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Membership Functions Definition in the Fuzzy Semantic Model

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Abstract: The Fuzzy Semantic Model (FSM) is a database model that has been recently proposed. In addition to supporting the fuzziness at the attribute level, FSM authorizes also an entity to be partially member of its class according to a given degree of membership that reflects the level to which the entity verifies the extent properties of this class. It also supports subclass/superclass membership functions permitting to calculate the range to which a class is a subclass of another class. In this paper, we provide the ways the membership function is defined for all the constructs of the FSM. We also introduce the definition of membership function at subclass/superclass relationships levels. The paper includes also the first results of an ongoing implementation of our works at the automotive company PSA Peugeot Citroen.

Key words: Fuzzy semantic model, Fuzzy database, Membership functions definition

1 Introduction

In database context, there are several tentatives to develop database models that support the fuzziness and impreciseness of real-world [10,7]. Most efforts have been oriented towards the extension of relational database models [3,5,15,4]. We enumerate also some extensions of object-oriented and semantic database models [18,19,1]. However, most of these extensions introduce fuzziness only at the attribute level and consider that entities are fully encapsulated into their classes.

There are however some recent extensions of object oriented database models to support the fuzziness of real-world at the class definition level [8,19,12]. We also enumerate some extensions of semantic database models [11].

In [6,16,2], the authors have proposed a new database model, the Fuzzy Semantic Model (FSM), that authorizes an entity to be partially member of its class according to a given degree of membership that reflect the level to which the entity verifies the extent properties of his class. FSM supports also subclass/superclass membership functions permitting to calculate the range to which a class is a subclass of another class. In this paper, we provide the ways the membership function is defined for all the constructs of the model. We also introduce the definition of membership function at subclass/superclass relationships levels.

We notice that several illustrative examples are provided in this paper and that most of them rely on the database example GALAXY illustrated in Figure 4 in the end of the paper. Readers are thus invited to refer frequently to this figure to better appreciate the examples.

The next section briefly introduces the FSM and than presents the ways the membership functions are defined. Section 3 presents the first results of an ongoing implementation of our works at the automotive company PSA Peugeot Citroen. Section 4 concludes the paper.

2 Membership functions in FSM

In FSM (see [6,16,2] for a full description of the FSM), a fuzzy class K in the space of entities E is a collection of fuzzy entities. Mathematically, we write $K=\{(e,\mu_k(e)):e\in E;\mu_K(e)\succ 0\}$. μ_k is a characteristic or membership function and $\mu_k(e)$ represents the degree of membership of fuzzy entity e in fuzzy class K.

2.1 Entity/Class membership functions

In FSM, membership functions of fuzzy classes are defined as follows. As it is underlined above, a fuzzy class is a collection of many fuzzy entities having some similar properties. Fuzziness is thus induced whenever an entity verifies only (partially) some of these properties. We denote by $P_K=\{p_1,p_2,\ldots,p_n\}$ (with $n\geq 1$) the set of these properties for a given fuzzy class K. P_k is called the *extent* of class K. The extent properties may be derived from the attributes of the class and/or from common semantics. For example, the fuzzy class STAR in Figure 4 may have two extent properties based on the attributes *luminosity* and *weight*. The extent to which each of theses properties determines the class K is not the same. Indeed, there are some properties that are more discriminative than others. To ensure this, we associate to each extent property p_i a non negative weight w_i reflecting its importance in deciding whether or not an entity e is a member of a given class K. We also impose that $\sum_{i=1}^{n} w_i \succ 0$.

On the other hand, an entity may verify fully or partially extent properties for a given fuzzy class. Let D^i be the basic domain of extent property p_i values and P_i is a subset of D^i, which represents the set of possible values of property p_i. The *partial membership function* of an extent property value is $\rho_{P_k^i}$, which maps elements of D_i into [0,1]. For any attribute value υ_i in D^i, $\rho_{P_k^i}(\upsilon_i)=0$ means that the fuzzy entity e violates property p_i and $\rho_{P_k^i}(\upsilon_i)=1$ means that this entity verifies fully the property. The number υ_i is the value of the attribute of entity e on which the property p_i is defined. More generally, the value of $\rho_{P_k^i}(\upsilon_i)$ represents the extent to which entity e verifies property p_i of fuzzy class K. Thus, the global d.o.m of fuzzy entity e in fuzzy class K is :

$$\mu_K(e)=\frac{\sum_i^n \rho_{p_k^i}(\upsilon_i)\cdot\omega_i}{\sum_i^n \omega_i} \qquad (1)$$

