

# Multiboson Correlation Interferometry with arbitrary single-photon pure states

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We provide a compact full description of multiboson correlation measurements of arbitrary order  $N$  in passive linear interferometers with arbitrary input single-photon pure states. This allows us to physically analyze the novel problem of *multiboson correlation sampling* at the output of random linear interferometers. Our results also describe general multiboson correlation landscapes for an arbitrary number of input single photons and arbitrary interferometers. In particular, we use two different schemes to demonstrate, respectively, arbitrary-order quantum beat interference and 100% visibility entanglement correlations even for input photons distinguishable in their frequencies.

**Motivation.** Multiboson interference based on correlated measurements is a fundamental phenomenon in atomic, molecular and optical physics with numerous applications in quantum information processing [1, 2], quantum metrology [3–5], and imaging [6]. The well-known two-boson interference “dip” [7–10] is recorded when two single bosons impinge on a balanced beam splitter and joint detections are performed at the output channels. The dip is a manifestation of the destructive quantum interference between the two-boson quantum paths corresponding to both bosons being reflected or transmitted. Recent works [11–26] have demonstrated the feasibility of multiboson experiments based on higher-order correlation measurements well beyond two-boson experiments, which are crucial towards quantum networks of arbitrary dimensions and the demonstration that boson sampling devices are probably hard to reproduce classically [27–29].

At the same time, the advent of fast detectors and the production of single photons with arbitrary temporal and spectral properties [30–32] make it possible to fully investigate the temporal dynamics of multiphoton interference via time-resolving correlation measurements [33] by using atom-cavity systems [34], nitrogen vacancy centers in diamonds [35, 36], atomic ions [37] and remote organic molecules [38]. Two-photon quantum interference as a function of the detection time has been observed [34] in the form of quantum beats for single photons even when the relative central frequency is larger than their bandwidths. Moreover, the possibility to encode and retrieve an entire time-dependent quantum alphabet with high fidelity [39, 40] within a given photon spectrum is important for cluster-state quantum computing [41], quantum cryptographic schemes [42], and enhanced time metrology [43, 44].

Finally, higher-order multiphoton interference based on polarization correlation measurements has been widely used for the generation of multiqubit entanglement [45, 46] and tests of quantum nonlocality [47]. This has triggered the implementation of many quantum information applications, including quantum dense coding

protocols [48], entanglement swapping, and teleportation [49, 50], entanglement distribution between distant matter qubits such as ions [51] and atomic ensembles [52].

Despite all these remarkable achievements, there is still no full quantum optical description of time and/or polarization-resolving correlation measurements of arbitrary order in linear multiboson interferometers with input bosons in an arbitrary internal state. In this letter, we wholly perform such a description and unravel the intimate connection between the fundamental physics of multiboson interference and its computational power.

Although here we consider photonic networks, our results are relevant for any interferometric network with bosonic sources, including atoms [9, 10], plasmons [53] and mesoscopic many-body systems [54], and can be easily extended to Fock states of an arbitrary number of bosons [55] as well as to different input states [56–58].

**Multiboson Correlation Interferometry.** Let us introduce the following general multiphoton correlation experiment based on time- and polarization-resolving measurements (see Fig. 1):  $N$  single photons are prepared at the  $N$  input ports of a linear interferometer with  $2M \geq 2N$  ports [59]. At the output of the interferometer, we consider all possible correlated detection events, at given times and polarizations, of the  $N$  photons at any  $N$ -port sample  $\mathcal{D}$  of the  $M$  output ports. The case of boson bunching at the detectors is described in the Supplemental Material.

