# Robust Ordinal Regression and Stochastic Multiobjective Acceptability Analysis in Multiple Criteria Hierarchy Process for the Choquet integral preference model

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November 30, 2015

**Abstract**

The paper deals with two important issues of Multiple Criteria Decision Aiding: interaction between criteria and hierarchical structure of criteria. To handle interactions, we apply the Choquet integral as a preference model, and to handle the hierarchy of criteria, we apply the recently proposed methodology called Multiple Criteria Hierarchy Process. In addition to dealing with the above issues, we suppose that the preference information provided by the Decision Maker is indirect and has the form of pairwise comparisons of some criteria and some alternatives with respect to some criteria. In consequence, many instances of the Choquet integral are usually compatible with this preference information. These instances are identified and exploited by Robust Ordinal Regression and Stochastic Multiobjective Acceptability Analysis. To illustrate the whole approach, we show its application to a real world decision problem concerning the ranking of universities for a hypothetical Decision Maker.

**Keywords**: Multiple Criteria Decision Aiding; Hierarchy of criteria; Choquet integral preference model; Robust Ordinal Regression; Stochastic Multiobjective Acceptability Analysis, University ranking.

## 1 Introduction

Multiple Criteria Decision Aiding (MCDA) helps Decision Makers in solving choice, ranking and sorting problems concerning a set of alternatives evaluated on multiple criteria (see [15] for a collection of state-of-the-art surveys on MCDA). Taking into account preferences of a particular Decision Maker (DM), in choice problems, a subset of best alternatives has to be chosen; in ranking problems, alternatives have to be partially or totally rank ordered from the best to the worst, while in sorting problems each alternative has to be assigned to one or more contiguous preferentially ordered classes. In order to deal with any of these problems, the evaluations of the alternatives on the considered criteria have to be aggregated by a preference model, which can be either a value function [33], or

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an outranking relation [8, 17], or a set of decision rules [29, 43].
Nowadays, MCDA is facing three important methodological challenges: handling a complex structure of criteria, dealing with interactions between criteria, and reducing the cognitive effort of the DMs in interaction with MCDA methods. These challenges are usually handled separately, however, they often concern the same decision problem.

In particular, with respect to the complex structure of criteria having the form of a hierarchy, the Analytic Hierarchy Process (AHP) [41], and then the Multiple Criteria Hierarchy Process (MCHP) [13] have been proposed. While AHP requires preference information at all levels of the hierarchy in the form of exhaustive pairwise comparisons, and provides recommendations at the comprehensive level only, MCHP accepts a partial preference information in form of pairwise comparisons of some alternatives at some levels of the hierarchy, and provides recommendations at all levels.

As to the challenge of interaction, it is present when evaluation criteria are not mutually preferentially independent [33]. To deal with interactions, MCDA methods use non additive integrals, such as the Choquet integral (see [9] for the Choquet integral definition, and [24] for the application of non additive integrals in MCDA), the Sugeno integral [46], and some of their generalizations [26, 28, 32, 36]). The preferential independence condition has also been smoothed in multiplicative and multilinear utility functions [33], but due to the high number of parameters that have to be elicited from the DM, their use has not been very successful in real world applications [45].
Moreover, the interaction between criteria has been recently considered in the ELECTRE methods [16] and in PROMETHEE methods [10]. It was also handled in artificial intelligence approaches, by weakening the preference independence condition in GAI-networks [23], as well as UCP-networks [7]. They are based on the concept of Generalized Additive Independence (GAI) decomposition introduced by Fishburn [18], which permits to aggregate performances on considered criteria through the sum of marginal utilities related to subsets of criteria. Yet another approach, recently proposed to deal with the interaction between criteria [31] is based on an enriched additive value function that is composed of the usual sum of marginal value functions related to each one of considered criteria and some additional terms expressing a bonus (in case of positive interaction) or a penalty (in case of negative interaction), incurred for interaction between some criteria. In this approach, the pairs of criteria for which there exists a positive or negative interaction are inferred through ordinal regression on the basis of preference information given by the DM on some reference alternatives.

The aforementioned aspects of hierarchy and interaction of criteria have been jointly analyzed and described in the hierarchical Choquet integral preference model [5]. Other studies devoted to modeling the hierarchy of criteria within the Choquet integral preference model can be found in [19, 20, 21, 22, 39, 40, 47]. Let us remark that their multi-step Choquet integral is different from our approach, since it requires the definition of a capacity at each node of the hierarchy of criteria. Consequently, their method considers Choquet integrals resulting from aggregation of Choquet integrals at the subsequent level of the hierarchy, which is not the case of our approach.

As to the challenge of reducing the cognitive effort of the DM, one can observe the trend of abandoning direct elicitation of preference model parameters in favor of an indirect elicitation of preferences. In the direct elicitation, the DM is expected to provide values of all parameters of the considered preference model, while in the indirect elicitation, the DM is expected to provide preference information in the form of pairwise comparisons between some alternatives or criteria. There are known two MCDA methodologies based on the indirect elicitation of preferences, which explore the whole set of preference model parameters compatible with the preference information provided by the DM. These are the Robust Ordinal Regression (ROR) (see [30] for the paper introducing ROR, and [11, 12] for surveys) and the Stochastic Multiobjective Acceptability Analysis (SMAA) (see [34] for the paper introducing SMAA, and [48] for a survey).

In this paper, we undertake all these three challenges together, combining the use of MCHP with the Choquet integral preference model on one hand and application of ROR and SMAA on the other hand. This combination is not straightforward, however, because it does not consist in chaining these three methods as they are, but in joint application of all of them, which needs some non-trivial adaptations. In this way, we extend the study presented in [5] by considering two new aspects:

- application of ROR to identify all instances of the Choquet integral preference model being compatible with the preference information provided by the DM; due to hierarchical structure of criteria, the DM can express preference information at a particular level of the hierarchical decomposition of the problem; in exchange, ROR provides robust recommendation in terms of necessary and possible preference relations at all levels of the hierarchy of criteria;

- application of SMAA to compute the frequency with which an alternative gets a particular position in the recommended ranking or the frequency with which an alternative is preferred to another one, at all levels of the hierarchy of criteria.

Let us observe that the methodology presented in this paper is not just a simple sum of the aforementioned three approaches, because MCHP requires that the Choquet integral preference model, SMAA and ROR are applied in all nodes of the hierarchy of criteria in a different way than in case of a flat structure of criteria; the hierarchy requires a coordination of calculations in particular nodes, and moreover, the preference information does not need to be given in all nodes. Moreover, the approach is really adaptive with respect to the complexity of the decision problem considered, since on one hand, it permits decomposition of complex problems due to hierarchical structure of criteria and, on the other hand, it permits to adapt the Choquet integral from 1-additive form (linear) to $k$-additive form, depending on the preference information provided by the DM. Another aspect that we would like to underline here and that will be clear in the next sections is that the extension of the MCHP to the Choquet integral preference model does not require more parameters than the application of the Choquet integral preference model in case of a flat structure of criteria. Indeed, the application of the Choquet integral in case of criteria structured in a hierarchical way requires only the definition of a capacity on the set of elementary criteria and not of a capacity on each node of the hierarchy. Indeed, the capacities on the different nodes of the hierarchy can be easily obtained by the capacity defined on the elementary criteria only.
The highlights characterizing the approach presented in this paper, are summarized briefly in the following paragraphs.

At the input, the DM is asked to provide the following preference information:

- comparisons related to importance and interaction of macro-criteria as well as between some elementary criteria, not necessarily belonging to the same macro-criterion;

- preference comparisons between alternatives at a comprehensive level as well as considering only a macro-criterion and, therefore, a particular aspect of the problem at hand.

At the output, the DM gets, we get the following results again with respect to each node of the hierarchy as well as at a comprehensive level:

- necessary and possible preference relations resulting from NAROR;

- all the probabilistic indices supplied by SMAA applied to the $k$-additive Choquet integral preference model;

- the rankings of the alternatives, by applying the Choquet integral preference model assuming the barycenter of the capacities compatible with the preference information provided by the DM.

The paper is organized as follows. In Section 2, we introduce some basic concepts relative to the Choquet integral preference model, MCHP, hierarchical Choquet integral preference model, ROR and SMAA. In Section 3, the proposed methodology, combining SMAA and ROR applied to the hierarchical Choquet integral preference model, is presented. A real world multicriteria problem, related to the ranking of universities, illustrates the considered methodology in Section 4. Conclusions are drawn and some future directions of research are provided in Section 5.