To better appreciate the way the global membership functions are defined and calculated, we consider the following example. Suppose that the fuzzy class YOUNG of young persons is defined through the attributes *age* and *height*. Accordingly, the extent set of this class is $P_{\text{Young}}=\{p_1, p_2\}$, where p_1 and p_2 properties are defined respectively on the *age* and *height* attributes. Clearly, the *age* attribute is more relevant in defining a young person. However, in many situations it is not possible to determine the exact age of that person and the *height* attribute will be a good indicator. To ensure this, we assign to p_1 and p_2 the weights of $w_1=0.8$ and $w_2=0.3$ respectively. Suppose now that we aim to calculate the global d.o.m of two persons e_1 and e_2 in the fuzzy class YOUNG. The two fuzzy properties of 'being young' and 'having average height' are shown in Figure 1. In this figure, it is easy to see that: $\rho_{p^1_{Young}}(e_1.age)=0.53$ and $\rho_{p^2_{Young}}(e_1.height)=0.9$. Thus by applying Equation (1), we have:

$$\mu_{Young}(e_1)=\frac{0.53\cdot 0.8+0.9\cdot 0.3}{0.8+0.3}=0.63.$$

In similar way, we have $\rho_{p^1_{Young}}(e_1.age)=0.8$ and $\rho_{p^2_{Young}}(e_1.height)=0.7$, which gives

$$\mu_{Young}(e_2)=\frac{0.8\cdot 0.8+0.7\cdot 0.3}{0.8+0.3}=0.78.$$

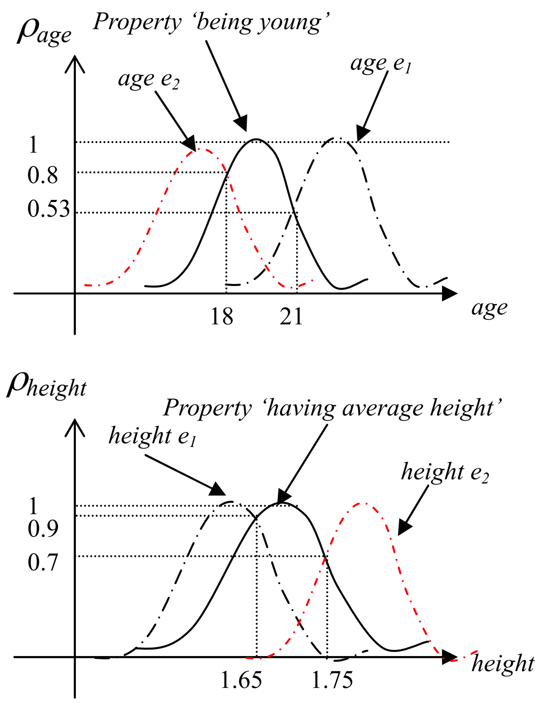

Figure 1. Partial membership functions

2.2 Membership functions for fuzzy interaction classes

An interaction (or association) relationship relates members of one fuzzy class to other members of one or many fuzzy classes. The interaction relationships may require or not the creation of new attributes that describe the interaction relationship. In the former case, a new *fuzzy interaction* class is generated. For instance, each member of the DISCOVERY fuzzy interaction class in Figure 4 associates one member (may be several members in the case when the discovery is accomplished by several scientists) from SCIENTISTS fuzzy class with one member from SUPERNOVAE fuzzy class. This relationship may be further described with two attributes *date-of-discovery* and *place-of-discovery* that permit to handle some information concerning the date and place of the discovery. In turn, the interaction relationship between LABORATORY and PERSON requires no attributes. (Although attributes such that *affiliation-start-date* and *affiliation-end-date* may be associated with it to indicate the period (s) of time during which the person is affiliated in the laboratory.)

