

Extension classification method for low-carbon product cases

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Abstract

In product low-carbon design, intelligent decision systems integrated with certain classification algorithms recommend the existing design cases to designers. However, these systems mostly dependent on prior experience, and product designers not only expect to get a satisfactory case from an intelligent system but also hope to achieve assistance in modifying unsatisfactory cases. In this article, we proposed a new categorization method composed of static and dynamic classification based on extension theory. This classification method can be integrated into case-based reasoning system to get accurate classification results and to inform designers of detailed information about unsatisfactory cases. First, we establish the static classification model for cases by dependent function in a hierarchical structure. Then for dynamic classification, we make transformation for cases based on case model, attributes, attribute values, and dependent function, thus cases can take qualitative changes. Finally, the applicability of proposed method is demonstrated through a case study of screw air compressor cases.

Keywords

Classification, extension theory, dependent function, transformation, low-carbon design

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Introduction

Low-carbon design is a design method for products; one considers the carbon emissions in product life cycle in meeting the basic function, performance, safety, and other requirements of the product.¹ Low-carbon design is driven by product low-carbon demands, and it inevitably produces various conflicting and contradictory issues in design processes.^{2,3} These issues arise from competing aspects of low-carbon performance and the original structure of products, routine performance, cost, and other factors. Consequently, resolving conflicts and contradictions in low-carbon design is an important research subject contributing to generate efficient design schemes. Design researchers have been conducting in-depth studies, research on concurrent design,^{4,5} collaborative design,^{6,7} and multi-objective optimization design methods,^{8,9} as well as case-based reasoning (CBR) method.^{10,11} Among these design

methodologies, CBR originated in the United States; its basic ideas and theories have been successfully applied to all kinds of intelligent systems to resolve new problems and inspire innovation on the basis of the prior experience.^{12,13} Ross et al.¹⁴ developed a CBR system, integrated with decision support method Graph Model for Conflict Resolution (GMCR), to aid the structuring and modeling of a conflict situation. Purvis and Pu¹⁵ pointed that a case-based reasoner was not just a retrieval and storage tool, and thus developed an adaptation

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methodology for case combination to address new problem requirements. Considering the poor semantic understanding ability in the existing CBR system, Guo et al.¹⁶ proposed an intelligent retrieval method by integrating the ontology technology into system for engineering design.

In future research for low-carbon design, we will build the product case database to support CBR system. But how to ensure an intelligent system can put forward the desirable case for users; an accurate categorization method research for cases is critical for the system output.

Previous researches about classification are mainly about three types, statistical method, artificial neural network (ANN) method,^{17–19} and rule-based method. Statistical classification method consists of *k*-nearest neighbor (KNN), Bayes method, and support vector machine (SVM). Bayes and KNN are widely applied in text classification with simple and efficient properties.^{20–22} Zhang and Zhou²³ proposed a multi-label learning approach based on KNN algorithm and finished natural scene classification. Denoeux²⁴ built a KNN classifier from the point of Dempster–Shafer theory for an unseen pattern, and it demonstrated the effectiveness compared with traditional KNN method. Due to the expensive computation in KNN implementation, Wu et al.²⁵ developed template condensing and preprocessing techniques to speed up classification while maintaining accurate results. Based on Bayes network and a basic adjustment model, Pavon et al.²⁶ presented a Bayes CBR system assist in automatic parameter tuning. Wu et al.²⁷ adopted immunity theory to search optimal attribute weight values, and thus proposed a new artificial immune system method for Bayes classification. Langseth and Nielsen²⁸ focused on hierarchical naive Bayes model research, resolving the inherent problems, attributes' interaction omission, and "information double-counting." SVM, based on the structural risk minimization principle in statistical learning theory, was adeptly used in pattern recognition with finite samples.^{29–31} Abbasion et al.³² combined wavelet analysis and SVM method in multi-fault diagnosis research for rolling bearing fault detection. Zhang et al.³³ proposed a novel hybrid method for parameter optimization of SVM, to solve the local optimal solution and time-consuming problems. Rule-based method mainly composed of decision tree approach and rough set theory; based on the classification rules, a complex multi-class categorization can be converted into several simple classification solutions.^{34–36} Goel et al.³⁷ evaluated the classification potential of decision tree method to discriminate different growth scenarios in a cornfield with hyperspectral data. Hashemi et al.³⁸ advocated one-versus-all decision trees for data stream classification. He et al.³⁹ integrated ant

colony algorithm and rough set theory into application of toxicity mechanism classification.

The three types of classification methods have their own advantages and are applied in text classification, pattern recognition, and other fields. However, when it comes to low-carbon design, designers expect the CBR system not only outputs desirable cases but also offers modification suggestion, that is, the intelligent system can give advice on how to redesign unsatisfactory cases to meet special demands. For satisfactory cases, designers hope to discriminate the best case without considering subjective factors of scheme evaluation. Therefore, in this article, we introduce an extension classification method from the perspective of operations research. Extension classification method is based on extension theory which is a new discipline belonging to domain of artificial intelligence and operations research. In this method, it comprises static classification and dynamic classification, and it reveals the classification process by a hierarchical structure for cases.

The remainder of this article is organized as follows. Introduce the basic concepts including extension theory, comprehensive basic-element set, similar basic-element set, similar basic-element cut set, and basic-element extension set. The next sections are static classification and dynamic classification. The proposed classification method is demonstrated through a case study. A discussion is given and conclusions and future research are offered in the end.

Basic concepts

In this study, some basic concepts need to be given about extension theory, comprehensive basic-element set, similar basic-element set, similar basic-element cut set, and basic-element extension set.

Extension theory

Extension theory consists of three branch theories: basic-element theory, extension set theory, and extension logic.⁴⁰ In this study, we use basic-element theory to describe attributes of product cases in a qualitative and quantitative way. Extension set theory includes extension set and dependent function; the former is used to construct a set for similar product cases and the latter is applied to establish a similarity function, or to distinguish product cases. In dynamic classification, we adopt transformation method based on extension logic.

Comprehensive basic-element set

The comprehensive basic-element set of low-carbon products, S_{PLCD} , is composed of whole product cases in design case library, and each product case $Z_{\text{PLCD_object}}$

is expressed in a basic-element formation. The representations of S_{PLCD} and $Z_{\text{PLCD_object}}$ are as follows⁴⁰

$$S_{\text{PLCD}} = \{Z_{\text{PLCD_object}}^i\} \\ = \{Z_{\text{PLCD_object}}^i | Z_{\text{PLCD_object}}^i = (\text{Case_Product}^i, C, V)\} \quad (1)$$

In this equation

$$C = [\text{Pro_Identity}^i, \text{Pro_Name}^i, \dots, \text{Pro_Attribute}^i, \text{Pro_Require}^i]^T \\ V = [v_1^i, v_2^i, \dots, \{B_{\text{Pro_Attribute}}^i\}, \{B_{\text{Pro_Require}}^i\}]^T$$

and $Z_{\text{PLCD_object}}^i$ denotes the i th product case in the library. Case_Product^i , C , and V are the three elements in basic-element formation. C denotes the attributes of a product case, and it consists of identity number, name, product attributes, customer requirements, and so on. V denotes the values of the attributes in C accordingly.

Similar basic-element set

The similar basic-element set of low-carbon products, $S_{\text{PLCD}}^{\text{sim}}$, is composed of product cases with the primitive retrieval in design case library, and the retrieval is on the basis of S_{PLCD} . The representation of $S_{\text{PLCD}}^{\text{sim}}$ is as follows

$$S_{\text{PLCD}}^{\text{sim}} = \left\{ Z_{\text{PLCD_object}}^i | Z_{\text{PLCD_object}}^i \xrightarrow{PR_i \cup \text{sim}_{l,i}} x_i, x_i \in (0, 1] \right\} \quad (2)$$

Here, $\text{sim}_{l,i}(PR_i, P_i)$ is the similarity function constructed by dependent function. PR_i denotes the requirement value from customers, and P_i is the real value of i th product case in database. The value of P_i is stored in V in S_{PLCD} . The value of x_i indicates the level of similarity between PR_i and P_i , and its range domain is $(0, 1]$.

In set $S_{\text{PLCD}}^{\text{sim}}$, it may contain many similar cases $Z_{\text{PLCD_object}}$, thus a cut number δ is determined using normal distribution method, and a case similar basic-element cut set is given as follows

$$\tilde{S}_{\text{PLCD}}^{\text{sim}} = \left\{ Z_{\text{PLCD_object}}^i | Z_{\text{PLCD_object}}^i \xrightarrow{PR_i \cup \text{sim}_{l,i}} x_j \geq \delta, x_{j-1} \leq x_j \leq x_{j+1}, \delta \in [0, 1] \right\} \quad (3)$$

In set $\tilde{S}_{\text{PLCD}}^{\text{sim}}$, each product case is ordered by similarity value x_i in case library.