# 2 Basic concepts

In this section, we introduce some basic concepts used further in the paper. In subsection 2.1, we present the Choquet integral preference model. In subsections 2.2 and 2.3, we recall ROR applied to the Choquet integral (called NAROR), and SMAA, respectively, while in subsection 2.4, a description of the hierarchical Choquet integral preference model is presented together with an example (subsection 2.4.1).

## 2.1 The Choquet integral, preference model

Let $G = \{g_1, \ldots, g_n\}$ be the set of evaluation criteria and $2^G$ the set of all subsets of $G$; a capacity on $2^G$ is a function $\mu : 2^G \rightarrow [0,1]$ such that $\mu(\emptyset) = 0$, $\mu(G) = 1$ (normalization constraints) and $\mu(T) \leq \mu(R)$ for all $T \subseteq R \subseteq G$ (monotonicity constraints). The Möbius representation of the capacity $\mu$ is the function $m : 2^G \rightarrow \mathbb{R}$, such that, for all $R \subseteq G$,

$$\mu(R) = \sum_{T \subseteq R} m(T). \tag{1}$$

Let also $A$ be a set of alternatives. Given an alternative $a \in A$ and a capacity $\mu$, the Choquet integral of $a$ is defined as

$$C_\mu(a) = \sum_{i=1}^{n} \left[g_{(i)}(a) - g_{(i-1)}(a)\right] \mu(N_i),$$

where $_{(\cdot)}$ stands for a permutation of the indices of criteria, such that $0 = g_{(0)}(a) = g_{(1)}(a) \leq \ldots \leq g_{(n)}(a)$, with $N_i = \{(i), ...., (n)\}$, $i = 1, .., n$. In the following, we suppose that all criteria are of the gain type.
Using the Möbius representation of $\mu$, and without reordering the criteria, the Choquet integral of $a$ is therefore redefined as

$$C_\mu(a) = \sum_{T \subseteq G} m(T) \min_{i \in T} g_i(a).$$

Since in case of interacting criteria the importance of the criterion $i$, as well as its interaction with other criteria, do not depend on its importance as singleton only, but also on its contribution to all coalitions of criteria, we recall the *Shapley value* [42]

$$\varphi(\{i\}) = \sum_{T \subseteq G:\, i \notin T} \frac{(|G-T|-1)!|T|!}{|G|!} \left[\mu(T \cup \{i\}) - \mu(T)\right], \tag{2}$$

and the *interaction index* [38]

$$\varphi(\{i,j\}) = \sum_{T \subseteq G:\, i,j \notin T} \frac{(|G-T|-2)!|T|!}{(|G|-1)!} \left[\mu(T \cup \{i,j\}) - \mu(T \cup \{i\}) - \mu(T \cup \{j\}) + \mu(T)\right]. \tag{3}$$

Using the Möbius representation of capacity $\mu$, equations (2) and (3) can be formulated as follows [27]

$$\varphi(\{i\}) = \sum_{A \subseteq G:\, i \in A} \frac{m(A)}{|A|} \tag{4}$$

and

$$\varphi(\{i,j\}) = \sum_{\{i,j\} \subseteq A \subseteq G} \frac{m(A)}{|A|-1}. \tag{5}$$

A direct application of the Choquet integral preference model implies the elicitation of $2^{|G|}-2$ parameters $\mu(T)$ (one for each subset $T \subseteq G$, apart from $T=\emptyset$ and $T=G$, since $\mu(\emptyset)=0$ and $\mu(G)=1$). As the inference of all these parameters is cognitively hard, the concept of *$q$-additive* capacity has been defined in [25]. A capacity is $q$-additive if $m(T)=0$ for all $T \subseteq G$, such that $|T|>q$. In real world applications, it is enough considering 2-additive capacities only. The use of a 2-additive capacity involves knowledge of $n+\binom{n}{2}$ parameters only: a value $m(\{i\})$ for each criterion $i$ and a value $m(\{i,j\})$ for each couple of criteria $\{i,j\}$. Considering the Möbius representation $m$ of a 2-additive capacity $\mu$, normalization and monotonicity constraints have the following form

**1c)** $m(\emptyset)=0, \ \sum_{i \in G} m(\{i\}) + \sum_{\{i,j\} \subseteq G} m(\{i,j\}) = 1,$

**2c)** $\begin{cases} m(\{i\}) \geq 0, \ \forall i \in G, \\ m(\{i\}) + \sum_{j \in T} m(\{i,j\}) \geq 0, \ \forall i \in G \text{ and } \forall\, T \subseteq G \setminus \{i\},\, T \neq \emptyset, \end{cases}$

while the Choquet integral of $a \in A$ can be computed as

$$C_\mu(a) = \sum_{i \in G} m(\{i\})\, g_i(a) + \sum_{\{i,j\} \subseteq G} m(\{i,j\}) \min\{g_i(a), g_j(a)\}. \tag{6}$$

Equations (4) and (5) expressing the Shapley value and the interaction index can be, therefore, rewritten in the following way:

$$\varphi(\{i\}) = m(\{i\}) + \sum_{j \in G \setminus \{i\}} \frac{m(\{i,j\})}{2}, \tag{7}$$

$$\varphi(\{i,j\}) = m(\{i,j\}). \tag{8}$$

## 2.2 Non Additive Robust Ordinal Regression (NAROR)

NAROR [6] belongs to the family of ROR methods (see [11, 12, 30]). In NAROR, the DM is asked to give the following type of preference information on a subset $A^* \subseteq A$ of reference alternatives (s)he knows well:

- $a$ is preferred to $b$, denoted by $a \succ b$ (translated to the constraint $C_\mu(a) \geq C_\mu(b) + \varepsilon$);
- $a$ is indifferent to $b$, denoted by $a \sim b$ ($C_\mu(a) = C_\mu(b)$);

- $a$ is preferred to $b$ more than $c$ is preferred to $d$, denoted by $(a, b) \succ^* (c, d)$ $(C_\mu(a) - C_\mu(b) \geq C_\mu(c) - C_\mu(d) + \varepsilon$ and $C_\mu(c) - C_\mu(d) \geq \varepsilon)$;
- the intensity of preference between $a$ and $b$ is the same of the intensity of preference between $c$ and $d$, denoted by $(a, b) \sim^* (c, d)$ $(C_\mu(a) - C_\mu(b) = C_\mu(c) - C_\mu(d))$,

where $a, b, c, d \in A^*$.
Moreover, differently from other ROR methods, in NAROR the DM can provide also some preference information on criteria $i, j, l, k \in G$, such as:

- criterion $i$ is more important than criterion $j$, denoted by $g_i \succ g_j$ (translated to the constraint $\varphi(\{i\}) \geq \varphi(\{j\}) + \varepsilon$);
- criteria $i$ and $j$ are indifferent, denoted by $g_i \sim g_j$ $(\varphi(\{i\}) = \varphi(\{j\}))$;
- criteria $i$ and $j$ are positively (negatively) interacting $(\varphi(\{i, j\}) \geq \varepsilon$ $(\leq -\varepsilon))$;
- $i$ is preferred to $j$ more than $l$ is preferred to $k$, denoted by $(g_i, g_j) \succ^* (g_l, g_k)$ $(\varphi(\{i\}) - \varphi(\{j\}) \geq \varphi(\{l\}) - \varphi(\{k\}) + \varepsilon$ and $\varphi(\{l\}) - \varphi(\{k\}) \geq \varepsilon)$;
- the difference of importance between $i$ and $j$ is the same as the difference of importance between $l$ and $k$, denoted by$(g_i, g_j) \sim^* (g_l, g_k)$ $(\varphi(\{i\}) - \varphi(\{j\}) = \varphi(\{l\}) - \varphi(\{k\}))$.

In the above constraints, $\varepsilon$ is an auxiliary variable used to convert the strict inequalities into weak inequalities; for example $C_\mu(a) \geq C_\mu(b) + \varepsilon$ is the translation of $C_\mu(a) > C_\mu(b)$.

At the output of NAROR, two preference relations, one *necessary* $\succsim^N$ and another *possible* $\succsim^P$, are presented to the DM:

$$a \succsim^N b \text{ iff } C_\mu(a) \geq C_\mu(b) \text{ for all } \textit{compatible capacities},$$
$$a \succsim^P b \text{ iff } C_\mu(a) \geq C_\mu(b) \text{ for at least one } \textit{compatible capacity},$$

where a *compatible capacity* is a set of Möbius measures for which the preference information provided by the DM is restored.