The fuzzy interaction class should not have extent properties since its members are fully defined in terms of the extent properties of the participant fuzzy classes. However, the d.o.m of a member e of a fuzzy interaction class I relating m members $e_1, e_2, ...e_m$ from m fuzzy classes $K_1, K_2,..., K_m$ may be calculated as follows :

$$\mu_I(e) = \prod_{i=1}^{m} \mu_{Ki}(e_i). \qquad (2)$$

Consider, for instance, an entity e member of fuzzy class DISCOVERY that relates entities e_1 and e_2 from SCIENTISTS and SUPERNOVAE fuzzy classes, respectively. SCIENTIST is an exact class that all its members are true ones, i.e, $\forall\ e \in$ SCIENTIST, $\mu_{Scientist}(e) = 1$. Particularly, we have $\mu_{Scientist}(e_1) = 1$. The extent properties set of fuzzy class SUPERNOVAE is $P_{Supernovae}=\{p_1, p_2\}$ where extent properties p_1 and p_2 are based on *luminosity* and *weight* attributes. Let $\rho_{P^1_{Supernovae}}(e_2.luminosity) = 0.45$ and $\rho_{P^2_{Supernovae}}(e_2.weight) = 0.80$. We also assign to p_1 and p_2 the weights $w_1 = 0.6$ and $w_2 = 0.5$, respectively. Equation (1) above gives:

$$\mu_{Supernovae}(e_2) = \frac{0.45 \cdot 0.6 + 0.80 \cdot 0.5}{0.6 + 0.5} = 0.609.$$

By applying Equation (2), the d.o.m of entity e in fuzzy class DISCOVERY is equal to:

$$\mu_{Discovery}(e) = \mu_{Scientist}(e_1).\ \mu_{Supernovae}(e_2)$$

$$= 0.609.$$

Finally, we mention that other ways for calculating the d.o.m of e in I may also apply as for example:

$$\mu_I(e) = \min_{1 \le i \le m} \mu_{K_i}(e_i)$$

or

$$\mu_I(e) = \sum_{i=1}^{m} \mu_{K_i}(e_i) / m.$$

2.3 Membership function for fuzzy class relationships

FSM supports two types of inter-class relationships: generalization and specialization. (Several authors use the term of ISA relationships instead of generalization/specialization; e.g., each scientist is-a person.) The *generalization relationship* relates a fuzzy superclass to one or several simple or complex fuzzy subclasses. Such a relation advocates that all members of the fuzzy subclass are members of its fuzzy superclass. Any generalization relationship creates implicitly a specialization relationship, which relates a fuzzy subclass to a fuzzy superclass. The same superclass may have one, two or more subclasses (e.g. class PERSON in Figure 4 has three subclasses: SCIENTIST, TECHNICIAN and OFFICER) and the same subclass may have more than one fuzzy superclasses.

A fuzzy subclass may be attribute-defined, roster-defined or set-operation-defined. An attribute-defined fuzzy subclass has one or several attributes' values that are in accordance with some discriminative values that characterize perfectly its members. For instance the fuzzy subclass SUPERNOVAE in Figure 4 is a specialization of the fuzzy class STAR basing on the attribute *type-of-star*. The attribute-defined fuzzy subclasses inherit all attributes of their fuzzy superclasses.

A roster-defined fuzzy subclass is simply defined by an explicit enumeration of its members. These subclasses inherit all attributes of their superclasses. For instance, in Figure 4 the subclasses SCIENTIST, TECHNICIAN and OFFICER are three roster-defined classes of superclass PERSON.

A set-operation fuzzy subclass may be defined as the difference or the set-intersection of two or more fuzzy classes. Members of difference fuzzy subclass of two fuzzy superclasses are members of the first fuzzy class that are not members of the second fuzzy superclass. The difference fuzzy subclass inherits only attributes of the first fuzzy superclass. Members of a set-intersection fuzzy subclass of two or several fuzzy superclasses are members of each of these superclasses. The set-intersection fuzzy subclass inherits all attributes that are common to all the participant fuzzy superclasses.