If  $\mathcal{S}$  describes the set of occupied input ports, the  $N$ -photon input state is

$$|\mathcal{S}\rangle := \bigotimes_{s \in \mathcal{S}} |1[\xi_s]\rangle_s \bigotimes_{s \notin \mathcal{S}} |0\rangle_s, \quad (1)$$

where, using an arbitrary polarization basis  $\{e_1, e_2\}$ , the single-photon multimode states are defined as

$$|1[\xi_s]\rangle_s := \sum_{\lambda=1,2} \int_0^\infty d\omega (e_\lambda \cdot \xi_s(\omega)) \hat{a}_{s,\lambda}^\dagger(\omega) |0\rangle_s, \quad (2)$$

with the creation operator  $\hat{a}_{s,\lambda}^\dagger(\omega)$  for the frequency mode  $\omega$  and the polarization  $\lambda$  [60]. The direction, magnitude, and phase of the complex spectral distribution  $\xi_s(\omega)$  (with normalization condition  $\int d\omega |\xi_s(\omega)|^2 = 1$ ) define

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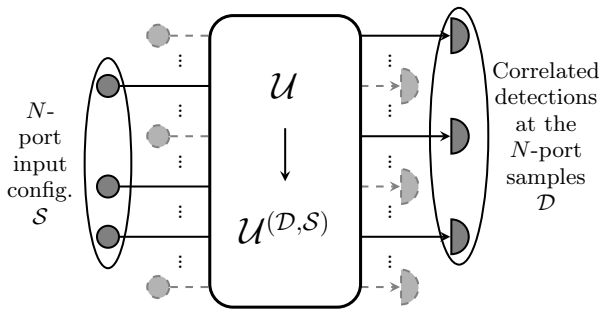


FIG. 1. General setup for multiboson correlation interferometry.  $N$  single bosons are injected into an  $N$ -port subset  $\mathcal{S}$  of the  $M$  input ports of a linear interferometer. They can be detected at the output in any possible sample  $\mathcal{D}$  containing  $N$  of the  $M$  output ports at  $N$  corresponding detection times  $\{t_d\}_{d \in \mathcal{D}}$ . For each output port sample  $\mathcal{D}$  and given input configuration  $\mathcal{S}$ , the evolution through the interferometer is fully described by a  $N \times N$  submatrix  $\mathcal{U}^{(\mathcal{D}, \mathcal{S})}$  of the original  $M \times M$  interferometer matrix  $\mathcal{U}$ . The correlated measurements can be performed in any bosonic degree of freedom, such as time, polarization and spin.

the polarization, the frequency spectrum, and the time of emission of the photon, respectively.

After the evolution in the interferometer, an  $N$ -photon detection can occur in any  $N$ -port sample  $\mathcal{D}$  at detection times  $\{t_d\}_{d \in \mathcal{D}}$  and in the polarizations  $\{\mathbf{p}_d\}_{d \in \mathcal{D}}$ . For simplicity, we consider input photon spectra in the narrow bandwidth approximation and a polarization-independent interferometric evolution with equal propagation time  $\Delta t$  for each possible path. The field operators  $\hat{\mathbf{E}}_d^{(+)}(t_d)$  at the detected ports  $d \in \mathcal{D}$  can then be written in terms of the operators  $\hat{\mathbf{E}}_s^{(+)}(t_d - \Delta t)$  at the input ports  $s \in \mathcal{S}$  as

$$\hat{\mathbf{E}}_d^{(+)}(t_d) = \sum_{s \in \mathcal{S}} \mathcal{U}_{d,s} \hat{\mathbf{E}}_s^{(+)}(t_d - \Delta t) \quad (3)$$

through the  $N \times N$  submatrix

$$\mathcal{U}^{(\mathcal{D}, \mathcal{S})} := [\mathcal{U}_{d,s}]_{\substack{d \in \mathcal{D} \\ s \in \mathcal{S}}}$$

of the  $M \times M$  unitary matrix  $\mathcal{U}$  describing the interferometer.

The rate of an  $N$ -fold detection event for ideal photodetectors is now given by the  $N$ th-order Glauber correlation function [61]

$$G_{\{t_d, \mathbf{p}_d\}}^{(\mathcal{D}, \mathcal{S})} := \langle \mathcal{S} | \prod_{d \in \mathcal{D}} (\mathbf{p}_d^* \cdot \hat{\mathbf{E}}_d^{(-)}(t_d)) (\mathbf{p}_d \cdot \hat{\mathbf{E}}_d^{(+)}(t_d)) | \mathcal{S} \rangle, \quad (4)$$

where  $\mathbf{p}_d \cdot \hat{\mathbf{E}}_d^{(+)}(t_d)$  is the component of the electric field operator in Eq. (3) in the detected polarization  $\mathbf{p}_d$ .