Denoting by $E^{DM}$ the set of above constraints translating the DM's preference information together with the monotonicity and normalization constraints **1c)** and **2c)**, the existence of a *compatible capacity* is checked by solving the following linear programming problem:

$$\varepsilon^* = \max \varepsilon, \text{ subject to } E^{DM}.$$

If $E^{DM}$ is feasible and $\varepsilon^* > 0$, then there exists at least one compatible capacity, otherwise there exists some inconsistency in the preferences provided by the DM that could be identified by using one of the methods presented in [37].

The two following sets of constraints,

$$\left.\begin{array}{l} C_\mu(b) \geq C_\mu(a) + \varepsilon, \\ E^{DM} \end{array}\right\} E^N(a, b), \qquad \left.\begin{array}{l} C_\mu(a) \geq C_\mu(b) \\ E^{DM} \end{array}\right\} E^P(a, b)$$

are used to compute the necessary and the possible preference relation between alternatives $a$ and $b$, $a, b \in A$. In particular, the necessary preference relation holds between $a$ and $b$ if $E^N(a, b)$ is infeasible or $\varepsilon^N \leq 0$, where $\varepsilon^N = \max \varepsilon$, subject to $E^N(a, b)$. Analogously, the possible preference relation holds between $a$ and $b$ if $E^P(a, b)$ is feasible and $\varepsilon^P > 0$, where $\varepsilon^P = \max \varepsilon$, subject to $E^P(a, b)$.

## 2.3 Stochastic Multiobjective Acceptability Analysis (SMAA)

SMAA [34, 35] is a family of MCDA methods which take into account uncertainty or imprecision on the evaluations and preference model parameters. In this section we describe SMAA-2 [35], since our proposed methodology also regards ranking problems.
The most frequently used value function in SMAA-2 is the linear one

$$U(a_k, w) = \sum_{i=1}^{n} w_i g_i(a_k)$$

where $w \in W = \{(w_1, \ldots, w_n) \in \mathbb{R}^n \colon w_i \geq 0 \text{ and } \sum_{i=1}^{n} w_i = 1\}$. In SMAA methods, the indirect preference information is composed of two probability distributions, $f_\chi$ and $f_W$, defined on the evaluation space $\chi$ and on the weight space $W$, respectively.
Defining the rank function

$$rank(k, \xi, w) = 1 + \sum_{h \neq k} \rho\left(U(\xi_h, w) > U(\xi_k, w)\right),$$

(where $\rho(false) = 0$ and $\rho(true) = 1$) that, for all $a_k \in A$, $\xi \in \chi$ and $w \in W$ gives the rank position of alternative $a_k$, SMAA-2 computes the set of weights of criteria for which alternative $a_k$ assumes rank $r = 1, 2, \ldots, n$, as follows:

$$W_k^r(\xi) = \{w \in W : rank(k, \xi, w) = r\}.$$

The following further indices are computed in SMAA-2:

- *The rank acceptability index* that measures the variety of different parameters compatible with the DM's preference information giving to the alternative $a_k$ the rank $r$:

  $$b_k^r = \int_{\xi \in \chi} f_\chi(\xi) \int_{w \in W_k^r(\xi)} f_W(w)\; dw\; d\xi;$$

  $b_k^r$ gives the probability that alternative $a_k$ has rank $r$, and it is within the range $[0, 1]$.

- *The central weight vector* that describes the preferences of a typical DM giving to $a_k$ the best position:

  $$w_k^c = \frac{1}{b_k^1} \int_{\xi \in \chi} f_\chi(\xi) \int_{w \in W^1(\xi)} f_W(w) w\; dw\; d\xi;$$

- *The pairwise winning index* that is defined as the frequency that an alternative $a_h$ is preferred to an alternative $a_k$ in the space of weight vectors:

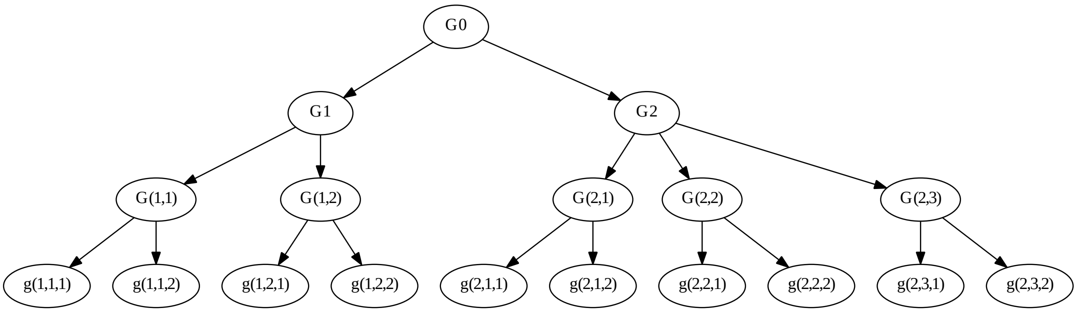
  $$p_{hk} = \int_{w \in W} f_W(w) \int_{\xi \in \chi : u(\xi_h, w) > u(\xi_k, w)} f_\chi(\xi) d\xi\; dw.$$

From a computational point of view, the multidimensional integrals defining the considered indices are estimated by using the Monte Carlo method. Let us note that, recently, the potentialities of SMAA and the Choquet integral preference model have been combined in [3] and further investigated in [4].

## 2.4 Multiple Criteria Hierarchy Process (MCHP) and the Choquet integral preference model

In MCHP, the evaluation criteria are not all considered at the same level but they are structured in a hierarchical way. This means that one considers a root criterion (the comprehensive objective) and a set of subcriteria branching successively, as shown in Figure 1.

Figure 1: Example of a hierarchy of criteria. $G_0$ is the root criterion, $G_1$ and $G_2$ are the first level subcriteria while 10 elementary criteria are in the last level of the hierarchy.




The following notation will be used in the paper:

- $\mathcal{G}$ denotes the set of all criteria at all considered levels, while $\mathcal{I}_{\mathcal{G}}$ is the set of indices of all criteria in the hierarchy;
- $EL$ is the set of indices of the elementary criteria, that is criteria located at the last level of the hierarchy and on which the alternatives are evaluated (in Figure 1, $EL = \{(1,1,1),(1,1,2),(1,2,1),(1,2,2),(2,1,1),(2,1,2),(2,2,1),(2,2,2),(2,3,1),(2,3,2)\}$);
- $G_{\mathbf{r}}$ is a generic criterion in the hierarchy, while $G_{(\mathbf{r},1)},\ldots,G_{(\mathbf{r},n(\mathbf{r}))}$ are the subcriteria of criterion $G_{\mathbf{r}}$ in the subsequent level (in Figure 1, $G_{(\mathbf{2},1)}$, $G_{(\mathbf{2},2)}$ and $G_{(\mathbf{2},3)}$ are the subcriteria of $G_{\mathbf{2}}$ in the subsequent level);
- $E(G_{\mathbf{r}})$ is the set of indices of elementary criteria descending from $G_{\mathbf{r}}$ (in Figure 1, $E(G_{\mathbf{1}}) = \{(\mathbf{1},1,1),(\mathbf{1},1,2),(\mathbf{1},2,1),(\mathbf{1},2,2)\}$);
- Given $\mathcal{F} \subseteq \mathcal{G}$, $E(\mathcal{F}) = \cup_{G_{\mathbf{r}}\in\mathcal{F}} E(G_{\mathbf{r}})$ is the set of all elementary criteria descending from at least one criterion in $\mathcal{F}$ (in Figure 1, considering $\mathcal{F} = \left\{G_{(\mathbf{1},1)}, G_{(\mathbf{2},3)}\right\}$, then $E(\mathcal{F}) = \{(\mathbf{1},1,1),(\mathbf{1},1,2),(\mathbf{2},3,1),(\mathbf{2},3,2)\}$);
- $\mathcal{G}_{\mathbf{r}}^{k}$ is the set of subcriteria of $G_{\mathbf{r}}$ located at level $k$ (in Figure 1, $\mathcal{G}_{\mathbf{1}}^{2} = \left\{G_{(\mathbf{1},1)}, G_{(\mathbf{1},2)}\right\}$, while $\mathcal{G}_{\mathbf{1}}^{3} = \left\{g_{(\mathbf{1},1,1)}, g_{(\mathbf{1},1,2)}, g_{(\mathbf{1},2,1)}, g_{(\mathbf{1},2,2)}\right\}$).