Fuzzy subclasses as well as superclasses have their own extent properties and the d.o.m of their members may be calculated through Equation (1). In [12] the authors distinguish two types of object/class relationships in object-oriented databases. The first is a direct object-class relationship, which apply when the object and the class have the same attributes. The second is an indirect object-class relationship and is specific for subclass/superclass relationships where an object belonging to the subclass must belong to the superclass since a subclass is a specialization of the superclass. The authors propose ways to calculate the d.o.m of a member of the subclass to the superclass. The idea may be adapted to our FSM as follows. Let S_1 be a subclass of S_2. The inheritance concept associated with subclass/superclass relationships advocates that S_1 inherits some attributes from S_2, overrides some others and adds some new ones. Then, let S_2 has the extent properties set $P_{S2} = \{p_1, p_2,...,p_k, p_{k+1},...,p_m\}$ and S_1 has extent properties set $P_{S1} = \{p_1, p_2,..., p_k, p'_{k+1},..., p'_m, p_{m+1}..., p_n\}$ where $p'_{k+1},..., p'_m$ are overridden from $p_{k+1},...,p_m$ (i.e. for all $i= k+1$ to m, p'_i is based on the same attribute on which p_i is based). Thus, the d.o.m of an entity e from fuzzy subclass S_1 in fuzzy superclass S_2 of S_1, written $\mu_{S_2}(e/S_1)$, is :

$$\mu_{s_2}(e/S_1) = \frac{\sum_{i=1}^{k} \rho_{Pi}(\upsilon_i)\cdot\omega_i + \sum_{J=k+1}^{m} \rho_{P'j}(\upsilon_j)\cdot\omega_j'}{\sum_{i=1}^{k} w_i + \sum_{j=k+1}^{m} w_j'}. \quad (3)$$

For i through 1 to m, $P^i \subset D^i$ represents the set of possible values of extent property p_i, w_i is the weight of p_i and v_i is the value of the attribute of entity e on which property p_i is based.

For example, suppose that $P_{Star} = \{p_1, p_2, p_3\}$ where extent properties p_1, p_2 and p_3 are based on *luminosity*, *weight* and *age* attributes, respectively; and $P_{Supernovae} = \{p_1', p_2'\}$ where extent properties p_1' and p_2' are based on *luminosity* and *weight* attributes, respectively. Let also $\omega_1 = \omega_1' = 0.7$, $\omega_2 = \omega_2' = 0.3$ and $\omega_3 = 0.2$. Suppose that exists a member e in SUPERNOVAE for which:

$\rho_{p^1_{Star}}(\text{e.}luminosity) = 0.56$

$\rho_{p^2_{Star}}(\text{e.weight}) = 0.34,$

$\rho_{p^3_{Star}}(\text{e.age}) = 0.12$,

$\rho_{p'^1_{Supernovae}}(\text{e.}luminosity) = 0.35$ and

$\rho_{p'^2_{Supernovae}}(\text{e.weight}) = 0.60$.

Then , the d.o.m of e in STAR is :

$$\mu_{Star}(e / Supernovae)$$

$$= \frac{0.35\cdot 0.7 + .60\cdot 0.3 + 0.12\cdot 0.2}{0.7 + 0.3 + 0.2} = 0.37.$$

2.4 Subclass/Superclass membership functions

In [12] the authors extend the notion of membership function to the subclass/superclass relationships in object-oriented database models. To calculate the d.o.m of a subclass in a superclass, they use a weighted sum of the inclusion degrees of the domains of the attributes of the subclass in the domains of the attributes of the superclass. In this paper we adopt a similar way. The only difference is that we will use the d.o.m of entities relatively to their direct classes as weights. Formally, the d.o.m of a fuzzy subclass S_j in its fuzzy superclass S_i, written $\mu(S_i, S_j)$, is equal to :

$$\mu(S_i, S_j) = \frac{\sum_{e_r \in S_j} \mu_{S_i}(e_r / S_j)\cdot \mu_{S\,j}(e_r)}{\sum_{e_r \in S_j} \mu_{S_j}(e_r)}, \quad (4)$$

where $\mu_S(e)$ is the d.o.m of entity e in fuzzy class S_j calculated as in Equation (1) and $\mu_{Si}(e/S_j)$ represents the d.o.m of entity e of S_j in S_i calculated as in Equation (3).

