Probing for Variation of Neutrino Mass with Current Observations

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With the latest astronomical data including Cosmic Microwave Background (WMAP three year, CBI, ACBAR, VSA), Type Ia Supernova ("gold sample"), Galaxy Clustering (SDSS 3-D matter power, Lyman-α forest and Baryon Acoustic Oscillating (BAO)), we make a global fitting to constrain the mass varying neutrinos. We find that the parameter δ, denoting time evolving of neutrino mass, is weakly constrained and the neutrino mass limit today can be relaxed at least by a factor of two. Adding data of $0\nu2\beta$ decay of Heidelberg-Moscow experiment to our analysis, we find that δ can be constrained tightly and mass varying neutrinos are favored at about 99.7% confidence level.

The mass varying neutrino which has been studied actively in the literature recently brings forth new theoretical challenges and abundant phenomenology. The predictions of the variation of the neutrino masses can be tested in the experiments of neutrino oscillation[1], short gamma ray bursts[2] and extremely high energy cosmic neutrino[3]. Cosmologically the mass varying neutrino plays an interesting role in determining the evolution of the universe[4] and has interesting implications in leptogenesis[5], Supernova[6], the Cosmic Microwave Background(CMB) and Large Scale Structure(LSS)[7].

Neutrino mass variation can be induced by the interaction between neutrino and scalar field[8, 9, 10, 11] in models of neutrino dark energy[1]. In the framework of ΛCDM the variation of neutrino mass could result from the coupling between neutrino and Ricci scalar in the form of $f(R)\bar{\nu}\nu$ and possibly some other unknown mechanism. In this paper we work in a model-independent way to constrain the time evolving of neutrino mass by parameterizing neutrino mass in the form of Eq.(1). For simplicity and to extract the effect of mass varying neutrinos we use the simplest dark energy model, the cosmological constant for the analysis in this paper.

We take a global fit to constrain the parameter δ denoting the mass variation of neutrinos and study its effect on the determination of other cosmological parameters. Our result shows that 1)with the mass variation, the cosmological limit on the neutrino mass is relaxed by roughly a factor of two; 2)if taking the limits on the neutrino mass from Heidelberg-Moscow experiment(HM) as a prior[14, 15, 16], we find that the variation of neutrino mass is tightly constrained and the mass varying neutrinos are favored at about 3 σ.

We assume that three species of neutrinos whose masses are degenerate and parameterize the neutrino mass in form of [2]

$$m_\nu(a) = m_{\nu0}[1 + \delta(1 - a)] \tag{1}$$

where $m_\nu(a)$ is the sum of neutrino masses which is a function of scale factor a, $m_{\nu0}$ is the sum of neutrino mass at present, δ is the dimensionless factor denoting the time-varying effect of neutrino masses. For the background,

$$\rho_\nu = \frac{1}{a^4}\int q^2 dq d\Omega \epsilon f_0(q), \tag{2}$$

with $\epsilon^2 = q^2 + m_\nu^2(a)a^2$, $q^i = ap^i$ is the comoving momentum. The pressure is

$$p_\nu = \frac{1}{3a^4}\int q^2 dq d\Omega f_0(q)\frac{q^2}{\epsilon}. \tag{3}$$

Thus,

$$\dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) = \frac{\partial \ln m_\nu(a)}{\partial a}\dot{a}(\rho_\nu - 3p_\nu). \tag{4}$$

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[1] In Ref.[12] Afshordi *et al.* concluded the neutrino dark energy model presented by Fardon *et al.*[9] is unstable due to the negative sound speed squared. However, their argument is based on the *ad hoc* model in [9] thus lack the generality. Generically, for the mass varying neutrino model originally proposed in [8], there exists large parameter space where the system is stable. For a very recent survey, see Ref.[13].

[2] The physical motivation for mass varying neutrino mainly stems from the coupling of neutrino and the dark energy thus we choose this parametrization which of similar form of widely-used parametrization for dark energy, namely, $w(a) = w_0 + w_1(1 - a)$. Here $m_{\nu0}$ and δ denote the neutrino mass presently and its derivative with respect to scale factor, say, $dm(a)/da$. In this sense, we work in a model-independent fashion. We stress that our parametric method using Eq.(1) is just one possible way to study the variation of neutrino mass. With the accumulation of high quality astronomical data, we can investigate the neutrino mass variation in a more model-independent way, such as using the technique of Principle Component Analysis(PCA).

