Appendix D

Theoretical Basis

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11 Appendix D: Theoretical Basis

11.1 General

The numerical study of this research is carried out by the commercial program *ELPLA* [11]. *ELPLA* [11] is a program to analyze single piles, pile groups, piled rafts and rafst using different subsoil models. This chapter presents a short description of the theoretical basis used for analyzing foundation elements in the study, which is considered in the program *ELPLA* [11]. This description is taken from user's manuals of the program. More details about the theoretical basis maybe found in user's manuals of the program.

11.2 Introduction

Analyzing piled raft is a complex task because it is a three-dimensional problem including many capabilities. The main capabilities that must be considered in the analysis are: the interaction between all piled raft and soil elements; taking into account the actual loading and geometry of the piled raft; representing the soil by a real model and treating the problem as nonlinear analysis. Considering all these capabilities requires great experience and effort. Besides, such a problem requires long computational time where huge size soil matrix is required for a large piled raft due to discretized nodes along piles and under the raft. For these reasons many authors suggested simplified methods in recent years to reduce the size of analysis.

Clancy & *Randolph* (1993) and (1994) developed the hybrid layer method to reduce the computing effort. *Ta* & *Small* (1997) approximated the surface displacement of the soil by a polynomial instead of generating flexibility factors, but the raft have to be square and of equal size. *Russo* (1998) presented an approximate numerical method for the analysis of piled raft where piles were modeled as interactive linear or non-linear springs. He used the interaction factor method and a preliminary BEM to model pile to pile interaction. *Poulos* (1999) described an approximate analysis for the response of a pile group. The analysis uses a simplified form of boundary element analysis to obtain single pile responses and interaction factors, and employs various simplifying assumptions to facilitate the computational process. *Lee* & *Xiao* (2001) presented a simplified analytical method for nonlinear analysis of the behavior of pile groups using a hyperbolic approach to describe the nonlinear relation between the shaft stress and displacement. They developed the method for pile groups under both rigid and flexible pile cap based on the load-transfer function. *Kitiyodom* & *Matsumoto* (2002) and (2003) developed a simplified method of numerical analysis of piled raft using a hybrid mode. Raft is modeled as thin plate, the piles as elastic beams and the soil as springs. *Mendonça* & *Paiva* (2003) presented BEM/ FEM formulation for the analysis of piled raft in which each pile is represented by a single element with three nodal points and the shear force along the shaft is approximated by a quadratic function. The soil is considered as half-space medium. *Jeong et al.* (2003) proposed a simple algorithm to analyze laterally loaded three-dimensional pile groups using beam column method. *Liang* & *Chen* (2004) presented a modified variational approach for analyzing piled raft by a simplified analytical solution to evaluate the pile-soil interaction. They applied the approach on piled rigid and flexible rafts resting on homogeneous soil. *Wong* & *Poulos* (2005) developed approximations for the settlement interaction factors between dissimilar piles via an extensive parametric study. *Lutz et al.* (2006) presented a simple method to estimate the load settlement behavior of piled raft based on the theory of elasticity and solutions for calculation of ultimate limit state. Most of the simplified analyses carried out by the methods mentioned previously approximated the soil model. However, several methods are available for analyzing this complex problem by a full three-dimensional analysis but they are time consuming even for fast computers of today.

In standard methods of analyzing piled raft based on elasticity theory, the entire soil stiffness matrix of the piled raft is assembled due to all elements of piles and raft. Then, settlements of piled raft elements are obtained directly by solving the global equations. Based on elasticity theory *El Gendy* (2007) presented more efficient analysis of single pile, pile group and piled raft by using composed coefficient technique to reduce the size of entire soil stiffness matrix. In the technique, the pile is treated as a rigid member having a uniform settlement on its nodes. This assumption enables to assemble pile coefficients in composed coefficients. It can be easily modeling the nonlinear response of single pile, pile groups or piled raft. The composed coefficient technique makes the size of the soil stiffness matrix of the piled raft equivalent to that of the raft alone without piles. The proposed analysis reduces considerably the number of equations that need to be solved. Raft can be analyzed as flexible, rigid or elastic on continuum soil medium. The advantage of the analysis is that there is no approximation when generating the flexibility coefficients of the soil. In the analysis a full interaction among piled raft elements is taken into account by generating the entire flexibility matrix of the piled raft. Using the composed coefficient technique enables to apply the nonlinear response of the pile by a hyperbolic relation between the load and settlement of the pile. *El Gendy* (2007) introduced also a direct hyperbolic function for nonlinear analysis of a single pile. Besides, an iteration method is developed to solve the system of nonlinear equations of pile groups or piled raft. This chapter presents numerical modeling single pile, pile groups and piled raft according to *El Gendy* (2007), which is implemented in the program *ELPLA*.