Given a capacity $\mu$ defined on the power set of $EL$, a criterion $G_{\mathbf{r}}$ with $\mathbf{r}\in\mathcal{I}_{\mathcal{G}}\cap\mathbb{N}^h$ (that is, $G_{\mathbf{r}}$ is a criterion located at level $h$ of the hierarchy) and $k = h+1,\ldots,l$, where $l$ is the number of levels in the hierarchy tree (for example, $l = 3$ in Figure 1), we can define a capacity on the power set of $\mathcal{G}_{\mathbf{r}}^{k}$

$$\mu_{\mathbf{r}}^{k} : 2^{\mathcal{G}_{\mathbf{r}}^{k}} \rightarrow [0,1] \tag{9}$$

such that

$$\mu_{\mathbf{r}}^{k}(\mathcal{F})=\frac{\mu\left(E\left(\mathcal{F}\right)\right)}{\mu\left(E\left(G_{\mathbf{r}}\right)\right)} \tag{10}$$

for all $\mathcal{F}\subseteq\mathcal{G}_{\mathbf{r}}^{k}$.

According to equation (10), the capacity $\mu_{\mathbf{r}}^{k}$ can be written in terms of the capacity $\mu$ defined on the power set of $EL$.

Considering criterion $G_{\mathbf{r}}$, $\mathbf{r}\in\mathcal{I}_{\mathcal{G}}\setminus EL$ at any but the last level of the hierarchy, and the capacity $\mu$ defined on the power set of $EL$, the Choquet integral of alternative $a\in A$ on criterion $G_{\mathbf{r}}$ can be computed as

$$C_{\mu_{\mathbf{r}}}(a)=\frac{C_{\mu}(a_{\mathbf{r}})}{\mu(E(G_{\mathbf{r}}))} \tag{11}$$

where $a_{\mathbf{r}}$ is a fictitious alternative having the same evaluations as $a$ on elementary criteria from $E(G_{\mathbf{r}})$ and null evaluation on elementary criteria from outside $E(G_{\mathbf{r}})$, i.e., $g_{\mathbf{t}}(a_{\mathbf{r}})=g_{\mathbf{t}}(a)$ if $\mathbf{t}\in E(G_{\mathbf{r}})$ and $g_{\mathbf{t}}(a_{\mathbf{r}})=0$ if $\mathbf{t}\notin E(G_{\mathbf{r}})$.

Starting from equations (2) and (3), and considering a criterion $G_{\mathbf{r}}$, $\mathbf{r}\in\mathcal{I}_{\mathcal{G}}\setminus\{EL\}$, we can define the Shapley value of criterion $G_{(\mathbf{r},w)}$ and the interaction index between criteria $G_{(\mathbf{r},w_1)}$ and $G_{(\mathbf{r},w_2)}$, with $G_{(\mathbf{r},w)}$, $G_{(\mathbf{r},w_1)}$, $G_{(\mathbf{r},w_2)}\in\mathcal{G}_{\mathbf{r}}^{k}$, as follows:

$$\varphi_{\mathbf{r}}^{k}\left(\left\{G_{(\mathbf{r},w)}\right\}\right)=\sum_{T\subseteq\mathcal{G}_{\mathbf{r}}^{k}\setminus\left\{G_{(\mathbf{r},w)}\right\}}\frac{\left(|\mathcal{G}_{\mathbf{r}}^{k}\setminus T|-1\right)!|T|!}{|\mathcal{G}_{\mathbf{r}}^{k}|!}\left[\mu_{\mathbf{r}}^{k}\left(T\cup\left\{G_{(\mathbf{r},w)}\right\}\right)-\mu_{\mathbf{r}}^{k}\left(T\right)\right], \tag{12}$$

$$\varphi_{\mathbf{r}}^{k}\left(\left\{G_{(\mathbf{r},w_1)},G_{(\mathbf{r},w_2)}\right\}\right)=\sum_{T\subseteq\mathcal{G}_{\mathbf{r}}^{k}\setminus\left\{G_{(\mathbf{r},w_1)},\,G_{(\mathbf{r},w_2)}\right\}}\frac{\left(|\mathcal{G}_{\mathbf{r}}^{k}\setminus T|-2\right)!|T|!}{(|\mathcal{G}_{\mathbf{r}}^{k}|-1)!}. \tag{13}$$

As for the capacity $\mu$ defined on the power set of $G$, we can define analogously the Möbius representation $m_{\mathbf{r}}^{k}:2^{\mathcal{G}_{\mathbf{r}}^{k}}\rightarrow[0,1]$ of the capacity $\mu_{\mathbf{r}}^{k}$, such that

$$\mu_{\mathbf{r}}^{k}\left(\mathcal{F}\right)=\sum_{T\subseteq\mathcal{F}}m_{\mathbf{r}}^{k}\left(T\right) \tag{14}$$

for all $\mathcal{F}\subseteq\mathcal{G}_{\mathbf{r}}^{k}$.

By considering the Möbius representation $m_{\mathbf{r}}^{k}$ of the capacity $\mu_{\mathbf{r}}^{k}$, equations (12) and (13) can be rewritten as follows:

$$\varphi_{\mathbf{r}}^{k}\left(\left\{G_{(\mathbf{r},w)}\right\}\right)=\sum_{\mathcal{F}\subseteq\mathcal{G}_{\mathbf{r}}^{k}:\,G_{(\mathbf{r},w)}\in\mathcal{F}}\frac{m_{\mathbf{r}}^{k}\left(\mathcal{F}\right)}{|\mathcal{F}|} \tag{15}$$

$$\varphi_{\mathbf{r}}^{k}\left(\left\{G_{(\mathbf{r},w_1)},G_{(\mathbf{r},w_2)}\right\}\right)=\sum_{\mathcal{F}\subseteq\mathcal{G}_{\mathbf{r}}^{k}:\,G_{(\mathbf{r},w_1)},\,G_{(\mathbf{r},w_2)}\in\mathcal{F}}\frac{m_{\mathbf{r}}^{k}\left(\mathcal{F}\right)}{|\mathcal{F}|-1} \tag{16}$$

where $G_{(\mathbf{r},w)},G_{(\mathbf{r},w_1)},G_{(\mathbf{r},w_2)}\in\mathcal{G}_{\mathbf{r}}^{k}$.

The Möbius transformation $m_{\mathbf{r}}^{k}$ of the capacity $\mu_{\mathbf{r}}^{k}$ can be written in terms of the Möbius transformation $m$ of the capacity $\mu$, as stated in the following proposition.

**Proposition 2.1.** *Let $\mu$ be, a capacity defined on $2^{EL}$, and $m$ its Möbius representation. Let $G_{\mathbf{r}}\in\mathcal{G}$, $\mathbf{r}\in\mathcal{I}_{\mathcal{G}}\setminus\{EL\}$ with $\mu_{\mathbf{r}}^{k}$ being a capacity defined on $2^{\mathcal{G}_{\mathbf{r}}^{k}}$ and $m_{\mathbf{r}}^{k}$ its Möbius representation; then for all $\mathcal{F}=\left\{G_{(\mathbf{r},w_1)},\ldots,G_{(\mathbf{r},w_\alpha)}\right\}\subseteq\mathcal{G}_{\mathbf{r}}^{k}$,*

$$m_{\mathbf{r}}^k(\mathcal{F}) = m_{\mathbf{r}}^k\left(\left\{G_{(\mathbf{r},w_1)},\ldots,G_{(\mathbf{r},w_\alpha)}\right\}\right) = \frac{\displaystyle\sum_{\substack{T_1\subseteq E\left(G_{(\mathbf{r},w_1)}\right),\, T_1\neq\emptyset\\ \cdots \\ T_\alpha\subseteq E\left(G_{(\mathbf{r},w_\alpha)}\right),\, T_\alpha\neq\emptyset}} m\left(\{T_1,\ldots,T_\alpha\}\right)}{\mu\left(E\left(G_{\mathbf{r}}\right)\right)}.$$

*Proof.* See Appendix. □

**Example 2.1.** *Let* $\mathcal{F}=\left\{G_{(\mathbf{1},1)}\right\}\subseteq\mathcal{G}_{\mathbf{1}}^2$ *be, as shown in Figure 1; considering the Möbius representation* $m_{\mathbf{1}}^2$ *of the capacity* $\mu_{\mathbf{1}}^2$*, we have that*

$$m_{\mathbf{1}}^2\left(\left\{G_{(\mathbf{1},1)}\right\}\right) = \frac{\displaystyle\sum_{T_1\subseteq E\left(G_{(\mathbf{1},1)}\right),\, T_1\neq\emptyset} m(\{T_1\})}{\mu\left(E\left(G_{\mathbf{1}}\right)\right)} =$$

$$= \frac{1}{\mu\left(E\left(G_{\mathbf{1}}\right)\right)}\left[m\left(\left\{g_{(\mathbf{1},1,1)}\right\}\right)+m\left(\left\{g_{(\mathbf{1},1,2)}\right\}\right)+m\left(\left\{g_{(\mathbf{1},1,1)},g_{(\mathbf{1},1,2)}\right\}\right)\right].$$