Basic-element extension set

The basic-element extension set $J(Z_{\text{PLCD_object}})$ is a combination of basic-element theory and extension set, and it is a qualitative and quantitative tool to describe

variability of product cases. The representation of $J(Z_{\text{PLCD_object}})$ is as follows⁴⁰

$$J(Z_{\text{PLCD_object}}) \\ (T) = \{ (Z_{\text{PLCD_object}}^i, Y, Y') | Z_{\text{PLCD_object}}^i \in T_{S_{\text{PLCD}}^{\text{sim}}} \tilde{S}_{\text{PLCD}}^{\text{sim}} Y \\ = K(Z_{\text{PLCD_object}}^i, PR_i), Y' = T_K K(Z_{\text{PLCD_object}}^i, PR_i) \} \quad (4)$$

Here, T denotes the transformation, Y and Y' denote the dependent function, and $T_{S_{\text{PLCD}}^{\text{sim}}}$ and T_K denote the transformation of $\tilde{S}_{\text{PLCD}}^{\text{sim}}$ and dependent function K , respectively. If there is no transformation, then $Y = Y'$.

Static classification for product cases

Traditional classification method lacks an analysis of qualitative changes in cases and cannot effectively reveal classification law under demand changes. Extension classification method takes classifying procedure as a dynamic research process, including both quantitative and qualitative analyses by synthesis.

Low-carbon demand mainly consists of carbon footprint demand E , cost demand C , and performance demand P of a product. The static classification method for low-carbon demand divides cases into three states, satisfying demand state (i.e. positive field), not satisfying demand state (i.e. negative field), and critical state. Figure 1 shows the classification states.

In Figure 1(a), V_+ denotes a positive field, V_- denotes a negative field, and V_0 denotes the critical state.

Figure 1(b) shows the dynamic classification states. $V_{+.}$ denotes the field of positive qualitative change, namely, the product case from negative field to positive field. V_{-} denotes the field of negative qualitative change, namely, the product case from positive field to negative field.

Suppose $Z_{\text{PLCD_object}}^j$ is the j th product case, with n attributes, S_1 and S_2 are desirable interval and acceptable interval, K_{n-D} represents n -dimensions' dependent function. Then three kinds of static classification are as follows:

1. $Z_{\text{PLCD_object}}^j$ belongs to the class meeting product demand, that is, in positive field V_+ . To study which interval subclass in positive field is the sensitive type, this class is divided into at least three subclasses. Here, sensitive type refers to the class interval that responds quickly and undergoes a qualitative change under certain transformation conditions.

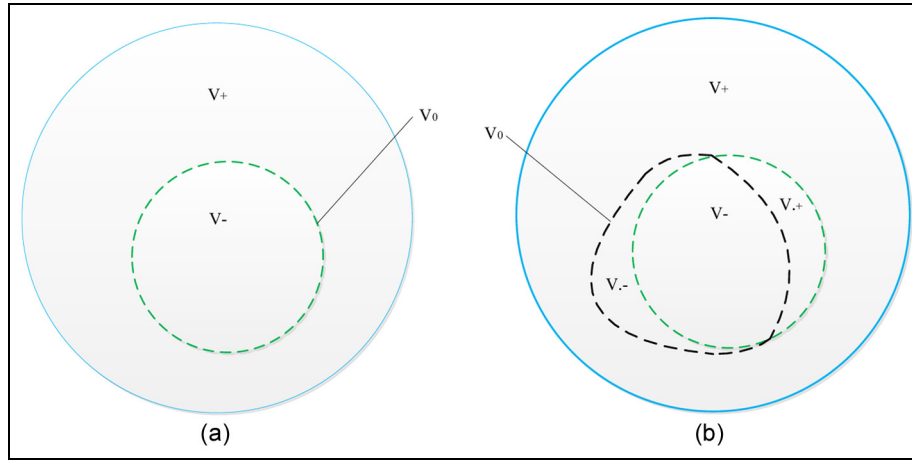


Figure 1. Classification states: (a) static classification states and (b) dynamic classification states.

Suppose there are two real numbers $\theta_1 < \theta_2 \in (0, +\infty)$. Then V_+ can be divided into three smaller positive fields

$$\begin{bmatrix} V_+^{11} \\ V_+^{12} \\ V_+^{13} \end{bmatrix} = \begin{cases} \{Z_{PLCD_object^t} | Z_{PLCD_object^t} \in \tilde{S}_{PLCD}^{sim}, 0 < K_{3-D}\{(E, C, P), S_1, S_2\} < \theta_1\} \\ \{Z_{PLCD_object^t} | Z_{PLCD_object^t} \in \tilde{S}_{PLCD}^{sim}, \theta_1 < K_{3-D}\{(E, C, P), S_1, S_2\} < \theta_2\} \\ \{Z_{PLCD_object^t} | Z_{PLCD_object^t} \in \tilde{S}_{PLCD}^{sim}, \theta_2 < K_{3-D}\{(E, C, P), S_1, S_2\} < +\infty\} \end{cases} \quad (5)$$

- $Z_{PLCD_object^t}$ belongs to the class reaching the critical point of product demand, that is, classified into the boundary field V_0 . According to the property of dependent function, there is at least one attribute $K_{i-D}(Z_{PLCD_object^t}) = 0, i = \{1, 2, 3\}, K_{(3-i)-D}(Z_{PLCD_object^t}) > 0$. Thus, cases in this class can be attributed to the first class, that is, can be combined with V_+^{11}

$$V_+^{11} \cup V_0 = \{Z_{PLCD_object^t} | Z_{PLCD_object^t} \in \tilde{S}_{PLCD}^{sim}, 0 \leq K_{3-D}\{(E, C, P), S_1, S_2\} < \theta_1\} \quad (6)$$

- $Z_{PLCD_object^t}$ belongs to the class that has at least one attribute not conforming to product demand, that is, classified into the negative field V_- . Cases of this class can be divided into two types by the number of satisfactory attributes:
 - All the dependent function values of attributes are less than 0

$$V_-^{21} = \{Z_{PLCD_object^t} | Z_{PLCD_object^t} \in \tilde{S}_{PLCD}^{sim}, K_{n-D}(Z_{PLCD_object^t}) < 0, K_{(n-i)-D} < 0, K_{1-D}^i < 0, i = 1, \dots, n\} \quad (7)$$

In low-carbon design, life cycle of a product consists of eight stages: (1) raw material stage, (2)

transportation stage, (3) manufacturing stage, (4) loading/unloading stage, (5) use stage, (6) maintenance

stage, (7) reusable stage, and (8) recycling stage. According to the stage division, E is divided into eight sub-attributes, E_1-E_8 (denoting carbon footprint during each stage, respectively), C is divided into C_1-C_8 (denoting cost during each stage, respectively), and P is divided into P_1-P_8 (denoting performance during each

stage, respectively). Therefore, when the dependent function values of some sub-attributes are negative and those attributes can be dividable, then they can be further classified until they are completely undividable. Suppose attributes j_1 and j_2 have subsets, then

$$\left\{ \begin{array}{l} V_-^{21\dots 1} = \{Z_{\text{PLCD_object}^l} | Z_{\text{PLCD_object}^l} \in \tilde{S}_{\text{PLCD}}^{\text{sim}}, K^1_{n-D}(Z_{\text{PLCD_object}^l}) < 0, K^2_{(n-i)-D} < 0, K^{2,i}_{1-D} < 0, \\ K^{3,j_1}_{(m_1-t_1)-D} < 0, K^{3,t_1}_{t_1-D} \geq 0, K^{3,j_2}_{(m_2-t_2)-D} < 0, K^{3,t_2}_{t_2-D} \geq 0, \dots, K^{l,r_1}_{(o_1-r_1)-D} < 0, K^{l,r_1}_{1-D} < 0, \\ i = 1, \dots, n, t_1 = 1, \dots, m_1, t_2 = 1, \dots, m_2, \dots, r_1 = 1, \dots, o_1 \} \\ V_-^{21\dots 2} = \{Z_{\text{PLCD_object}^l} | Z_{\text{PLCD_object}^l} \in \tilde{S}_{\text{PLCD}}^{\text{sim}}, K^1_{n-D}(Z_{\text{PLCD_object}^l}) < 0, K^2_{(n-i)-D} < 0, K^{2,i}_{1-D} < 0, \\ K^{3,j_1}_{(m_1-t_1)-D} < 0, K^{3,t_1}_{t_1-D} \geq 0, K^{3,j_2}_{(m_2-t_2)-D} < 0, K^{3,t_2}_{t_2-D} \geq 0, \dots, K^{l,r_1}_{(o_1-r_1)-D} < 0, K^{l,r_1}_{1-D} \geq 0, \\ i = 1, \dots, n, t_1 = 1, \dots, m_1, t_2 = 1, \dots, m_2, \dots, r_1 = 1, \dots, o_1 \} \end{array} \right. \quad (8)$$

where in dependent function $K^l_{(n-i)-D}$, l denotes the hierarchy of sub-attributes contained in an attribute, and i denotes the dimensions of attributes.