11.3 Modeling single pile

11.3.1 Soil flexibility for single pile

In the analysis, the pile is divided into a number of shaft elements with *m* nodes, each acted upon by a uniform shear stress and a circular base having a uniform stress as shown in Figure 11.1a. To carry out the analysis, pile shaft elements are represented by line elements as indicated in Figure 11.1b. To consider the interaction between the pile and soil, the soil is represented as layered medium or isotropic elastic half-space medium. Considering a typical node *i* as shown in Figure 11.1b, the settlement s_i of the soil adjacent to the node *i* due to shear forces Qs_j on all *m* nodes and due to the base force *Qb* is expressed as:

$$s_i = \sum_{j=1}^{m} f_{i,j}\, Qs_j + f_{i,b}\, Qb \qquad (11.1)$$

where:

$f_{i,j}$ Flexibility coefficient of node *i* due to a unit shear force on a node shaft *j*, [m/kN].

$f_{i,b}$ Flexibility coefficient of node *i* due to a unit force on the base *b*, [m/kN].

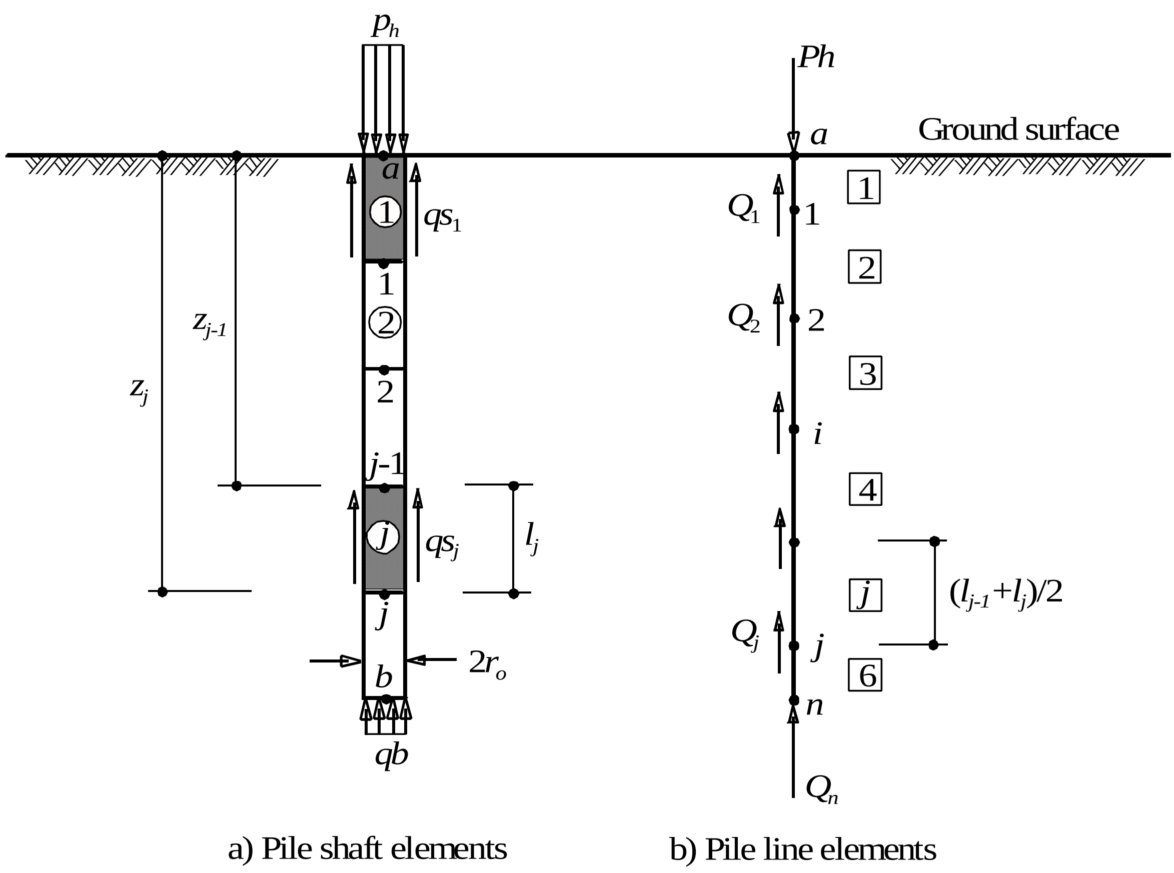

Figure 11.1 Pile geometry and elements

As a special case of Eq. (11.1) and by changing the index i to b, the settlement of the base s_b may be expressed as:

$$s_b = \sum_{j=1}^{m} f_{b,j}\, Qs_j + f_{b,b}\, Qb \tag{11.2}$$

where:

$f_{b,j}$ Flexibility coefficient of the base b due to a unit shear force on a node shaft j, [m/kN].