For example, suppose that the fuzzy subclass SUPERNOVAE contains three entities e_1, e_2 and e_3 with $\mu_{Supernovae}(e_1) = 0.55$, $\mu_{Supernovae}(e_2) = 1.0$ and $\mu_{Supernovae}(e_3) = 0.67$. We suppose also that these entities verify the following:

- $\mu_{Star}(e_1/Supernovae) = 0.37$.
- $\mu_{Star}(e_2/Supernovae) = 0.15$.
- $\mu_{Star}(e_3/Supernovae) = 0.90$.

Then, Equation (4) gives:

$\mu(Star, Supernovae) =$

$$= \frac{0.37\cdot 0.55 + 0.15\cdot 1.0 + 0.90\cdot 0.67}{0.55 + 1.0 + 0.67} = 0.430.$$

The subclass/superclass membership functions as calculated here differ from those in [12] in that sense that they are function of the entities currently present in the sublcass. This means that the value of $\mu(S_i, S_j)$ may evolve over time and it is not 'static' as in [12].

Since roster-defined subclasses are directly and explicitly controlled by the users [9], they may be considered---mainly when the subclass and its superclass do not share any extent property---as true members [2,6] of their superclasses. In that case, $\mu_{Si}(e_r/S_j)$ in Equation (4) will be equal to one and, consequently, $\mu(S_i, S_j) = 1$.

2.5 Membership functions for fuzzy composite classes

A *fuzzy composite* class is a strong fuzzy class that has other fuzzy classes as members. If the members of a fuzzy composite class are subclasses of the same fuzzy superclass, they are said to be homogenous. Otherwise, they are *heterogeneous*. The essential utility of fuzzy composite classes is that they permit to maintain some class attributes. For example, a fuzzy composite class SCIENTIST-TYPES may be defined on the SCIENTIST fuzzy class and may have ASTROPHYSICIST, ASTRONOMER and CHEMIST as subclasses. Further, a *total-number-of-scientists* may be associated with it as a class attribute.

A fuzzy composite class may be defined basing on a collection of attributes or simply by enumerating its members. Those, which are defined on a collection of attributes, share the same values for a subset of attributes. These attributes are called selection attributes of the fuzzy composite class. For instance, the fuzzy composite class SCIENTIST-TYPES may be defined on the *field-of-research* attribute of SCIENTIST. Selection attributes form also the attributes of the fuzzy composite class and they constitute the identifier of the members of this class. Since all the members of attribute–based fuzzy composite classes have exactly the same attributes, they are necessarily homogenous ones.

The d.o.m in attribute-based fuzzy composite classes as computed as follows. Let C be an attribute-based fuzzy composite class basing on the selection attributes $a_1, a_2, \ldots a_r$. The extent properties set of fuzzy composite class C is based on all or a subset of the selection attributes : $P_C = \{p_1,p_2,...p_q\}$ with $q \leq r$ and $p_1,p_2,...p_q$ are based respectively on attributes $a_1,a_2,...,a_q$. As mentioned above, members of an attribute-based fuzzy composite class C are themselves fuzzy classes K_1, K_2,...K_p that are subclasses of the same fuzzy superclass S. This means that each member e_i of C is in relation with all the members of one class, say K_i. Thus, since at least a subset of the selection attributes are common to the fuzzy composite class and to its members, the d.o.m of e_i in C may then be calculated as follows :

$$\mu_C(e_i) = \mu(C, K_i) \cdot \mu(S, K_i). \quad (5)$$

$\mu(C, K_i)$ and $\mu(S, K_i)$ are the d.o.m of class K_i in fuzzy composite class C and in its fuzzy superclass S, respectively. They may be computed in similar way to Equation (4), i.e.:

$$\mu(C,K_i) = \frac{\sum_{e_r \in Ki} \mu_C(e_r / K_i) \cdot \mu_{K_i}(e_r)}{\sum_{e_r \in Ki} \mu_{Ki}(e_r)}, \quad (6)$$

and

$$\mu(S,K_i) = \frac{\sum_{e_r \in K_i} \mu_S(e_r/K_i) \cdot \mu_{Ki}(e_r)}{\sum_{e_r \in K_i} \mu_{K_i}(e_r)}. \quad (7)$$