*Analogously, considering set* $\mathcal{F}=\left\{G_{(\mathbf{1},1)},G_{(\mathbf{1},2)}\right\}=\mathcal{G}_{\mathbf{1}}^2$*, we have that*

$$m_{\mathbf{1}}^2\left(\left\{G_{(\mathbf{1},1)},G_{(\mathbf{1},2)}\right\}\right) = \sum_{\substack{T_1\subseteq E\left(G_{(\mathbf{1},1)}\right),\, T_1\neq\emptyset\\ T_2\subseteq E\left(G_{(\mathbf{1},2)}\right),\, T_2\neq\emptyset}} m\left(\{T_1,T_2\}\right) =$$

$$= \frac{1}{\mu\left(E\left(G_{\mathbf{1}}\right)\right)}\left[m\left(\left\{g_{(\mathbf{1},1,1)},g_{(\mathbf{1},2,1)}\right\}\right)+m\left(\left\{g_{(\mathbf{1},1,1)},g_{(\mathbf{1},2,2)}\right\}\right)+m\left(\left\{g_{(\mathbf{1},1,2)},g_{(\mathbf{1},2,1)}\right\}\right)+\right.$$
$$+m\left(\left\{g_{(\mathbf{1},1,2)},g_{(\mathbf{1},2,2)}\right\}\right)+m\left(\left\{g_{(\mathbf{1},1,1)},g_{(\mathbf{1},1,2)},g_{(\mathbf{1},2,1)}+\right\}\right)+m\left(\left\{g_{(\mathbf{1},1,1)},g_{(\mathbf{1},1,2)},g_{(\mathbf{1},2,2)}\right\}\right)+$$
$$\left.+m\left(\left\{g_{(\mathbf{1},1,1)},g_{(\mathbf{1},2,1)},g_{(\mathbf{1},2,2)}\right\}\right)+m\left(\left\{g_{(\mathbf{1},1,2)},g_{(\mathbf{1},2,1)},g_{(\mathbf{1},2,2)}\right\}\right)\right].$$

As mentioned before, 2-additive capacities are in general sufficient for practical use. For this reason, in the last part of this section we concentrate on the application of MCHP to the 2-additive Choquet integral preference model. First, we provide a proposition stating that if $\mu$ is a $q$-additive capacity, then $\mu_{\mathbf{r}}^k$ is also $q$-additive for each subcriterion $G_{\mathbf{r}}$ in the hierarchy, while the second proposition expresses the Shapley value and the interaction index in case of 2-additive capacities.

**Proposition 2.2.** *Let* $\mu$ *be a* $q$*-additive capacity defined on* $2^{EL}$*, then* $\mu_{\mathbf{r}}^k$ *is a* $q$*-additive capacity defined on* $2^{\mathcal{G}_{\mathbf{r}}^k}$*, for all* $G_{\mathbf{r}}\in\mathcal{G}$ *with* $\mathbf{r}\in\mathcal{I}_{\mathcal{G}}\setminus\{EL\}$*.*

*Proof.* See Appendix. □

**Proposition 2.3.** *Let* $\mu$ *be a 2-additive capacity defined on* $2^{EL}$ *and* $G_{(\mathbf{r},w)},G_{(\mathbf{r},w_1)},G_{(\mathbf{r},w_2)}\in\mathcal{G}_{\mathbf{r}}^k$*, with* $\mathbf{r}\in\mathcal{I}_{\mathcal{G}}\setminus\{EL\}$*, then:*

*1.*

$$\varphi_{\mathbf{r}}^k\left(\left\{G_{(\mathbf{r},w)}\right\}\right) = \left[\sum_{\mathbf{t}\in E\left(G_{(\mathbf{r},w)}\right)} m\left(g_{\mathbf{t}}\right) + \sum_{\mathbf{t_1},\mathbf{t_2}\in E\left(G_{(\mathbf{r},w)}\right)} m\left(g_{\mathbf{t_1}},g_{\mathbf{t_2}}\right) + \sum_{\substack{\mathbf{t_1}\in E\left(G_{(\mathbf{r},w)}\right)\\ \mathbf{t_2}\in E\left(\mathcal{G}_{\mathbf{r}}^k\setminus\left\{G_{(\mathbf{r},w)}\right\}\right)}} \frac{m(g_{\mathbf{t_1}},g_{\mathbf{t_2}})}{2}\right]\frac{1}{\mu(E(G_{\mathbf{r}}))}, \tag{17}$$

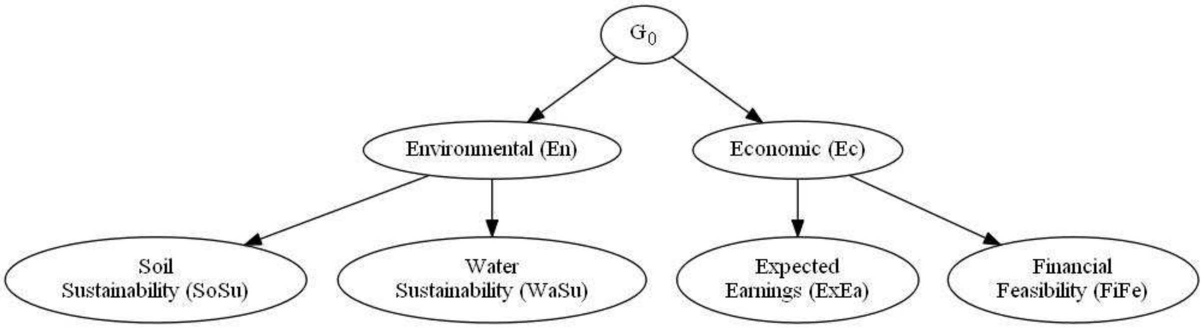



Figure 2: Hierarchy of criteria for evaluation of projects.

*2.*

$$\varphi_{\mathbf{r}}^{k}\left(\left\{G_{(\mathbf{r},w_1)},G_{(\mathbf{r},w_2)}\right\}\right)=\left[\sum_{\substack{\mathbf{t_1}\in E\left(G_{(\mathbf{r},w_1)}\right)\\ \mathbf{t_2}\in E\left(G_{(\mathbf{r},w_2)}\right)}} m(g_{\mathbf{t_1}},g_{\mathbf{t_2}})\right]\frac{1}{\mu(E(G_{\mathbf{r}}))}. \tag{18}$$

*Proof.* See Appendix. □

It is meaningful observing in equation (17) that the importance of a criterion $G_{(\mathbf{r},w)}$ depends on which criterion it is descending from. This means that if $G_{(\mathbf{r},w)}$ is a subcriterion of $G_{\mathbf{r}}$ and, in turn, $G_{\mathbf{r}}$ is a subcriterion of $G_{\mathbf{s}}$, then $G_{(\mathbf{r},w)}$ will get importance $\varphi_{\mathbf{r}}^{k}(G_{(\mathbf{r},w)})$, because it is subcriterion of $G_{\mathbf{r}}$, and importance $\varphi_{\mathbf{s}}^{k}(G_{(\mathbf{r},w)})$, because it is also subcriterion of $G_{\mathbf{s}}$. This is due to the fact that when computing $\varphi_{\mathbf{s}}^{k}(G_{(\mathbf{r},w)})$ one should take into account a greater number of interactions than when computing $\varphi_{\mathbf{r}}^{k}(G_{(\mathbf{r},w)})$.

### 2.4.1 Example

In this section, we provide a simple example to explain how to apply the hierarchical Choquet integral preference model, and to highlight some characteristics of the method. Let us suppose that four alternative projects are evaluated with respect to two macro-criteria, Environmental (En) and Economic (Ec), and that each of these macro-criteria is composed of two elementary criteria. In particular, Soil Sustainability (SoSu) and Water Sustainability (WaSu) are elementary criteria of En while Expected Earnings (ExEa) and Financial Feasibility (FiFe) are elementary criteria of Ec. The hierarchy of criteria is shown in Figure 2. The general notation concerning set of criteria $\mathcal{G}=\left\{G_1,G_2,g_{(1,1)},g_{(1,2)},g_{(2,1)},g_{(2,2)}\right\}$ corresponds now to $\mathcal{G}=\{$En,Ec,SoSu,WaSu,ExEa,FiFe$\}$. All criteria are defined on a common gain scale 10-30. The evaluations of the four projects with respect to the considered elementary criteria are shown in Table 1(a).