By using the Fourier transforms

$$\chi_s(t) := \mathcal{F}[\xi_s](t - \Delta t)$$

of the frequency distributions, defining the matrices

$$\mathcal{T}_{\{t_d, \mathbf{p}_d\}}^{(\mathcal{D}, \mathcal{S})} := [\mathcal{U}_{d,s} (\mathbf{p}_d \cdot \chi_s(t_d))]_{\substack{d \in \mathcal{D} \\ s \in \mathcal{S}}} \quad (5)$$

and applying the definition of the permanent of a matrix,

$$\text{perm } \mathcal{M} := \sum_{\sigma \in \Sigma_N} \prod_i \mathcal{M}_{i, \sigma(i)},$$

where the sum runs over all permutations  $\sigma$  in the symmetric group  $\Sigma_N$ , the  $N$ -photon probability rate in Eq. (4) can be easily expressed as

$$G_{\{t_d, \mathbf{p}_d\}}^{(\mathcal{D}, \mathcal{S})} = \left| \text{perm } \mathcal{T}_{\{t_d, \mathbf{p}_d\}}^{(\mathcal{D}, \mathcal{S})} \right|^2, \quad (6)$$

as shown in the Supplemental Material. Here, the permanent describes the coherent superposition of  $N!$  *detection probability amplitudes* each corresponding to a different  $N$ -photon quantum path from the input ports in  $\mathcal{S}$  to the output ports in  $\mathcal{D}$ . Each  $N$ -photon amplitude is the product of the  $N$  respective single-photon amplitudes, which are the entries of the matrix  $\mathcal{T}_{\{t_d, \mathbf{p}_d\}}^{(\mathcal{D}, \mathcal{S})}$  in Eq. (5). Therefore, the interference between the  $N!$  quantum paths depends strongly not only on the interferometric evolution but also on the spectral distributions defining the multiphoton state in Eq. (1) and on the detection times and polarizations associated with a measured correlation sample.

**Multiboson Correlation Sampling.** The probabilities in Eq. (6) allow us to physically describe the novel problem of multiboson correlation sampling, i.e. sampling by time and polarization resolving correlation measurements from the probability distribution at the interferometer output.

For approximately equal detection times  $t_d \approx t$  and equal polarizations  $\mathbf{p}_d = \mathbf{p}, \forall d \in \mathcal{D}$ , the multiphoton detection rate in Eq. (6) becomes

$$G_{t, \mathbf{p}}^{(\mathcal{D}, \mathcal{S})} = \left| \text{perm } \mathcal{U}^{(\mathcal{D}, \mathcal{S})} \right|^2 \prod_{s \in \mathcal{S}} |\mathbf{p} \cdot \chi_s(t)|^2. \quad (7)$$

which is not trivial if, for each input photon, the detection probability  $|\mathbf{p} \cdot \chi_s(t)|^2$  after free propagation is not vanishing at a given time  $t$ . Interestingly, all  $N$ -photon quantum paths in Eq. (7) are effectively indistinguishable even for non-identical input photons. Their interference depends, apart from an overall factor, only on the permanents of submatrices  $\mathcal{U}^{(\mathcal{D}, \mathcal{S})}$  of the interferometer transformation. In particular, for random linear interferometers with  $30 \lesssim N \ll M$  input photons, such permanents start to be not tractable with a classical computer [27]. Therefore, the physics of multiboson correlation sampling with non-identical input photons reveals a remarkable potential in quantum information processing [62–64].

**Multiboson correlation landscapes.** The general result obtained in Eq. (6) allows us also to describe the possible multiboson interference “landscapes” which