Because all the members of a subclass of a fuzzy composite class share exactly the same values for the selection attributes on which extent properties set is based, they belong to the composite class with the same d.o.m. This means that in Equation (6) above $\mu_C(e_r / K_i)$ is the same for all e_r in K_i. If we suppose that $\forall e_r$ in K_i, $\mu_C(e_r/Ki) = \alpha_i$ with $\alpha_i \in [0, 1]$, then Equation (6) above gives:

$$\mu(C,K_i) = \frac{\alpha_i \cdot \sum_{r \in Ki} \mu_{ki}(e_r)}{\sum_{r \in Ki} \mu_{K_i}(e_r)} = \alpha_i. \quad (8)$$

Consequently, Equation (5) becomes:

$$\mu_C(e_i) = \alpha_i \cdot (S, K_i) \quad (9)$$

To illustrate this, we suppose that PLANET-TYPES is a fuzzy composite class from fuzzy class PLANET basing on the *age* attribute. The members of PLANET-TYPES are four classes: VERY-OLD-PLANET, OLD-PLANET, YOUNG-PLANET and VERY-YOUNG-PLANET. Through Equation (5), a member c from PLANET-TYPES that maps to YOUNG-PLANETS will have a d.o.m equal to:

$\mu_{Planet\text{-}Types}(c) =$

$\mu(Planet\text{-}Types, Young\text{-}Planet) \cdot \mu(Planet, Young\text{-}Planet)$.

If we suppose that PLANET contains three young planets e_1, e_2 and e_3. Suppose also that for all $e_r \in$ YOUNG-PLANET we have $\mu_{Planet\text{-}Types}$ (e_r/*Young-Planet*) = 0.7. In addition, we suppose that:

- $\mu_{Planet}(e_1/Young\text{-}Planet) = 0.24$.
- $\mu_{Planet}(e_2/Young\text{-}Planet) = 0.50$.
- $\mu_{Planet}(e_3/Young\text{-}Planet) = 0.70$.
- $\mu_{Young\text{-}Planet}(e_1) = 0.53$.
- $\mu_{Young\text{-}Planet}(e_2) = 0.47$.
- $\mu_{Young\text{-}Planet}(e_3) = 1.0$.

By Equation (7), we have:

$\mu(Planet, Young\text{-}Planet)$

$$= \frac{0.24 \cdot 0.53 + 0.50 \cdot 0.47 + 0.70 \cdot 1.0}{0.53 + 0.47 + 1.0} = 0.531,$$

and, through Equation (9) we have :

$\mu_{Planet\text{-}Types}(c) = 0.7 \cdot 0.531 = 0.371$.

As mentioned earlier, a fuzzy composite class can also be defined by the enumeration of its members. These members may be homogenous or heterogeneous. Each member of an enumerated fuzzy composite class may have their own attributes in addition to the common ones. Enumerated fuzzy composite classes do not have extent properties. However, for the same reasons explained earlier in 2.4, members of an enumerated fuzzy subclass are though to be true members of their fuzzy composite classes. This means that $\forall e_r \in K_j$, $\mu_c(e_r/K_j)$ in Equation (6) is equal to 1. Consequently, Equation (5) above will be:

$$\mu_C(e_i) = \mu(S, K_i). \quad (10)$$

If in the exmplae above PLANET–TYPES is an enumerated fuzzy composite class, then Equation (10) gives:

$\mu_{Planet\text{-}Types}(c) = \mu(Planet, Young\text{-}Planet) = 0.531$.

2.6 Membership functions for fuzzy grouping classes

A fuzzy grouping class is a collection of members from other fuzzy classes. We may distinguish two types of fuzzy grouping classes: aggregate or grouping. A member of a fuzzy aggregate class is a heterogeneous collection of fuzzy classes, in which each member (or aggregate) is composed of one member from different fuzzy classes that are called components. In other words, members of a fuzzy aggregate class are (a subset of) the cartesian product of the members of its components. A fuzzy grouping class is an homogenous collection of members (or groups) from the same fuzzy class that is called component. In both cases, members of the fuzzy grouping or aggregate class are unique collections of the component classes. In other words, the addition or the elimination of one member from the collection creates a new group or a new aggregate. For example, GALAXY is a fuzzy aggregate class whose members are unique collections of members from COMETS, STARS, and PLANETS fuzzy grouping classes. In the other hand, COMETS, STARS, and PLANETS are homogenous collections of members from fuzzy classes COMET, STAR, and PLAENT.

