Extensions for Benders cuts and new valid inequalities for solving the European day-ahead electricity market clearing problem efficiently

Gökhan Ceyhan[a], Murat Köksalan[b], Banu Lokman[c,*]

[a] *Department of Industrial Engineering, Middle East Technical University, 06800, Ankara, Turkey*
[b] *Ross School of Business, University of Michigan, 701 Tappan Ave, Ann Arbor, MI 48109, USA*
[c] *Department of Operations and Systems Management, Centre for Operational Research and Logistics, University of Portsmouth, Portsmouth, PO1 3DE, United Kingdom*

Abstract

We study the day-ahead electricity market clearing problem under the prevailing market design in the European electricity markets. We revisit the Benders decomposition algorithm that has been used to solve this problem. We develop new valid inequalities that substantially improve the performance of the algorithm. We generate instances that mimic the characteristics of past bids of the Turkish day-ahead electricity market and conduct experiments. We use two leading mixed-integer programming solvers, IBM ILOG Cplex and Gurobi, in order to assess the impact of employed solver on the algorithm performance. We compare the performances of our algorithm, the primal-dual algorithm, and the Benders decomposition algorithm using the existing cuts from the literature. The extensive experiments we conduct demonstrate that the price-based cuts we develop improve the performance of the Benders decomposition algorithm and outperform the primal-dual algorithm.

Keywords: OR in energy, Day-ahead electricity market clearing problem, Mixed-integer linear programming, Benders decomposition

1. Introduction

In this study, we develop an algorithm to solve the market clearing problem in European day-ahead electricity markets (DAMs) based on the Benders decomposition (BD) algorithms developed for the same problem in the literature. European DAMs are spot markets that are organized to trade electricity between sellers and buyers one day prior to the actual generation and consumption.

*Corresponding author
Email address: banu.lokman@port.ac.uk (Banu Lokman)

Market participants can submit combinations of different types of bids with different prices and quantities for different periods of the delivery day. Market operators (MOs) solve the market clearing problem, and find the surplus maximizing electricity trade and the market clearing prices (MCPs).

European DAM is an *exchange-type* electricity market which is fundamentally different from the *pool-type* electricity markets (Van Vyve et al., 2011). In the latter, system operators pool all the generation and consumption assets as well as the transmission system elements to determine the optimal unit commitment and the economic dispatch. We see many pool-type design applications in U.S. electricity markets, Pennsylvania-New Jersey-Maryland (PJM), Midcontinent ISO (MISO) and Electricity Reliability Council of Texas (ERCOT), to name a few.

European MOs have undertaken a major market coupling process in the last decade to create a single pan European DAM, called Single Day-Ahead Coupling (SDAC). It was initiated by eight power exchanges and now accounts for 95% of the EU consumption. The value of the daily traded electricity is around 200 million Euros on average (NEMO Committee, 2020). The resulting problem is a large scale mixed-integer linear or quadratic program (depending on the types of bids available in the market) and the problem needs to be solved in about 10 minutes in order to implement the results within the tight time frame the market is operating in.

BD is the most studied solution approach for this problem in the literature (Martin et al., 2014; Madani & Van Vyve, 2014; Madani & Van Vyve, 2015; Madani & Van Vyve, 2018; Euphemia, 2019). This is mainly because of the complexity of solving a compact formulation when there are many binary variables, complex bid types and equilibrium constraints that have to be satisfied. The BD algorithm reduces the complexity by solving simpler models and introducing constraints (cuts) as necessary to enforce the feasibility of the original model. The performances of the BD algorithms are not up to the task of solving the problem within the required time frame. This is mainly due to the use of "no-good" cuts (Martin et al., 2014) that cause weak relaxation bounds or locally-valid cuts that can only be used in the sub-trees (Madani & Van Vyve, 2015; Madani & Van Vyve, 2018). SDAC uses the Euphemia algorithm that associates heuristic cuts with the aim of generating a high-quality solution within the time limit (Euphemia, 2019).

The Iberian and the Italian markets differ substantially from the rest of the European DAMs in bid types and pricing rules. Fernández-Blanco et al. (2016) and Madani & Van Vyve (2017) develop mixed-integer linear programs (MILPs) to model the Iberian markets that allow the bidders to specify minimum income conditions and ramping constraints. Savelli et al. (2017) model the Italian DAM problem that has uniform purchase prices and zonal selling prices as a bilevel nonlinear program, formulate an equivalent MILP, and solve for MCPs.

Van Vyve et al. (2011) combine the best properties of the pricing mechanisms of the European and the U.S. electricity markets to develop a new pricing mechanism for the European DAMs. Madani et al. (2018) also exploit the pricing mechanisms used in the U.S. DAMs to the exchange-type market setting and compare the results with the current practices in the European DAMs. In addition to studies that focus on alternative pricing mechanisms, there is research that propose new or modified bid types for the European DAMs (see, for example, Vlachos et al., 2016; Madani & Van Vyve, 2018).

In this paper, we develop *price-based* cuts utilizing the MCPs associated with an integer solution and incorporate them into a BD algorithm. We prove that the price-based cuts are valid and stronger than the "no-good" cuts. We test the performance of our algorithm on practical-sized instances and show that our algorithm is superior to the existing BD algorithms, as well as the primal-dual (PD) approach. The improved performance implies substantial surplus increases in European DAMs with millions of Euros of daily trade and provides an efficient algorithm for MOs that operate under strict timelines. We also evaluate the performances of our algorithm using two leading commercial mixed-integer programming (MIP) solvers, IBM ILOG Cplex and Gurobi. We show that our algorithm outperforms the compared algorithms with both solvers, and performs best when Gurobi is employed.

The organization of the paper is as follows. In the next section, we briefly introduce the main aspects of the surplus maximization problem and elaborate on the properties of its solutions (see the electronic companion for more information on DAMs and European DAMs). In Sections 3 and 4, we present a PD and a BD formulation of the surplus maximization problem under pricing constraints, respectively. We next develop new valid inequalities in Section 5 and present their extensions to the markets having bids of different types in Section 6. In Section 7, we test the new cuts on practical-sized problem instances. We conclude in Section 8.

2. Surplus maximization problem

In this section, we provide a MIP formulation of the surplus maximization problem. We examine the linear relaxation of the model and investigate its implications on the market equilibrium. We then elaborate on the MIP model and address the cases when the duality-gap is not zero.

In the rest of the paper, we concentrate on hourly and (profile) block bids, the most commonly-used bid types in the European DAMs, for brevity of the expositions of formulations and findings, while capturing the essence of the auctions. We generalize our findings to more sophisticated bid types in Section 6.

2.1. Hourly bids

An hourly bid, h, is a price-quantity pair, (p_h, q_h). $q_h < 0$ ($q_h > 0$) implies a *supply (demand)* bid that the bidder is willing to sell (buy) an additional amount $|q_h|$ to (from) the market at a minimum (maximum) price of p_h. A bidder can specify a sequence of hourly bids in the increasing order of the price for supply bids and decreasing order of the price for demand bids. Such bids form step functions that indicate the amount of energy the bidder is willing to sell (buy) at different MCPs.

In Nordic (NordPool) and Turkish (EXIST) markets, the MO also accepts a piece-wise linear function instead of a step function. In fact, EXIST only accepts piece-wise linear hourly bids. In this case, each hourly bid is defined by two price values and a quantity. A set of hourly bids, as a piece-wise linear function of MCPs indicates the exact amounts of energy the bidder is willing to sell or buy at those MCPs. We initially restrict our discussions to hourly bids that are represented by step functions. We show in Section 6 that our findings are applicable to piece-wise linear hourly bid functions as well.

2.2. Block bids

Block bids are collections of single hourly bids offered for consecutive time periods. For a block bid, a single price applies to all periods it is offered for. However, the quantities for different periods need not be the same. A block bid needs to be either accepted or rejected as a whole (at full quantity for each period). There is no partial acceptance of a block bid in terms of quantity or the set of time periods.

2.3. Problem formulation

We use the following sets, parameters, and decision variables in our problem formulation:

Sets and parameters:

- T, H, B: sets of time periods, hourly bids, and block bids, respectively
- $q_{h,t}$, p_h: quantity for time period $t \in T$ and price, respectively, for an hourly bid $h \in H$ ($q_{h,t} = 0$, $\forall t \in T, t \neq t'$ for a particular period t')
- $q_{b,t}$, p_b: quantity for time period $t \in T$ and price, respectively, for a block bid $b \in B$
- T_b: the set of time periods spanned by block bid $b \in B$, $T_b \subseteq T$, $(q_{b,t} = 0, \forall t \notin T_b)$

Decision variables:

- x_h: the accepted fraction of hourly bid $h \in H$, $x_h \in [0, 1]$
- y_b: takes a value of 1 if block bid $b \in B$ is accepted, and 0 if rejected.

Denoting the hourly and block bid decision variable vectors as $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively, the market surplus can be calculated as follows:

$$S_L(\boldsymbol{x}, \boldsymbol{y}) = \sum_{t \in T} \left\{ \sum_{h \in H} p_h q_{h,t} x_h + \sum_{b \in B} p_b q_{b,t} y_b \right\} \tag{1}$$

In Equation (1), quantities are negative (positive) for all supply (demand) bids. Therefore, the right-hand side of the equation shows the difference between the total value assigned by the buyers to the accepted demand bids and the total value assigned by the sellers to the same quantity of accepted supply bids. The market surplus function is linear in $\boldsymbol{x}$ and $\boldsymbol{y}$. We formulate the surplus-maximizing MILP as follows:

$$\begin{aligned}
\textbf{(SMILP)}: \quad \text{Max} \quad & S_L(\boldsymbol{x}, \boldsymbol{y}) & \\
\text{s.to.} \quad & \sum_{h \in H} q_{h,t} x_h + \sum_{b \in B} q_{b,t} y_b = 0 & \forall t \in T \\
& x_h \leq 1 & \forall h \in H \\
& x_h \geq 0 & \forall h \in H \\
& y_b \in \{0, 1\} & \forall b \in B
\end{aligned}$$

The first constraint balances the supply and demand in each period. The next two inequalities force x variables to be between 0 and 1. The last constraint ensures that a block bid cannot be accepted partially.

The linear relaxation of **(SMILP)** and the properties of its optimal solutions have been studied in the literature (see for example Martin et al., 2014). For the sake of the completeness, we also present our version of the model in Appendix A. The properties of the optimal solution ensure that each bidder who participates in the DAM will have a non-negative surplus. This guarantees that none of the accepted demand bids is overvalued and none of the accepted supply bids is undervalued by the MO. Furthermore, none of the in-the-money bids are rejected, implying that there is no missed potential for additional surplus for any bidders. These properties jointly show that the market equilibrium is achieved.

When we enforce the binary restrictions on $\boldsymbol{y}$, the market equilibrium may not hold at the optimal solution of **(SMILP)**. Under a marginal pricing scheme, we may first solve **(SMILP)** and fix the values of the binary variables obtained in **(SMILP)** (O'Neill et al., 2005). Let $\bar{\boldsymbol{y}}$ be such a binary vector and B_0 and B_1 be a partition of B such that $B_0 = \{b \in B : \bar{y}_b = 0\}$ and $B_1 = \{b \in B : \bar{y}_b = 1\}$. Let $(\mathbf{SMLP}(\bar{\boldsymbol{y}}))$ denote the linear program obtained by fixing the binary variables to $\bar{\boldsymbol{y}}$. Let dual variables π_t and s_h denote the MCP for period t and the surplus associated with hourly bid h, respectively, and s_b, l_b, and m_b denote the surplus, the loss, and the missed surplus associated with block bid b, respectively.

$$
\begin{aligned}
(\mathbf{SMLP}(\bar{\boldsymbol{y}})): \quad & \text{Max} \quad S_L(\boldsymbol{x}, \boldsymbol{y}) & & \\
& \text{s.to.} \quad \sum_{h \in H} q_{h,t} x_h + \sum_{b \in B} q_{b,t} y_b = 0 & \forall t \in T \quad & [\pi_t] \\
& x_h \leq 1 & \forall h \in H \quad & [s_h] \\
& y_b \leq 1 & \forall b \in B \quad & [s_b] \\
& -y_b \leq -1 & \forall b \in B_1 \quad & [l_b] \\
& y_b \leq 0 & \forall b \in B_0 \quad & [m_b] \\
& x_h \geq 0 & \forall h \in H \quad & \\
& y_b \geq 0 & \forall b \in B \quad &
\end{aligned}
$$

Let $\boldsymbol{\pi}^* \in \mathbb{R}^{|T|}$ represent the value of the MCP vector in an optimal solution to $(\mathbf{SMLP}(\bar{\boldsymbol{y}}))$

and $\boldsymbol{q}_b \in \mathbb{R}^{|T|}$ and $\boldsymbol{p}_b \in \mathbb{R}^{|T|}$ be the quantity and price vectors for a given bid b, respectively. At this optimal solution, some rejected block bids may end up being in-the-money or some accepted block bids may end up being out-of-the-money. These bids are called *paradoxically* rejected and accepted bids, respectively (Martin et al., 2014).