For the sake of this example, we assume that Möbius parameters of the Choquet integral are known (see Table 1(b)). They have been obtained by ordinal regression technique which will be explained in the next section. Values of the Choquet integral for the four projects can now be computed with respect to the totality of criteria (equation 6), as well as with respect to each of the two considered macro-criteria (equation 11). These values are given in Table 2. At the same time, one can also compute the Shapley values of different criteria (see Table 3).

Looking at Table 2, we can observe that even if project $a$ is better than project $b$ with respect to En and Ec ($C_{\mu_{\mathbf{1}}}(a)>C_{\mu_{\mathbf{1}}}(b)$ and $C_{\mu_{\mathbf{2}}}(a)>C_{\mu_{\mathbf{2}}}(b)$), $b$ is preferred to $a$ with respect to the totality of criteria ($C_{\mu}(b)>C_{\mu}(a)$). An analogous situation can be observed for projects $c$ and $d$ with $c$ being preferred to $d$ on the two macro-criteria and $d$ being preferred to $c$ with respect to the totality of

Table 1: Evaluations of projects and Möbius parameters

(a) Evaluations of projects

| | En | | Ec | |
|---|---|---|---|---|
| Projects | SoSu | WaSu | ExEa | FiFe |
| $a$ | 17 | 14 | 13 | 18 |
| $b$ | 14 | 15 | 18 | 15 |
| $c$ | 11 | 21 | 11 | 20 |
| $d$ | 15 | 14 | 15 | 14 |

(b) Möbius parameters

| | |
|---|---|
| $m$(SoSu) | 0.3793 |
| $m$(WaSu) | 0.1724 |
| $m$(ExEa) | 0.0507 |
| $m$(FiFe) | 0.1562 |
| $m$(SoSu, WaSu) | −0.1724 |
| $m$(SoSu, ExEa) | −0.0507 |
| $m$(SoSu, FiFe) | −0.1562 |
| $m$(WaSu, ExEa) | 0.6168 |
| $m$(WaSu, FiFe) | 0.0039 |
| $m$(ExEa, FiFe) | 0 |

Table 2: Values of the Choquet integral for the four projects

(a) At the comprehensive level

| | $G_1$ (**En**) | | $G_2$ (**Ec**) | | Choquet integral values |
|---|---|---|---|---|---|
| | SoSu | WaSu | ExEa | FiFe | $C_\mu(\cdot)$ |
| $a$ | 17 | 14 | 13 | 18 | $C_\mu(a) = 14.67$ |
| $b$ | 14 | 15 | 18 | 15 | $C_\mu(b) = 15.15$ |
| $c$ | 11 | 21 | 11 | 20 | $C_\mu(c) = 14.16$ |
| $d$ | 15 | 14 | 15 | 14 | $C_\mu(d) = 14.37$ |

(b) At the level of macro-criteria

| | $G_1$ (**En**) | | $G_2$ (**Ec**) | | Choquet integral values |
|---|---|---|---|---|---|
| | SoSu | WaSu | ExEa | FiFe | $C_\mu(\cdot)/\mu(E(G_\mathbf{r}))$ |
| $a_1$ | 17 | 14 | 0 | 0 | $C_{\mu_1}(a) = 17$ |
| $a_2$ | 0 | 0 | 13 | 18 | $C_{\mu_2}(a) = 16.77$ |
| $b_1$ | 14 | 15 | 0 | 0 | $C_{\mu_1}(b) = 14.45$ |
| $b_2$ | 0 | 0 | 18 | 15 | $C_{\mu_2}(b) = 15.73$ |
| $c_1$ | 11 | 21 | 0 | 0 | $C_{\mu_1}(c) = 15.54$ |
| $c_2$ | 0 | 0 | 11 | 20 | $C_{\mu_2}(c) = 17.79$ |
| $d_1$ | 15 | 14 | 0 | 0 | $C_{\mu_1}(d) = 15$ |
| $d_2$ | 0 | 0 | 15 | 14 | $C_{\mu_2}(d) = 14.24$ |

criteria. Even if this could seem an unlikely situation at a first sight, it is justifiable if we observe that in computing the Choquet integral of a project with respect to En (analogously with respect to Ec), we take into account only the interactions between the elementary criteria of En (Ec), while in computing the Choquet integral of a project with respect to the totality of criteria, we consider the interactions between all four elementary criteria.

Table 3: Shapley values

(a) Shapley values of each elementary criterion with respect to the considered macro-criterion

| | **En** | | **Ec** | |
|---|---|---|---|---|
| | SoSu | WaSu | ExEa | FiFe |
| $\varphi^2_\mathbf{r}(G_{(\mathbf{r},w)})$ | 0.7727 | 0.2272 | 0.2450 | 0.7549 |

(b) Shapley values of each elementary criterion with respect to the totality of criteria

| | $\varphi^2_\mathbf{0}(G_{(\mathbf{r},w)})$ |
|---|---|
| SoSu | 0.1896 |
| WaSu | 0.3965 |
| ExEa | 0.3337 |
| FiFe | 0.080 |

Looking at Table 3, one can observe another phenomenon characteristic for the hierarchical Choquet integral preference model. Indeed, SoSu is more important than WaSu when they are considered as subcriteria of En, while the opposite is true when they are considered as subcriteria of the root criterion $G_0$. Analogous situation could be observed for the elementary criteria ExEa and FiFe, where FiFe is more important than ExEa when they are considered as subcriteria of Ec, while the opposite is true when they are considered as elementary criteria of the root criterion $G_0$. Also this phenomenon could seem unlikely, but this can be explained as before. In fact, when computing the importance of SoSu with respect to En one has to take into account only the interaction between SoSu and WaSu, while computing the importance of SoSu with respect to the root criterion $G_0$, one has to take into account its interaction with WaSu and the two elementary criteria of Ec.

# 3 Robust Ordinal Regression (ROR) and Stochastic Multiobjective Acceptability Analysis (SMAA) applied to the hierarchical Choquet integral preference model

According to Section 2.4, to apply the hierarchical Choquet integral preference model, one has to define the Möbius representation of a capacity defined on the power set of $EL$, that is $m(\{g_{\mathbf{t}}\})$ for each elementary criterion $g_{\mathbf{t}}$, and $m(\{g_{\mathbf{t_1}}, g_{\mathbf{t_2}}\})$ for each couple of elementary criteria $\{g_{\mathbf{t_1}}, g_{\mathbf{t_2}}\}$. These values will be calculated using an ordinal regression technique from some indirect preference information. Below, we explain this technique in detail.

Given a criterion $G_{\mathbf{r}}$, $\mathbf{r} \in \mathcal{I}_{\mathcal{G}} \setminus \{EL\}$, the DM is requested to provide the following type of preference information:

- $a$ is preferred to $b$ on criterion $G_{\mathbf{r}}$, denoted by $a \succ_{\mathbf{r}} b$ (translated to the constraint $C_{\mu_{\mathbf{r}}}(a) \geq C_{\mu_{\mathbf{r}}}(b) + \varepsilon$);
- $a$ is indifferent to $b$ on criterion $G_{\mathbf{r}}$, denoted by $a \sim_{\mathbf{r}} b$ ($C_{\mu_{\mathbf{r}}}(a) = C_{\mu_{\mathbf{r}}}(b)$);
- on criterion $G_{\mathbf{r}}$, $a$ is preferred to $b$ more than $c$ is preferred to $d$, denoted by $(a, b) \succ^*_{\mathbf{r}} (c, d)$, ($C_{\mu_{\mathbf{r}}}(a) - C_{\mu_{\mathbf{r}}}(b) \geq C_{\mu_{\mathbf{r}}}(c) - C_{\mu_{\mathbf{r}}}(d) + \varepsilon$ and $C_{\mu_{\mathbf{r}}}(c) - C_{\mu_{\mathbf{r}}}(d) \geq \varepsilon$);
- on criterion $G_{\mathbf{r}}$ the intensity of preference between $a$ and $b$ is the same as the intensity of preference between $c$ and $d$, denoted by $(a, b) \sim^*_{\mathbf{r}} (c, d)$ ($C_{\mu_{\mathbf{r}}}(a) - C_{\mu_{\mathbf{r}}}(b) = C_{\mu_{\mathbf{r}}}(c) - C_{\mu_{\mathbf{r}}}(d)$).