- (b) Not all the dependent function values of attributes are less than 0

Dynamic classification based on case model transformation

In a basic-element formation,⁴¹ product cases can be expressed as equation (11), O denotes the case model and c denotes the case attribute; in low-carbon design,

$$V_-^{22} = \{Z_{\text{PLCD_object}^l} | Z_{\text{PLCD_object}^l} \in \tilde{S}_{\text{PLCD}}^{\text{sim}}, K^1_{n-D}(Z_{\text{PLCD_object}^l}) < 0, K^2_{(n-i)-D} < 0, K^i_{1-D} \geq 0, i = 1, \dots, n\} \quad (9)$$

Hence, when an attribute of a case with dimensions of $(n - i)-D$ contains sub-attributes, it can be subdivided into smaller fields. Suppose an attribute contains l -layer subfactors, then

$$\left\{ \begin{array}{l} V_-^{22\dots 1} = \{Z_{\text{PLCD_object}^l} | Z_{\text{PLCD_object}^l} \in \tilde{S}_{\text{PLCD}}^{\text{sim}}, K^1_{n-D}(Z_{\text{PLCD_object}^l}) < 0, K^2_{(n-i)-D} < 0, K^i_{1-D} \geq 0, \\ K^{3,j_1}_{(m_1-t_1)-D} < 0, K^{3,t_1}_{1-D} < 0, \dots, K^{l,r_1}_{(o_1-r_1)-D} < 0, K^{l,t_2}_{1-D} < 0, K^{l,r_1}_{1-D} \geq 0, \\ i = 1, \dots, n, t_1 = 1, \dots, m_1, \dots, t_2 = 1, \dots, (o_1 - r_1), r_1 = 1, \dots, o_1 \} \\ V_-^{22\dots 2} = \{Z_{\text{PLCD_object}^l} | Z_{\text{PLCD_object}^l} \in \tilde{S}_{\text{PLCD}}^{\text{sim}}, K^1_{n-D}(Z_{\text{PLCD_object}^l}) < 0, K^2_{(n-i)-D} < 0, K^i_{1-D} \geq 0, \\ K^{3,j_1}_{(m_1-t_1)-D} < 0, K^{3,t_1}_{1-D} \geq 0, \dots, K^{l,r_1}_{(o_1-r_1)-D} < 0, K^{l,t_2}_{1-D} < 0, K^{l,r_1}_{1-D} \geq 0, \\ i = 1, \dots, n, t_1 = 1, \dots, m_1, \dots, t_2 = 1, \dots, (o_1 - r_1), r_1 = 1, \dots, o_1 \} \end{array} \right. \quad (10)$$

Therefore, from the hierarchical classification, designers can reveal bottom subfactors that are not conforming to special demands, that is, subfactors $V_-^{21\dots 1}$, $V_-^{21\dots 2}$, $V_-^{22\dots 1}$, and $V_-^{22\dots 2}$. The framework of hierarchical classification for product cases is shown in Figure 2. Modifying these subfactors, designers can rapidly put forward satisfying design schemes or even get innovative products.

Dynamic classification for product cases

When static classification result fails to live up to expectation, namely, there are many product cases in V_+ , or most of the cases cannot satisfy demands. In this process, for the former situation, customer demands should be detailed and updated to achieve the best product case. For the latter situation, which is the main study in the article, we make transformations for product cases in V_- and restart static classification; product cases in V_+ are expected. We call this process as dynamic classification, and it consists of four aspects as follows.

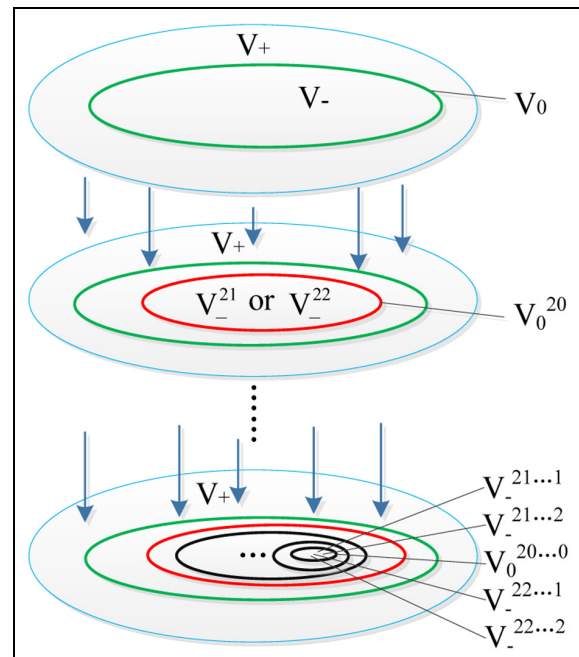


Figure 2. Hierarchical classification for product cases.

it mainly includes carbon footprint c_E , cost c_C , performance c_P , and v denotes the attribute value

$$\begin{bmatrix} O_{\text{PLCD_object}^t}, & c_E(\text{Pro_Attribute}^{i1}), & v_E^{i1} \\ & c_C(\text{Pro_Attribute}^{i2}), & v_C^{i2} \\ & c_P(\text{Pro_Attribute}^{i3}), & v_P^{i3} \\ & \vdots & \vdots \end{bmatrix} \quad (11)$$

$$= [O_{\text{PLCD_object}^t}, \{c(\text{Pro_Attribute}^{ij})\}, \{v^{ij}\}]$$

1. Case model transformation is correlated with its attributes and values, then

$$(\mu O_{\text{PLCD_object}^t} = O_{\text{PLCD_object}^{t1}}) \Rightarrow$$

$$\begin{cases} U_1^1 = [\mu T_{c,1} c(\text{Pro_Attribute}^{ij}) = c(\text{Pro_Attribute}^{it_2})_{j \neq t_2} = \emptyset] \Rightarrow (\mu \cdot \mu T_{c,1} \cdot c_{,1} T_v v^{ij} = v^{it_2} = \emptyset)_{j \neq t_2} \\ \Rightarrow [\mu \cdot \mu T_{c,1} \cdot c_{,1} T_v \cdot v^{i_2} T_{v(E/C/P)} v_2(E/C/P) = v_2'(E/C/P) \neq v_2(E/C/P)] \\ U_1^2 = [\mu T_{c,2} c(\text{Pro_Attribute}^{ij}) = c(\text{Pro_Attribute}^{it_3})_{j \neq t_3} \neq \emptyset] \Rightarrow (\mu \cdot \mu T_{c,2} \cdot c_{,2} T_v v^{ij} = v^{it_3} \neq \emptyset)_{j \neq t_3} \\ \Rightarrow [\mu \cdot \mu T_{c,2} \cdot c_{,2} T_v \cdot v^{i_3} T_{v(E/C/P)} v_3(E/C/P) = v_3'(E/C/P) \neq v_3(E/C/P)] \\ U_1^3 = [\mu T_{c,3} c(\text{Pro_Attribute}^{ij})] = \{\mu T_{c,3,t_4} c(\text{Pro_Attribute}^{ij}) = c(\text{Pro_Attribute}^{i(j+t_4)})\}_{t_4=1, \dots, n} \\ \Rightarrow \{\mu \cdot \mu T_{c,3} \cdot \mu T_{c,3,t_4} \cdot c_{,3} T_v v^{ij} = v^{i(j+t_4)} \neq \emptyset\} \end{cases} \quad (12)$$

Equation (12) explains that case model $O_{\text{PLCD_object}^t}$ takes an active transformation μ , replaced by model $O_{\text{PLCD_object}^{t1}}$. Due to the active transformation, there are conductive transformations for case attributes and values. U_1^1 denotes the case field with deletion transformation, U_1^2 denotes the case field with substitution transformation, and U_1^3 denotes the case field with duplication transformation.

Where $\mu T_{c,1}$ and $c_{,1} T_v$ are the conductive deletion transformations of an attribute and its value, respectively. $v^{i_2} T_{v(E/C/P)}$ is the conductive substitution transformation of related attributes such as carbon footprint, cost, or performance. $\mu T_{c,2}$ and $c_{,2} T_v$ are the conductive substitution transformations of an attribute and its value, respectively. $v^{i_3} T_{v(E/C/P)}$ is the conductive substitution transformation of related attributes such as carbon footprint, cost, or performance. $\mu T_{c,3}$ and $c_{,3} T_v$ are the conductive duplication transformation of an attribute and its value, respectively. $\mu T_{c,3,t_4}$ is the conductive substitution transformation of additional attributes.

2. Case model transformation is only correlated with its attribute value, then

$$(\mu O_{\text{PLCD_object}^t} = O_{\text{PLCD_object}^{t1}}) \Rightarrow \mu T_{v_{ij}} v [c(\text{Pro_Attribute}^{ij})] = v' [c(\text{Pro_Attribute}^{ij})] \quad (13)$$

where $\mu T_{v_{ij}}$ denotes conductive expansion or contraction transformation of j th attribute value arising from active transformation of i th product case.

Dynamic classification based on attribute transformation

In low-carbon design, since E and C are both single factors in each stage, and P is integrated by multiple factors in each of eight stages of product life cycle,⁴² in this section we mainly study attribute transformation of performance P .