Definition 1. ***Paradoxically accepted bid (PAB)*** *Bid b is paradoxically accepted if it is accepted and $(\boldsymbol{p}_b - \boldsymbol{\pi}^*)^T \boldsymbol{q}_b < 0$.*

Definition 2. ***Paradoxically rejected bid (PRB)*** *Bid b is paradoxically rejected if it is rejected and $(\boldsymbol{p}_b - \boldsymbol{\pi}^*)^T \boldsymbol{q}_b > 0$.*

The paradoxes are that the accepted quantities of a PAB generate a negative surplus (loss) and the rejected quantities of a PRB forgo a positive surplus (missed surplus).

At a feasible solution to **(SMILP)**, both PABs and PRBs may occur among the block bids. The MCPs and the surplus maximizing quantities for hourly bids are at equilibrium since the problem is a linear program once the block bid decisions are fixed. Since a single price vector may not achieve market equilibrium, the auctioneer may have to come up with a partial-equilibrium solution with sub-optimal market surplus. The common approach in European DAMs is the PD approach that associates pricing variables and constraints into the surplus maximization problem and determines the allocations and prices simultaneously (Euphemia, 2019; Madani & Van Vyve, 2014, 2017; Derinkuyu et al., 2019) (see Liberopoulos & Andrianesis, 2016, for a critical review of different pricing schemes in markets with non-convex costs).

The MO may also choose to compensate PABs by paying as much as the associated loss and PRBs by paying as much as the missed surplus to establish a market equilibrium. Such payments made by the MO are called the *uplift payments* (Gribik et al., 2007). Uplift payments create a budget deficit for the MO since it pays more to sellers and receives less from the buyers than accounted for by the solution. Furthermore, due to the uplift payments, the MO deviates from uniform pricing since different bidders may be settled with different energy prices for the same unit of energy. Although this is not an issue in markets with non-uniform pricing schemes, it creates an unfair energy pricing between market participants in markets that are designed with uniform pricing schemes.

Let B_{pab} and B_{prb} be the sets of PABs and PRBs, respectively. The total uplift payment for

PABs is:

$$TU_{pab} = \sum_{b \in B_{pab}} (\boldsymbol{\pi}^* - \boldsymbol{p}_b)^T \boldsymbol{q}_b \tag{2}$$

Similarly, the total uplift payment for PRBs is:

$$TU_{prb} = \sum_{b \in B_{prb}} (\boldsymbol{p}_b - \boldsymbol{\pi}^*)^T \boldsymbol{q}_b \tag{3}$$

We next define and formulate the surplus maximization problem under constraints that prevent or limit TU_{pab} or TU_{prb}.

3. Surplus maximization problem under pricing constraints

Madani & Van Vyve (2014) develop a PD formulation of the surplus maximization problem (see **(E-SMILP)** in Appendix B). We develop the *generalized uplift* problem, **(SMILP-GU)**, based on **(E-SMILP)** by imposing upper bounds, $\overline{TU}_{pab}$ and $\overline{TU}_{prb}$, on TU_{pab} and TU_{prb}, respectively. We will refer to these upper bounds as the *pricing constraints*. **(SMILP-GU)** can be used to enforce market design rules such as rejecting all out-of-the-money bids (as in the EU markets) or accepting all in-the-money bids (as in the Turkish market). Similarly, it can be used to limit the total market loss associated with the accepted out-of-the-money bids, TU_{pab}, or to limit total missed surplus associated with the rejected in-the-money bids, TU_{prb}.

(SMILP-GU) :

$$\begin{aligned}
\text{Max} \quad & S_L(\boldsymbol{x}, \boldsymbol{y}) & \\
\text{s.to.} \quad & \sum_{h \in H} q_{h,t} x_h + \sum_{b \in B} q_{b,t} y_b = 0 & \forall t \in T \\
& x_h \leq 1 & \forall h \in H \\
& y_b \leq 1 & \forall b \in B \\
& s_h \geq \sum_{t \in T} (p_h - \pi_t) q_{h,t} & \forall h \in H \\
& s_b - l_b + m_b \geq \sum_{t \in T} (p_b - \pi_t) q_{b,t} & \forall b \in B \\
& m_b \leq M_b (1 - y_b) & \forall b \in B \\
& l_b \leq M_b y_b & \forall b \in B \\
& S_L(\boldsymbol{x}, \boldsymbol{y}) \geq \sum_{h \in H} s_h + \sum_{b \in B} s_b - \sum_{b \in B} l_b &
\end{aligned}$$

$$\sum_{b \in B} l_b \leq \overline{TU}_{pab}$$

$$\sum_{b \in B} m_b \leq \overline{TU}_{prb}$$

$$x_h, s_h \geq 0 \qquad \forall h \in H$$

$$s_b, l_b, m_b \geq 0, \quad y_b \in \{0,1\} \qquad \forall b \in B$$

We represent the feasible set of **(SMILP-GU)** with Ω. **(SMILP-GU)** with $\overline{TU}_{pab} = 0$ and $\overline{TU}_{prb} > M = \sum_{b \in B} M_b$ corresponds to a special case, **(SMILP-NoPAB)**, which we define next. In this case, the market loss is prevented by eliminating solutions with PABs and there is no binding constraint on the market missed surplus.

$$\textbf{(SMILP-NoPAB)}: \quad \text{Max} \quad S_L(\boldsymbol{x}, \boldsymbol{y})$$
$$\text{s.to.} \quad (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\pi}, \boldsymbol{s}, \boldsymbol{l}, \boldsymbol{m}) \in \Omega$$
$$\sum_{b \in B} l_b \leq 0$$

(SMILP-NoPAB) is always feasible since any optimal solution to **(SMLP($\bar{\boldsymbol{y}}$))** for $\bar{\boldsymbol{y}} = \boldsymbol{0}$ will have zero loss for all block bids. Let $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ be the optimal values of the primal variables. $TU_{prb} = \min \left\{ \sum_{b \in B} m_b : (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\pi}, \boldsymbol{s}, \boldsymbol{l}, \boldsymbol{m}) \in \Omega, l_b \leq 0, \forall b \in B, (\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{x}^*, \boldsymbol{y}^*) \right\}$ is the minimum total missed surplus, and $TU_{pab} = 0$ is the total loss. In European markets, the MO does not pay uplift to PRBs and TU_{prb} is regarded as a foregone opportunity. In a similar manner, **(SMILP-NoPRB)** can be stated as **(SMILP-GU)** with $\overline{TU}_{pab} > M$ and $\overline{TU}_{prb} = 0$.

4. Benders decomposition

In this section, we first present a BD algorithm (Benders, 1962) for **(SMILP-GU)**. We then consider a special case, **(SMILP-NoPAB)**, that has been developed in the literature (see, for example Martin et al., 2014; Madani & Van Vyve, 2015). We also address the special case **(SMILP-NoPRB)** as well as the case of introducing network constraints to the problem, **(SMILP-MultiNode)**.

*4.1. Solving **(SMILP-GU)***

Since the surplus maximization problem is easy to solve in the absence of pricing variables and constraints, BD has the potential to perform well in solving **(SMILP-GU)**. We define **(SMILP)** as the *master problem* and the problem of finding an MCP vector satisfying the pricing constraints as the *subproblem.* A feasible solution to the master problem will also be feasible to **(SMILP-GU)** if there exists a set of prices in the subproblem that satisfy the hourly bid equilibrium and the pricing constraints. Otherwise, the solution cannot be feasible to **(SMILP-GU)** and must be eliminated by adding an appropriate cut. Every feasible solution of **(SMILP-GU)** should satisfy a valid cut and the cut should eliminate the solution identified as infeasible, at the minimum. We next present the master **(MP)** and sub **(SP)** problems:

$$
\begin{aligned}
\textbf{(MP)}: \quad & \text{Max} \quad S_L(\boldsymbol{x}, \boldsymbol{y}) & \\
& \text{s.to.} \quad \sum_{h \in H} q_{h,t} x_h + \sum_{b \in B} q_{b,t} y_b = 0 & \forall t \in T \\
& \qquad x_h \leq 1 & \forall h \in H \\
& \qquad x_h \geq 0 & \forall h \in H \\
& \qquad y_b \in \{0, 1\} & \forall b \in B
\end{aligned}
$$

Let Z be the set of feasible points of **(SMILP)**, $\boldsymbol{\rho} \in P$ be the projection of $\boldsymbol{\omega} \in \Omega$ onto Z and $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \in Z$ be such that $\bar{\boldsymbol{x}}$ is the optimal solution of $\mathbf{(SMLP(\bar{\boldsymbol{y}}))}$. For a given $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})$, the solution of the linear program $\mathbf{(SP(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}))}$ reveals if feasible MCPs that satisfy the pricing constraints associated with block bids and hourly bid equilibrium constraints exist. Since $\mathbf{(SP(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}))}$ is a feasibility problem, an arbitrary objective would be sufficient. We simply set the objective function to zero. The variables in brackets show the dual variables associated with each constraint of the subproblem.

$$
\begin{aligned}
\mathbf{(SP(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}))}: \quad & \text{Min} \quad 0 & & \\
& \text{s.to.} \quad s_h + \sum_{t \in T} \pi_t q_{h,t} \geq \sum_{t \in T} p_h q_{h,t} & \forall h \in H \quad & [x_h^d] \\
& \qquad s_b - l_b + \sum_{t \in T} \pi_t q_{b,t} \geq \sum_{t \in T} p_b q_{b,t} & \forall b \in B_1 \quad & [y_b^{d,1}] \\
& \qquad s_b + m_b + \sum_{t \in T} \pi_t q_{b,t} \geq \sum_{t \in T} p_b q_{b,t} & \forall b \in B_0 \quad & [y_b^{d,0}] \\
& \qquad -\sum_{h \in H} s_h - \sum_{b \in B} s_b + \sum_{b \in B_1} l_b \geq -S_L(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) & & [\phi] \\
& \qquad -\sum_{b \in B_1} l_b \geq -\overline{TU}_{pab} & & [\alpha]
\end{aligned}
$$

$$- \sum_{b \in B_0} m_b \geq -\overline{TU}_{prb} \qquad [\beta]$$

$$s_h \geq 0 \qquad \forall h \in H$$

$$s_b, l_b, m_b \geq 0 \qquad \forall b \in B$$

Madani & Van Vyve (2015) derive the necessary and sufficient condition for feasible MCPs in (**SMILP-NoPAB**) using the Farkas lemma. They develop classical Benders infeasibility cuts as well as locally valid cuts that are only added to the branch-and-bound tree node where the subproblem corresponding to an integer feasible solution at that node is infeasible. In contrast, we consider the generalized problem, (**SMILP-GU**), here and use the following dual subproblem to detect and cut the integer solutions that lead to infeasible subproblems:

$$\begin{aligned}
\mathbf{(DSP(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}))}: \quad \text{Max} \quad & S_L(\boldsymbol{x^d}, \boldsymbol{y^d}) - \phi S_L(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) - \alpha \overline{TU}_{pab} - \beta \overline{TU}_{prb} \\
\text{s.to.} \quad & x_h^d - \phi \leq 0 && \forall h \in H \\
& y_b^{d,1} - \phi \leq 0 && \forall b \in B_1 \\
& y_b^{d,0} - \phi \leq 0 && \forall b \in B_0 \\
& - y_b^{d,1} + \phi - \alpha \leq 0 && \forall b \in B_1 \\
& y_b^{d,0} - \beta \leq 0 && \forall b \in B_0 \\
& \sum_{h \in H} q_{h,t} x_h^d + \sum_{b \in B_1} q_{b,t} y_b^{d,1} + \sum_{b \in B_0} q_{b,t} y_b^{d,0} = 0 && \forall t \in T \\
& x_h^d, y_b^{d,1}, y_b^{d,0}, \phi, \alpha, \beta \geq 0
\end{aligned}$$

where $S_L(\boldsymbol{x^d}, \boldsymbol{y^d}) = \sum_{t \in T} \left\{ \sum_{h \in H} p_h q_{h,t} x_h^d + \sum_{b \in B_1} p_b q_{b,t} y_b^{d,1} + \sum_{b \in B_0} p_b q_{b,t} y_b^{d,0} \right\}$, $\boldsymbol{y^d} = (\boldsymbol{y^{d,1}}, \boldsymbol{y^{d,0}})$.

$\mathbf{(DSP(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}))}$ is feasible since there exists a trivial feasible solution: $\boldsymbol{x^d} = \boldsymbol{y^{d,1}} = \boldsymbol{y^{d,0}} = \mathbf{0}$, $\phi = \alpha = \beta = 0$. The optimal objective function value of this trivial solution is non-negative. Hence, $\mathbf{(SP(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}))}$ is infeasible if and only if $\mathbf{(DSP(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}))}$ is unbounded.