The extent properties set of a fuzzy aggregate class is the union of the extent properties sets of its components. Mathematically $P_A = \bigcup_{i=1}^{m} P_{Ki}$, where A is an aggregation of m fuzzy class K_1, K_2,..., K_m. The d.o.m of an entity e of an aggregate fuzzy class A that maps to m entities $e_1, e_2, ... e_m$ of m fuzzy classes K_1, K_2,..., K_m is calculated as follows :

$$\mu_A(e) = \prod_{i=1}^{m} \mu_{Ki}(e_i). \qquad (11)$$

Equation (11) will provide a value equal to zero if at least one component has a d.o.m equal to 0. This is in accordance with the fact that a fuzzy aggregate class is unique collection of exactly one member from different components. An example for computing the d.o.m of members of fuzzy aggregate classes is given below. Since the fuzzy aggregate class and its components may share no attributes, the d.o.m of an entity e form a fuzzy subclass K_i in the aggregate class A is simply equal to the d.o.m of e in K_i :

$$\mu_A(e/K_i) = \mu_{Ki}(e) \qquad (12)$$

Fuzzy grouping classes are homogeneous collections of members from the same fuzzy classes. The d.o.m of an entity e of a fuzzy grouping class G that groups m entities e_2, e_2,...e_m from fuzzy class K may be calculated in at least three ways :

$$\mu_G(e) = max\ (\mu_k(e_1),...\mu_K(e_m)). \qquad (13)$$

or

$$\mu_G(e) = min\ (\mu_k(e_1),...\mu_K(e_m)). \qquad (14)$$

or

$$\mu_G(e) = \sum_{i=1}^{m} \frac{\mu_K(e_i)}{m}. \qquad (15)$$

For each entity in the component fuzzy class that belongs to the group fuzzy class G, $\mu_G(e/K) = \mu_K(e)$. The d.o.m of fuzzy class K in G is $\mu(G, K) = 1$.

Through Equation (13), the d.o.m of a member e_1 from fuzzy grouping class PLANETS grouping three entities e_{11}, e_{12} and e_{13} from PLANET is:

$$\mu_{Planets}(e_1) = max\ (\mu_{Planet}(e_{11}), \mu_{Planet}(e_{12}), \mu_{Planet}(e_{13})).$$

If we suppose that $\mu_{Planet}(e_{11}) = 0.25$, $\mu_{Planet}(e_{12}) = 0.57$ and $\mu_{Planet}(e_{13}) = 0.88$, then $\mu_{Planets}(e_1) = 0.25$. With the same data, equation (15) gives $\mu_{Planets}(e_1) = 0.56$.

Now if we suppose that an entity e from GALAXY aggregates three entities e_1, e_2 and e_3 from fuzzy grouping classes PLANETS, STARS and COMETS; respectively, and we suppose also that

$\mu_{Planets}(e_1) = 0.56$, $\mu_{Stars}(e_2) = 0.12$ and $\mu_{Comets}(e_3) = 0.34$, then Equation (12) gives: $\mu_{Galaxy}(e_1) = \mu_{Planets}(e_1) \cdot \mu_{Stars}(e_2) \cdot \mu_{Comets}(e_1) = 0.022$.

3 Illustrative example

In this section, we present a concrete example to show the utility of the FSM in practice. However, due to the confidentiality of the data, only a general idea of this work is provided here. The problem we are concerned with is relative to the management and modelling of knowledge at the French PSA Peugeot Citroen Company. In fact, knowledge is defined as "an *information that has been organized and analyzed to make it understandable to problem solving and decision-making*" [17]. In practice, knowledge is perceived and interpreted differently according to the decision-makers competences'. Thus, the knowledge is a fuzzy concept. In the model of Figure 3, we associate to the fuzzy class KNOWLEDGE the extent properties set $P_{knowledge} = \{p_1, p_2\}$ where p_1 and p_2 are based on *level-of-tacit* and *degree-of-maturity* attributes, respectively. The experts associate the weights of $w_1 = 3$, $w_2 = 0.4$ and $w_3 = 0.5$. The model of Figure 3 contains also several other fuzzy classes: ACTOR, PROJECT, EXPICIT-KNOWLEDGE, TACIT-KNOWLEDGE. The d.o.m of an entity to the fuzzy class EXPICIT-KNOWLEDGE is given by the PSA Experts'.

