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Modelling Volatility Using High, Low, Open and Closing Prices: Evidence from Four S&P Indices

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Abstract

This paper uses several models (Alizadeh, Brandt and Diebold, 1999; Parkinson, 1980; Garman and Klass, 1980; Rogers and Satchell, 1991) for the calculation of volatility based on high, low, open and closing prices. We use recent daily data from four S&P indices, namely S&P 100, S&P 400, S&P 500 and S&P Small Cap 600. The results show that a simple measure of volatility (defined as the first logarithmic difference between the high and low prices) overestimates the other three measures.

Keywords: Volatility, S&P indices, High price, Low price, Open Price, Closing Price, US.
JEL Classification Codes: C14, C15, G13, G15.

1. Introduction

Financial theories are often based on assumptions concerning the structure of price data (stock returns, exchange and interest rates), see Andreou et al (2001). For example, Efficient market hypothesis (EMH) assumes that speculative prices can be modelled as random walks (Fama, 1970) while CAPM and Black-Scholes option pricing model assume that returns are Normally distributed and follow the probabilistic assumptions of uncorrelatedness and stationarity.

Empirical evidence of the time series of daily stock returns include (1) leptokurtosis (fat tails relative to the normal distribution), (2) skewness, and (3) volatility clustering (large returns are expected to follow large returns, and small returns to follow small returns). According to Pagan (1996), volatility is related to uncertainty and shows how much asset prices are moving around[1].

There has been a significant emphasis on time series models to explain the empirical observation of volatility clustering. Empirical models include the Generalised Autorgressive Conditional Heteroskedasticity (GARCH) models developed by Engle (1982) and Bollerslev (1986), and stochastic volatility (SV) models (see Hwang and Satchell, 2000).

[1] Financial time series exhibit periods where the volatility is consistently low that alternate with periods of consistently high volatility. This variation of volatility can be linked to the arrivals of information (see Mandelbrot and Taylor, 1967) and trading volume (see Karpoff, 1987).

A number of research papers model stock index returns to test the performance of GARCH and SV models (Heyen and Kat, 1994; Andersen, 1994; Kim et al., 1998; Andersen, 1996; Chan and Lien, 2003) and report that making the choice between GARCH and SV models is not easy[2].

For this article, we consider four volatility measures, other than GARCH and SV, for US stock indices following the works of Alizadeh, Brandt and Diebold (1999), Gallant, Hsu and Tauchen (1999), Parkinson (1980), Garman and Klass (1980), and Rogers, Satcell and Yoon (1994). We model daily volatility using opening, closing, high and low prices from four S&P indices, namely S&P 100, S&P 400, S&P 500 and S&P Small Cap 600.

The purpose of this paper is to re-evaluate the performance of several volatility measures using recent daily range data from the US. Our aim is twofold: (i) we examine the performance of volatility estimates when using US data from S&P stock market, and (ii) we test if volatility estimators based on opening, high, low and closing prices are efficient measures for S&P indices.

The paper is organised as follows: Section 2 provides the methodology and data information. Section 3 presents the main empirical results, while Section 4 concludes the paper and summarises our findings.

2. Methodology and Data Description

Numerous recent studies have been directed at modelling the stock market volatility using time series modelling (Canarella and Pollard, 2007; Floros, 2007; Floros and Vougas, 2006). However, they only use closing prices, and therefore, their examinations fail to consider a full range of prices (high, low, open as well as closing prices) in each trading day. To further test the efficiency of volatility measures in our data, we model the non-constant volatility parameter using four models based on the opening, closing, high and low prices[3].

Volatility measures (Chan and Lien, 2003)

Let O_t, C_t, H_t, L_t denote the opening, closing, high and low prices at day t, respectively.

A simple measure of volatility is defined as the first logarithmic difference between the high and low prices (Alizadeh, Brandt and Diebold, 1999; Gallant, Hsu and Tauchen, 1999):

$$V_{S,t} = \ln(H_t) - \ln(L_t) \tag{1}$$

Parkinson (1980) proposes a volatility measure assuming an underlying geometric Brownian motion with no drift for the prices:

$$V_{P,t} = 0.361R_t^2 = 0.361[\ln(H_t / L_t)]^2 \tag{2}$$

According to Chan and Lien (2003), $V_{P,t}$ could be as much as 8.5 times more efficient than log-squared returns.

A further volatility measure is based on opening and closing prices. Garman and Klass (1980) suggest the following measure:

$$V_{GK,t} = \frac{1}{2}[\ln(H_t) - \ln(L_t)]^2 - [2\ln 2 - 1][\ln(C_t) - \ln(O_t)]^2 \tag{3}$$

According to Chan and Lien (2003), both measures are unbiased when the sample data are continuously observed with $V_{GK,t}$ being more efficient than $V_{P,t}$.