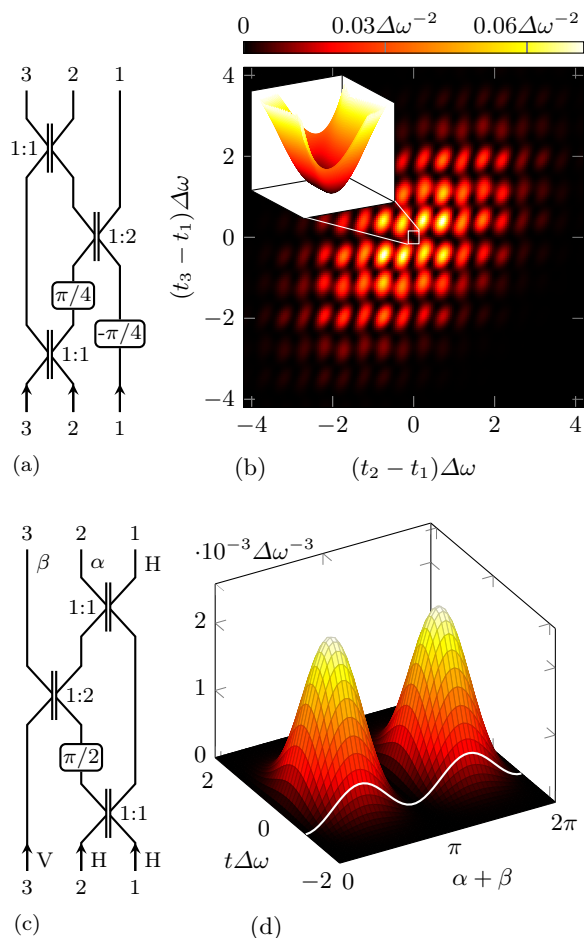


FIG. 2. (Color online) Correlation landscapes at the output of a tritter for three input photons with Gaussian spectra with identical variance  $\Delta\omega^2$  but different central frequencies ( $\omega_{0,2} - \omega_{0,1} = 8.0\Delta\omega$ ,  $\omega_{0,3} - \omega_{0,1} = 12.7\Delta\omega$ ). In panel (b), we observe three-photon quantum beats in the threefold coincidence rate measured at the output of the tritter in panel (a) for equally polarized input photons and polarization-independent detections. The inset magnifies the three-dimensional “dip” occurring at equal detection times. In panel (c),  $H$ -,  $H$ -, and  $V$ -polarized input photons impinge on a symmetric tritter. The emergence of  $W$ -state-type correlations for joint detections at approximately equal times  $t$  is shown in panel (d) (here, for simplicity, the propagation time is  $\Delta t \cong 0$ ): if a  $H$ -polarized photon is detected in one output port we observe in the remaining two ports 100% visibility correlations in the sum  $\alpha + \beta$  of the detected polarization angles typical of a Bell state (indicated for an arbitrary time  $t$  by the white curve).

arise from correlation measurements in given degrees of freedom (time, polarization, spin, etc.) depending on the internal state of the input bosons and on the interferometer transformation. As an example, we consider  $N = 3$ -photon correlation measurements in the extreme case of three completely distinguishable Gaussian single-

photon pulses (with identical variances  $\Delta\omega^2$  and relative central frequencies  $\omega_{0,2} - \omega_{0,1} = 8.0\Delta\omega$ ,  $\omega_{0,3} - \omega_{0,1} = 12.7\Delta\omega$ ) impinging in two different  $2M = 6$ -port interferometers.

*Quantum beats.* The first interferometer, shown in Fig. 2a, is a tritter characterized by a unitary transformation

$$\mathcal{U} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & i & -i \\ i & (1 - \sqrt{3})/2 & -(1 + \sqrt{3})/2 \\ i & (\sqrt{3} + 1)/2 & (\sqrt{3} - 1)/2 \end{bmatrix}$$

with permanent zero, implying, for three input photons with equal spectra and polarization, a vanishing threefold coincidence rate independent of the detection times. Does completely destructive interference occur also for input photons distinguishable in their frequencies? Remarkably, the answer is yes at approximately equal detection times according to Eq. (7). Indeed, the multiphoton landscape depicted in Fig. 2b, corresponding to the three-photon detection rate in Eq. (6) as a function of the relative detection times  $t_2 - t_1$  and  $t_3 - t_2$  for polarization-independent detections, reveals a three-dimensional “dip” at the origin. Although the photons are fully distinguishable in their frequencies, the three-photon quantum paths for equal detection times are completely indistinguishable and thereby interfere. More interestingly, departing from the dip at  $t_1 \cong t_2 \cong t_3$  we observe three-dimensional quantum beats in the two relative detection times. These beats emerge from the superposition according to the interferometric evolution of several (in general of the order of  $(N!)^2$ ) interfering terms in Eq. (6) oscillating with periodicity determined by the frequency differences of the input photons. The beats show a Gaussian damping along both diagonals with a width characterized by the photon coherence time  $\Delta\omega^{-1}$ . In general, a plethora of  $N$ -photon interference landscapes can be obtained by tuning different physical parameters, such as the input internal states, the interferometer evolution and the measurement observables. The emerging multiphoton interference landscapes are thereby a powerful tool to extract information about all these parameters simultaneously.