If $\mathbf{(DSP(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}))}$ is unbounded, then there exist feasible solutions, $(\bar{\boldsymbol{x}}^{\boldsymbol{d}}, \bar{\boldsymbol{y}}^{\boldsymbol{d,1}}, \bar{\boldsymbol{y}}^{\boldsymbol{d,0}}, \bar{\phi}, \bar{\alpha}, \bar{\beta})$, such that $S_L(\bar{\boldsymbol{x}}^{\boldsymbol{d}}, \bar{\boldsymbol{y}}^{\boldsymbol{d}}) - \bar{\phi} S_L(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) - \bar{\alpha} \overline{TU}_{pab} - \bar{\beta} \overline{TU}_{prb} > 0$. For $\bar{\phi} > 0$, $\boldsymbol{d} = (\bar{\boldsymbol{x}}^{\boldsymbol{d}}, \bar{\boldsymbol{y}}^{\boldsymbol{d,1}}, \bar{\boldsymbol{y}}^{\boldsymbol{d,0}}, \bar{\phi}, \bar{\alpha}, \bar{\beta})$ is a direction of unboundedness. Then, $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \in Z$ has to satisfy

$$S_L(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \geq \frac{1}{\bar{\phi}} S_L(\bar{\boldsymbol{x}}^{\boldsymbol{d}}, \bar{\boldsymbol{y}}^{\boldsymbol{d}}) - \frac{\bar{\alpha}}{\bar{\phi}} \overline{TU}_{pab} - \frac{\bar{\beta}}{\bar{\phi}} \overline{TU}_{prb} \qquad (4)$$

if $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \in P$. By setting $\phi = 1$ and dropping the fixed term $S_L(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})$ in the objective function, we can rewrite the dual subproblem as a bounded dual subproblem, **(BDSP($\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}$))**, as follows:

$$
\begin{aligned}
\textbf{(BDSP}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})\textbf{)}: \quad & \text{Max} \quad S_L(\boldsymbol{x^d}, \boldsymbol{y^d}) - \alpha \overline{TU}_{pab} - \beta \overline{TU}_{prb} & \\
& \text{s.to.} \quad x_h^d \leq 1 & \forall h \in H \\
& \qquad y_b^{d,1} \leq 1 & \forall b \in B_1 \\
& \qquad y_b^{d,0} \leq 1 & \forall b \in B_0 \\
& \qquad \alpha \geq 1 - y_b^{d,1} & \forall b \in B_1 \\
& \qquad \beta \geq y_b^{d,0} & \forall b \in B_0 \\
& \qquad \sum_{h \in H} q_{h,t} x_h^d + \sum_{b \in B_1} q_{b,t} y_b^{d,1} + \sum_{b \in B_0} q_{b,t} y_b^{d,0} = 0 & \forall t \in T \\
& \qquad x_h^d, y_b^{d,1}, y_b^{d,0}, \alpha, \beta \geq 0 &
\end{aligned}
$$

In this case, the objective function of **(BDSP($\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}$))** is bounded from above and there is no direction of unboundedness. If there exist $(\bar{\boldsymbol{x}}^{\boldsymbol{d}}, \bar{\boldsymbol{y}}^{\boldsymbol{d,1}}, \bar{\boldsymbol{y}}^{\boldsymbol{d,0}}, \bar{\alpha}, \bar{\beta})$ such that $S_L(\bar{\boldsymbol{x}}^{\boldsymbol{d}}, \bar{\boldsymbol{y}}^{\boldsymbol{d}}) - \bar{\alpha}\overline{TU}_{pab} - \bar{\beta}\overline{TU}_{prb} > S_L(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})$, then **(DSP($\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}$))** becomes unbounded in the direction $\boldsymbol{d} = (\bar{\boldsymbol{x}}^{\boldsymbol{d}}, \bar{\boldsymbol{y}}^{\boldsymbol{d,1}}, \bar{\boldsymbol{y}}^{\boldsymbol{d,0}}, 1, \bar{\alpha}, \bar{\beta})$. Otherwise, the problem cannot be unbounded for any other $\phi \neq 1$ either and we can conclude that $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \in P$. Let $(\boldsymbol{x^{d*}}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}), \boldsymbol{y^{d*}}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}), \alpha^*(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}), \beta^*(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}))$ be an optimal solution to **(BDSP($\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}$))**. We can rewrite equation (4) as:

$$
S_L(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \geq S_L(\boldsymbol{x^{d*}}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}), \boldsymbol{y^{d*}}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})) - \alpha^*(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})\overline{TU}_{pab} - \beta^*(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})\overline{TU}_{prb} \tag{5}
$$

Any solution to **(MP)** must satisfy inequality (5) if it leads to a feasible solution to **(SMILP-GU)** with upper bounds $\overline{TU}_{pab}$ and $\overline{TU}_{prb}$ on the market loss and market missed surplus, respectively. We next construct the necessary condition, $\overline{TU}_{pab} = \overline{TU}_{prb} = 0$, to have market equilibrium.

Proposition 1. *Let $\overline{TU}_{pab} = \overline{TU}_{prb} = 0$ and $\boldsymbol{\omega} \in \Omega$. Then, $\boldsymbol{\rho} \in P$ is optimal to **(SMLP)**.*

Proof. Let $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) = \boldsymbol{\rho} \in P$ and consider **(BDSP($\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}$))**. Since $\overline{TU}_{pab} = \overline{TU}_{prb} = 0$, $\alpha \geq 1 - y_b^{d,1}, \forall b \in B_1$ and $\beta \geq y_b^{d,0}, \forall b \in B_0$ constraints become redundant and can be removed from the problem together with α and β variables. Then, **(BDSP($\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}$))** is equivalent to **(SMLP)**. This implies that $S_L(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \geq S_L(\boldsymbol{x^*}, \boldsymbol{y^*}), \forall \boldsymbol{\rho} = (\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \in P$, where $(\boldsymbol{x^*}, \boldsymbol{y^*})$ is the optimal solution of **(SMLP)**. ■

Proposition 1 implies that the MO can achieve equilibrium only when the duality gap of **(SMILP)** is zero. Otherwise, there exists a positive market loss or a market missed surplus for the optimal allocation, and the optimal allocation is not surplus maximizing.

For any $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}) \in Z$ such that $\hat{\boldsymbol{y}} = \bar{\boldsymbol{y}}$, $S_L(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}) = S_L(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})$ and the optimal objective function value of **(BDSP($\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}$))** is equal to that of **(BDSP($\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}$))**. Hence, if $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \notin P$ the following inequality is valid for **(SMILP-GU)**.

$$\sum_{b \in B_1} (1 - y_b) + \sum_{b \in B_0} y_b \geq 1 \tag{6}$$

Inequality (6) ensures that for a feasible solution of **(SMILP-GU)**, at least one of the rejected block bids at $\bar{\boldsymbol{y}}$ must be accepted or at least one of the accepted block bids at $\bar{\boldsymbol{y}}$ must be rejected. These so-called "no-good" cuts have been frequently used in the literature for solving MILP problems, and Martin et al. (2014) and Madani & Van Vyve (2015) also use these cuts in their proposed BD algorithms to solve **(SMILP-NoPAB)**.

*4.2. Solving **(SMILP-NoPAB)***

In this section, we examine a special case of **(SMILP-GU)** in which $\overline{TU}_{pab} = 0$ and $\overline{TU}_{prb} > M$. At a feasible solution of this problem, there are no PABs so that every bidder will be settled with the same unit energy price, the MCP.

Reconsidering **(BDSP($\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}$))**, let $(\boldsymbol{x^{d*}}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}), \boldsymbol{y^{d*}}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}), \alpha^*(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}), \beta^*(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}))$ be an optimal solution to **(BDSP($\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}$))**. $\beta^*(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) = 0$ when $\overline{TU}_{prb} > M$. As a result, $\boldsymbol{y^{d,0*}}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) = \mathbf{0}$. In addition, $\boldsymbol{y^{d,1*}}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})$ is not restricted by $\alpha^*(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})$ since $\overline{TU}_{pab} = 0$. Using these properties, we can simplify **(BDSP($\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}$))** as follows:

$$
\begin{aligned}
\textbf{(BDSP}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})\textbf{-NoPAB)}: \quad & \text{Max} \quad S_L(\boldsymbol{x}^d, \boldsymbol{y}^d) \\
& \text{s.to.} \quad x_h^d \leq 1 && \forall h \in H \\
& \qquad y_b^{d,1} \leq 1 && \forall b \in B_1 \\
& \qquad y_b^{d,0} = 0 && \forall b \in B_0 \\
& \qquad \sum_{h \in H} q_{h,t} x_h^d + \sum_{b \in B_1} q_{b,t} y_b^{d,1} = 0 && \forall t \in T \\
& \qquad x_h^d, y_b^{d,1} \geq 0
\end{aligned}
$$

For **(SMILP-NoPAB)**, $S_L(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \leq S_L(\boldsymbol{x}^{\boldsymbol{d}*}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}), \boldsymbol{y}^{\boldsymbol{d}*}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}))$ since $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \in Z$ is a feasible solution to **(BDSP($\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}$)-NoPAB)**. If $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \in P$, $S_L(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \geq S_L(\boldsymbol{x}^{\boldsymbol{d}*}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}), \boldsymbol{y}^{\boldsymbol{d}*}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}))$ must be satisfied, and this implies that $S_L(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) = S_L(\boldsymbol{x}^{\boldsymbol{d}*}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}), \boldsymbol{y}^{\boldsymbol{d}*}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}))$, $\forall (\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \in P$.

In Madani & Van Vyve (2015), a strengthened version of inequality (6), inequality (7), is shown to be valid in the sub-tree associated with the node solution $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})$ of **(MP)**. Hence, inequality (7) can be added to each node of the sub-tree if solution $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})$ is infeasible for **(SMILP-NoPAB)**.

$$\sum_{b \in B_1} (1 - y_b) \geq 1 \tag{7}$$

For a more general case, Madani & Van Vyve (2018) extend (7) into a globally valid inequality when the optimal objective function value of **(BDSP($\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}$)-NoPAB)** is greater than that of **(SMILP)**. Given $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}) \in Z$ and $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ is an optimal solution to **(SMILP)**, they show that if $S_L(\boldsymbol{x}^*, \boldsymbol{y}^*) < S_L(\boldsymbol{x}^{\boldsymbol{d}*}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}), \boldsymbol{y}^{\boldsymbol{d}*}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}))$, then $\sum_{b \in B_1} (1 - y_b) \geq 1$ is a globally valid inequality for **(SMILP-NoPAB)**.

Solving **(SMILP)** before solving **(SMILP-NoPAB)** and incorporating inequality (7) as a globally-valid inequality could be beneficial if **(SMILP)** is an easy problem to solve.

*4.3. Solving **(SMILP-NoPRB)***

Surplus maximization with no PRB is the current market rule in the Turkish market (EXIST, 2016a). The PABs are settled from the bid prices instead of the market clearing prices through the uplift payments so that their "loss" is fully compensated. This creates a budget deficit for the MO, but supports equilibrium in the market. There is no foregone opportunity for any bidder and the allocations made under the MCPs and uplift payments not only maximize the market surplus but also maximize the surplus of individual bidders.

There are several algorithms developed to solve the Turkish DAM clearing problem (see for example Derinkuyu, 2015; Yörükoğlu et al., 2018; Derinkuyu et al., 2019). In order to address the no PRB problem, they integrate the price variables into the surplus maximization that leads to bigM formulations or nonlinear programs. To the best of our knowledge, our study is the first to employ a BD algorithm to solve this problem.

We can generalize the BD algorithm in order to solve **(SMILP-NoPRB)**. The basic combinatorial cuts of the form (6) are also valid for this problem as they only eliminate the current integer variable vector from the feasible space of block bid decisions. We can rewrite the dual subproblem, $\mathbf{(BDSP(\bar{\boldsymbol{x}},\bar{\boldsymbol{y}}))}$, for this case as follows:

$$
\begin{aligned}
\textbf{(BDSP}(\bar{\boldsymbol{x}},\bar{\boldsymbol{y}})\textbf{-NoPRB)}: \quad & \text{Max} \quad S_L(\boldsymbol{x^d},\boldsymbol{y^d}) \\
& \text{s.to.} \quad x_h^d \le 1 && \forall h \in H \\
& \qquad\quad y_b^{d,1} = 1 && \forall b \in B_1 \\
& \qquad\quad y_b^{d,0} \le 1 && \forall b \in B_0 \\
& \qquad\quad \sum_{h\in H} q_{h,t} x_h^d + \sum_{b\in B_1} q_{b,t} y_b^{d,1} = 0 && \forall t \in T \\
& \qquad\quad x_h^d, y_b^{d,0} \ge 0
\end{aligned}
$$

Suppose that $(\bar{\boldsymbol{x}},\bar{\boldsymbol{y}}) \notin P$. In this case, similar to the logic in the no PAB case, one cannot get a feasible solution to **(SMILP-NoPRB)** in a sub-tree of a $(\bar{\boldsymbol{x}},\bar{\boldsymbol{y}})$ unless at least one of the rejected block bids is accepted in the new solution. Hence, cut (8) is a valid inequality in the subtree of the node associated with $(\bar{\boldsymbol{x}},\bar{\boldsymbol{y}})$.

$$\sum_{b\in B_0} y_b \ge 1 \tag{8}$$

4.4. Network-constrained markets

We can extend the BD algorithm to handle the market coupling as well. European DAMs have undergone a coupling process in the last decade with the intention of creating a single "pan-european" power market (Euphemia, 2019). To account for the constraints imposed by the capacity of the transmission network elements on the flow of electricity, European power exchanges have adopted the *zonal-pricing* methodology. In this methodology, the capacity constraints for only a set of critical transmission lines are included in the DAM clearing problem. The transmission system operator defines *bidding zones* that are separated by those critical transmission lines. The configuration of the bidding zones are to be determined in such a way that the intra-zonal transmission capacity constraints are non-binding for any possible production-consumption schedule in the day-ahead stage. In case the inter-zonal transmission lines are congested, the DAM clearing prices may differ between bidding zones.