Considering criteria $G_{\mathbf{r_1}}, G_{\mathbf{r_2}}, G_{\mathbf{r_3}}, G_{\mathbf{r_4}}$, with $\mathbf{r_1}, \mathbf{r_2}, \mathbf{r_3}, \mathbf{r_4} \in \mathcal{G}^k_{\mathbf{r}}$, the DM can provide the following preference information:

- criterion $G_{\mathbf{r_1}}$ is more important than criterion $G_{\mathbf{r_2}}$, denoted by $G_{\mathbf{r_1}} \succ G_{\mathbf{r_2}}$ (translated to the constraint $\varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_1}}\}) \geq \varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_2}}\}) + \varepsilon$);
- criteria $G_{\mathbf{r_1}}$ and $G_{\mathbf{r_2}}$ are equally important, denoted by $G_{\mathbf{r_1}} \sim G_{\mathbf{r_2}}$ ($\varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_1}}\}) = \varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_2}}\})$);
- criteria $G_{\mathbf{r_1}}$ and $G_{\mathbf{r_2}}$ are positively interacting ($\varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_1}}, G_{\mathbf{r_2}}\}) \geq \varepsilon$);
- criteria $G_{\mathbf{r_1}}$ and $G_{\mathbf{r_2}}$ are negatively interacting ($\varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_1}}, G_{\mathbf{r_2}}\}) \leq -\varepsilon$);
- the interaction between criteria $G_{\mathbf{r_1}}$ and $G_{\mathbf{r_2}}$ is greater than the interaction between criteria $G_{\mathbf{r_3}}$ and $G_{\mathbf{r_4}}$
  - if there is positive interaction between both pairs of criteria, then the constraint translating this preference are $\varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_1}}, G_{\mathbf{r_2}}\}) - \varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_3}}, G_{\mathbf{r_4}}\}) \geq \varepsilon$ and $\varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_3}}, G_{\mathbf{r_4}}\}) \geq \varepsilon$
  - if there is negative interaction between both pairs of criteria, then the constraint translating this preference are $\varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_1}}, G_{\mathbf{r_2}}\}) - \varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_3}}, G_{\mathbf{r_4}}\}) \leq -\varepsilon$ and $\varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_3}}, G_{\mathbf{r_4}}\}) \leq -\varepsilon$;

- $G_{\mathbf{r_1}}$ is preferred to $G_{\mathbf{r_2}}$ more than $G_{\mathbf{r_3}}$ is preferred to $G_{\mathbf{r_4}}$, denoted by $(G_{\mathbf{r_1}}, G_{\mathbf{r_2}}) \succ^* (G_{\mathbf{r_3}}, G_{\mathbf{r_4}})$ $\left(\varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_1}}\}) - \varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_2}}\}) \geq \varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_3}}\}) - \varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_4}}\}) + \varepsilon \text{ and } \varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_3}}\}) - \varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_4}}\}) \geq \varepsilon\right)$;

- the difference of importance between $G_{\mathbf{r_1}}$ and $G_{\mathbf{r_2}}$ is the same of the difference of importance between $G_{\mathbf{r_3}}$ and $G_{\mathbf{r_4}}$, denoted by $(G_{\mathbf{r_1}}, G_{\mathbf{r_2}}) \sim^* (G_{\mathbf{r_3}}, G_{\mathbf{r_4}})$ $(\varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_1}}\}) - \varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_2}}\}) = \varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_3}}\}) - \varphi^k_{\mathbf{r}}(\{G_{\mathbf{r_4}}\}))$.

Similarly to Section 2.2, $\varepsilon$ is an auxiliary variable used to convert the strict inequalities into weak ones. Moreover, like in Section 2.2, $E^{DM}$ denotes the set of constraints translating the DM's preference information together with the monotonicity and normalization constraints.
To check if there exists at least one compatible capacity, one has to solve the following linear programming problem:

$$\varepsilon^* = \max \varepsilon, \text{ subject to } E^{DM}.$$

If $E^{DM}$ is feasible, and $\varepsilon^* > 0$ then there exists at least one compatible capacity, otherwise some inconsistency arised, which has to be identified [37].
Considering criterion $G_{\mathbf{r}}$ located at a not last level of the hierarchy, and the two following sets of constraints,

$$\left.\begin{array}{l} C_{\mu_{\mathbf{r}}}(b) \geq C_{\mu_{\mathbf{r}}}(a) + \varepsilon, \\ E^{DM}. \end{array}\right\} E^N_{\mathbf{r}}(a,b), \qquad \left.\begin{array}{l} C_{\mu_{\mathbf{r}}}(a) \geq C_{\mu_{\mathbf{r}}}(b) \\ E^{DM} \end{array}\right\} E^P_{\mathbf{r}}(a,b)$$

the necessary preference relation with respect to criterion $G_{\mathbf{r}}$ holds for alternatives $a$ and $b$ if $E^N_{\mathbf{r}}(a,b)$ is infeasible or $\varepsilon^N_{\mathbf{r}} \leq 0$, where $\varepsilon^N_{\mathbf{r}} = \max \varepsilon$, subject to $E^N_{\mathbf{r}}(a,b)$. Analogously, the possible preference relation with respect to criterion $G_{\mathbf{r}}$ holds for alternatives $a$ and $b$ if $E^P_{\mathbf{r}}(a,b)$ is feasible and $\varepsilon^P_{\mathbf{r}} > 0$, where $\varepsilon^P_{\mathbf{r}} = \max \varepsilon$, subject to $E^P_{\mathbf{r}}(a,b)$.
In practice, it is very likely that, given an available preference information, $a$ is possibly preferred to $b$ and $b$ is possibly preferred to $a$. Nevertheless, the number of compatible capacities for which $a$ is preferred to $b$ could be very different from the number of compatible capacities for which $b$ is preferred to $a$. For this reason, in order to estimate how good an alternative is compared to others and how often it is preferred over another alternative, we propose to apply the SMAA methodology. This technique applied to the hierarchical Choquet integral preference model is explained in detail below.
The set of linear constraints in $E^{DM}$ defines a convex set of Möbius parameters. To explore this set of parameters the Hit-And-Run (HAR) method can be applied [44, 49, 50]. HAR samples iteratively a set of Möbius parameters satisfying $E^{DM}$ until a stopping condition is met. For each sampled set of Möbius parameters and a given criterion $G_{\mathbf{r}}$, one can compute values of the Choquet integral for all considered alternatives. These values rank the alternatives with respect to $G_{\mathbf{r}}$. Having as many rankings as the samples, one can compute the indices typical to the SMAA methodology recalled in Section 2.2:

- the rank acceptability index $b^l_{k,\mathbf{r}}$, being the frequency with which alternative $a_k$ gets position $l$ in the ranking obtained with respect to criterion $G_{\mathbf{r}}$,

- the pairwise winning index $p_{\mathbf{r}}(a,b)$, giving the frequency of the preference of $a$ over $b$ on criterion $G_{\mathbf{r}}$.

Moreover, by using the rank acceptability indices, other two indices recently introduced in [2] can be computed:

- the downward cumulative rank acceptability index $b^{\leq l}_{k,\mathbf{r}}$, being the frequency that alternative $a_k$ will get a position not greater than $l$ on criterion $G_{\mathbf{r}}$,

$$b^{\leq l}_{k,\mathbf{r}} = \sum_{s=1}^{l} b^{s}_{k,\mathbf{r}},$$

- the upward cumulative rank acceptability index $b^{\geq l}_{k,\mathbf{r}}$, being the frequency that alternative $a_k$ will get a position not lower than $l$ on criterion $G_{\mathbf{r}}$,

$$b^{\geq l}_{k,\mathbf{r}} = \sum_{s=l}^{n} b^{s}_{k,\mathbf{r}}.$$

It is worth stressing that at the comprehensive level, represented by criterion $G_{\mathbf{0}}$, we also get the necessary and possible preference relations on one hand, and the SMAA indices on the other hand.

# 4 An illustrative real world decision making problem

In this section, we apply the proposed methodology to a real world decision making problem [1]. 220 European universities from 30 countries have been evaluated on a 1-5 scale (1-weak, 2-below average, 3-average, 4-good, 5-very good) with respect to criteria structured in a hierarchical way, as shown in Figure 3. The three macro-criteria are Teaching & Learning (TL), Research (R) and Knowledge Transfer (KT), and they are further decomposed to more detailed elementary criteria. For macro-criteron TL, these are:

- Masters Graduation Rate (MGR),
- Masters Graduating on Time (MGOT).

Macro-criterion R is decomposed to:

- Number of Research Publications (NRP),
- Citation Rate (CR),
- Proportion of Top Cited Publications (PTCT),

and macro-criterion KT is decomposed to:

- Number of Patents Awarded (NPA),
- Number of Spin-Offs (NSO),
- Research and Knowledge Transfer Revenues (RKTR).