$f_{b,b}$ Flexibility coefficient of the base b due to a unit force on the base b, [m/kN].

11.3.2 Elastic analysis of single pile

11.3.2.1 Soil settlement

Equations (11.1) and (11.2) for settlements of the soil adjacent to all nodes of the pile may be written in a matrix form as:

$$\{w\} = [Is]\{Q\} \tag{11.3}$$

where:

$\{w\}$ n settlement vector.

$\{Q\}$ n contact force vector.

$[Is]$ $n*n$ soil flexibility matrix.

11.3.2.2 Pile displacement

The finite element method is used for analyzing the pile. Only the axial compression of the pile is considered in determining displacements of pile elements. Assuming full compatibility between pile displacement δ_i and soil settlement s_i, the following equation can be obtained:

$$[[kp] + [ke]]\{\delta\} = \{P\} \tag{11.4}$$

Solving the above system of linear equations, gives the displacement at each node, which equal to the soil settlement at that node.

11.3.3 Rigid analysis of single pile

For a rigid pile, the settlement will be uniform. Therefore, the unknowns of the problem are n contact forces Q_j and the rigid body translation wo., which given by:

$$Q_i = \frac{Ph \sum_{j=1}^{n} k_{i,j}}{\sum_{i=1}^{n} \sum_{j=1}^{n} k_{i,j}} \tag{11.5}$$

and

$$wo = \frac{Ph}{\sum_{i=1}^{n}\sum_{j=1}^{n} k_{i,j}} \tag{11.6}$$

where:

$k_{i,j}$ Coefficients of the soil stiffness matrix $[ks] = [Is]^{-1}$

Ph Load on pile head, $Ph = \sum_{i=1}^{n} Q_i$

11.4 Modeling pile groups (freestanding rigid raft)

11.4.1 Soil stiffness for pile groups

Deriving equations for freestanding raft on piles requires taking into account the interaction effect among the pile groups. For doing that, the simple freestanding raft on pile groups shown in Figure 11.2 as an example is considered, which having n_p = 4 piles and total nodes of n = 23. Due to the high rigidity of the pile in its length direction, the settlement in every pile itself is considered as a uniform. This assumption can establish the relationship between the uniform pile settlement and the force on the pile head in the pile groups. It can be done by equating all settlements in each pile by a uniform settlement.