The algorithm that is used to clear the European single DAM, Euphemia, embeds a network model in which each bidding zone is a node and the nodes are connected to each other via *arcs*. In the early days of the market coupling, the flow capacity on each arc was determined making sure that the energy imbalance in any node is equal to the sum of the flows on the arcs connected to that node. This model is known as the Available Transmission Capacity (ATC) model. This model totally ignored the physical laws of electricity flow, and was replaced by the *Flow-Based* model (FB) that approximates the physical flow of electricity between the nodes better. In this model, power transmission distribution factors (PTDFs) that disclose the marginal energy change on an arc due to a unit energy exchange between two nodes are used. These factors can take different values for different arcs associated with a node, in contrast with the all-equal assumption of the ATC model. Nevertheless, both network designs can be modelled as a set of linear constraints and can be embedded in the surplus maximization problem as follows:

$$
\textbf{(SMILP-MultiNode)}: \quad \begin{aligned}
\text{Max} \quad & \sum_{n \in N} \sum_{t \in T} \left\{ \sum_{h \in H} p_h q_{h,n,t} x_h + \sum_{b \in B} p_b q_{b,n,t} y_b \right\} & \\
\text{s.to.} \quad & \sum_{h \in H} q_{h,n,t} x_h + \sum_{b \in B} q_{b,n,t} y_b + \delta_{n,t} = 0 & \forall n \in N, \forall t \in T \\
& \sum_{n \in N} \sigma^a_{n,t} \delta_{n,t} \leq C^a_t & \forall a \in A, \forall t \in T \\
& \sum_{n \in N} \delta_{n,t} = 0 & \forall t \in T \\
& x_h \leq 1 & \forall h \in H \\
& x_h \geq 0 & \forall h \in H \\
& y_b \in \{0,1\} & \forall b \in B
\end{aligned}
$$

In this model, $\delta_{n,t}$ represents the net energy export of node n at time period t and $\sigma^a_{n,t}$ is the PTDF associated with node n, time period t, and arc a. The total energy flow induced on arc a is calculated as $\sum_{n \in N} \sigma^a_{n,t} \delta_{n,t}$, and is restricted by arc capacity C^a_t.

The network capacity constraints presented above define a convex set and preserve the convexity of the surplus maximization problem. Complementary slackness constraints can be satisfied by the net energy export variables for any given $\bar{\boldsymbol{y}}$, similar to the case of hourly bids, and therefore the net energy export values and the zonal prices $\pi_{n,t}$ will be at equilibrium for any set of accepted block bids. For example, under the ATC model, the price in the exporting node is at most as big as the price in the importing node, and the prices of two nodes connected by an arc must be equal if the arc capacity constraint is non-binding.

The structure of the bounded dual subproblem is the same as that of **(SMILP-MultiNode)** except for the additional variable bounds imposed by $\bar{\boldsymbol{y}}$ found by the master problem. Since the net energy export variables are continuous, it can be shown that the Benders cuts are directly applicable for this case as well.

5. New valid inequalities: price-based cuts

In this section, we develop new valid inequalities and incorporate them into the BD algorithm discussed in the previous section. We solve the master problem employing a MIP solver. During the branching process, we activate a lazy-constraint callback function as soon as an integer feasible solution is obtained at a node of the search tree. The callback function solves the subproblem associated with the obtained integer feasible solution, adding the newly developed cuts whenever the subproblem indicates that the solution is not feasible to the original problem.

5.1. The ***(SMILP-NoPAB)*** case

Let B^s and B^d be the set of supply and demand block bids, respectively. Let indicator variable $\delta_{b,\hat{b}} = 1$ denote that the block bids b and $\hat{b}$ have at least one common period, and 0 otherwise. For each $\hat{b}$, we add inequality (9), if it is a supply PAB, and inequality (10), if it is a demand PAB, to **(MP)**.

$$(1 - y_{\hat{b}}) + \sum_{b \in B^s : \bar{y}_b = 1, \delta_{b,\hat{b}} = 1} (1 - y_b) + \sum_{b \in B^d : \bar{y}_b = 0, \delta_{b,\hat{b}} = 1} y_b \geq 1 \tag{9}$$

$$(1 - y_{\hat{b}}) + \sum_{b \in B^s : \bar{y}_b = 0, \delta_{b,\hat{b}} = 1} y_b + \sum_{b \in B^d : \bar{y}_b = 1, \delta_{b,\hat{b}} = 1} (1 - y_b) \geq 1 \tag{10}$$

The idea here is that once we have a supply (demand) PAB at the new incumbent solution obtained at a node of the master problem branch-and-bound tree, we must either reject this bid or increase (decrease) the average MCP of the periods for which this bid is offered. In order to increase (decrease) the average MCP for those periods, we must either reject at least one accepted supply (demand) block bid in one of those periods or accept at least one rejected demand (supply) block bid in one of those periods.

If there are alternate optimal solutions on the MCP vector, then a block bid may turn out to be PAB or not, depending on which of those alternate optimal solutions is chosen. In this case, there may not be clarity on the PAB status of a block bid. On the other hand, if the MCP vector is unique, we can exactly determine whether a block bid is PAB or not. To identify if an MCP vector is unique, we solve $(\mathbf{SMLP}(\bar{\boldsymbol{y}}))$. The resulting MCP vector is equal to the optimal value of the dual variable vector, $\boldsymbol{\pi}^{*}$. If the optimal solution to $(\mathbf{SMLP}(\bar{\boldsymbol{y}}))$ is non-degenerate, then the optimal dual solution, and hence the MCP vector, is unique, and we can identify the PAB status of a block bid. Therefore, we generate a price-based cut for $\hat{b}$ only if the MCP vector is unique over $T_{\hat{b}}$, as identified from the optimal solution of $(\mathbf{SMLP}(\bar{\boldsymbol{y}}))$.

In order to check the degeneracy, we decompose the multi-period problem, $(\mathbf{SMLP}(\bar{\boldsymbol{y}}))$, into single period problems since all block bid decisions are fixed in $(\mathbf{SMLP}(\bar{\boldsymbol{y}}))$ and define the following LP for each period $t \in T$:

$$
\begin{aligned}
(\mathbf{SMLP}(\bar{\boldsymbol{y}}, t)): \quad \text{Max} \quad & \sum_{h \in H_t} p_h q_{h,t} x_h \\
\text{s.to.} \quad & \sum_{h \in H_t} q_{h,t} x_h = -Q_{\bar{\boldsymbol{y}},t} \quad [\pi_t] \\
& 0 \le x_h \le 1, \quad \forall h \in H_t
\end{aligned}
$$

where $Q_{\bar{\boldsymbol{y}},t} = \sum_{b \in B} q_{b,t} \bar{y}_b$ represents the total accepted block bid quantities in period t associated with the block bid decision vector $\bar{\boldsymbol{y}}$ and $H_t = \{h \in H : q_{h,t} \neq 0\}$ such that $H_{t'} \cap H_{t''} = \emptyset$, for $t' \neq t''$, as hourly bids are given only for a single period. We denote the optimal solution by $\boldsymbol{x^{*}(t)}$. If $\nexists h \in H_t$ such that $0 < x^{*}_{h}(t) < 1$, then $x^{*}_{h}(t) \in \{0, 1\}$, $\forall h \in H_t$. In this case, the problem has to have one basic variable that has to be either at its upper bound or its lower bound. Hence, $\boldsymbol{x^{*}(t)}$ is a degenerate optimal solution to $(\mathbf{SMLP}(\bar{\boldsymbol{y}}, t))$ and there may exist alternative MCPs, π^{*}_{t}, if $\nexists h \in H_t$ such that $0 < x^{*}_{h}(t) < 1$.

Corollary 1. *π^{*}_{t} is unique if and only if there exists an optimal solution, $\boldsymbol{x^{*}(t)}$, to $(\mathbf{SMLP}(\bar{\boldsymbol{y}}, t))$ such that $0 < x^{*}_{h}(t) < 1, \exists h \in H_t$.*

Proof. Part 1. If π^{*}_{t} is unique, then there must exist a non-degenerate optimal solution to $(\mathbf{SMLP}(\bar{\boldsymbol{y}}, t))$. For a non-degenerate optimal solution $(\mathbf{SMLP}(\bar{\boldsymbol{y}}, t))$, at least one of the hourly bids must be partially accepted $0 < x^{*}_{h}(t) < 1, \exists h \in H_t$.

Part 2. Let $\bar{h} \in H_t : 0 < x^{*}_{\bar{h}}(t) < 1$. Then, by the complementary slackness, $(\mathbf{CS\text{-}SMLP}(\bar{\boldsymbol{y}}))$, $s^{*}_{\bar{h}} = 0$ and $\pi^{*}_{t} = p_{\bar{h}}$. Hence, π^{*}_{t} is unique. ∎

Using Corollary 1, we determine if the MCP vector corresponding to $\bar{\boldsymbol{y}}$ is unique. After solving $(\mathbf{SMLP}(\bar{\boldsymbol{y}}, t))$, if the optimal solution returned by the solver has partially accepted hourly bids, we conclude that π_t^* is unique. If not, we check if an alternate optimal solution with partially accepted hourly bids exists in period t. An alternate optimal solution with partially accepted hourly bids exists if one of the following two conditions holds: 1) the prices of the lowest-priced rejected supply hourly bid and the highest-priced rejected demand bid are equal 2) the prices of the highest-priced accepted supply hourly bid and the lowest-priced accepted demand bid are equal. We show in Proposition 2 that inequality (9) are both valid and stronger than the "no-good" cuts under unique $\boldsymbol{\pi^*}$. It is straightforward to show the same results for inequalities (10) for demand PABs with a similar proof.

Proposition 2. *Let $\hat{b} \in B$ be a supply PAB for unique MCPs $\pi_t^* \in T_{\hat{b}}$ associated with $\bar{\boldsymbol{y}}$. Then, inequality (9) is valid for* **(SMILP-NoPAB)** *and stronger than the "no-good" cuts.*

Proof. Part 1. We first show that inequality (9) is a valid inequality for **(SMILP-NoPAB)**. If a supply bid $\hat{b}$, $\boldsymbol{q}_{\hat{b}} < 0$, is PAB for $\boldsymbol{\pi^*}$, then $\bar{l}_{\hat{b}} = -\sum_{t\in T}(p_{\hat{b}} - \pi_t^*)q_{\hat{b},t} > 0$. On the other hand, $l'_{\hat{b}} = 0$ for $\boldsymbol{\omega'} \in \Omega$. We check if every $\boldsymbol{\omega'} \in \Omega$ satisfies inequality (9) by considering the following two cases:

Case 1. If $y'_{\hat{b}} = 0$, $(1 - y'_{\hat{b}}) = 1$ and inequality (9) hold since $\sum_{b\in B^s:\bar{y}_b=1,\delta_{b,\hat{b}}=1}(1 - y'_b) + \sum_{b\in B^d:\bar{y}_b=0,\delta_{b,\hat{b}}=1} y'_b \geq 0$ for any $\boldsymbol{y'}$.

Case 2. If $y'_{\hat{b}} = 1$, $l'_{\hat{b}} = 0$ and $-\sum_{t\in T}(p_{\hat{b}} - \pi'_t)q_{\hat{b},t} \leq 0 < \bar{l}_{\hat{b}} = -\sum_{t\in T}(p_{\hat{b}} - \bar{\pi}_t)q_{\hat{b},t}$. That is, $\sum_{t\in T}\pi'_t q_{\hat{b},t} < \sum_{t\in T}\pi_t^* q_{\hat{b},t}$. This implies that there must exist $\hat{t} \in T_{\hat{b}}$ such that $\pi'_{\hat{t}} > \pi^*_{\hat{t}}$ since $q_{\hat{b},t} \leq 0, \forall t \in T_{\hat{b}}$.