Suppose the j_3 th performance attribute of a product case $P_{ij_1j_2j_3}$ on the third layer of the i th stage can be

transformed. It includes three scenarios: (1) deletion transformation of $P_{ij_1j_2j_3}$, (2) increase transformation of $P_{ij_1j_2j_3}$, and (3) substitution transformation of $P_{ij_1j_2j_3}$:

The deletion transformation of $P_{ij_1j_2j_3}$.

$$\begin{cases} \mu P_{ij_1j_2j_3} = \emptyset \Rightarrow \\ \left\{ \begin{array}{l} \mu T_v v(P_{ij_1j_2j_4}) = v'(P_{ij_1j_2j_4}) \\ [\mu T_{Z_{\text{LCSB}}} Z_{\text{LCSB}}^i = \emptyset] \Rightarrow \left\{ \begin{array}{l} [Z_{\text{LCSB}}^i T_{E_{ij_1j_2j_3}} v(E_{ij_1j_2j_3}) = v(E'_{ij_1j_2j_3})] \\ [Z_{\text{LCSB}}^i T_{C_{ij_1j_2j_3}} v(C_{ij_1j_2j_3}) = v(C'_{ij_1j_2j_3})] \end{array} \right. \end{array} \right. \end{cases} \quad (14)$$

Equation (14) indicates that attribute $P_{ij_1j_2j_3}$ takes the deletion transformation, and it results in conductive transformation for $P_{ij_1j_2j_4}$ and its correlated Z_{LCSB}^i .

Where μT_v denotes the conductive substitution transformation of $P_{ij_1j_2j_4}$, $\mu T_{Z_{\text{LCSB}}}^i$ denotes the conductive deletion transformation of the low-carbon structural element related to performance attribute $P_{ij_1j_2j_3}$. $Z_{\text{LCSB}}^i T_{E_{ij_1j_2j_3}}$ and $Z_{\text{LCSB}}^i T_{C_{ij_1j_2j_3}}$ denote the conductive contraction transformations of related carbon footprint and cost, respectively. The subscript Z_{LCSB}^i is the definition of a low-carbon structural element, referring to the smallest structural unit that realizes low-carbon function and it contains three factors, that is, carbon footprint E , cost C , and performance P .

As the transformation, dependent function of a product case will change, and the following scenarios may occur:

1. $K_{3-D}[(v'(P_{ij1j2j3}), v(E'_{ij1j2j3}), v(C'_{ij1j2j3})), S_1, S_2] \geq 0$:
 - (a) Suppose $\{K_{3-D}^{ij1j2j3}\}_{t_3 \neq j_3} \geq 0$, then $K_{3-D}^{ij1j2} \geq 0 \Rightarrow K_{3-D}^{ij1} \geq 0 \Rightarrow K_{3-D}^i \geq 0$, that is, this case exhibits a qualitative change such that its field changes from V_- to $V_{.+}$.
 - (b) Suppose $\{K_{3-D}^{ij1j2t_3}\}_{t_3 \neq j_3} < 0$, then $K_{3-D}^{ij1j2} < 0$. Therefore, this case exhibits only a quantitative change, but a qualitative change is realized for three attributes of the low-carbon structural element Z_{LCSB}^i .
2. $K_{3-D}[(v'(P_{ij1j2j3}), v(E'_{ij1j2j3}), v(C'_{ij1j2j3})), S_1, S_2] < 0$:
 - (a) Suppose $K_{2-D}[v(E'_{ij1j2j3}), v(C'_{ij1j2j3}), S_1, S_2] < 0$ and $\{K_{1-D}[v(E'_{ij1j2j3})], K_{1-D}[v(C'_{ij1j2j3})]\} < 0$, then for this case, attributes only take a quantitative change within the field V_-^{2111} or V_-^{2112} .
 - (b) Suppose $K_{2-D}[v(E'_{ij1j2j3}), v(C'_{ij1j2j3}), S_1, S_2] < 0$ and $K_{1-D}[v(E'_{ij1j2j3})] \cdot K_{1-D}[v(C'_{ij1j2j3})] < 0$, then carbon footprint or cost at this hierarchy exhibits a qualitative change, while the other factor exhibits only a quantitative change.

The increase transformation of $P_{ij1j2j3}$.

$$\begin{cases} P_{ij1j2j3} = \emptyset \\ \mu P_{ij1j2j3} \neq \emptyset \Rightarrow \\ \left\{ \begin{array}{l} \mu T_{v(P_{ij1j2j3})} v(P_{ij1j2j3}) = v'(P_{ij1j2j3}) \Rightarrow v(P_{ij1j2j3}) T_{v(P_{ij1j2j4})} v(P_{ij1j2j4}) = v'(P_{ij1j2j4}) \\ [\mu T_{Z_{LCSB}^i} Z_{LCSB}^i \neq \emptyset] \Rightarrow \left\{ \begin{array}{l} [Z_{LCSB}^i T_{E_{ij1j2j3}} v(E_{ij1j2j3}) = v(E'_{ij1j2j3})] \\ [Z_{LCSB}^i T_{C_{ij1j2j3}} v(C_{ij1j2j3}) = v(C'_{ij1j2j3})] \end{array} \right. \end{array} \right. \end{cases} \quad (15)$$

Equation (15) indicates that attribute $P_{ij1j2j3}$ takes the increase transformation, and it results in conductive transformation for $P_{ij1j2j4}$ and its correlated Z_{LCSB}^i .

Where $\mu T_{v(P_{ij1j2j3})}$ denotes the conductive increase transformation arising from active transformation of attribute $P_{ij1j2j3}$, and $v(P_{ij1j2j3}) T_{v(P_{ij1j2j4})}$ denotes the conductive substitution transformation of $P_{ij1j2j4}$ arising from transformation of $P_{ij1j2j3}$.

As the increase transformation, dependent function will change, and the scenarios are the same with those in deletion transformation.

The substitution transformation of $P_{ij1j2j3}$.

$$\begin{aligned} (\mu P_{ij1j2j3} = P_{ij1j2j4})_{j_3 \neq j_4} &\Rightarrow (\mu T_{Z_{LCSB}^i} Z_{LCSB}^i = Z_{LCSB}^i)_{P_{ij1j2j4} \sim Z_{LCSB}^i} \\ &\Rightarrow \begin{cases} Z_{LCSB}^i T_{v(P_{ij1j2j3})} v(P_{ij1j2j3}) = v(P_{ij1j2j4}) \\ Z_{LCSB}^i T_{v(E_{ij1j2j3})} v(E_{ij1j2j3}) = v(E_{ij1j2j4}) \\ Z_{LCSB}^i T_{v(C_{ij1j2j3})} v(C_{ij1j2j3}) = v(C_{ij1j2j4}) \end{cases} \quad (16) \end{aligned}$$

Equation (16) indicates that attribute $P_{ij1j2j3}$ takes the substitution transformation, and it results in conductive transformation for its correlated Z_{LCSB}^i .

Where $P_{ij1j2j4} \sim Z_{LCSB}^i$ indicates that attribute $P_{ij1j2j4}$ belongs to Z_{LCSB}^i . $Z_{LCSB}^i T_{v(P_{ij1j2j3})}$, $Z_{LCSB}^i T_{v(E_{ij1j2j3})}$ and $Z_{LCSB}^i T_{v(C_{ij1j2j3})}$ denote the conductive substitution transformation of performance, carbon footprint, and cost, respectively.

As the substitution transformation, dependent function will change. When $K_{1-D}(v(P_{ij1j2j4})) \geq 0$, qualitative change in the performance is achieved. When $K_{1-D}(v(P_{ij1j2j4})) < 0$ and $K_{1-D}(v(P_{ij1j2j4})) \geq K_{1-D}(v(P_{ij1j2j3}))$, then further transformation is needed, and transformation of this class is reserved. When $K_{1-D}(v(P_{ij1j2j4})) < 0$ and $K_{1-D}(v(P_{ij1j2j4})) \leq K_{1-D}(v(P_{ij1j2j3}))$, transformation of this class has to be deleted.

Dynamic classification based on attribute values' transformation

In this section, we take costs in the later four stages, that is, use, maintenance, reuse, and recycling stages, as transformation object. As cost is the most active factor in these stages, it can cause conductive transformation for performance and carbon footprint:

1. Cost transformation at a stage leads to reduction in cost at another stage or a few other stages

$$\begin{aligned} (\mu C_i = C'_i) \cup (C_i \in \{C_j\}_{j=1, \dots, 8}) \\ \Rightarrow \mu T_{\{C_j\}_{j \neq i}} \{C_j\}_{j \neq i} = \{C'_j\} \\ \Rightarrow \{C_j\}_{j \neq i} T_C C = C' = \sum_{j=1}^t C'_j + \sum_{l=1}^{7-t} C_l < C \quad (17) \\ \Rightarrow {}_C T_{K_{3-D}} K_{3-D}(E, C, P) = K'_{3-D}(E, C, P) \end{aligned}$$

Equation (17) indicates that cost C_i at a stage takes active transformation, and it results in conductive transformation in cost C_j at another stage or a few other stages. In the end, cost in full life cycle C and dependent function K_{3-D} are changed.

The classification of this transformation is as follows:

When $K_{3-D}' \geq K_{3-D} \geq 0$, $Z_{PLCD_Object}^j \in V_+$, when $K_{3-D}' \geq 0 \geq K_{3-D}$, $Z_{PLCD_Object}^j \in V_{.+}$, and when $0 \geq K_{3-D}' \geq K_{3-D}$, $Z_{PLCD_Object}^j \in V_-$.