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[2] Heyen and Kat (1994) show that GARCH models outperform SV models in modelling exchange rates, while Kim et al. (1998) report that SV models are superior to GARCH models. Further, Hwang and Satchell (2000) argue that GARCH models are more suitable for describing volatility.

[3] In the current literature there are other popular specifications, the GARCH models and the SV models; in this article, we only adopt the volatility framework based on the opening, closing, high and low prices. Our aim is to see which volatility measure (other than GARCH and SV) dominates the other.

When the drift term is not zero, neither the Parkinson nor the Garman-Klass measures are efficient (Chan and Lien, 2003). Hence, an alternative measure with independent drift is required. Rogers and Satchell (1991) and Rogers, Satchell and Yoon (1994) propose a volatility measure which is subject to a downward bias problem:

$$V_{RS,t} = [\ln(H_t) - \ln(O_t)][\ln(H_t) - \ln(C_t)] + [\ln(L_t) - \ln(O_t)][\ln(L_t) - \ln(C_t)] \quad (4)$$

The data employed in this study comprise 2010 daily observations on the S&P stock indices: S&P 100, S&P 400, S&P 500 and S&P Small Cap 600. The data covers the period 3 January 2000 - 31 December 2007. Closing, Open, High and Low prices for stock indices were obtained from *Datastream International* and *Bloomberg*. Table 1 gives the descriptive statistics for daily stock prices. We present three statistics which are calculated using the observations in the full sample: Skewness, Kurtosis, and Jarque-Bera. Almost all US series have negative skewness implying that the distribution has a long left tail (only S&P 500 index shows a negative skewness). The values for kurtosis are less than three in all cases. In other words, the distributions are not peaked relative to normal. Moreover, the Jarque-Bera test rejects normality at the 5% level for all distributions. So, the sample has all financial characteristics: volatility clustering and platykurtosis. Furthermore, the results from the ADF unit root tests (not reported here) indicate that all series are I(1), and therefore, quantitative models can be used to measure daily volatility.

Table 1: Descriptive Statistics (Prices)

H S&P 100	**Close**	**High**	**Low**	**Open**
Mean	596.2284	600.3405	591.9310	596.2746
Median	575.1450	578.0400	572.4600	575.1450
Maximum	832.6500	846.4000	827.4100	832.6500
Minimum	392.6900	402.8800	384.9600	392.6900
Std. Dev.	95.01544	95.72924	94.33992	95.08405
Skewness	0.430767	0.468055	0.392205	0.431512
Kurtosis	2.698530	2.697492	2.706670	2.697226
Jarque-Bera	69.77435	81.05446	58.73724	70.05546
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2010	2010	2010	2010
I S&P 500	**Close**	**High**	**Low**	**Open**
Mean	1213.119	1220.938	1204.723	1213.111
Median	1203.755	1209.755	1198.240	1203.755
Maximum	1565.150	1576.090	1555.460	1564.980
Minimum	776.7600	798.5500	768.6300	776.7600
Std. Dev.	186.5732	186.9500	186.2163	186.5805
Skewness	-0.129782	-0.110655	-0.152416	-0.129977
Kurtosis	2.241587	2.214698	2.268541	2.241918
Jarque-Bera	53.81478	55.75046	52.59116	53.78962
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2010	2010	2010	2010
J S&P 600	**Close**	**High**	**Low**	**Open**
Mean	285.7010	287.6307	283.4392	285.5959
Median	268.8200	270.2400	265.5600	268.6050
Maximum	445.1900	445.8200	442.3600	445.1900
Minimum	170.7300	174.9800	169.6400	170.7300
Std. Dev.	79.01810	79.35722	78.61554	79.00761
Skewness	0.435326	0.440219	0.429534	0.437204
Kurtosis	1.774186	1.775190	1.772044	1.776510
Jarque-Bera	189.1413	190.3693	187.9048	189.2135
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2008	2008	2008	2008
K S&P 400	**Close**	**High**	**Low**	**Open**
Mean	607.9489	611.8262	603.3863	607.7454
Median	567.1300	570.6250	562.9250	566.8450
Maximum	926.2300	926.6700	921.3800	926.2300
Minimum	372.8800	385.3400	370.8300	372.8800
Std. Dev.	146.7333	146.9058	146.4490	146.6712
Skewness	0.537715	0.547049	0.527681	0.539286
Kurtosis	2.038998	2.047717	2.029388	2.042080
Jarque-Bera	173.8595	175.8505	171.8370	173.9310
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2006	2006	2006	2006

Notes:

- Skewness is a measure of asymmetry of the distribution of the series around its mean.
- Kurtosis measures the peakedness or flatness of the distribution of the series.
- Jarque-Bera is a test statistic for testing whether the series is normally distributed.