where the overdot represents the derivative with respect to the conformal time. For the evolution of perturbation, working in the synchronous gauge and following the treatment of Ref.[17], we find that the Boltzmann equations Eq.(40) in [17] don't relate directly to the mass varying terms $\partial \ln m_\nu / \partial a$, and therefore the hierarchy equations for the massive neutrinos Eq.(56) in [17] remain unchanged. In our numerical calculation we integrate these hierarchy equations rather than use the approximate strategy suggested in [18] to assure a high precision. Using a modified code of `CosmoMC`[19], we make a global fit to constrain the mass varying neutrinos. Assuming a flat universe, we set the most general cosmological parameter space as:

$$\mathbf{P} \equiv (\omega_b, \omega_c, \Theta_S, \tau, f_\nu, \delta, n_s, \log[10^{10}A_s]) \ , \tag{5}$$

where $\omega_b = \Omega_b h^2$ and $\omega_c = \Omega_c h^2$ are the physical baryon and cold dark matter densities relative to the critical density, Θ_S characterizes the angular scale of sound horizon, τ is the optical depth to the last scattering surface and A_s is defined as the amplitude of initial power spectrum, f_ν is the dark matter neutrino fraction at present, namely,

$$f_\nu = \frac{\rho_\nu}{\rho_{DM}} = \frac{m_{\nu 0}}{93.105 \ eV \ \Omega_c h^2} \ \ , \tag{6}$$

where ρ_ν and ρ_{DM} denote the energy density of neutrino and dark matter at present respectively. We sample in above an eight dimensional parameter space and fit the theoretical output to the observation using Markov Chain Monte Carlo algorithm[24, 25, 26]. We take the weak priors as: $\tau < 0.8$, $0.5 < n_s < 1.5$, $0 < f_\nu < 0.5$, a cosmic age tophat prior as 10 Gyr$< t_0 <$20 Gyr. To keep the positivity of neutrino mass we take $-1 < \delta < 10$. Furthermore, we make use of the Hubble space telescope (HST) measurement of the Hubble parameter $H_0 = 100h$ km s^{-1}Mpc^{-1} [31] by multiplying the likelihood by a Gaussian likelihood function centered around $h = 0.72$ and with a standard deviation $\sigma = 0.08$. We impose a weak Gaussian prior on the baryon density $\Omega_b h^2 = 0.022 \pm 0.002$ (1σ) from the Big Bang nucleosynthesis[32].

For CMB data, we use the three-year WMAP (WMAP-3) Temperature-Temperature (TT) and Temperature-Polarization (TE) power spectrum with the routine for computing the likelihood supplied by the WMAP team[3] [20, 21, 22, 23] as well as ACBAR [27], CBI [28, 29] and VSA [30] data. To break the degeneracies of the cosmological parameters, we add non-CMB data into our analysis. For supernova type Ia (SN Ia) of "Riess gold sample" [33], we have marginalized over the nuisance parameter[35] in the calculation of SN Ia likelihood. For LSS information, we have used the 3D matter power spectrum of SDSS[37] and 2dFGRS [36], Lyman-α forest data (Lyα) from SDSS [39] and the recent measurement of the baryon acoustic oscillation (BAO) feature in the 2-point correlation function of SDSS [40]. To be conservative but more robust, we only use the first 14 bins of the SDSS 3D matter power spectrum, which are well within the linear regime[41]. For Lyα likelihood, we modify the interpolating code[4] to incorporate our models. For BAO likelihood, we use the constraint [40]:

$$A \equiv D_V(0.35) \frac{\sqrt{\Omega_m H_0^2}}{0.35c} = 0.469 \pm 0.017 \ \ , \tag{7}$$

$$D_V(z) = \left[D_M(z)^2 \frac{cz}{H(z)} \right]^{1/3} \ \ , \tag{8}$$

where $H(z)$ is the Hubble parameter, c is the speed of light and $D_M(z)$ is the comoving angular diameter distance at a specific redshift z. Moreover, the Heidelberg-Moscow experiment uses the half time of $0\nu 2\beta$ decay to constrain the effective Majorana mass and this translates to the constraint on the sum of neutrino masses under some assumptions[42]:

$$m_{\nu 0} \sim 1.8 \pm 0.6 \ eV \ (2\sigma) \ . \tag{9}$$

Given that the Heidelberg-Moscow experiment is controversial for the time being, we just make a tentative fit choosing the HM prior.