*Entanglement correlations.* The second interferometer is a symmetric tritter described by the unitary transformation

$$\mathcal{U} = \frac{1}{\sqrt{3}} [\exp(i \frac{2\pi}{3} d \cdot s)]_{d=1,2,3}^{s=1,2,3}$$

(see Fig. 2c) with two input photons horizontally polarized and the third vertically polarized. For input photons identical in their frequency-temporal spectra, a three-fold coincidence measurement would be only sensitive to the entangled state

$$|W\rangle = \frac{1}{\sqrt{3}} (|H, H, V\rangle + |H, V, H\rangle + |V, H, H\rangle)$$

(so called  $W$  state [65]) independently of the detection time. This is evidently not the case for input photons

with different spectral distributions, which are relevant from an experimental point of view. However, here we demonstrate 100% visibility  $W$ -state correlations even for input photons completely distinguishable in their frequencies. The emergence of such entanglement correlations is shown in Fig. 2d for joint detections at approximately equal times  $t$  according to Eq. (7): if an  $H$ -polarized photon is detected in one output port, we observe in the remaining two ports, at any time  $t$ , correlations typical of a Bell state in the sum of the detected polarization angles. These correlations arise from the physics of time-resolving correlation measurements: at approximately equal detection times all the multiphoton detection amplitudes fully interfere even for input photons distinguishable in their frequencies. Moreover, similarly to Ref. [66], this scheme has the potential to implement more general entanglement correlations both in frequency/time and polarization, with the advantage of not requiring entangled sources.

**Non-resolving correlation measurements.** We now consider the case of correlation measurements which do not resolve the detection times and polarizations, resulting in an average over these degrees of freedom. In this case, we obtain the probability

$$P_{\text{av}}(\mathcal{D}; \mathcal{S}) := \sum_{\{\mathbf{p}_d\} \in \{\mathbf{e}_1, \mathbf{e}_2\}^{\otimes N}} \int_{-\infty}^{\infty} \left( \prod_{d \in \mathcal{D}} dt_d \right) G_{\{t_d, \mathbf{p}_d\}}^{(\mathcal{D}, \mathcal{S})} \quad (8)$$

to detect the  $N$  photons coming from the input ports  $\mathcal{S}$  in the output ports  $\mathcal{D}$ , where  $\{\mathbf{e}_1, \mathbf{e}_2\}$  is an arbitrary polarization basis.

As we show in the Supplemental Material, by defining the *overlap factors*

$$f_\rho(\mathcal{S}) := \prod_{s \in \mathcal{S}} \int_{-\infty}^{\infty} dt \chi_s(t) \cdot \chi_{\rho(s)}(t) \quad (9)$$

for the interfering  $N$ -photon detection amplitudes in Eq. (6) and the *interference-type matrices*

$$\mathcal{A}_\rho^{(\mathcal{D}, \mathcal{S})} := [\mathcal{U}_{d,s}^* \mathcal{U}_{d,\rho(s)}]_{\substack{d \in \mathcal{D} \\ s \in \mathcal{S}}}, \quad (10)$$

the probability of an  $N$ -fold detection in the sample  $\mathcal{D}$  can be expressed concisely as

$$P_{\text{av}}(\mathcal{D}; \mathcal{S}) = \sum_{\rho \in \Sigma_N} f_\rho(\mathcal{S}) \text{perm } \mathcal{A}_\rho^{(\mathcal{D}, \mathcal{S})}. \quad (11)$$

The probability for each configuration  $(\mathcal{D}; \mathcal{S})$  in Eq. (11) describes the multiphoton interference in a boson sampling device and represents a generalization of the two-photon “dip” interference [7, 8] to a general number  $N$  of single photons in a linear interferometer with  $2M \geq 2N$  ports [67]. If we assume Gaussian temporal distributions  $\chi_s(t)$  which only differ by a time shift, this result reduces to the one obtained for  $N = 3$  in Refs. [23, 24] which relies on the use of immanants [68].