2. Cost transformation at a stage leads to reduction in carbon footprint

$$\begin{aligned}
 & (\mu C_i = C'_i) \cup (C_i \in \{C_j\}_{j=1, \dots, 8}) \\
 & \Rightarrow \begin{cases} \mu T_C C = C'_i + (C_j)_{j \neq i} \leq C \\ \mu T_C C = C'_i + (C_j)_{j \neq i} = (1 + \alpha)C, \alpha > 0 \end{cases} \quad (18) \\
 & \Rightarrow c T_{\{E_t\}} \{E_t\}_{t=1, \dots, 8} = \{E'_t\} \leq \{E_t\} \\
 & \Rightarrow \{E_t\} T_E E = E' = \{E'_t\} + \{E_{8-t}\} < E \\
 & \Rightarrow {}_E T_{K_{3-D}} K_{3-D} = K'_{3-D} > K_{3-D}
 \end{aligned}$$

Equation (18) indicates that cost C_i at a stage takes active transformation, and it results in conductive transformation in carbon footprint E_t . In the end, cost C , carbon footprint E in full life cycle, and the dependent function K_{3-D} are changed. Classification of this transformation is the same as Step 1.

3. Cost transformation at a stage leads to conductive transformation of performance and reduction in carbon footprint

$$\begin{aligned}
 & (\mu C_i = C'_i) \cup (C_i \in \{C_j\}_{j=1, \dots, 8}) \\
 & \Rightarrow \begin{cases} \mu T_C C = C'_i + (C_j)_{j \neq i} \leq C \\ \mu T_C C = C'_i + (C_j)_{j \neq i} = (1 + \alpha)C, \alpha > 0 \end{cases} \\
 & \Rightarrow c T_{\{P_t\}} \{P_t\}_{t=1, \dots, 8} = \{P'_t\} \quad (19) \\
 & \Rightarrow \{P_t\} T_{\{E_t\}} \{E_t\} = \{E'_t\} \leq \{E_t\} \\
 & \Rightarrow \{E_t\} T_E E = E' = \{E'_t\} + \{E_{8-t}\} < E \\
 & \Rightarrow {}_E T_{K_{3-D}} K_{3-D} = K'_{3-D} > K_{3-D}
 \end{aligned}$$

Equation (19) indicates that cost C_i at a stage takes active transformation, and it results in conductive transformation in performance P_t which also contributes to the transformation of carbon footprint E_t . In the end, cost C , performance P , carbon footprint E in full life cycle, and dependent function K_{3-D} are changed. Classification of this transformation is the same as Step 1.

Dynamic classification based on dependent function transformation

The main purpose of dynamic classification is to choose product cases in V_+ and $V_{.+}$ after transformations, namely, to change the dependent function values. In

the previous section, according to customer requirements, we take dynamic classification based on product case model, attributes, and values, and finally change the dependent function value. In this section, we take dynamic classification based on the dependent function itself.

As equation (20), it is a simple one-dimensional dependent function,⁴¹ and for the multi-dimensional dependent function, it can be calculated through the operation of dimensionality reduction^{43,44}

$$\begin{aligned}
 & k_{1-D}(x) = \\
 & \begin{cases} \frac{\rho(x, X_0)}{D(x, X_0, X)} - 1, & \rho(x, X_0) = \rho(x, X) \text{ and } x \notin X_0 \\ \frac{\rho(x, X_0)}{D(x, X_0, X)}, & \text{others} \end{cases} \quad (20)
 \end{aligned}$$

In equation, x is any point in real axis, and X_0 and X are the correlated intervals, $X_0 \subseteq X$. $\rho(x, X_0)$ and $\rho(x, X)$ denote the extension distance between point x and intervals X_0 and X , respectively. $D(x, X_0, X)$ denotes the place value in extension theory; it describes the relation between point x and intervals X_0 and X . Thus, when the intervals change, the dependent function value will change.

For the multi-dimensional parameters in low-carbon design, desirable interval S_1 and acceptable interval S_2 , $S_1 \subseteq S_2$ can also be changed, namely, modify the customer requirements, and it can be divided into three transformation scenarios:

1. The desirable interval S_1 is changed to be S'_1 , while S'_1 is still contained by the acceptable interval S_2 , and S_2 remains the same interval. This transformation can be expressed as $\{(\mu S_1 = S'_1) \cup (S'_1 \leq S_2)\}$.
2. The desirable interval S_1 is changed to be S'_1 , and the acceptable interval S_2 is changed to be S'_2 with the conductive transformation μT_{S_2} , while S'_1 is contained by S'_2 . This transformation can be expressed as $\{(\mu S_1 = S'_1) \cup (S'_1 \leq S_2)\} \cup \{(\mu T_{S_2} S_2 = S'_2) \cup (S'_1 \leq S'_2)\}$.
3. The acceptable interval S_2 is changed to be S'_2 , and S_1 stays the same. This transformation can be expressed as $\{(\mu S_2 = S'_2) \cup (S_1 \leq S'_2)\}$.

The classification is as follows:

When $K_{n-D}' \geq K_{n-D} \geq 0$, $Z_{PLCD_Object}^j \in V_+$, when $K_{n-D}' \geq 0 \geq K_{n-D}$, $Z_{PLCD_Object}^j \in V_{.+}$, and when $0 \geq K_{n-D}' \geq K_{n-D}$, $Z_{PLCD_Object}^j \in V_-$.

Table 1. Similarity retrieval results based on some screw air compressor cases.

Case	$K_{n-D} 1$	$K_{n-D} 2$	$K_{n-D} 3$	Sim	Case	$K_{n-D} 1$	$K_{n-D} 2$	$K_{n-D} 3$	Sim
1. JN-1	-0.005	-0.249	1	0.931	11. SB-1	-0.389	-0.249	1	0.773
2. JN-2	0.166	0.251	1	1.000	12. SB-2	-0.266	-0.249	1	0.817
3. JN-3	0.182	-0.072	1	0.979	13. SB-3	0.297	-0.249	1	0.934
4. LG-1	-0.593	-0.527	1	0.653	14. KH-1	-0.417	-0.499	1	0.712
5. LG-2	-0.165	-0.527	1	0.801	15. KH-2	-0.365	-0.249	1	0.781
6. LG-3	0.143	-0.527	1	0.877	16. KH-3	-0.308	0.251	1	0.868
7. SA-1	-0.368	-0.499	1	0.728	17. KH-4	0.383	0.308	1	1.000
8. SA-2	-0.424	-0.249	1	0.761	18. WD-1	-1.873	-0.245	1	0.511
9. SA-3	-0.237	-0.263	1	0.825	19. WD-2	-1.925	0.271	1	0.573
10. SA-4	0.448	-0.263	1	0.931	20. WD-3	-1.919	0.308	1	0.573

A case study

Static classification

In this article, we perform similarity retrieval for screw air compressors based on acquired screw compressor cases shown in Table 1.

In the retrieval, we suppose the j th demand for the product is

$$\text{sim}_{j,t}(PR_j, P_t) = \sum_{i=1}^3 \omega_i \text{sim}_{j,t}^i(PR_j, P_t) \quad (22)$$

Here, l denotes the l th retrieval, $l = 1, 2, 3$, and $w_1 = 0.5$, $w_2 = 0.3$, and $w_3 = 0.2$. P_t is the t th case product in database.

The truncation interval is determined to be [0.71, 1] using normal distribution method. Based on the cut number, the basic-element cut set for screw compressor cases obtained is

$$\begin{aligned} \tilde{S}_{\text{PLCD}}^{\text{sim}} &= \{Z_{\text{PLCD_CASE}^j} | Z_{\text{PLCD_CASE}^j} \xrightarrow{PR_j \cup \text{sim}_{i,j}} x_j \geq 0.71, x_{j-1} \leq x_j \leq x_{j+1}, j = 1, \dots, 16\} \\ &= \{\text{CASE2, CASE17, CASE3, CASE13, CASE1, CASE10, CASE6, CASE16, CASE9,} \\ &\quad \text{CASE12, CASE5, CASE15, CASE11, CASE8, CASE7, CASE14}\} \\ &= \{\text{CASE}_{i,j}^j, j = 1, \dots, 16 \cup i = 1, \dots, 17 \cup i \neq 4\} \end{aligned}$$

$$PR_j = \begin{bmatrix} O_{PR_j} & \text{exhaust pressure} & [0.9, 1.1] \text{ MPa} \\ & \text{exhaust volume} & [3.8, 5.5] \text{ m}^3/\text{min} \\ & \text{noise} & [60, 70] \text{ dB} \\ & \text{buying cost} & [45, 000, 63, 000] \text{ yuan} \\ & \text{carbon footprint used} & [140, 000, 160, 000] \text{ kgCO}_2\text{e} \\ & \text{carbon footprint of marketed product} & [18, 000, 24, 000] \text{ kgCO}_2\text{e} \end{bmatrix} = \begin{bmatrix} PR_j^1 \\ PR_j^2 \\ PR_j^3 \\ PR_j^4 \\ PR_j^5 \\ PR_j^6 \end{bmatrix}$$

In the table, $K_{n-D}|1$ denotes the dependent function value in first retrieval, including cost attribute PR_j^4 and carbon footprint attributes PR_j^5 and PR_j^6 . $K_{n-D}|2$ denotes the dependent function value in second retrieval, including performance attributes PR_j^1 , PR_j^2 , and PR_j^3 . $K_{n-D}|3$ denotes the dependent function value in third retrieval, including the special demands for products. As there is no special demand, we suppose the cases can meet this requirement, the value of $K_{n-D}|3$ being 1 for each case. The similarity values at each stage can be calculated using following equation

$$\text{sim}_{j,t}^l(PR_j, P_t) = \begin{cases} 1, & K_{n-D}(P_t) \geq 0 \\ e^{K_{n-D}(P_t)}, & K_{n-D}(P_t) < 0 \end{cases} \quad (21)$$