3. Empirical Results

According to Cheung et al. (2009), daily highs and lows of stock indices do not diverge over time. The same applies for opening and closing prices for our study[4].

The results from equations (1)-(4) are presented in Table 2. In all cases Vs overestimates Vgk, Vp and Vrs, and it ranges from 1.5% (S&P 100) to 1.36% (S&P 500). Furthermore, S&P 100 has the highest Vgk, S&P Small Cap 600 shows a high Vp, S&P 100 has a high Vrs and S&P Small Cap 600

[4] The cointegration framework for daily high-low and open-close prices show strong evidence of long-run relationships (the results are available upon request).

has a high Vs. Hence, both S&P 100 and S&P Small Cap 600 show an increase in volatility measures when specific models are used.

Table 2: Volatility Estimates

A S&P 100	**Vgk**	**Vp**	**Vrs**	**Vs**
Mean	9.05E-05	0.000102	8.96E-05	0.014207
Median	4.55E-05	5.13E-05	4.02E-05	0.011919
Maximum	0.002770	0.002812	0.003666	0.088257
Minimum	2.23E-06	2.09E-06	0.000000	0.002406
Std. Dev.	0.000148	0.000160	0.000176	0.009003
Skewness	7.846783	5.819569	9.128437	1.882482
Kurtosis	112.0735	64.64777	139.6175	9.011545
Jarque-Bera	1017003.	329633.1	1591053.	4213.766
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2010	2010	2010	2010
B S&P 500	**Vgk**	**Vp**	**Vrs**	**Vs**
Mean	8.09E-05	9.29E-05	7.96E-05	0.013674
Median	4.32E-05	4.86E-05	3.77E-05	0.011600
Maximum	0.002777	0.002595	0.003584	0.084792
Minimum	1.59E-06	2.07E-06	0.000000	0.002392
Std. Dev.	0.000133	0.000146	0.000158	0.008384
Skewness	8.997905	6.602327	10.24632	1.996139
Kurtosis	143.3613	79.92530	175.0667	10.31993
Jarque-Bera	1677105.	510193.7	2514753.	5822.274
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2010	2010	2010	2010
C S&P 600	**Vgk**	**Vp**	**Vrs**	**Vs**
Mean	8.35E-05	0.000106	8.00E-05	0.015166
Median	5.21E-05	6.36E-05	4.19E-05	0.013272
Maximum	0.003447	0.002583	0.005044	0.084585
Minimum	2.93E-06	2.94E-06	0.000000	0.002854
Std. Dev.	0.000128	0.000138	0.000166	0.007892
Skewness	11.82871	6.243402	15.31244	1.843349
Kurtosis	261.9472	78.54830	407.3596	9.732854
Jarque-Bera	5656980.	490576.7	13758527	4929.897
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2008	2008	2008	2008
D S&P 400	**Vgk**	**Vp**	**Vrs**	**Vs**
Mean	8.44E-05	0.000102	8.32E-05	0.014588
Median	4.78E-05	5.69E-05	4.10E-05	0.012559
Maximum	0.004034	0.003003	0.005651	0.091203
Minimum	2.47E-06	2.59E-06	0.000000	0.002678
Std. Dev.	0.000146	0.000154	0.000190	0.008285
Skewness	12.72504	7.440915	15.74074	2.153297
Kurtosis	291.3111	101.7833	401.4898	12.05380
Jarque-Bera	7001858.	834128.9	13355379	8401.627
Probability	0.000000	0.000000	0.000000	0.000000
Observations	2006	2006	2006	2006

Notes:

- Skewness is a measure of asymmetry of the distribution of the series around its mean.
- Kurtosis measures the peakedness or flatness of the distribution of the series.
- Jarque-Bera is a test statistic for testing whether the series is normally distributed.