Differently from [23, 24], our result is valid for any value  $N$ , for any single-photon spectra, and depends only on “multiphoton interference” permanents.

We now consider two limiting scenarios:

*a. Absence of  $N$ -boson interference:* All  $N$ -photon quantum paths are distinguishable, corresponding to overlap factors in Eq. (9)  $f_\rho(\mathcal{S}) \approx 0 \forall \rho \neq \mathbf{1}$ . Therefore, the probability in Eq. (11) is given by the completely incoherent superposition

$$P_{\text{av}}(\mathcal{D}; \mathcal{S}) \approx \text{perm } \mathcal{A}_{\rho=\mathbf{1}}^{(\mathcal{D}, \mathcal{S})}$$

with the non-negative matrix  $\mathcal{A}_{\rho=\mathbf{1}}^{(\mathcal{D}, \mathcal{S})} = [|\mathcal{U}_{d,s}|^2]_{\substack{d \in \mathcal{D} \\ s \in \mathcal{S}}}$ , whose permanent can be efficiently estimated [69]. Since, in this case, no multiphoton interference occurs the problem is computationally feasible.

*b. Complete  $N$ -boson interference:* All  $N!$   $N$ -photon quantum paths are indistinguishable,  $f_\rho(\mathcal{S}) = 1 \forall \rho$ , and interfere. Thereby, Eq. (11) reduces to

$$P_{\text{av}}(\mathcal{D}; \mathcal{S}) \approx \sum_{\rho \in \Sigma_N} \text{perm } \mathcal{A}_\rho^{(\mathcal{D}, \mathcal{S})} = \left| \text{perm } \mathcal{U}^{(\mathcal{D}, \mathcal{S})} \right|^2.$$

The fact that only in this case the output probabilities are determined by permanents of complex matrices is at the heart of the demonstration of the complexity of boson sampling devices based on non-resolving correlation measurements given in Ref. [27].

In the two limits considered, we recover the well-known results [13, 14, 70] describing the detection probabilities for full multiboson distinguishability and indistinguishability. In addition, the general result in Eq. (11) allows us to fully describe all possible experimental scenarios of partial multiphoton distinguishability. This description triggers exciting questions about the complexity of these scenarios from an experimental point of view.

**Discussion.** We provided a compact full description of multiphoton interferometry based on correlated measurements in time and polarization of any order for arbitrary states of the input photons.

We have physically analyzed the novel problem of multiboson correlation sampling at the output of random linear interferometers. This is fundamental towards a deeper understanding of the full potential of multiboson quantum interference in quantum information processing in the case of non-identical photons, which is of interest from an experimental point of view.

Moreover, we demonstrated how multiphoton correlation measurements lead to arbitrary-order multiphoton landscapes, which can be tuned with respect to different physical parameters, such as the input internal states, the interferometer evolution and the measured physical observables. These results pave the way for the use of multiphoton interference as a powerful tool for the characterization of the spectral distribution [71] of an arbitrary number of single photons and their distinguishability [72] after the interferometric evolution, which is essential in multiphoton quantum networks [47].



We also showed that even with non-identical input photons it is possible to achieve entanglement correlations with 100% visibility for an arbitrary number of photons. This result may lead to real-world applications in quantum information processing with non-identical photons, such as sampling of bosonic qubits [73], nondeterministic nonlinear gates [2, 74, 75], entanglement of an arbitrary number of distant qubits [34, 36, 76], time-bin qubit networks [39, 40], quantum teleportation [77] and quantum communication protocols [78].

Finally, we provided a full description of arbitrary-order interferometry based on correlation measurements not sensitive to the detected polarizations and times. This description can be applied to the optimization of multiphoton metrology schemes [4] with non-identical single-photon sources for applications in biomedical

physics [79].

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