order in slow-roll. It is not possible for us to calculate the next-order corrections without also deriving the slow-roll correction to tensor amplitudes. This correction has not yet been calculated because it requires one to go beyond the approximation of a separable wave equation in the bulk for a de Sitter brane.

From an end-user's perspective our results are significant in establishing rigorously the validity of assumptions made in earlier brane-world literature. The most significant difference between the prediction for the spectral indices in the brane-world at high energy and in standard inflation with the same potential arises at first order in slow-roll from the modified evolution of the Hubble parameter and scalar field, leading to different values of the slow-roll parameters when the number of e-foldings of inflation is fixed [6, 9, 10]. It is remarkable that the predictions from inflation theories should be so robust that this result hold in spite of the leading-order change to the solutions of the classical equations of motion.

Acknowledgments

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