For each regular calculation, we run 6 independent chains comprising of 150,000-300,000 chain elements and spend thousands of CPU hours to calculate on a cluster. The average acceptance rate is about 40%. We eliminate the first

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[3] Available at http://lambda.gsfc.nasa.gov/product/map/current/
[4] Available at http://www.cita.utoronto.ca/~pmcdonal/LyaF/public.lyafchisq.tar.gz

TABLE I: Mean and 1σ constraints on the cosmological parameters. For the weakly constrained parameters, such as $m_{\nu 0}$ and δ for some data combinations, we quote the 95% upper limits instead. Upper part of the table is for ΛCDM + neutrinos with constant mass while in the lower part we free δ to study the mass varying neutrinos. "ALL" denotes WMAP3+ACBAR+VSA+CBI+RIESS+SDSS+2dF+Lyα throughout this paper.

$\delta = 0$	ALL	ALL+BAO	ALL+BAO+HM
$m_{\nu 0}$ (eV)	< 0.616	< 0.393	$0.760^{+0.093}_{-0.104}$
Ω_m	0.317 ± 0.021	0.280 ± 0.015	$0.303^{+0.016}_{-0.017}$
σ_8	0.832 ± 0.024	0.834 ± 0.024	$0.795^{+0.025}_{-0.026}$
δ free	ALL	ALL+BAO	ALL+BAO+HM
δ	< 6.661	< 8.562	< -0.713
$m_{\nu 0}$ (eV)	< 1.619	< 0.776	$1.568^{+0.143}_{-0.141}$
Ω_m	0.319 ± 0.024	0.281 ± 0.014	0.298 ± 0.015
σ_8	0.829 ± 0.027	$0.835^{+0.025}_{-0.024}$	$0.790^{+0.022}_{-0.023}$
$\Delta\chi^2$	0.198	0.222	9.752

10% chain elements for "build in", and for the convergence test typically we get the chains satisfy the Gelman and Rubin[43] criteria where $R - 1 < 0.1$.

We summarize our main results of the mass varying neutrinos in the lower part of Table I. For comparison, we also study the ΛCDM model with neutrinos of constant mass. The first discovery is that the neutrino mass limit at present epoch can be relaxed dramatically if neutrino mass varies during evolution. Without Heidelberg-Moscow data, we find that the neutrino mass limit can be relaxed by a factor of 2.6 for "All" data and 1.8 for "All+BAO"[5]. Adding Heidelberg-Moscow prior, we find that the mean value of neutrino mass rises up from 0.760 to 1.568 while reducing χ^2 by 9.752. This is expected since the Heidelberg-Moscow prior is in great tension with the cosmological observations. For example, the authors of Ref.[46] argued that given the current cosmological constraint on the (constant) neutrino mass the HM prior can be excluded. However, this controversy can be resolved if neutrino mass varies. For our *ad hoc* parametrization (1), we see that the "All+BAO+HM" prior can put stringent limit on δ, namely, $-1 < \delta < -0.713$. The best fit value of δ is about -0.9. It means that, in order to be consistent with all the cosmological observations, the neutrino mass must be very small in the past, but has grown recently in order to agree with the Heidelberg-Moscow data.

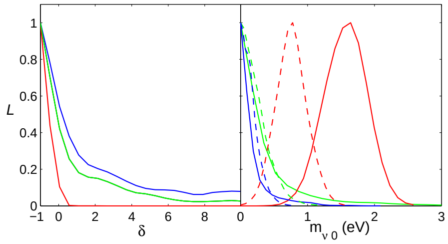

FIG. 1: One dimensional posterior distribution of neutrino mass today m_0 and its time variation δ. Solid curves denotes the mass varying models (see text) while dashed lines show the models with constant neutrino mass. Different data combinations are distinguished by color. Green: All; Blue: All+BAO; Red: All+BAO+HM prior.

Mass varying neutrinos lessen the tension between the HM prior and the cosmological data thus are more favored at nearly 3σ than neutrinos with constant mass. These results are shown graphically in Fig.1. From the left panel we see that δ is weakly constrained unless we add HM prior while the right panel shows explicitly the modification of posterior distribution of neutrino mass today if we allow it to vary with cosmic time. Furthermore, from Fig.2 we see that the current neutrino mass is correlated with its time variation. This correlation mainly stems from the LSS

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[5] It's true that the bounds on the neutrino mass at current epoch can be relaxed by considering the correlation among neutrino mass with dark energy parameters[44] and with inflationary parameters[45], however if the neutrino mass varies with time, its mass limit can be much looser.

data. We know that the galaxy survey is powerful to weigh neutrinos by detecting the suppression on small scales due to the free streaming effect of neutrinos[47]. The free streaming scale of mass varying neutrinos $\lambda_{FS-\nu}$ can be roughly estimated as:

$$\lambda_{FS-\nu} \simeq 20(\frac{m_\nu(a_{NR})}{30eV})^{-1}\ Mpc \quad , \tag{10}$$

where a_{NR} is the scale factor when neutrinos become non-relativistic. We have seen from Eqs.(10) and (1) that $\lambda_{FS-\nu}$ is determined by the neutrino mass today and its evolution behavior, which lead to the correlation among $m_{\nu 0}$, δ and $m_{\nu 0} * \delta$. In Fig.2 we find that $m_{\nu 0}$ is anti-correlated with δ and $m_{\nu 0} * \delta$ as expected.