We get the final similarity values according to

where \cup indicates that the values of i and j are a correlated pair, for instance, CASE_{11}^{13} indicates that the value of CASE11 in the basic-element cut set is numbered 13. Now with screw compressor case CASE_9^9 as an example, its basic-element extension set is constructed as follows

$$\begin{aligned} J(Z_{\text{PLCD_CASE}_9^9})(T) &= \{(Z_{\text{PLCD_CASE}_9^9}, Y, Y') | Z_{\text{PLCD_CASE}_9^9} \in T_{\text{PLCD}}^{\text{sim}}, \tilde{S}_{\text{PLCD}}^{\text{sim}}, Y = -0.263, \\ Y' = K_{n-D}(\{T_{V_i(B_{\text{Pro-Attribute}_i}^i)} v_i(B_{\text{Pro-Attribute}_i}^i)\}, T_{S_1} S_1, T_{S_2} S_2)\} \end{aligned}$$

This case belongs to the scenario that carbon footprint PR_j^5 and noise attribute PR_j^3 at use stage do not satisfy low-carbon demand. Hence, this case belongs to the layer-1 negative field (i.e. $Z_{\text{PLCD_CASE}_9^9} \in V_-$), and

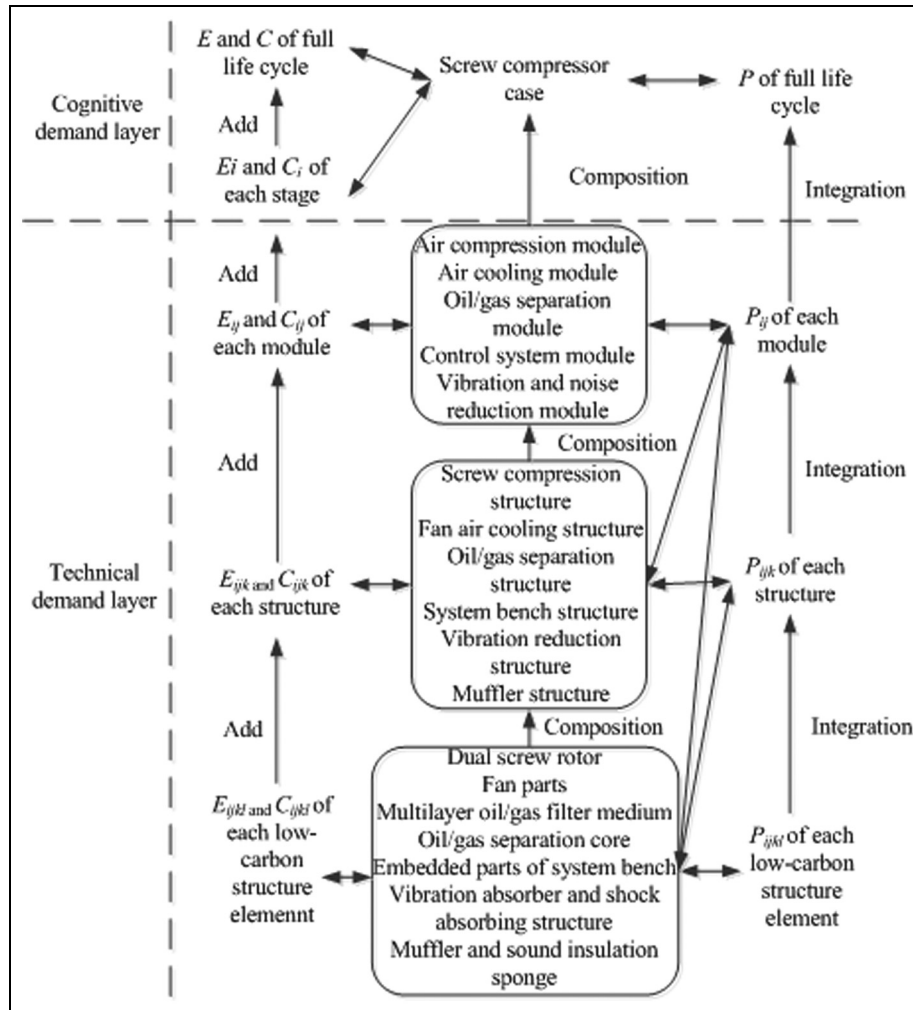


Figure 3. Internal correlation between hierarchical structure and E, C, and P.

the cost demand is $K_{1-D}^1\{C(CASE_9^9), S_1, S_2\} > 0$, thus $Z_{PLCD_CASE_9^9} \subset V_-^{22}$. Based on the influence of each submodule on previous hierarchy module in conjunction with Figure 3, three sub-attributes' nonconforming scenarios were discovered:

1. Carbon footprint in use stage mainly comes from three power consumption modules' carbon footprints: air compression module carbon footprint (E_{51}), control system module carbon footprint (E_{52}), and cooling module carbon footprint (E_{53}). E_{51} and E_{53} are the main factors contributing to carbon footprint. The noise is composed of air compression module noise (P_{51}), cooling module noise (P_{52}), and vibration and noise reduction module noise (P_{53}). P_{51} and P_{52} cannot be changed after products enter the market, whereas P_{53} is the decisive factor to

noise performance. Hence, the layer-3 classification at this stage is expressed as follows

$$\begin{aligned}
 &Z_{PLCD_CASE_9^9} \subset V_-^{222} \\
 &= \{K_{3-D}^1 < 0, K_{2-D}^2(E, P) < 0, K_{1-D}^2(C) > 0, K_{3-D}^{3,j_1}(E_{51}, E_{53}, P_{53}) < 0, \\
 &K_{3-D}^{3,3}(E_{52}, P_{51}, P_{52}) > 0, j_1 = 1, 2, 3\}
 \end{aligned}$$

2. E_{51} consists of air compression structure carbon footprint (E_{511}) and lubricant oil consumption carbon footprint (E_{512}), and the former accounts for large percentage of carbon footprint in the module. E_{53} mainly comprises fan air cooling structure carbon footprint (E_{531}), and E_{531} does not conform to requirement. P_{53} consists of noise reduction structure noise (P_{531}) and vibration reduction module noise (P_{532}), with P_{531} being the decisive factor to noise.

Table 2. Case attributes for case model transformation.

Required characteristic	Case model							
	BLT-1	BLT-2	BLT-3	BLT-4	BJ-1	BJ-2	BJ-3	SCK
CT1 (MPa)	0.7	0.8	1.0	1.2	0.7	0.8	1.0	0.7
CT2 (m ³ /min)	6.80	6.28	5.60	4.60	6.80	6.20	5.60	6.4
CT3 (dB)	69	69	70	70	68	69	69	72
CT4 (yuan)	43,500	44,000	44,000	44,500	45,000	45,500	45,500	42,000
CT5 (kgCO ₂ e)	160,438	160,678	160,802	160,971	165,076	167,315	168,940	160,884
CT6 (kgCO ₂ e)	18,622	18,804	18,849	18,976	18,153	18,455	18,639	19,096

Table 3. Dependent function values for test samples.

Case	BLT-1	BLT-2	BLT-3	BLT-4	BJ-1	BJ-2	BJ-3	SCK
K_{n-D}	-0.015	-0.226	-0.027	-0.031	-0.169	-0.250	-0.298	-0.100
V	V ₋	V ₋	V ₋	V ₋	V ₋	V ₋	V ₋	V ₋

Hence, the layer-4 classification at this stage is expressed as follows

$$Z_{PLCD_CASE_9^0} \subset V_{-}^{22222}$$

$$= \{K_{3-D}^1 < 0, K_{2-D}^2(E, P) < 0, K_{1-D}^{2,1}(C) > 0, K_{3-D}^{3,j_1}(E_{51}, E_{53}, P_{53}) < 0, K_{3-D}^{3,3}(E_{52}, P_{51}, P_{52}) > 0, K_{3-D}^{4,t_1}(E_{511}, E_{531}, P_{531}) < 0, K_{2-D}^{4,2}(E_{512}, P_{532}) > 0, j_1 = 1, 2, 3; t_1 = 1, 2, 3\}$$

- E_{511} consists of dual-screw rotor structure carbon footprint (E_{5111}) and electric rotor structure carbon footprint (E_{5112}), with E_{5112} being the main factor. E_{531} mainly comprises fan parts' structure carbon footprint (E_{5311}). P_{531} comprises muffler structural element of the gas inlet noise (P_{5311}) and muffler cover structural element of screw compressor housing noise (P_{5312}); neither P_{5311} nor P_{5312} satisfies the requirement. Hence, the layer-5 classification at this stage is expressed as follows

$$Z_{PLCD_CASE_9^0} \subset V_{-}^{22222}$$

$$= \{K_{3-D}^1 < 0, K_{2-D}^2(E, P) < 0, K_{1-D}^{2,1}(C) > 0, K_{3-D}^{3,j_1}(E_{51}, E_{53}, P_{53}) < 0, K_{3-D}^{3,3}(E_{52}, P_{51}, P_{52}) > 0, K_{3-D}^{4,t_1}(E_{511}, E_{531}, P_{531}) < 0, K_{2-D}^{4,2}(E_{512}, P_{532}) > 0, K_{4-D}^{5,t_2}(E_{5112}, E_{5311}, P_{5311}, P_{5312}) < 0, K_{1-D}^{5,1}(E_{5111}) > 0, j_1 = 1, 2, 3; t_1 = 1, 2, 3; t_2 = 1, 2, 3, 4\}$$

Based on this method, $CASE_9^0$ can be classified into a more detailed and precise layer in the negative field. By analogy, $CASE_2^1$ and $CASE_{17}^2$ are classified into

$$J(Z_{PLCD_object^l})$$

$$= \{(Z_{PLCD_object^l}, Y, Y') | Y = Y' = K_{n-D}(\{v_i(B_{Pro_Attribute^l}^i)\}, S_1, S_2) < 0\}$$

$$= \{CASE_{i,j}^l, j = 3, \dots, 16 \cup i = 1, \dots, 16 \cup i \neq 2 \text{ and } 4, CASE_{i,l}^l, l = 1, \dots, 8\}$$

positive fields, whereas the other 18 cases are in negative field.