Let $Q_{\boldsymbol{y'},\hat{t}} = \sum_{b\in B} q_{b,\hat{t}}y'_b$ and $Q_{\bar{\boldsymbol{y}},\hat{t}} = \sum_{b\in B} q_{b,\hat{t}}\bar{y}_b$ be the total accepted block bid quantities in period $\hat{t} \in T$. Similarly, let $Q_{\boldsymbol{x'},\hat{t}} = \sum_{h\in H} q_{h,\hat{t}}x'_h$ and $Q_{\bar{\boldsymbol{x}},\hat{t}} = \sum_{h\in H} q_{h,\hat{t}}\bar{x}_b$ be the total accepted hourly bid quantities in period $\hat{t} \in T$ for $\boldsymbol{x'}$ and $\bar{\boldsymbol{x}}$, respectively. Since $\pi^*_{\hat{t}}$ is unique, there exists a $\bar{h} \in H_{\hat{t}}$ such that $0 < \bar{x}_{\bar{h}} < 1$ by Corollary 1. For $\pi'_{\hat{t}} > \pi^*_{\hat{t}}$, $x'_{\bar{h}} = 1$ if $\bar{h}$ is a supply hourly bid, $x'_{\bar{h}} = 0$ if it is a demand hourly bid. Then, $Q_{\boldsymbol{x'},\hat{t}} < Q_{\bar{\boldsymbol{x}},\hat{t}}$. Since $Q_{\boldsymbol{y'},\hat{t}} + Q_{\boldsymbol{x'},\hat{t}} = 0$ and $Q_{\bar{\boldsymbol{y}},\hat{t}} + Q_{\bar{\boldsymbol{x}},\hat{t}} = 0$, $Q_{\boldsymbol{x'},\hat{t}} < Q_{\bar{\boldsymbol{x}},\hat{t}}$ implies that $Q_{\boldsymbol{y'},\hat{t}} > Q_{\bar{\boldsymbol{y}},\hat{t}}$. This can only be achieved if either at least one accepted supply block bid covering period $\hat{t}$ is rejected or at least one rejected demand block bid covering period $\hat{t}$ is accepted. That is, $\sum_{b\in B^s:\bar{y}_b=1,\delta_{b,\hat{b}}=1}(1 - y_b) + \sum_{b\in B^d:\bar{y}_b=0,\delta_{b,\hat{b}}=1} y_b \geq 1$.

Either *Case 1* or *Case 2* must hold for every feasible solution of **(SMILP-NoPAB)**. This proves that inequality (9) is a valid inequality for **(SMILP-NoPAB)**.

Part 2. We next show that inequality (9) is stronger than inequality (6). For any $\boldsymbol{y'}$, the left-hand side of inequality (6) is at least as large as the left-hand side of inequality (9). If $\boldsymbol{y'}$

violates inequality (9), it also violates inequality (6). However, the reverse is not true when none of the accepted supply bids at $\bar{\boldsymbol{y}}$ covering a period in $T_{\hat{b}}$ is rejected at $\boldsymbol{y'}$, or none of the rejected demand bids at $\bar{\boldsymbol{y}}$ covering a period in $T_{\hat{b}}$ is accepted at $\boldsymbol{y'}$. ■

Proposition 3. *Let $\hat{b} \in B$ be a demand PAB for unique MCPs $\pi^*_t \in T_{\hat{b}}$ associated with $\bar{\boldsymbol{y}}$. Then, inequality (10) is valid for **(SMILP-NoPAB)** and stronger than the "no-good" cuts.*

Proof. Proof is similar to that of Proposition 2. ■

We summarize our algorithm in Figure 1. The left box shows the flow of operations inside our lazy constraint callback function. The flow starts by solving the subproblem for the integer solution under consideration. If the solution satisfies the pricing constraints, it becomes the new incumbent solution and we update the lower bound, LB. Otherwise, we impose price-based cuts to eliminate the solution that violates the pricing constraints as demonstrated in the right box of Figure 1. More specifically, let $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})$ be the integer feasible solution under consideration. We solve **(SMLP($\bar{\boldsymbol{y}}, t$))**, $\forall t \in T$, to find the MCPs, $\boldsymbol{\pi^*}$, and to identify the resulting PABs. In case there are no PABs, implying that there exist alternative MCPs, we add cut (6). Otherwise, we add cut (9) or (10) depending on whether the PAB is a supply or a demand bid, respectively.

*5.2. The **(SMILP-NoPRB)** and **(SMILP-MultiNode)** cases*

We extend the cuts we developed for the **(SMILP-NoPAB)** case to the **(SMILP-NoPRB)** and **(SMILP-MultiNode)** cases.

In **(SMILP-NoPRB)**, under unique MCPs, once we have a supply (demand) PRB, $\hat{b}$, we must either accept this bid or decrease (increase) the average MCP of the periods for which this bid is offered. To decrease (increase) the average MCP for those periods, we must either accept at least one rejected supply (demand) block bid or reject at least one accepted demand (supply) block bid in one of those periods. The logic in this case is very similar to that of the **(SMILP-NoPAB)** case. Hence, the following price-based cuts are valid for **(SMILP-NoPRB)**:

$$y_{\hat{b}} + \sum_{b \in B^s : \bar{y}_b = 0, \delta_{b,\hat{b}} = 1} y_b + \sum_{b \in B^d : \bar{y}_b = 1, \delta_{b,\hat{b}} = 1} (1 - y_b) \geq 1 \tag{11}$$

$$y_{\hat{b}} + \sum_{b \in B^s : \bar{y}_b = 1, \delta_{b,\hat{b}} = 1} (1 - y_b) + \sum_{b \in B^d : \bar{y}_b = 0, \delta_{b,\hat{b}} = 1} y_b \geq 1 \tag{12}$$

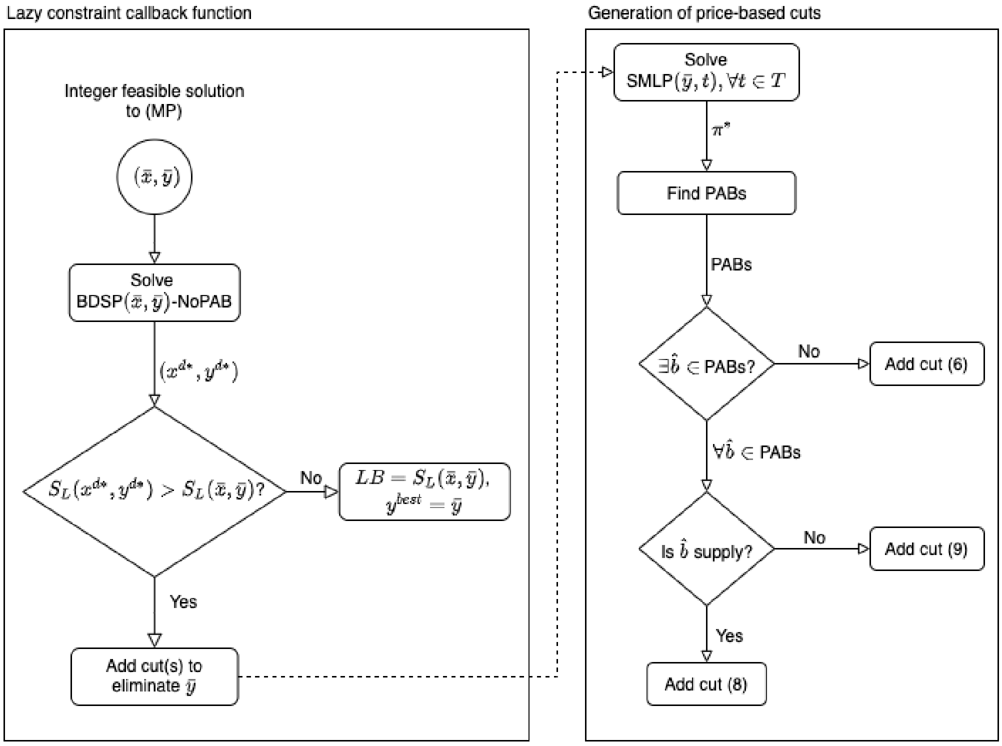

Figure 1: Flow in our lazy constraint callback function (left) and generation of price-based cuts (right)

As discussed before, the condition for infeasible subproblems in the BD algorithm does not change for the **(SMILP-MultiNode)** problem. The price-based cuts are still valid for eliminating an infeasible integer solution provided that the market clearing prices are unique. However, the existence of a partially accepted hourly bid is not a necessary condition for having a unique MCP for each period anymore. Consider a period where all capacity constraints are non-binding and the MCP for a node is unique. In this case, the MCPs for all other nodes are also unique due to the complementary slackness conditions between the capacity constraints and the MCPs. If, on the other hand, there are binding capacity constraints, we need to consider whether the net energy exports of the nodes are unique or not. If they are unique, the multi-node problem can be decomposed into multiple single-node problems. In each single node problem, the net energy export value of the node is captured in the right-hand-side of the corresponding supply-demand balance constraint. Then, the uniqueness of MCPs in each of the created single-node problems can be identified using Corollary 1. When the net energy exports are not unique, it is not straightforward to check the uniqueness of MCPs and the validity of the price-based cuts. When this is the case, we may revert to using cut (6).

6. More sophisticated bid types

In the previous sections, we concentrated on hourly and block bids in order to keep the presentation simple. We now show that the BD algorithm with all the cuts reviewed in Section 4 and the price-based cuts we developed in Section 5 are valid for the European DAM clearing problem under the existence of piece-wise linear hourly bids, linked block bids, exclusive block bids and flexible bids.

If there are piece-wise linear hourly bids in the market, the market surplus function becomes a quadratic function. We denote the set of such hourly bids with H^q. Let p_h^0 and p_h^1 be the starting and ending prices, respectively, for an hourly bid such that $p_h^1 > p_h^0$ for supply bids and $p_h^0 > p_h^1$ for demand bids. Then, equation (13) gives the market surplus function:

$$S_Q(\boldsymbol{x}, \boldsymbol{y}) = S_L(\boldsymbol{x}, \boldsymbol{y}) + \sum_{t \in T} \sum_{h \in H^q} p_h^0 q_{h,t} x_h + \frac{1}{2}(p_h^1 - p_h^0) q_{h,t} x_h^2 \tag{13}$$

In this case, the surplus maximization problem is a mixed-integer quadratic program (MIQP) instead of a MILP. Once the integrality requirements are relaxed, strong duality still applies as the problem is still convex. Similarly, $(\mathbf{BDSP}(\boldsymbol{x}, \boldsymbol{y}))$ becomes an MIQP as well for any $(\boldsymbol{x}, \boldsymbol{y}) \in Z$. Essentially, nothing changes apart from the type of the master and subproblem solved in the BD algorithm, and all the cuts are still valid. We should also note that if all the hourly bids are piece-wise linear, then the corresponding MCPs will be unique since the objective function becomes strictly convex with respect to MCPs. This ensures that any existing PABs or PRBs in a solution can be uniquely determined. On the other hand, the PD formulation becomes a mixed integer quadratically-constrained program (MIQCP) and it is harder to solve compared to MILPs or MIQPs.

If block bid $\hat{b}$ is linked to block bid b, then $\hat{b}$ can only be accepted when b is accepted. This can be modelled by adding $y_{\hat{b}} - y_b \leq 0$ to the constraint set for each such linked block bid pair. In this relation, we refer block bids b and $\hat{b}$ as the parent and the child bids, respectively. A parent bid can have multiple child bids but the reverse is not possible. Also, a parent bid can be a child bid of another parent. A parent bid with its all descendant bids form a link family. Similarly, if E represents a set of exclusive block bids in the same group, then we need to add $\sum_{b \in E} y_b \leq 1$ to the model for each such exclusive block bid group. A flexible bid can also be regarded as a set of exclusive block bids, e.g., creating as many single period block bids as the number of periods and adding a constraint to ensure that at most one of them can be accepted. We can generalize these constraints by assuming a constraint of the form:

$$\sum_{b \in B} \boldsymbol{a}_b y_b \leq \boldsymbol{e} \tag{14}$$

Assuming there are m such constraints, $\boldsymbol{a}_b \in \mathbb{Z}^m$ with entries $a_{b,i} \in \{-1, 0, 1\}, i = 1, \ldots, m$, and $\boldsymbol{e} \in \mathbb{Z}^m_{\geq 0}$ where each entry is either 0 or 1, $e_i = \{0, 1\}, \forall i = 1, \ldots, m$. Following similar steps to those in Section 4 for the surplus maximization problem with the additional constraint (14), we end up with the following bounded dual subproblem for the no-PAB case:

$$
\begin{aligned}
\textbf{(BDSP}(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})\textbf{-NoPAB)}: \quad & \text{Max} \quad S_L(\boldsymbol{x^d}, \boldsymbol{y^d}) \\
& \text{s.to.} \quad x_h^d \leq 1 && \forall h \in H \\
& \qquad y_b^{d,1} \leq 1 && \forall b \in B_1 \\
& \qquad y_b^{d,0} = 0 && \forall b \in B_0 \\
& \qquad \sum_{b \in B_1} \boldsymbol{a}_b y_b^{d,1} \leq \boldsymbol{e} \\
& \qquad \sum_{h \in H} q_{h,t} x_h^d + \sum_{b \in B_1} q_{b,t} y_b^{d,1} = 0 && \forall t \in T \\
& \qquad x_h^d, y_b^{d,1} \geq 0
\end{aligned}
$$

In this case, **(BDSP($\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}$)-NoPAB)** has additional constraint $\sum_{b \in B_1} \boldsymbol{a}_b y_b^{d,1} \leq \boldsymbol{e}$. All the valid inequalities ((6), (7), (9), (10)) presented above are still valid. However, the definition of PAB changes slightly for a parent block bid that has its child block bids. An accepted parent block bid is PAB only if the surplus generated by the acceptance of this bid and all accepted descendant bids is negative. Hence, a parent bid can be accepted as long as the link family generates non-negative surplus. In contrast, a child block bid cannot be accepted if its acceptance generates negative surplus even though its parent bid generates sufficient surplus to cover the loss of the child. That is, as stated in Euphemia (2019), "the child can save the parent with its surplus, but not the opposite". This is because of the additional non-positive terms ($a_{b,i} = -1$ if b is the parent bid in link constraint i) in the dual feasibility constraint related to accepted parent block bid b, and hence, $l_b = 0$ is feasible even if $\sum_{t \in T} (p_b - \pi_t) q_{b,t} < 0$ (see Appendix C for the updated PD model).