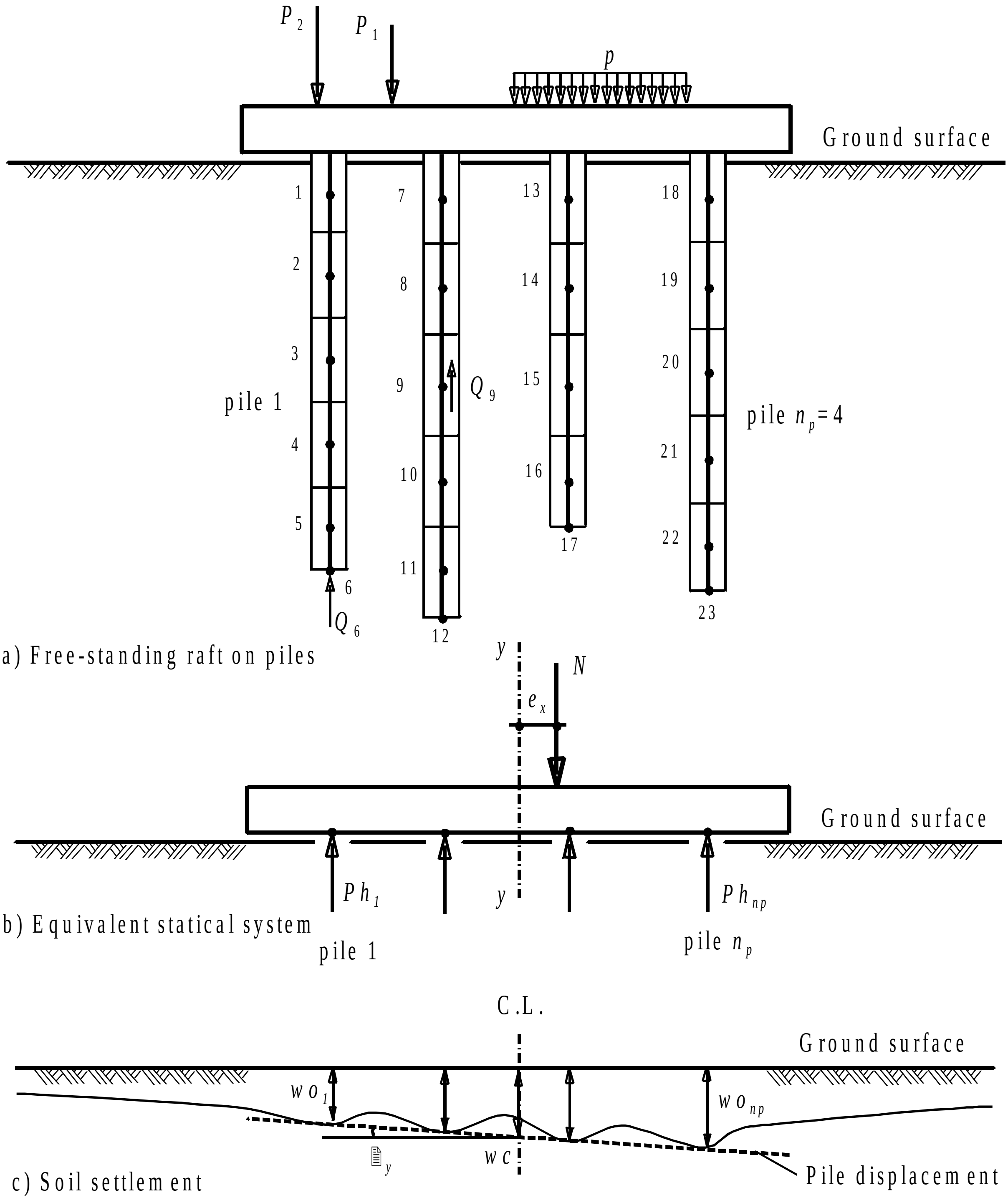

Figure 11.2 Modeling freestanding raft on pile groups

Accordingly, pile load-settlement relation can be written for the simple pile groups in Figure

11.2 in composed coefficients as:

$$\begin{Bmatrix} Ph_1 \\ Ph_2 \\ Ph_3 \\ Ph_4 \end{Bmatrix} = \begin{bmatrix} K_{1,1} & K_{1,2} & K_{1,3} & K_{1,4} \\ K_{2,1} & K_{2,2} & K_{2,3} & K_{2,4} \\ K_{3,1} & K_{3,2} & K_{3,3} & K_{2,4} \\ K_{4,1} & K_{4,2} & K_{4,3} & K_{2,4} \end{bmatrix} \begin{Bmatrix} wo_1 \\ wo_2 \\ wo_3 \\ wo_4 \end{Bmatrix} \tag{11.7}$$

where:

wo_i Settlement in pile *i*, [m].

$K_{i,j}$ Composed coefficient, [kN/m]. In general $K_{i,j} = \sum_{n=n1}^{n2} \sum_{m=m1}^{m2} k_{n,m}$

Ph_i Force on the head of pile *i*, which is equal to the summation of all contact forces in that pile, [kN]. In general $Ph_i = \sum_{n=n1}^{n2} Q_n$

$n1 = 1 + \sum_{l=1}^{i-1} nn(l)$, $n2 = \sum_{l=1}^{i} nn(l)$, $m1 = 1 + \sum_{l=1}^{j-1} nn(l)$ and $m2 = \sum_{l=1}^{j} nn(l)$

$nn(l)$ Number of nodes in pile *l*.

11.4.2 Analysis of pile groups

In general case of a completely rigid raft, the linear settlement of the raft at any point is defined by the vertical displacement *wc* of the center and by two rotations θ_x and θ_y about *x*- and *y*-axes, respectively. The settlement of the pile *i*, having coordinates x_i and y_i referred to the center, must be compatible with the raft settlement at that point. Determining values of displacement *wc* and rotations θ_x and θ_y, allows to find the unknown pile head forces and settlements. The settlement wo_i in the general case of an eccentric load at any pile *i* that has coordinates x_i and y_i from the geometry centroid is given by:

$$wo_i = wc + x_i \tan \theta_y + y_i \tan \theta_x \tag{11.8}$$

while the force on the pile head is given by:

$$Ph_i = wc \sum_{j=1}^{n_p} K_{i,j} + \tan\theta_y \sum_{j=1}^{n_p} x_i\, K_{i,j} + \tan\theta_x \sum_{j=1}^{n_p} y_i\, K_{i,j} \tag{11.9}$$

and the soil stiffness ks_i adjacent to the pile *i* in the pile groups is given by:

$$ks_i = \frac{Ph_i}{wc + x_i \tan\theta_y + y_i \tan\theta_x} \tag{11.10}$$

11.5 Modeling piled raft

11.5.1 Soil stiffness for piled raft

For a complete analysis of piled raft foundation, pile-soil-raft and raft-soil-raft interactions must be taken into account in addition to pile-soil-pile interaction. To illustrate how to formulate the composed coefficient technique for piled raft, the simple piled raft shown in Figure 11.3 is considered, which having n_p = 4 piles and a total n_{pr} = 33 contact nodes of raft and piles with the soil. If the raft is analyzed alone without piles, the number of its nodes will be n_r = 14. In the analysis, the contact area is divided for the raft into triangular and/or rectangular elements, while that for pile shafts into cylindrical elements and that for pile bases into circular elements. The contact pressure under the raft, on pile shafts or on pile bases is represented by a series of contact forces on nodes. For the set of 33 nodes of the piled raft, the relation between soil settlements and contact forces in composed coefficients is expressed as:

$$
\begin{Bmatrix} Q_1 \\ Q_2 \\ Ph_1 \\ Q_9 \\ Q_{10} \\ Ph_2 \\ \ldots \\ Q_{33} \end{Bmatrix} =
\begin{bmatrix}
k_{1,1} & k_{1,2} & K_{1,p1} & k_{1,9} & k_{1,10} & K_{1,p2} & \cdots & k_{133} \\
k_{2,1} & k_{2,2} & K_{2,p1} & k_{2,9} & k_{2,10} & K_{2,p2} & \cdots & k_{2,33} \\
K_{p1,1} & K_{p1,2} & K_{p1,p1} & K_{p1,9} & K_{p1,10} & K_{p1,p2} & \cdots & K_{p2,33} \\
k_{9,1} & k_{9,2} & K_{9,p1} & k_{9,9} & k_{9,10} & K_{9,p2} & \cdots & k_{9,33} \\
k_{10,1} & k_{10,2} & K_{10,p1} & k_{10,9} & k_{10,10} & K_{10,p2} & \cdots & k_{10,33} \\
K_{p2,1} & K_{p2,2} & K_{p2,p1} & K_{p2,9} & K_{p2,10} & K_{p2,p2} & \cdots & K_{p2,33} \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
k_{33,1} & k_{33,2} & K_{33,p1} & k_{33,9} & k_{33,10} & K_{33,p2} & \cdots & k_{33,33}
\end{bmatrix}
\begin{Bmatrix} w_1 \\ w_2 \\ wo_1 \\ w_9 \\ w_{10} \\ wo_2 \\ \ldots \\ w_{33} \end{Bmatrix}
\tag{11.11}
$$

where $K_{pi,\,pj}$, $K_{i,\,pj}$ and $K_{pi,\,j}$ are composed coefficients of the piled raft, [kN/m].

Based on Eq. (11.52), the relationship between settlements and contact forces of the piled raft can be written in general compacted matrix form as:

$$\{Q\} = [kb]\{w\} \tag{11.12}$$

where:

$\{w\}$ n_r settlement vector.

$\{Q\}$ n_r contact force vector.

$[kb]$ $n_r * n_r$ soil stiffness matrix of the piled raft.