Dynamic classification

- Dynamic classification based on case model transformation. Here, $CASE_{11}^{13}$ was chosen as a case for transformation. The main attributes basically include exhaust pressure (CT1), exhaust volume (CT2), motor power, noise (CT3), weight, exhaust interface, overall dimensions, buying cost (CT4), use cost, recycling cost, carbon footprint used (CT5), and carbon footprint of marketed product (CT6), as listed in Table 2.

Here, we just list the required attributes CT1–CT6, which respond to the six attributes ($PR_j^1, PR_j^2, PR_j^3, PR_j^4, PR_j^5$, and PR_j^6). With back propagation (BP) neural network, we take 16 product cases as training samples, the six attributes as the input parameters, and the dependent function values as the output. The eight new case models are taken as testing samples, and the corresponding dependent function values are obtained, as listed in Table 3.

Assume that screw compressor cases in Table 3 are expressed as $CASE_l^i, l = 1, \dots, 8$, then basic-element extension set for the negative field of screw compressor cases after transformation is

$$\begin{aligned}
\begin{cases} P_{431} = \emptyset \\ \mu P_{431} \neq \emptyset \end{cases} &\Rightarrow \begin{cases} \mu T_{v(P_{431})} v(P_{431}) = v'(P_{431}) \Rightarrow v'(P_{431}) T_{v(P_{\text{Noise}})} v(P_{\text{Noise}}) = v'(P_{\text{Noise}}) \\ [\mu T_{Z_{\text{LCSB}}} Z_{\text{LCSB}} \neq \emptyset] \Rightarrow \begin{cases} Z_{\text{LCSB}} T_{v(E_{\text{Sell}})} v(E_{\text{Sell}}) = v'(E_{\text{Sell}}) \\ Z_{\text{LCSB}} T_{v(C_{\text{Buy}})} v(C_{\text{Buy}}) = v'(C_{\text{Buy}}) \\ Z_{\text{LCSB}} T_{v(E_{\text{Use}})} v(E_{\text{Use}}) = v'(E_{\text{Use}}) \end{cases} \end{cases} \\
&\Rightarrow \begin{cases} K_{3-D}(v'(P_{\text{Noise}}, P_{\text{PP}}, P_{\text{PV}}), S_1, S_2) > 0 > K_{3-D} \\ K_{3-D}(v'(E_{\text{Sell}}, C_{\text{Buy}}, E_{\text{Use}}), S_1, S_2) > 0 > K_{3-D} \end{cases} \\
&\Rightarrow K_{6-D}(v'(P_{\text{Noise}}, P_{\text{PP}}, P_{\text{PV}}, E_{\text{Sell}}, C_{\text{Buy}}, E_{\text{Use}}), S_1, S_2) > 0 > K_{6-D}
\end{aligned}$$

2. *Dynamic classification based on attribute transformation: take CASE₅¹¹ as an example.*

- (a) This screw compressor case is noisy in use stage, and increase transformation T_{add} is chosen for attribute transformation, that is, adding one muffler (Ingsoland, model 80064843). In accordance with the calculation method used for sound deadening capacity of the muffler, the obtained noise after transformation was 68.5 dB, such that $K_{3-D}(v'(P_{\text{Noise}}, P_{\text{PP}}, P_{\text{PV}}), S_1, S_2) > 0 > K_{3-D}$, where P_{PP} and P_{PV} denote the exhaust pressure and exhaust volume, respectively. Hence, performance indicators of this case exhibited a qualitative change.
- (b) The buying cost of the screw compressor changed to 45,100 yuan and the market carbon footprint changed to 19,370.2 kgCO₂e, such that

$$\begin{aligned}
&\begin{cases} K_{1-D}(v(C_{\text{Buy}})) > K_{1-D}(Z_{\text{LCSB}} T_{v(C_{\text{Buy}})} v(C_{\text{Buy}})) > 0 \\ K_{1-D}(v(E_{\text{Sell}})) > K_{1-D}(Z_{\text{LCSB}} T_{v(E_{\text{Sell}})} v(E_{\text{Sell}})) > 0 \end{cases} \\
&\Rightarrow 0 < K_{2-D}(v'(C_{\text{Buy}}, E_{\text{Sell}}), S_1, S_2) < K_{2-D}
\end{aligned}$$

where Z_{LCSB} is the additive low-carbon structure basic of the muffler module, and C_{Buy} and E_{Sell} denote the buying cost and carbon footprint of marketed products, respectively. Hence, both C_{Buy} and E_{Sell} experienced a conductive transformation and a quantitative change.

- (c) After the muffler was added, motor working energy consumption decreased, and carbon footprint at use stage could be reduced; its value is 157,497.5 kgCO₂e after transformation, such that $K_{1-D}(Z_{\text{LCSB}} T_{v(E_{\text{Use}})} v(E_{\text{Use}})) > 0 > K_{1-D}$. Hence, attribute E_{Use} exhibited a qualitative change, from not satisfying low-carbon demand to satisfying low-carbon demand. The transformation process of CASE₅¹¹ is expressed as follows

Therefore, CASE₅¹¹ qualitatively changed from a negative case field V_- to a case field of positive qualitative change $V_{.+}$.

3. *Dynamic classification based on attribute values' transformation: take CASE₁₅¹² as an example.*

If cost expenditure at maintenance stage is increased (e.g. an additional 3500 yuan is spent for maintenance cost C_6), that is, $T_{\text{add}}C_6 = C_6' = C_6 + 3500$, then both energy consumption and carbon footprint at use stage could be reduced (with C_5 decreasing by about 4680 yuan and E_5 decreasing by about 10,384.1 kgCO₂e).

The combinational transformation based on case field of the screw compressor can add more product cases that are consistent with low-carbon requirements. It greatly improves the competition for product attributes and provides a good foundation for further dynamic classification.

The dynamic classification results for screw compressor cases are listed in Table 4, in which K_0 denotes the static classification, K_1 denotes the dynamic classification based on case field transformation, and $K_i - 1$ and K_i denote the dynamic classification based on dependent function transformation.

The result of dynamic classification in Table 4 shows that transformations of case field based on case model, attributes, and attribute values greatly affect the original static classification. For instance, cases 5, 6, 9, 10, 16, 63, 64, 65, and 67 all undergo a positive qualitative change, from V_- to V_+ . Cases 2 and 17 undergo a positive quantitative change, whereas cases 1, 3, 7, 8, 11, 12, 13, 14, 15, 61, 62, 66, and 68 all undergo a negative quantitative change.

4. *Dynamic classification based on dependent function transformation.* Make transformation for desirable interval S_1 and acceptable interval S_2 ; suppose $(i - 1)$ th and i th demand for screw compressor cases are

Table 4. Classification for screw compressor cases.