Figure 1: S&P 100

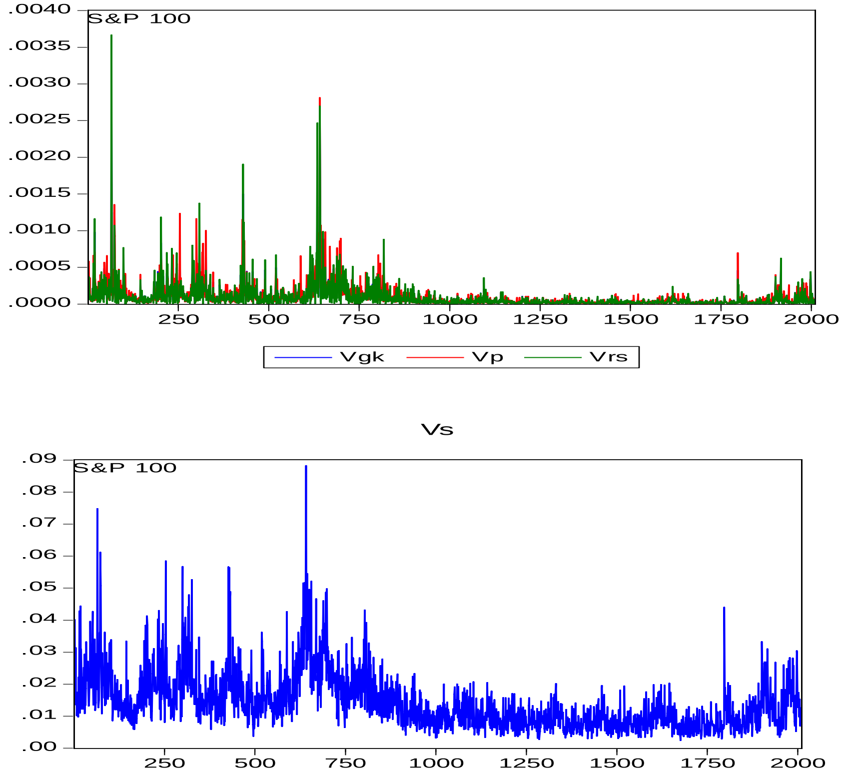


Figure 2: S&P 500

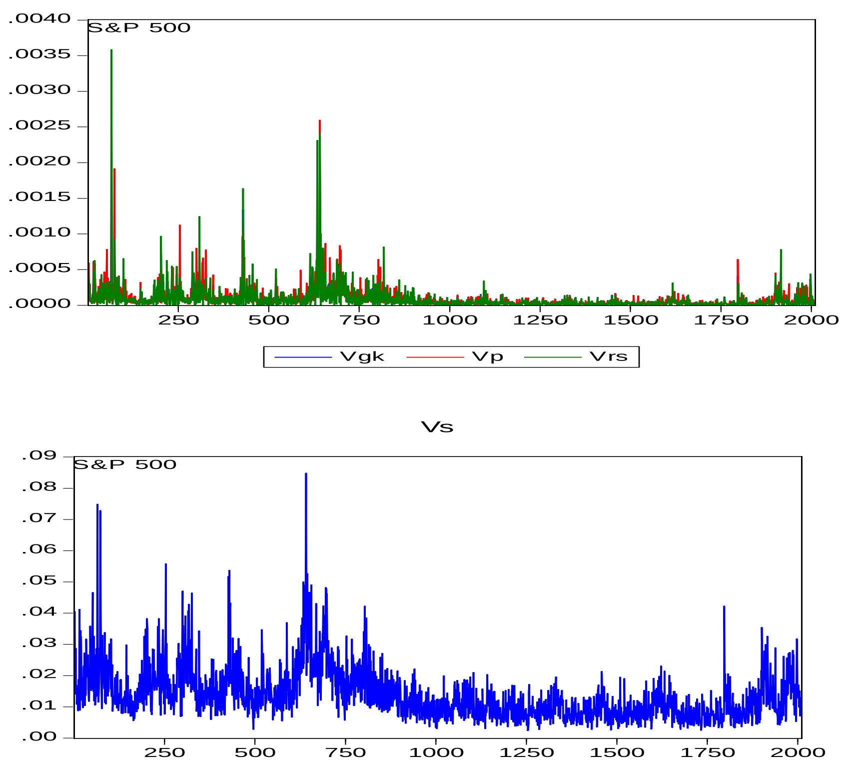


Figure 3: S&P Small Cap 600

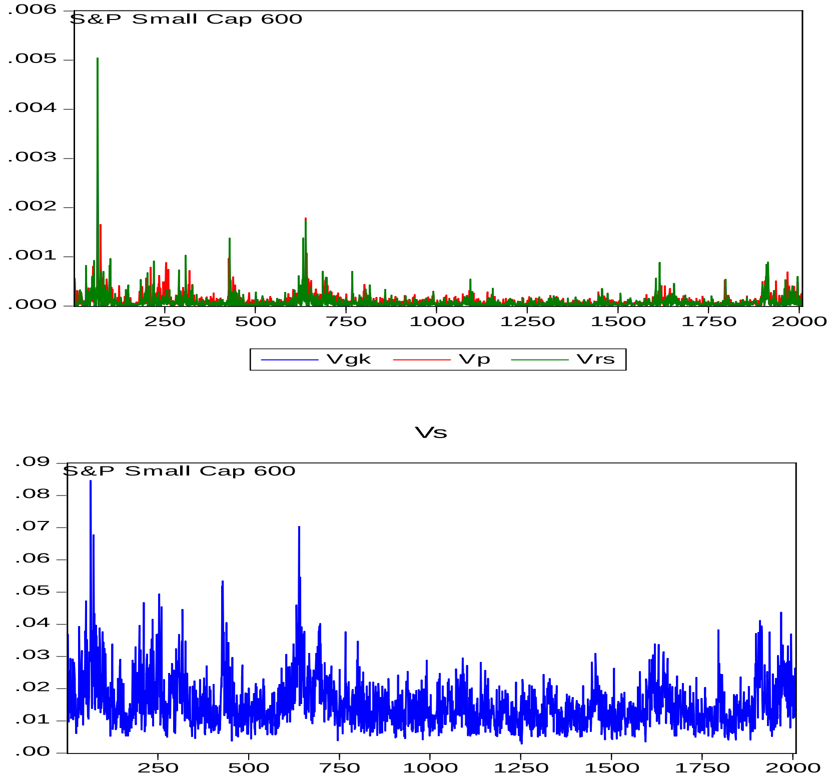
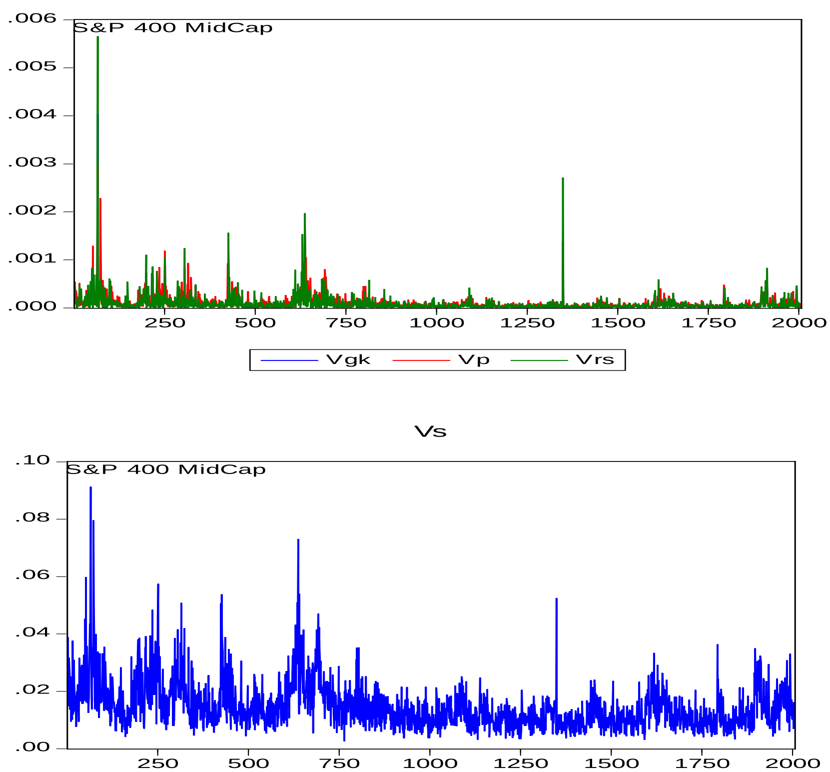


Figure 4: S&P 400



4. Summary and Conclusions

Volatility in financial markets has attracted growing attention by practitioners, policy makers and researchers as it is a measurement of risk. The results reported in this paper show estimates of volatility in the US. We model volatility using four models based on open, closing, high and low daily prices. We consider daily data from four US stock indices (S&P 100, S&P 400, S&P 500 and S&P Small Cap 600) to test which measure dominates each other.

First, we find strong evidence that daily prices can be characterised by volatility models. In particular, we report that the prices have all financial characteristics: volatility clustering, platykurtosis and nonstationarity. Finally, we use four models to calculate daily volatility. The results show that Vs, a simple measure of volatility defined as the first logarithmic difference between the high and low prices, overestimates Vgk, Vp and Vrs.

These findings are strongly recommended to risk managers and modellers dealing with the US financial indices. Future research should examine the performance of stochastic volatility methods to describe both volatility and market risk of major stock indices.

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