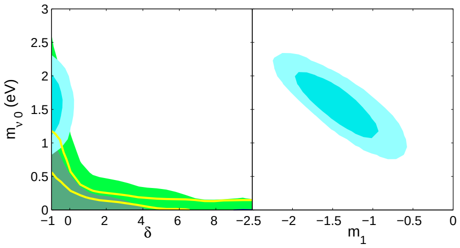

FIG. 2: Contour plots of parameters related to mass varying effect of neutrino. m_1 is the neutrino mass changing with time, say, $m_1 = m_0 * \delta$. Different data combinations are distinguished by color. Green: All; Yellow: All+BAO; Cyan: All+BAO+HM prior. 68% and 95% C.L. contours are illustrated from inside out.

From Table I, we find that the mean value and the error bars of Ω_m and σ_8 do not change much if neutrino mass varies. At first glance this seems at odds since the neutrino mass limit today has been significantly relaxed by its time variation and we know that the neutrino mass today is strongly correlated with matter density and σ_8 as illustrated in Fig.(3). However since δ is anti-correlated with $m_{\nu 0}$, the aforementioned effect is counteracted, leaves Ω_m and σ_8 nearly unchanged. This means mass varying neutrinos can hardly be excluded by data sensitive to Ω_m and σ_8, such as SN Ia, CMB and LSS.

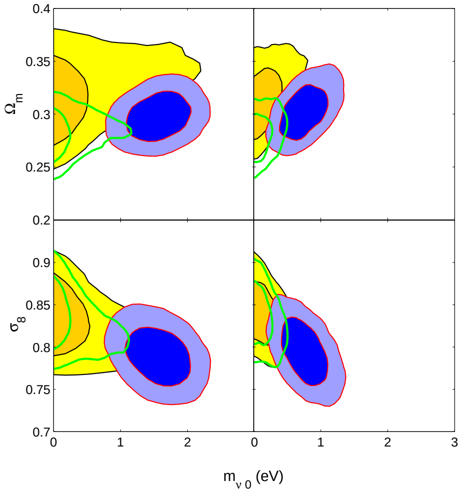

FIG. 3: Contour plots of sum of neutrino mass at present versus Ω_m and σ_8. Left panel: Neutrino mass varies as Eq.1; Right panel: Constant neutrino mass. Different data combinations are distinguished by color. Yellow: All; Green: All+BAO; Blue: All+BAO+HM prior. 68% and 95% C.L. contours are illustrated from inside out.

In Fig.(4), we show the evolution of neutrino mass with 1 and 2 σ error using all data mentioned in this paper. we see that the neutrino mass is best measured at an intermediate redshift rather than now due to the sensitive SN data in this redshift range.

In summary, in this paper, we for the first time study the cosmological implications of the time variation of neutrino mass in a model-independent fashion. We find that numerically time variation of neutrino mass can relax the current

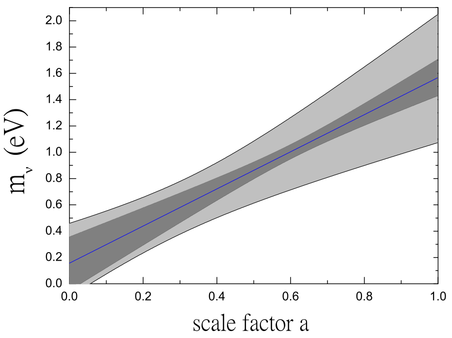

FIG. 4: Evolution of neutrino mass with respect to the scale factor. From inside out, we show the models with mean parameter value (central blue line), 68% C.L.(dark gray) and 95% C.L. (bright gray) fitting with the data combination of All+BAO+HM prior.

mass limit significantly. This result has some interesting and important implications. It could resolve the tension between HM prior, if confirmed, and cosmological data, while it does not spoil the common prediction of Ω_m and σ_8, and it might also be possible to revive the models of warm dark matter which has been shown to be excluded[48].

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