Case	K0	K1	Ki - 1	Ki	Case	K0	K1	Ki - 1	Ki
1. JN-1	V ₋	V ₋ /V ₋	V ₋ /V ₋	V ₋	14. KH-1	V ₋	V ₋ /V ₋	V ₋ /V ₋	V ₋
2. JN-2	V ₊	V ₊ /V ₊	V ₋ /V ₋	V ₋	15. KH-2	V ₋	V ₋ /V ₋	V ₋ /V ₋	V ₋
3. JN-3	V ₋	V ₋ /V ₋	V ₋ /V ₋	V ₋	16. KH-3	V ₋	V ₊ /V ₊	V ₋ /V ₋	V ₋
5. LG-2	V ₋	V ₊ /V ₊	V ₋ /V ₋	V ₋	17. KH-4	V ₊	V ₊ /V ₊	V ₋ /V ₋	V ₋
6. LG-3	V ₋	V ₊ /V ₊	V ₋ /V ₋	V ₋	61. BLT-1	V ₋	V ₋ /V ₋	V ₋ /V ₋	V ₋
7. SA-1	V ₋	V ₋ /V ₋	V ₋ /V ₋	V ₋	62. BLT-2	V ₋	V ₋ /V ₋	V ₋ /V ₋	V ₋
8. SA-2	V ₋	V ₋ /V ₋	V ₋ /V ₋	V ₋	63. BLT-3	V ₋	V ₊ /V ₊	V ₋ /V ₋	V ₋
9. SA-3	V ₋	V ₊ /V ₊	V ₋ /V ₋	V ₋	64. BLT-4	V ₋	V ₊ /V ₊	V ₋ /V ₋	V ₋
10. SA-4	V ₋	V ₊ /V ₊	V ₋ /V ₋	V ₋	65. BJ-1	V ₋	V ₊ /V ₊	V ₋ /V ₋	V ₋
11. SB-1	V ₋	V ₋ /V ₋	V ₋ /V ₋	V ₋	66. BJ-2	V ₋	V ₋ /V ₋	V ₋ /V ₋	V ₋
12. SB-2	V ₋	V ₋ /V ₋	V ₋ /V ₋	V ₋	67. BJ-3	V ₋	V ₊ /V ₊	V ₋ /V ₋	V ₋
13. SB-3	V ₋	V ₋ /V ₋	V ₋ /V ₋	V ₋	68. SCK	V ₋	V ₋ /V ₋	V ₋ /V ₋	V ₋

$$\left\{ \begin{array}{l} PR_{i-1} = \\ PR_i = \end{array} \right. \left[\begin{array}{l} O_{PR_{i-1}} \text{ exhaust pressure} \\ O_{PR_{i-1}} \text{ exhaust volume} \\ O_{PR_{i-1}} \text{ noise} \\ O_{PR_{i-1}} \text{ buying cost} \\ O_{PR_{i-1}} \text{ carbon footprint used} \\ O_{PR_{i-1}} \text{ carbon footprint of marketed product} \\ O_{PR_i} \text{ exhaust pressure} \\ O_{PR_i} \text{ exhaust volume} \\ O_{PR_i} \text{ noise} \\ O_{PR_i} \text{ buying cost} \\ O_{PR_i} \text{ carbon footprint used} \\ O_{PR_i} \text{ carbon footprint of marketed product} \end{array} \right] = \left[\begin{array}{l} [1, 1.3] \text{ MPa} \\ [5.0, 6] \text{ m}^3/\text{min} \\ [65, 68] \text{ dB} \\ [42, 000, 43, 000] \text{ yuan} \\ [145, 000, 158, 000] \text{ kgCO}_2\text{e} \\ [18, 000, 18, 500] \text{ kgCO}_2\text{e} \\ [1, 1] \text{ MPa} \\ [5.0, 5.6] \text{ m}^3/\text{min} \\ [65, 69] \text{ dB} \\ [42, 000, 45, 000] \text{ yuan} \\ [150, 000, 155, 000] \text{ kgCO}_2\text{e} \\ [18, 000, 20, 000] \text{ kgCO}_2\text{e} \end{array} \right] = \left[\begin{array}{l} PR_{i-1}^1 \\ PR_{i-1}^2 \\ PR_{i-1}^3 \\ PR_{i-1}^4 \\ PR_{i-1}^5 \\ PR_{i-1}^6 \\ PR_i^1 \\ PR_i^2 \\ PR_i^3 \\ PR_i^4 \\ PR_i^5 \\ PR_i^6 \end{array} \right]$$

Comparing two low-carbon demands, we take screw compressor case BJ-2 as an example. As customer demand is modified, the desirable and acceptable interval of dependent function will change. Under $(i - 1)$ th demand, the dependent function value is $K_{6-D}^{i-1} = -0.7500$, and under i th demand, the dependent function value is $K_{6-D}^i = -0.8000$. Hence, this case exhibited a negative quantitative change.

Dynamic classification for screw compressor cases is a process under approximately two low-carbon demand transformations, namely, take $(i - 1)$ th classification result as the static classification (i.e. V_{+} can be regarded as V_{+}), and it is the basis for i th dynamic classification. Table 4 lists the dynamic classification result for the $(i - 1)$ th and i th demands of all cases. To reflect classification clearly, we choose four cases, that is, cases 2, 10, 17, and 68 with different classification processes, to make a histogram comparative analysis, as shown in Figure 4.

In the histogram, under $(i - 1)$ th demand (with gray parts), the result indicates that cases have low similarity with customer requirements. For i th demand, the dependent function value of each case increases, namely, the similarity is improved. Among the four cases, cases 2 and 10 have a closer expected value,

especially case 2 exhibits a more active property and has a higher similarity.

Discussion

In the article, we proposed extension classification method for low-carbon design, including static classification and dynamic classification. But there are some critical problems need to be discussed.

In static classification, designers can get detailed information about cases in negative field, and it favors modification for unsatisfactory cases. But one case in one demand may belong to negative field while in another demand it might belong to positive field, and in a database numerous cases get retrieved at the same time. Therefore, constructing a dynamic algorithm procedure to exhibit cases information in different demands is necessary.

In dynamic classification, we take transformation for case model, attributes, and attribute values in order, but in a real product design, it might take these three transformations together. Moreover, in attributes' transformation, deletion, increase, and substitution transformations are researched independently, but in

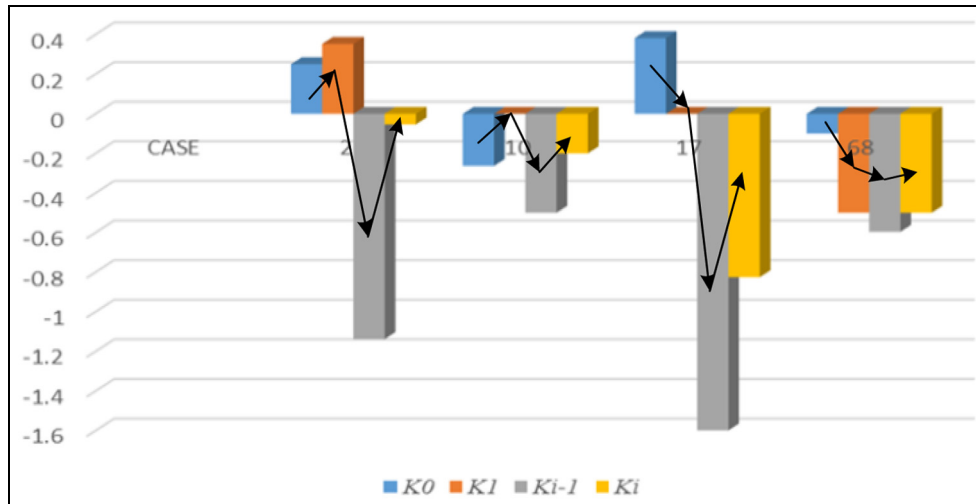


Figure 4. Dynamic classification for cases under variable low-carbon demands.

application, they also might be used together. In attribute values' transformation, we only study cost values' transformation, carbon footprint and performance follow the same approach.

For extension classification, we adopt dependent function to build the similarity function or to discriminate attributes. Thus, computational accuracy of dependent function, especially for a high-dimension state, is important. Researchers have been studying computational methods for multi-dimension dependent function and improving work still needs to be done.

Conclusion and future research

In low-carbon design, we proposed extension classification method based on extension theory. In static classification, with dependent function and similarity function, product cases are categorized by a hierarchical structure, and it assists designers in modifying cases. In dynamic classification, transformations based on case model, attributes, attribute values, and dependent function are studied, and through these transformations cases are modified to meet special demands. In case study, we take some screw compressor cases to demonstrate the effectiveness of the proposed classification method.

Research on product cases' classification is the foundation of CBR system for low-carbon design. Carbon footprint is one of the primary evaluation indicators, and there is still no normal method to make an accurate estimation. In the classification, case attributes in a same stage or in a different stage have complex correlations; when we make transformation for one attribute, we should take all the related factors into consideration. Therefore, in future research, we focus on constructing a computation model for carbon footprint in a product life cycle based on the activity-based costing

(ABC) method, and we also study the representation for attributes with complex correlations.

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Appendix I

Notation

		T	transformation
		T_x	transformation for attribute x
		${}_xT_y$	conductive transformation, transformation for attribute y arising from the transformation for attribute x
C	case attribute representation in basic-element formation		case attribute value representation in basic-element formation
C	cost in low-carbon attributes, C_C		positive field
Case_Product	case identity representation in basic-element formation	V	negative field
E	carbon footprint in low-carbon attributes, C_E	V_+	field of positive qualitative change
$J(Z_{PLCD_object})$	basic-element extension set	V_-	field of negative qualitative change
K	dependent function	$V_{.+}$	critical state
K_{n-D}	n -dimension dependent function		$Y = K$, dependent function value
O_{PLCD_object}	case model	$V_{.-}$	$Y' = T_K K$, dependent function value after transformation
P	performance in low-carbon attributes, C_P	V_0	low-carbon structural element, referring to the smallest structural unit including E , C , and P
P	product case attributes respond to PR	Y	product case in S_{PLCD}
		Y'	
PR	product requirement		
S_1	desirable interval	Z_{LCSB}	
S_2	acceptable interval		
S_{PLCD}	comprehensive basic-element set of product low-carbon design	Z_{PLCD_object}	
\tilde{S}_{PLCD}^{sim}	similarity basic-element set		
\tilde{S}_{PLCD}^{sim}	similarity basic-element cut set	μ	active transformation
Sim	similarity value		