7. Computational Results

In this section, we present the computational results for the approaches from the literature as well as our approach. In particular, we evaluate the surplus maximization problem with no PAB, **(SMILP-NoPAB)**, employing:

1. Primal-dual formulation: **PD**
2. Benders decomposition with
    (a) *No-Good* cut, cut (6), used in Martin et al. (2014): **BD-NG**
    (b) *Locally Valid* cut, cut (7), used in Madani & Van Vyve (2015): **BD-LV**
    (c) *Price-Based* cuts we developed, cuts (9) and (10): **BD-PB**

We run our models using two leading commercial solvers available, IBM ILOG Cplex and Gurobi to assess the impact of the MIP solver on the performance of the solution methods, if any. Our aim is to reveal the best-performing solution approach to solve the DAM clearing problem with no-PAB constraints and to investigate whether the performances of the approaches depend on the solver used. We also compare the BD using the price-based cuts we developed with the available BD algorithms in the literature for the problem we address.

We run each solution approach on a test set of 20 instances generated based on the real market data published by EXIST on the transparency platform (EXIST, 2016c). EXIST publishes the full set of hourly bids submitted to the auction daily. In order to create representative bids, we take 20 separate instances from different months of 2017 and 2018. In terms of block bids, EXIST only publishes total supply and total demand volumes of block bids accepted and rejected on an hourly basis. To generate the block bids for the selected days, we use market report EXIST (2018) and the aggregate data to approximate the real block bid data as discussed below.

The yearly market statistics report, (EXIST, 2018), contains the daily average number of supply and demand block bids in the auction as well as the share of the block bid volume in the total volume. Additionally, there are market rules limiting the price, the quantities, and the number of periods of the bids that can be submitted into the auction (EXIST, 2016b). We generate block bids for our experiments maintaining all these properties. For each instance of our experiments, we generate 15,000 hourly and 150 block bids that resemble the actual bids.

We programmed our solution approaches in Python 3.5 and used Python APIs of the solvers to create the models and solve the instances. In the experiments, we used IBM ILOG Cplex

12.8.0 and Gurobi 8.0.1. We conducted the tests on a MacBook Pro with 2.6 GHz Intel Core i5 processor and 8 GB 1600 MHz DDR3 memory with a time limit of 600 seconds and 10^{-6} relative MIP gap tolerance. In the Turkish DAM instances, the average market surplus is around 5×10^8 Turkish liras. A solution within 10^{-6} relative gap will have market surplus within 500 Turkish liras (less than 100 USD as of 2020) of the maximum surplus on average, which is an immaterial amount within the context of this problem. In order to examine the impact of parallel tree search on the performances of the approaches, we conducted tests with both single and multi-thread (4 threads that were available in the computation environment) settings of the solvers. We could have conducted experiments on multiple threads only but some solver-method combinations do not work in this case and the comparisons would not have been comprehensive.

Madani & Van Vyve (2015) report favorable computational performance in exploring the zero values of the binary variables first in the branching strategy for problems similar to ours. As fine-tuning the parameter settings of the solvers is beyond the scope of this study, we did not explore different strategies and used the default configurations of all the parameters, except from the MIP gap tolerance and the number of threads mentioned above. We provide our source codes and test instances on GitHub (`https://github.com/gokhanceyhan/modam`).

We use the following abbreviations for the solution statuses of the problem instances in Tables 1-3:

- the number of cases an optimal solution (a feasible solution satisfying the relative gap tolerance) was found (*Opt*),
- the number of cases an optimal solution could not be found but a feasible solution (that does not satisfy the relative gap tolerance) was found (*Feas*), and
- the number of cases a feasible solution could not be found within the time limit (*Inf*).

In our results to follow, we display average and maximum run times as well as the relative gaps. Run time statistics are calculated only over the instances where the solution status is *Opt*, and relative gap statistics are calculated only over the instances where the solution status is *Feas* under both solvers.

We present the results of different approaches over 20 different instances of the surplus maximization problem under no-PAB constraints **(SMILP-NoPAB)** using two different solvers. We should note that in none of the instances did we encounter a case with alternative MCPs. In other words, all the integer solutions we found yielded unique MCPs, enabling the application of the new cuts we developed. The first part of Table 1 presents the results with single thread (Threads=1) execution of the solvers. When using the **PD** approach, we found an optimal solution in 17 and 14 of the 20 instances with Cplex and Gurobi, respectively. The 14 instances Gurobi solved to optimality turned out to be a subset of the 17 instances Cplex solved to optimality. Over those 14 instances, Cplex solved faster than Gurobi both on average and in the worst case (Cplex solved the remaining 3 instances to optimality in 446.56 seconds at most) and found higher quality solutions for the instances neither solver could solve to optimality. Gurobi could not find a feasible solution within the time limit for two of the instances, whereas Cplex found high quality feasible solutions (with relative gaps below 15×10^{-6}) for all three instances it could not solve to optimality. In multi-thread (Threads=4) execution of **PD**, the results are about the same.

For the BD-based approaches (**BD-NG**, **BD-LV**, and **BD-PB**) we present in Table 1, there are differences in the implementations of the solvers due to the differences in their interfaces. In Cplex, we solve the master problem and use a *LazyConstraint* callback to call the subproblem whenever an integer feasible solution is found at a node of the search tree. Cplex automatically disables some problem reduction routines when a lazy constraint callback is included. This is necessary since the model is not complete without the lazy constraints and the problem reductions on the incomplete model can cut off the true optimal solution. In addition, it defaults to single thread in the existence of lazy constraint callbacks since Cplex does not guarantee thread-safety of these types of callbacks. Hence, there is no straightforward multi-thread implementation of BD approaches with Cplex.

In Gurobi, we implement a *MIPSOL* callback to initiate the subproblem and add the cuts when necessary. Since Gurobi Python API does not provide the capability to add locally valid inequalities in a node of the branch-and-bound tree, we cannot present test results of inequality (7) on the Gurobi solver. We set *LazyConstraints=1* to disable problem reductions.

With **BD-NG**, the number of instances for which an optimal solution was found is roughly the same for the two solvers. Gurobi was substantially faster than Cplex in the 5 instances for which both solvers found an optimal solution. On another 3 instances where both solvers could only find feasible solutions, Gurobi was able to find slightly higher quality solutions compared to

Table 1: Comparison of PD and the BD-based approaches for (SMILP-NoPAB)

Threads	Approach	Solver	Solution			Run Time (secs)			Relative Gap ($\times 10^{-6}$)		
			Opt	Feas	Inf	BO[a]	Avg	Max	BF[b]	Avg	Max
Single	PD	Cplex	17	3	0	14	20.30	108.66	2	10	11
		Gurobi	14	4	2		76.67	341.86		18	24
	BD-NG	Cplex	5	15	0	5	90.33	449.08	3	86	245
		Gurobi	7	3	10		2.70	12.68		83	142
	BD-LV[c]	Cplex	11	9	0	11	60.42	254.61	9	11,320	27,893
	BD-PB	Cplex	19	1	0	18	56.48	320.29	1	33	33
		Gurobi	18	2	0		30.73	151.29		792	792
Multi[d]	PD	Cplex	16	4	0	15	21.53	79.78	2	3	3
		Gurobi	15	2	3		64.02	462.29		36	67
	BD-NG[e]	Gurobi	7	6	7	7	29.22	143.78	6	38	88
	BD-PB[e]	Gurobi	20	0	0	20	18.49	166.29	0	-	-

[a] BO (Both Optimal): Number of instances both solvers solved to optimality (*Opt* status)

[b] BF (Both Feasible): Number of instances both solvers found a feasible solution but neither reached optimality (*Feas* status)

[c] Gurobi results are not available as Gurobi MIPSOL callback API does not support adding local cuts.

[d] The number of threads is equal to 4.

[e] Cplex results are not available as Cplex disables parallel MIP search when there exist lazy constraint callbacks.

Cplex. However, Gurobi could not find a feasible solution in half of the instances. The quality of the solutions is rather poor when we consider all 15 instances Cplex found only a feasible solution for. The average and the maximum relative gaps are 1.74×10^{-3} and 1.13×10^{-2}, respectively, over these 15 instances.

We report only Cplex tests for **BD-LV** as Gurobi MIPSOL callback API does not support local cuts. Cplex found an optimal solution for 11 of the instances and found a feasible solution for the remaining 9 instances. The solution quality is rather poor for the 9 instances for which only a feasible solution could be generated.

With the **BD-PB** approach we developed, Cplex and Gurobi found an optimal solution in 19 and 18 of the 20 instances, respectively. The quality of the feasible solution Cplex found in the remaining instance was good with a relative gap of 33×10^{-6}. On the other hand, when we compare the solvers on 18 instances for which both solvers found an optimal solution, Gurobi outperforms Cplex substantially in the run time. Furthermore, when Gurobi is run with four threads, it solves all 20 instances to optimality under 20 seconds on average and about 3 minutes at maximum.

Among the BD-based approaches, we observe that **BD-NG** and **BD-LV** do not perform as well as **PD** in finding an optimal solution within the imposed time limit. However, **BD-PB** we developed outperforms all other approaches in terms of all performance measures. We observe that Gurobi is much faster than Cplex when BD is implemented, whereas the performance is flipped when **PD** is implemented, considering the problems solved to optimality in all cases.

To put the relative gaps into perspective, recall that 10^{-6} corresponds to a little under \$100, on average, and the problem is solved daily. Based on this, the worst case performance obtained with **BD-LV** represents a magnitude in the order of roughly $100 \times 27,893 = \$2,789,300$. The average absolute gap of **BD-LV** is about \$1,132,000 over the 9 problems for which it could find a feasible solution. The corresponding values for **BD-NG** with Cplex are around \$8,600 on average over the 3 problems and \$24,500 in the worst case. Similarly, for **BD-NG** with Gurobi and single-thread execution, the average and maximum relative gap over the 3 problems correspond to about \$8,300 and \$14,200.

In the computational tests presented above, the problem size represents the market size of the Turkish DAM. In order to test the performance of the algorithms for larger problem sizes, we create larger instances extrapolating the instances from the Turkish DAM. In order to create instances that are similar to central-western European market instances that have around 50,000 hourly bids and 600 block bids (used by Madani & Van Vyve (2015)), we merged four instances of the Turkish DAM. We generated 10 large instances in such a way that any pair of large instances have half their bids the same and the other half different.

In Table 2, we present the results of the created large instances for **PD** and **BD-PB**. We did not test the performances of **BD-NG** and **BD-LV** on large-sized problem instances as those performed poorly on the instances from the Turkish DAM. We run the solvers with four threads, and hence, only Gurobi results are available for **BD-PB**. Out of 10 instances, **BD-PB** was able to solve 8 instances to optimality compared to 6 instances by **PD** approach, with both solvers. Over the 5 instances solved to optimality under all settings, **BD-PB** solved the instances in a fraction of the time of **PD**. In addition to the average and maximum computational times, **BD-PB** outperforms **PD** in every single instance that was solved to optimality by both approaches. The remaining 3 instances solved to optimality by **BD-PB** took 152.40 seconds on average, and 441.11 seconds at maximum. The average and maximum relative gaps of the four feasible solutions generated by **PD** are 2×10^{-6} and 4×10^{-6} with Cplex and 6×10^{-6} and 9×10^{-6} with Gurobi. Whereas the relative gaps for the two feasible solutions generated by **BD-PB** are 2×10^{-6} and 3×10^{-6}, respectively.