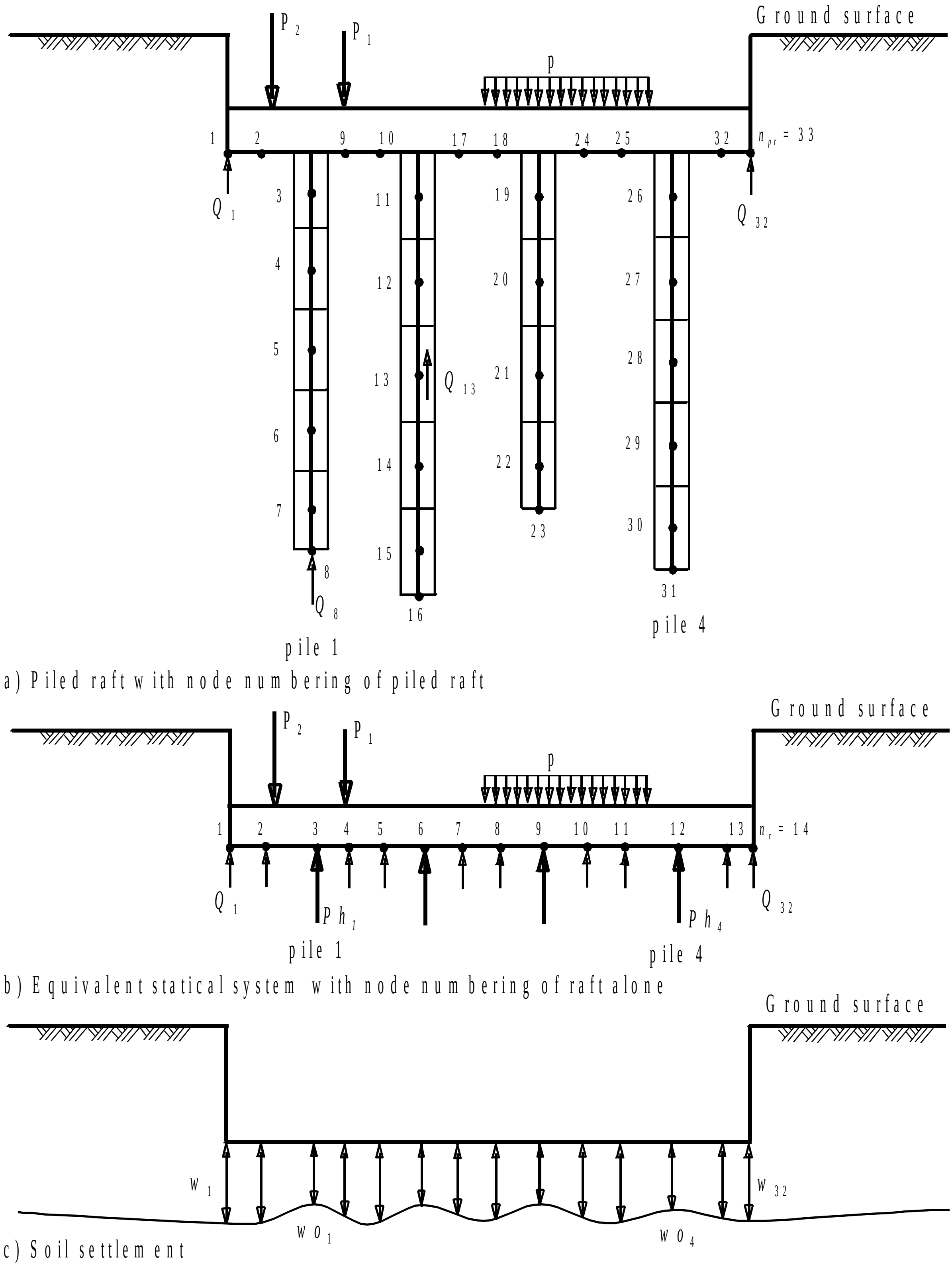
Ground surface
P 2
P 1
p
n pr = 33
Q 1
Q 32
Q 13
Q 8
pile 1
pile 4
a) Piled raft with node numbering of piled raft
Ground surface
P 2
P 1
p
n r = 14
Q 1
Q 32
Ph 1
Ph 4
pile 1
pile 4
b) Equivalent statical system with node numbering of raft alone
Ground surface
w 1
w 32
wo 1
wo 4
c) Soil settlement

Figure 11.3 Modeling piled raft

11.5.2 Analysis of piled flexible raft

In case of analyzing full flexible raft, the contact force vector {*Q*} on raft nodes is known. Only settlements are required. The advantage of the composed coefficient technique is that the composed soil stiffness matrix can be inverted to get a composed flexibility matrix. Accordingly, a relationship between contact forces under the flexible raft besides forces on pile heads and nodal settlements is expressed as:

$$\{w\} = [Cb]\{Q\} \tag{11.13}$$

where [*Cb*] is n_r*n_r flexibility matrix of the piled raft, $[Cb]=[kb]^{-1}$.

11.5.3 Analysis of piled rigid raft

For piled rigid raft, unknowns of the interaction problem are n_r contact forces Q_i, the rigid body translation of the piled raft *wc*, and the rigid body rotations θ_x and θ_y of the piled raft about axes of geometry centroid. These are determined by considering n_r compatibility equations of rigid piled raft deflection and the displacement of subsoil at n_r nodal points in addition to the three equations of overall equilibrium. The displacement and rotations the piled rigid raft can be given from the following linear system of equations:

$$\{N\} = [X][kb][X]^T\{\Delta\} \tag{11.14}$$

where:

{Δ} 3 vector of translation *wc* and rotations tan θ_y and tan θ_x

$[X]^T$ $3*n_r$ matrix of {1, x_i, y_i }. x_i, y_i are coordinates of node *i*.

{*N*} 3 vector of resultant and moments of applied loads acting on the piled raft.

11.5.4 Analysis of piled elastic raft

It is possible to treat the raft as an elastic plate on rigid piles from the finite element analysis of the plate. Considering compatibility between piled raft displacement δ_i and soil settlement

s_i, the following linear system of equations of the piled elastic raft can be obtained:

$$\left[[kp] + [kr] \right] \{\delta\} = \{P\} \qquad (11.15)$$

where:

$\{P\}$ $3*n_r$ vector of applied loads and moments on the raft nodes.

$[kr]$ $3\ n_r*3\ n_r$ plate stiffness matrix.

$\{\delta\}$ $3*n_r$ deformation vector of the raft.