Basing on the information provided by experts of PSA Peugeot Citroen [14,13], we have defined the partial d.o.m functions of properties p_1 and p_2 as in Figure 2.

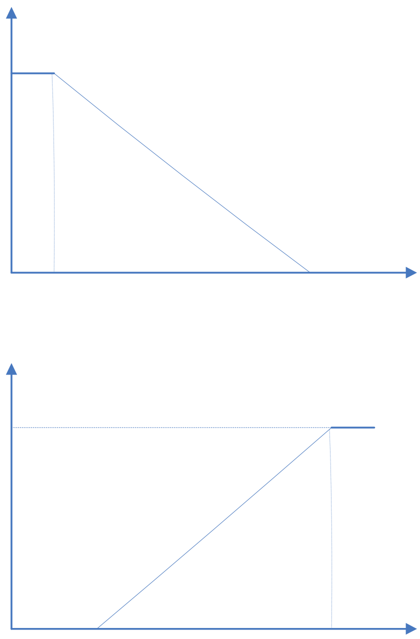

Figure 2. Partial d.o.m functions

Initially, the problem was modelled using the UML (*Unified Modelling Language*) with no support of fuzziness. With this first modelling, it was difficult to response to many queries such as:

Q1: Retrieve the list of knowledge which are relevant for project SFAP.
Q2: Retrieve the list of persons possessing crucial knowledge for project SFAP.

The common SQL-like query languages are not enable to response the above queries since this last ones necessities the consideration of fuzziness at both attributes and class levels. In fact, the experts often provide imprecise attributes values. Table 1 provides some imprecise values for our illustrative example.

By using our model and the associated SQL-like query language [16,2], it easy to response to the two above mentioned queries:

Q1:
FROM *knowledge* WITH DOM > 0.8, *project*
RETRIEVE *knowledge-id*
WHERE *knowledge-contribute* = *project-id*

Q2 :
FROM *actor, knowledge* WITH DOM > 0.9
RETRIEVE *actor-name*
WHERE *knowledge-possessed* = *actor-id*

K-Number	level-of-tacit	degree-of-maturity
Knowledge 1	20%	[60%, 70%]
Knowledge 2	80%	50%
Knowledge 3	[50%, 70%]	high
Knowledge 4	low	average

Table 1. Data of the example

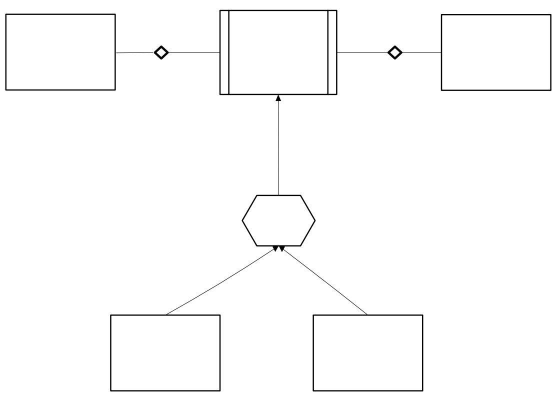

Figure 3. Graphical representation of the illustrative example database

4 Conclusion

The FSM is a database model that has been recently proposed. FSM uses basic concepts of classification, generalization, aggregation and association that are commonly used in semantic modeling and supports the fuzziness of real-world at attribute, entity, class and inter- and intra-classes levels. Hence, it provides tools to formalize and conceptualize real-world within a manner adapted to its perception and conceptualization by humans.

Currently, the FSM is being implemented. In addition, an SQL-like querying language adapted to the model is briefly described in [16] and fully detailed in [2].

One drawback of FSM is related to the compensatory nature of the weighted sum technique used to define and calculate the global membership functions. Indeed, the low values of one or many partial membership function may be compensated with high values of one or many other partial membership function. Thus, other, non compensatory operators are required. At the present, we investigate the aggregation based on the concordance/non-discordance concepts used in multicriteria analysis.

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