Table 2: Comparison of PD and BD-PB for (SMILP-NoPAB) on a large instance set[a]

Approach	Solver	Solution			Run Time (secs)			Relative Gap (x 10^{-6})		
		Opt	Feas	Inf	AO[b]	Avg	Max	AF[c]	Avg	Max
PD	Cplex	6	4	0		96.82	239.03		6	6
	Gurobi	6	4	0	5	85.10	160.54	1	2	2
BD-PB[d]	Gurobi	8	2	0		11.51	49.61		3	3

[a] Executed with parallel MIP search utilizing 4 threads.

[b] AO (All Optimal): Number of instances all approach-solver combinations solved to optimality (*Opt* status)

[c] AF (All Feasible): Number of instances all approach-solver combinations found a feasible solution but neither reached optimality (*Feas* status)

[d] Cplex results are not available as Cplex disables parallel MIP search when there exist lazy constraint callbacks.

We observe from all results that **BD-PB** outperforms all other approaches both in terms of solution quality and computational performance. Its computational performance is a small fraction of its competitors, especially when solved using Gurobi.

We further test the performance of our algorithm incorporating additional block bid constraints and piece-wise linear hourly bids. We modified the above 20 instances and created two new instance sets each having 20 instances. In the first set, we add link constraints to approximately 20% of the existing block bids, on average, and add flexible bids that are 3% of the block bids in number. In the second set, we take the instances in the first set and transform all hourly bids that are in the form of step functions to piece-wise linear functions. We run both **BD-PB** and **PD** on the two instance sets separately with the Gurobi solver and report the results in Table 3.

BD-PB outperforms **PD** in terms of the number of MILP and MIQP instances solved to

Table 3: Comparison of PD and BD-PB for MILP and MIQP instances[a,b]

Problem	Approach	Solution			Run Time (secs)		
		Opt	Feas	Inf	BO[c]	Avg	Max
MILP	PD	17	1	2	17	47.08	450.59
	BD-PB	19	1	0		3.86	28.59
MIQP[c]	PD	7	0	13	7	182.77	505.95
	BD-PB	19	1	0		36.12	161.49

[a] Executed with parallel MIP solver of Gurobi 9.1.0 utilizing 4 threads.

[b] We did not include the relative gap results in this table since BD-PB finds an optimal solution for each instance PD can find only a feasible solution.

[c] BO (Both Optimal): Number of instances both approaches solved to optimality (*Opt* status)

[d] The problem becomes MIQCP in the PD formulation.

optimality as well as their run times. In the MILP instance set, **BD-PB** solves 19 of 20 instances to optimality. Based on the 17 instances both algorithms could solve to optimality, **BD-PB** takes an average of less than 4 seconds, and at most about 30 seconds. The relative gap of the only instance **BD-PB** could not solve to optimality within the time limit is 20×10^{-6}. On the other hand, **PD** cannot find any feasible solution to 2 of the 20 instances within the allocated time limit. Furthermore, the solution quality on the one problem for which **PD** found a feasible (but not optimal) solution is relatively poor (with a relative gap value of $1,637 \times 10^{-6}$) and solution times are orders of magnitude larger than those of **BD-PB**, based on the problems both algorithms solved to optimality.

The superiority of **BD-PB** to **PD** is even more pronounced in terms of solution quality on the MIQP instances, for which the latter fails to reach to a feasible solution in 13 of the 20 instances within the allocated time frame. In contrast, **BD-PB** finds the optimal solution in 19 of the instances and reaches a very high quality feasible solution in the remaining instance (with a relative gap value of 1.74×10^{-6}). As before, the solution times of **BD-PB** are also much better than those of **PD** in the 7 instances for which both algorithms found the optimal solution.

The **BD-PB** MILP results in Table 3 show that the superiority of **BD-PB** over **PD** extends to the cases with additional block bid types and flexible bids. In fact, the relative run time superiority of **BD-PB** over **PD** becomes even bigger with the addition of these new bid times, compared to those where there are only regular hourly and block bids.

8. Conclusions

In this paper, we studied the market clearing problem in the exchange-type electricity market design, the preferred market mechanism by many European countries. Although the surplus maximizing MILP problem can be solved efficiently by today's most powerful solvers, the optimal solution may include some accepted block bids that bring negative profits, paradoxically-accepted bids (PABs), to their bidders at the market prices. This implies a non-equilibrium market outcome. To prevent solutions with PABs, the MO typically imposes additional constraints, settling for a lower total market surplus. With the addition of such constraints, the computational burden increases substantially.

We developed new valid inequalities that use the MCPs over the periods of a PAB to find the set of block bid variables of which at least one needs to be changed to eliminate the PAB. We call these price-based cuts and show that they are stronger than the cuts proposed by Martin et al. (2014). The computational results on practical-size instances from the Turkish DAM show that using our price-based cuts in the BD algorithm (**BD-PB**) outperforms using the no-good cuts of Martin et al. (2014) and the locally-valid cuts of Madani & Van Vyve (2015). **BD-PB** solved all instances to optimality within about one minute when Gurobi solver was used. The tests on larger instances also showed that **BD-PB** not only found feasible solutions for all instances but also solved more instances to optimality in a fraction of time of **PD**.

In practice, MOs operate under a time restriction to solve these problems and to announce MCPs (typically around 10 minutes). Currently, the BD algorithm with heuristic cuts is used as the approach with the aim of finding high quality feasible solutions for the European market coupling problem (Euphemia, 2019). Furthermore, the BD algorithm is the only practical approach in the existence of piece-wise linear hourly bids that lead to a quadratic objective function. These bids are allowed in some of the markets such as Nord-Pool and Turkish market. Based on all these, our price-based cuts appear to be a promising contribution to solving such problems in practice.

We also developed extensions for the market designs where no paradoxically-rejected bid (PRB) constraints replace no PAB constraints or when the market includes more sophisticated bid types and transmission network constraints. We discuss that the price-based cuts are valid under all these market designs. The superior performance of **BD-PB** also extends to the instances that contain flexible bids as well as block bid constraints such as links and mutually exclusive groups. We believe that, implementing **BD-PB** will improve the solution quality for the market clearing problems in the exchange-type electricity market designs. We also believe that, these developments will trigger new research in this area to further improve the results. As future research, we intend to extend the price-based cuts to complex bid types in the Iberian market and to the PUN orders in the Italian market, and to conduct tests for the cases of using no-PRB constraints and network constraints.

Appendix A. Surplus maximizing linear program

Let **(SMLP)** be the linear relaxation of **(SMILP)**. We next present the dual problem of **(SMLP)**, **(D-SMLP)**, and provide the complementary slackness conditions, **(CS-SMLP)**. In **(D-SMLP)**, π_t is the dual variable (*MCP*) associated with the supply-demand balance constraint at period t, and s_h and s_b are the dual variables (*bid surpluses*) associated with the upper bound constraints on x_h and y_b in **(SMLP)**, respectively.

$$\textbf{(D-SMLP)}: \quad \text{Min} \quad \sum_{h \in H} s_h + \sum_{b \in B} s_b$$

$$\text{s.to.} \quad s_h \geq \sum_{t \in T} (p_h - \pi_t) q_{h,t} \qquad \forall h \in H$$

$$s_b \geq \sum_{t \in T} (p_b - \pi_t) q_{b,t} \qquad \forall b \in B$$

$$s_h \geq 0 \qquad \forall h \in H$$

$$s_b \geq 0 \qquad \forall b \in B$$

Based on the primal and dual problems, the following complementary slackness conditions must hold at the optimal solution of **(SMLP)**:

$$\textbf{(CS-SMLP)}: \quad s_h(1 - x_h) = 0 \qquad \forall h \in H$$

$$s_b(1 - y_b) = 0 \qquad \forall b \in B$$

$$x_h(s_h - \sum_{t \in T} (p_h - \pi_t) q_{h,t}) = 0 \qquad \forall h \in H$$

$$y_b(s_b - \sum_{t \in T} (p_b - \pi_t) q_{b,t}) = 0 \qquad \forall b \in B$$

We next define the positioning of bid k with respect to the MCPs, $\boldsymbol{\pi^*} \in \mathbb{R}^{|T|}$, and present the properties implied by **(CS-SMLP)** for an optimal solution to **(SMLP)**:

Definition 3. *Bid k is in-the-money if $(\boldsymbol{p}_k - \boldsymbol{\pi^*})^T \boldsymbol{q}_k > 0$, at-the-money if $(\boldsymbol{p}_k - \boldsymbol{\pi^*})^T \boldsymbol{q}_k = 0$, and out-of-the-money if $(\boldsymbol{p}_k - \boldsymbol{\pi^*})^T \boldsymbol{q}_k < 0$.*

1. If an hourly bid is fully rejected, then it is not in-the-money.

$$x_h^* = 0 \implies \sum_{t \in T} (p_h - \pi_t^*) q_{h,t} \leq 0 \tag{A.1}$$

2. If an hourly bid is partially accepted, then it is at-the-money.

$$0 < x_h^* < 1 \implies \sum_{t \in T} (p_h - \pi_t^*) q_{h,t} = 0 \tag{A.2}$$

3. If an hourly bid is fully accepted, then it is not out-of-the-money.

$$x_h^* = 1 \implies \sum_{t \in T} (p_h - \pi_t^*) q_{h,t} \geq 0 \tag{A.3}$$

4. If an hourly bid is in-the-money, then it must be fully accepted.

$$\sum_{t \in T} (p_h - \pi_t^*) q_{h,t} > 0 \implies x_h^* = 1 \tag{A.4}$$

5. If an hourly bid is out-of-the-money, then it must be fully rejected.

$$\sum_{t \in T} (p_h - \pi_t^*) q_{h,t} < 0 \implies x_h^* = 0 \tag{A.5}$$

6. If a block bid is rejected, then it is not in-the-money.

$$y_b^* = 0 \implies \sum_{t \in T} (p_b - \pi_t^*) q_{b,t} \leq 0 \tag{A.6}$$

7. If a block bid is accepted, then it is not out-of-the-money.

$$y_b^* = 1 \implies \sum_{t \in T} (p_b - \pi_t^*) q_{b,t} \geq 0 \tag{A.7}$$

8. If a block bid is in-the-money, then it must be accepted.

$$\sum_{t \in T} (p_b - \pi_t^*) q_{b,t} > 0 \implies y_b^* = 1 \tag{A.8}$$

9. If a block bid is out-of-the-money, then it must be rejected.

$$\sum_{t \in T} (p_b - \pi_t^*) q_{b,t} < 0 \implies y_b^* = 0 \tag{A.9}$$

Appendix B. Primal-dual formulation of the surplus maximization problem (Madani & Van Vyve, 2014)

Let $\bar{\boldsymbol{y}}$ be a given commitment vector for the set of block bids and consider the following dual program:

$$
\begin{aligned}
\textbf{(D-SMLP}(\bar{\boldsymbol{y}})\textbf{)}: \quad & \text{Min} \quad \sum_{h\in H} s_h + \sum_{b\in B} s_b - \sum_{b\in B_1} l_b \\
& \text{s.to.} \quad s_h \geq \sum_{t\in T}(p_h - \pi_t) q_{h,t} && \forall h \in H \\
& \quad s_b - l_b \geq \sum_{t\in T}(p_b - \pi_t) q_{b,t} && \forall b \in B_1 \\
& \quad s_b + m_b \geq \sum_{t\in T}(p_b - \pi_t) q_{b,t} && \forall b \in B_0 \\
& \quad s_h \geq 0 && \forall h \in H \\
& \quad s_b, l_b, m_b \geq 0 && \forall b \in B
\end{aligned}
$$

If $(\boldsymbol{x}^*, \bar{\boldsymbol{y}})$ is a feasible solution to $\textbf{(SMLP}(\bar{\boldsymbol{y}})\textbf{)}$, then $\textbf{(D-SMLP}(\bar{\boldsymbol{y}})\textbf{)}$ is also feasible. Then, $(\boldsymbol{x}^*, \bar{\boldsymbol{y}})$ is an optimal solution to $\textbf{(SMLP}(\bar{\boldsymbol{y}})\textbf{)}$ if it satisfies the following complementary slackness constraints.

$$
\begin{aligned}
\textbf{(CS-SMLP}(\bar{\boldsymbol{y}})\textbf{)}: \quad & s_h(1 - x_h^*) = 0 && \forall h \in H \\
& s_b(1 - \bar{y}_b) = 0 && \forall b \in B \\
& l_b(1 - \bar{y}_b) = 0 && \forall b \in B_1 \\
& m_b \bar{y}_b = 0 && \forall b \in B_0 \\
& x_h^*(s_h - \sum_{t\in T}(p_h - \pi_t) q_{h,t}) = 0 && \forall h \in H \\
& \bar{y}_b(s_b - l_b - \sum_{t\in T}(p_b - \pi_t) q_{b,t}) = 0 && \forall b \in B_1 \\
& \bar{y}_b(s_b + m_b - \sum_{t\in T}(p_b - \pi_t) q_{b,t}) = 0 && \forall b \in B_0
\end{aligned}
$$