11.6 Nonlinear analysis

11.6.1 Nonlinear rigid analysis of single pile

Nonlinear analysis is an important consideration since piles may be loaded close to their full capacity, even under working condition. The nonlinear relation between the load and settlement of pile may be determined by considering a hyperbolic relation between load and settlement. Figure 11.4 shows a typical nonlinear curve of load-settlement for a wide range of soils. The curve can be approximated through a hyperbolic interpolation formula where several equation forms are available to verify this curve.

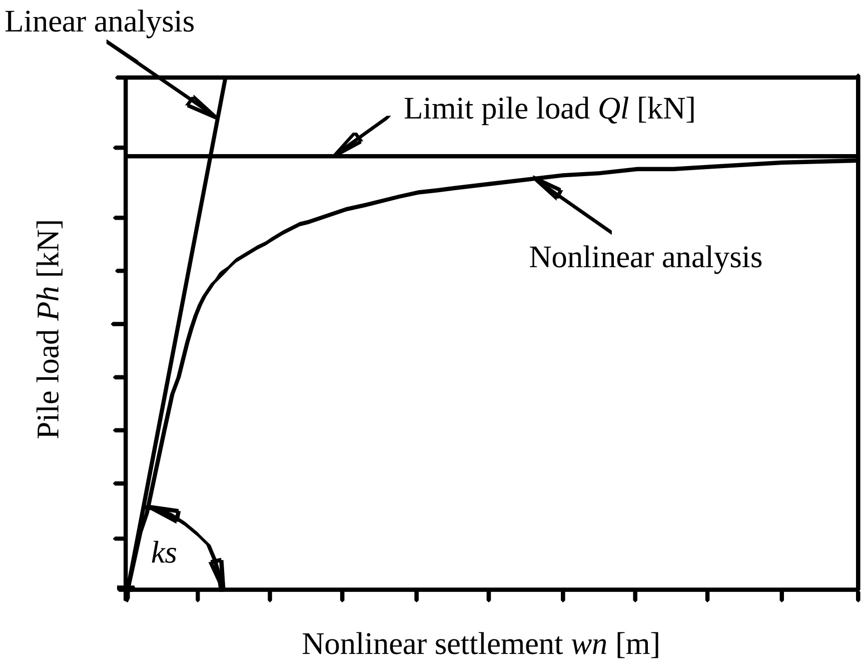

Figure 11.4 Load-settlement curve of a single pile (hyperbolic relation)

Many methods were developed to study pile-soil systems with nonlinear response using a hyperbolic relation between the load and settlement. *Fleming* (1992) developed a method to analyze and predict load-deformation behavior of a single pile using two hyperbolic functions describing the shaft and base performance individually under applied load. Analyzing nonlinear behavior by hyperbolic function was used by *Mandolini* & *Viggiani* (1997) for pile groups and was used by *Russo* (1998) for piled raft. They considerd piles as nonlinear interacting springs based on the method of interaction factors. *Basile* (1999) assumed soil *Young's* modulus varies with the stress level at the pile-soil interface using a hyperbolic stress-strain relationship.

Available nonlinear analysis of foundation on *Winkler's* soil medium was presented by *Baz* (1987) for grid and *Hasnien* (1993) for raft. *El Gendy* (1999) extended this analysis to be applicable for raft on continuum soil medium. The composed coefficient technique described in the previous sections enables to apply this analysis on pile problems. The nonlinear behavior of the pile head force-settlement at the piled raft-soil interface may be represented as:

$$Ph = \frac{wn}{\frac{1}{ks} + \frac{wn}{Ql}} \tag{11.16}$$

where:

wn Nonlinear settlement of the pile, [m].

Ql Limit pile load, [kN].

ks Composed coefficient equal to the sum of all coefficients of the soil stiffness matrix of the pile, $ks = \sum_{i=1}^{n} \sum_{j=1}^{n} k_{i,j}$

In Figure 11.4 and Eq. (11.16), the initial tangent modulus for single pile is easily obtained from linear analysis of the pile, which is equal to the modulus of soil stiffness *ks*. The limit pile load *Ql* is a geometrical parameter of the hyperbolic relation. In some cases the value of

Ql is different from the actual ultimate pile load. For a single pile, the force on the pile head *Ph* is known. Therefore, Eq. (11.16) gives directly the nonlinear settlement of the pile *wn*.

11.6.2 Nonlinear analysis of pile groups, elastic piled raft and rigid piled raft

The nonlinear analysis of the piled raft is also based on the hyperbolic relation. The initial tangent modulus of the hyperbolic relation may be obtained from the linear analysis of the piled raft as:

$$ks_i = \frac{Ph_i^o}{wo_i^o} \qquad (11.17)$$

where:

Ph_i^o Force on the pile head obtained from the linear analysis, [kN].

wo_i^o Pile settlement obtained from the linear analysis, [m].

i Pile number.

o Index denotes to the first analysis in the iteration (linear analysis).

11.6.3 Iterative Procedure

An iteration method is presented to solve the system of nonlinear equations of the piled raft. The main idea of this method is that the stiffness matrix [*kb*] for rigid raft or [*kp*] for elastic raft is converted to a diagonal stiffness matrix [*ke*]. Stiffness coefficients of this matrix, which represent nodal raft stiffness and pile stiffness coefficients, are determined from the contact force and its corresponding settlement. Only the pile stiffness is modified at each cycle from the iteration process. Using the equivalent diagonal matrix, equations of the piled raft are solved for each iteration cycle until the compatibility between raft, piles and soil is achieved. Figure 11.5 shows the iteration cycle and the flow chart of the iteration process.

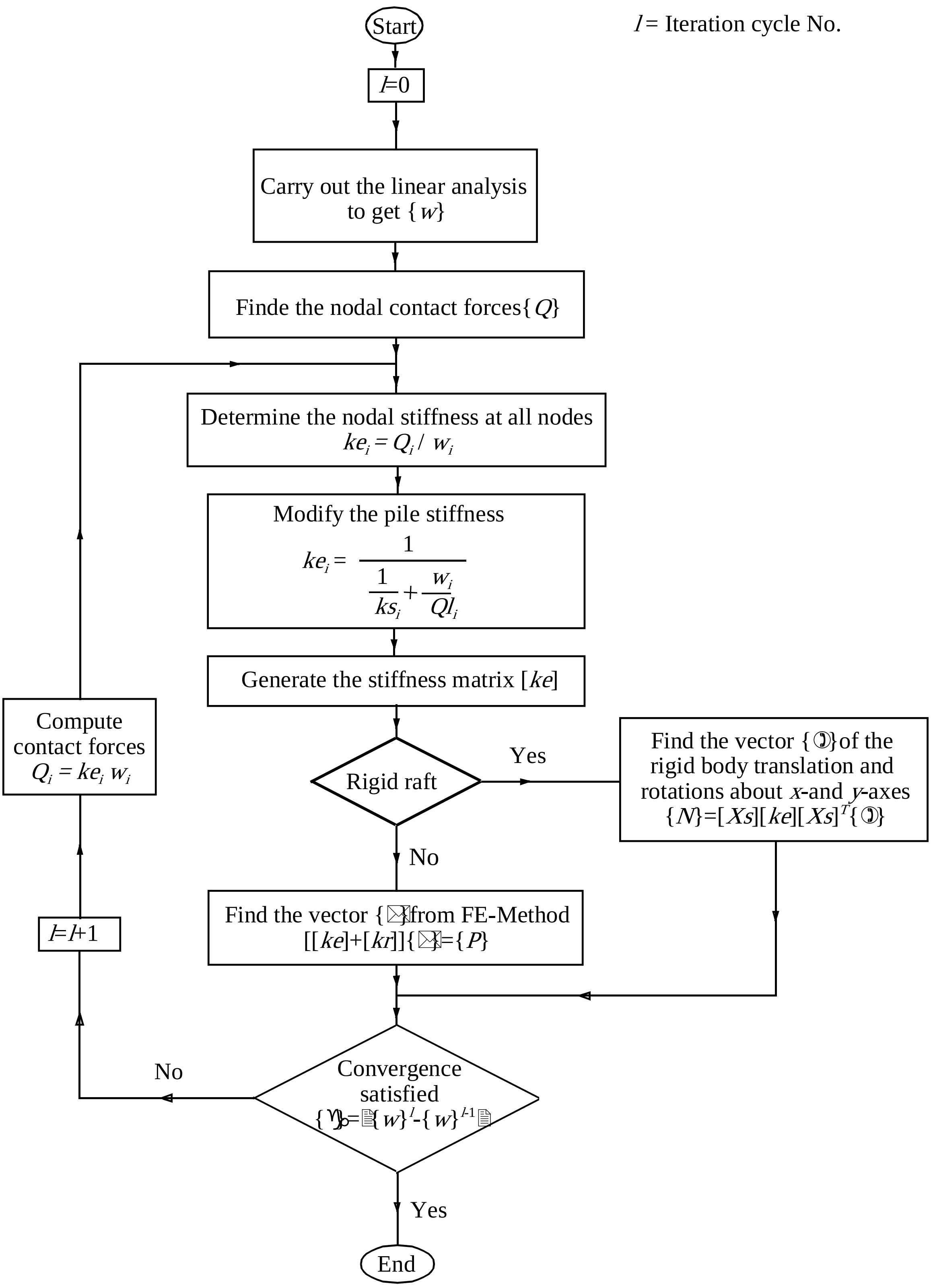

Figure 11.5 Flow chart of the iteration process