The complementary slackness constraints ensure that $l_b m_b = 0, \forall b \in B$. Madani & Van Vyve (2014) show that l_b and m_b are upper bounds on the loss and missed surplus of bid b, respectively. In addition, they replace the complementary slackness constraints with the strong duality constraint and state the following mathematical program with equilibrium constraints:

$$
\begin{aligned}
\textbf{(E-SMILP)}: \quad & \text{Max} \quad S_L(\boldsymbol{x},\boldsymbol{y}) && \\
& \text{s.to.} \quad \sum_{h\in H} q_{h,t}x_h + \sum_{b\in B} q_{b,t}y_b = 0 && \forall t \in T \\
& x_h \leq 1 && \forall h \in H \\
& y_b \leq 1 && \forall b \in B \\
& s_h \geq \sum_{t\in T}(p_h - \pi_t)q_{h,t} && \forall h \in H \\
& s_b - l_b + m_b \geq \sum_{t\in T}(p_b - \pi_t)q_{b,t} && \forall b \in B \\
& m_b \leq M_b(1-y_b) && \forall b \in B \\
& l_b \leq M_b y_b && \forall b \in B \\
& S_L(\boldsymbol{x},\boldsymbol{y}) \geq \sum_{h\in H} s_h + \sum_{b\in B} s_b - \sum_{b\in B} l_b && \\
& x_h, s_h \geq 0 && \forall h \in H \\
& s_b, l_b, m_b \geq 0, \quad y_b \in \{0,1\} && \forall b \in B
\end{aligned}
$$

In this model, the dual problem constraints associated with B_1 and B_0 are combined and the property that $l_b m_b = 0, \forall b \in B$ is modeled by the big-M constraints where M_b is an appropriate upper bound on the loss or missed surplus of a block bid b. Denoting the maximum and the minimum allowable bid prices as p^{max} and p^{min}, respectively, $M_b \geq \sum_{t\in T} |q_{b,t}|(p^{max} - p^{min})$ is an upper bound on the loss or missed surplus of a block bid, and is a sufficiently large big-M value.

Appendix C. Extension of the primal-dual formulation with additional bid types

The primal-dual formulation given by (Madani & Van Vyve, 2014) considers block bids and hourly bids with step functions or piece-wise linear functions. The primal-dual formulation becomes an MIQCP when there are hourly bids with piece-wise linear functions. In that case, the objective function is equal to $S_Q(\boldsymbol{x},\boldsymbol{y})$ given in Section 6 and the left-hand-side of the strong duality constraint of **(E-SMILP)** given in Appendix B is replaced with $S_Q(\boldsymbol{x},\boldsymbol{y})$.

We extend **(E-SMILP)** when there are linked block bids, mutually exclusive block bids and flexible bids in the bid set in addition to the hourly bids and regular block bids. We consider a flexible bid as a set of mutually exclusive block bids as many as the number of periods. Then, we generalize the requirements of these bid types by associating the following constraint to the surplus maximization problem:

$$\sum_{b \in B} \boldsymbol{a}_b y_b \leq \boldsymbol{e} \tag{C.1}$$

where $\boldsymbol{a}_b \in \mathbb{Z}^m$ with entries $a_{b,i} \in \{-1, 0, 1\}, \forall i \in I = \{1, \ldots, m\}$, and $\boldsymbol{e} \in \mathbb{Z}^m_{\geq 0}$ with entries $e_i = \{0, 1\}, \forall i \in I$. Then, the restricted linear relaxation problem, its dual, the complementarity conditions and the primal-dual model are updated as follows:

$$
\begin{aligned}
\textbf{(SMLP}(\bar{\boldsymbol{y}})\textbf{)}: \quad \text{Max} \quad & S_L(\boldsymbol{x}, \boldsymbol{y}) & & \\
\text{s.to.} \quad & \sum_{h \in H} q_{h,t} x_h + \sum_{b \in B} q_{b,t} y_b = 0 & \forall t \in T \quad & [\pi_t] \\
& x_h \leq 1 & \forall h \in H \quad & [s_h] \\
& y_b \leq 1 & \forall b \in B \quad & [s_b] \\
& \sum_{b \in B} a_{b,i} y_b \leq e_i & \forall i \in I \quad & [u_i] \\
& -y_b \leq -1 & \forall b \in B_1 \quad & [l_b] \\
& y_b \leq 0 & \forall b \in B_0 \quad & [m_b] \\
& x_h \geq 0 & \forall h \in H \quad & \\
& y_b \geq 0 & \forall b \in B \quad &
\end{aligned}
$$

$$
\begin{aligned}
\textbf{(D-SMLP}(\bar{\boldsymbol{y}})\textbf{)}: \quad \text{Min} \quad & \sum_{h \in H} s_h + \sum_{b \in B} s_b - \sum_{b \in B_1} l_b + \sum_{i \in I} e_i u_i & \\
\text{s.to.} \quad & s_h \geq \sum_{t \in T} (p_h - \pi_t) q_{h,t} & \forall h \in H \\
& s_b - l_b + \sum_{i \in I} a_{b,i} u_i \geq \sum_{t \in T} (p_b - \pi_t) q_{b,t} & \forall b \in B_1 \\
& s_b + m_b + \sum_{i \in I} a_{b,i} u_i \geq \sum_{t \in T} (p_b - \pi_t) q_{b,t} & \forall b \in B_0 \\
& s_h \geq 0 & \forall h \in H \\
& s_b, l_b, m_b \geq 0 & \forall b \in B
\end{aligned}
$$

$$u_i \geq 0 \qquad \forall i \in I$$

$$\textbf{(CS-SMLP}(\bar{\boldsymbol{y}})\textbf{)} : \quad s_h(1 - x_h^*) = 0 \qquad \forall h \in H$$

$$s_b(1 - \bar{y}_b) = 0 \qquad \forall b \in B$$

$$l_b(1 - \bar{y}_b) = 0 \qquad \forall b \in B_1$$

$$m_b \bar{y}_b = 0 \qquad \forall b \in B_0$$

$$u_i(e_i - \sum_{b \in B} a_{b,i} y_b) = 0 \qquad \forall i \in I$$

$$x_h^*(s_h - \sum_{t \in T}(p_h - \pi_t) q_{h,t}) = 0 \qquad \forall h \in H$$

$$\bar{y}_b(s_b - l_b + \sum_{i=1}^{m} a_{b,i} u_i - \sum_{t \in T}(p_b - \pi_t) q_{b,t}) = 0 \qquad \forall b \in B_1$$

$$\bar{y}_b(s_b + m_b + \sum_{i=1}^{m} a_{b,i} u_i - \sum_{t \in T}(p_b - \pi_t) q_{b,t}) = 0 \qquad \forall b \in B_0$$

$$\begin{aligned}
\textbf{(E-SMILP)} : \quad \text{Max} \quad & \sum_{t \in T} \left\{ \sum_{h \in H} p_h q_{h,t} x_h + \sum_{b \in B} p_b q_{b,t} y_b \right\} \\
\text{s.to.} \quad & \sum_{h \in H} q_{h,t} x_h + \sum_{b \in B} q_{b,t} y_b = 0, \quad \forall t \in T \\
& x_h \leq 1, \quad \forall h \in H \\
& y_b \leq 1, \quad \forall b \in B \\
& \sum_{b \in B} a_{b,i} y_b \leq e_i, \quad \forall i \in I \\
& s_h \geq \sum_{t \in T}(p_h - \pi_t) q_{h,t}, \quad \forall h \in H \\
& s_b - l_b + m_b + \sum_{i \in I} a_{b,i} u_i \geq \sum_{t \in T}(p_b - \pi_t) q_{b,t}, \quad \forall b \in B \\
& m_b \leq M_b(1 - y_b), \quad \forall b \in B \\
& l_b \leq M_b y_b, \quad \forall b \in B \\
& \sum_{t \in T} \left\{ \sum_{h \in H} p_h q_{h,t} x_h + \sum_{b \in B} p_b q_{b,t} y_b \right\} \geq \sum_{h \in H} s_h + \sum_{b \in B} s_b - \sum_{b \in B} l_b + \sum_{i \in I} e_i u_i
\end{aligned}$$

$$x_h, s_h \geq 0, \quad \forall h \in H$$

$$s_b, l_b, m_b \geq 0, \quad y_b \in \{0,1\}, \quad \forall b \in B$$

$$u_i \geq 0, \quad \forall i \in I$$

References

Benders, J. F. (1962). Partitioning procedures for solving mixed-variables programming problems. *Numerische Mathematik*, *4*, 238–252.

Derinkuyu, K. (2015). On the determination of European day ahead electricity prices: The Turkish case. *European Journal of Operational Research*, *244*, 980–989.

Derinkuyu, K., Tanrisever, F., Kurt, N., & Ceyhan, G. (2019). Optimizing day-ahead electricity market prices: Increasing the total surplus for energy exchange istanbul. *Manufacturing & Service Operations Management*, . (ePub ahead of print Nov 5 https://doi.org/10.1287/msom.2018.0767).

Euphemia (2019). Euphemia Public Description. http://www.nemo-committee.eu/assets/files/190410_Euphemia%20Public%20Description%20version%20NEMO%20Committee.pdf. Online; accessed 16 May 2020.

EXIST (2016a). Day-ahead electricity market clearing algorithm. https://www.epias.com.tr/en/day-ahead-market/matching/. Online; accessed 17 June 2020.

EXIST (2016b). Day-ahead market offers. https://www.epias.com.tr/en/day-ahead-market/offers. Online; accessed 17 June 2020.

EXIST (2016c). Energy Market Transparency Platform. https://seffaflik.epias.com.tr/transparency/. Online; accessed 17 June 2020.

EXIST (2018). 2017 Electricity Market Report. https://www.epias.com.tr/en/spot-electricity-market/electricity-market-reports/. Online; accessed 17 June 2020.

Fernández-Blanco, R., Arroyo, J. M., & Alguacil, N. (2016). On the solution of revenue-and network-constrained day-ahead market clearing under marginal pricing—part i: An exact bilevel programming approach. *IEEE Transactions on Power Systems*, *32*, 208–219.

Gribik, P. R., Hogan, W. W., & Pope, S. L. (2007). Market-clearing electricity prices and energy uplift. *Cambridge, MA*, .

Liberopoulos, G., & Andrianesis, P. (2016). Critical review of pricing schemes in markets with non-convex costs. *Operations Research*, *64*, 17–31.

Madani, M., Ruiz, C., Siddiqui, S., & Van Vyve, M. (2018). Convex hull, ip and european electricity pricing in a european power exchanges setting with efficient computation of convex hull prices. *arXiv preprint arXiv:1804.00048*, .

Madani, M., & Van Vyve, M. (2014). Minimizing opportunity costs of paradoxically rejected block orders in european day-ahead electricity markets. In *11th International Conference on the European Energy Market (EEM14)* (pp. 1–6). IEEE.

Madani, M., & Van Vyve, M. (2015). Computationally efficient MIP formulation and algorithms for European day-ahead electricity market auctions. *European Journal of Operational Research*, *242*, 580–593.

Madani, M., & Van Vyve, M. (2017). A mip framework for non-convex uniform price day-ahead electricity auctions. *EURO Journal on Computational Optimization*, *5*, 263–284.

Madani, M., & Van Vyve, M. (2018). Revisiting minimum profit conditions in uniform price day-ahead electricity auctions. *European Journal of Operational Research*, *266*, 1072–1085.

Martin, A., Müller, J. C., & Pokutta, S. (2014). Strict linear prices in non-convex European day-ahead electricity markets. *Optimization Methods and Software*, *29*, 189–221.

NEMO Committee (2020). Single Day-Ahead Coupling (SDAC). `http://www.nemo-committee.eu/sdac`. Online; accessed 16 May 2020.

O'Neill, R. P., Sotkiewicz, P. M., Hobbs, B. F., Rothkopf, M. H., & Stewart Jr, W. R. (2005). Efficient market-clearing prices in markets with nonconvexities. *European journal of operational research*, *164*, 269–285.

Savelli, I., Giannitrapani, A., Paoletti, S., & Vicino, A. (2017). An optimization model for the electricity market clearing problem with uniform purchase price and zonal selling prices. *IEEE Transactions on Power Systems*, *33*, 2864–2873.

Van Vyve, M. et al. (2011). *Linear prices for non-convex electricity markets: models and algorithms*. Technical Report CORE.

Vlachos, A., Dourbois, G., & Biskas, P. (2016). Comparison of two mathematical programming models for the solution of a convex portfolio-based european day-ahead electricity market. *Electric Power Systems Research*, *141*, 313–324.

Yörükoğlu, S., Avşar, Z. M., & Kat, B. (2018). An integrated day-ahead market clearing model: Incorporating paradoxically rejected/accepted orders and a case study. *Electric Power Systems Research*, *163*, 513–522.