Description of the elementary criteria is given in Table 4.

Many of these universities dominate[1] the others and, at the same time, many universities are dominated by others. For this reason, following a procedure well known from the evolutionary multiobjective optimization method, called NSGA-II [14], we ordered the universities in nondominated fronts. We put in the first front all nondominated universities; then, after removing these universities

[1]An alternative $a$ dominates an alternative $b$ with respect to criteria $\{g_1, \ldots, g_n\}$ if, supposing that all criteria are of the gain type, $g_i(a) \geq g_i(b)$ for all $i = 1, \ldots, n$, and there exists at least one $j \in \{1, \ldots, n\}$, such that $g_j(a) > g_j(b)$.

Figure 3: Hierarchical structure of criteria considered in the case study

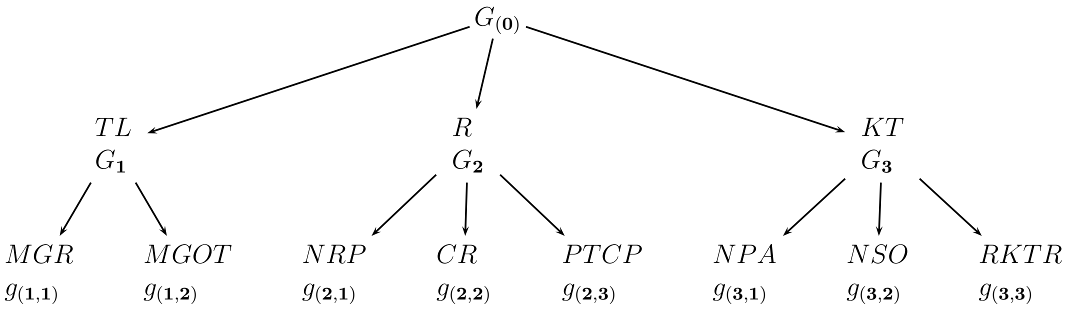



Table 4: Description of the elementary criteria

| **Elementary criterion** | **Description** |
|---|---|
| Masters Graduation Rate (MGR) | The percentage of new entrants that successfully completed their master programs |
| Masters Graduating on Time (MGOT) | The percentage of graduates that graduated within the time expected (normative time) for their masters programs |
| Number of Research Publications (NRP) | The number of research publications indexed in the Web of Science database, where at least one author is affiliated to the university (relative to the number of students) |
| Citation Rate (CR) | The average number of times that the university department's research publications (over the period 2008-2011) get cited in other research, adjusted (normalized) at the global level to take into account differences in publication years and to allow for differences |
| Proportion of Top Cited Publications (PTCP) | The proportion of the university's research publications that, compared to other publications in the same field and in the same year, belong to the top 10% most frequently cited |
| Number of Patents Awarded (NPA) | The number of patents assigned to (inventors working in) the university (over the period 2001-2010) |
| Number of Spin-Offs (NSO) | The number of spin-offs (i.e. firms established on the basis of a formal knowledge transfer arrangement between the institution and the firm) recently created by the institution (per 1,000 fte academic staff) |
| Research and Knowledge Transfer Revenues (RKTR) | Research revenues and knowledge transfer revenues from private sources (incl. not-for profit organizations), excluding tuition fees. Measured in €1,000s using Purchasing Power Parities. Expressed per fte academic staff. |

from the list of universities, we put in the second front the universities nondominated among the remaining ones, and so on. In this way, the universities belonging to the same front are more ore less similar, in the sense that there is not any strong evidence for the preference of one university over another. Consequently, it is meaningful from the DM's point of view to get a ranking recommendation with respect to universities from the same front. In this section, we shall focus our attention on the first nondominated front but, of course, a similar analysis could be done also with respect to another nondominated front, or with respect to any subset of universities considered as most interesting for a particular DM. The evaluations of the universities belonging to the first nondominated front on the eight elementary criteria are provided in Table 5.

Table 5: Evaluations of the universities belonging to the first nondominated front on the considered elementary criteria

| | | $G_{(\mathbf{0})}$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | **TL** ($G_{(\mathbf{1})}$) | | **R** ($G_{(\mathbf{2})}$) | | | **KT** ($G_{(\mathbf{3})}$) | | |
| **University** | **Country** | **MGR** ($g_{(\mathbf{1},\mathbf{1})}$) | **MGOT** ($g_{(\mathbf{1},\mathbf{2})}$) | **NRP** ($g_{(\mathbf{2},\mathbf{1})}$) | **CR** ($g_{(\mathbf{2},\mathbf{2})}$) | **PTCP** ($g_{(\mathbf{2},\mathbf{3})}$) | **NPA** ($g_{(\mathbf{3},\mathbf{1})}$) | **NS0** ($g_{(\mathbf{3},\mathbf{2})}$) | **RKTR** ($g_{(\mathbf{3},\mathbf{3})}$) |
| Bocconi University ($U_{25}$) | Italy | 5 | 4 | 2 | 5 | 5 | 1 | 1 | 5 |
| Budapest U Tech & Economics ($U_{35}$) | Hungary | 5 | 3 | 3 | 3 | 3 | 2 | 4 | 2 |
| U Cordoba ($U_{51}$) | Spain | 3 | 5 | 3 | 3 | 3 | 2 | 3 | 5 |
| Tech U Denmark ($U_{61}$) | Denmark | 4 | 4 | 5 | 5 | 5 | 5 | 5 | 5 |
| Dublin Inst. Tech ($U_{64}$) | Ireland | 2 | 5 | 2 | 5 | 5 | 2 | 4 | 2 |
| U Limerick ($U_{108}$) | Ireland | 4 | 5 | 2 | 5 | 4 | 4 | 3 | 5 |
| Lomonosow Moscow State U ($U_{117}$) | Russia | 5 | 5 | 5 | 2 | 2 | 2 | 5 | 5 |
| Mondragon U ($U_{129}$) | Spain | 4 | 5 | 2 | 5 | 5 | 1 | 5 | 5 |
| Newcastle U ($U_{136}$) | United Kingdom | 4 | 5 | 5 | 5 | 5 | 5 | 2 | 5 |
| U Salamanca ($U_{170}$) | Spain | 5 | 4 | 4 | 3 | 3 | 2 | 2 | 4 |
| U Trieste ($U_{196}$) | Italy | 5 | 2 | 5 | 4 | 4 | 3 | 3 | 3 |
| WHU School of Management ($U_{216}$) | Germany | 5 | 5 | 2 | 4 | 4 | 1 | 5 | 5 |

Suppose that the DM specifies the following preference information on the considered elementary criteria and on the macro-criteria. Within parentheses, we write the constraints translating the corresponding piece of preference information provided by the DM:

- R is more important than KT that, in turn, is more important than TL

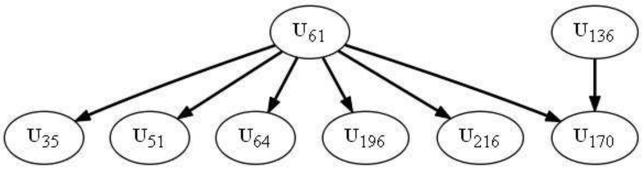
$(\varphi_{\mathbf{0}}(R) \geq \varphi_{\mathbf{0}}(KT) + \varepsilon$ and $\varphi_{\mathbf{0}}(KT) \geq \varphi_{\mathbf{0}}(TL) + \varepsilon)$,

- With respect to TL, MGOT is more important than MGR $(\varphi^2_{\mathbf{2}}(\{MGOT\}) \geq \varphi^2_{\mathbf{2}}(\{MGR\}) + \varepsilon)$,
- With respect to KT, RKTR is more important than NSO that, in turn, is more important than NPA $(\varphi^2_{\mathbf{2}}(\{RKTR\}) \geq \varphi^2_{\mathbf{2}}(\{NSO\}) + \varepsilon$ and $\varphi^2_{\mathbf{2}}(\{NSO\}) \geq \varphi^2_{\mathbf{2}}(\{NPA\}) + \varepsilon)$,
- At a comprehensive level, PTCP is more important than RKTR that, in turn, is more important than MGT $(\varphi^2_{\mathbf{0}}(\{PTCP\}) \geq \varphi^2_{\mathbf{0}}(\{RKTR\}) + \varepsilon$ and $\varphi^2_{\mathbf{0}}(\{RKTR\}) \geq \varphi^2_{\mathbf{0}}(\{MGT\}) + \varepsilon)$,
- TL and R are positively interacting $(\varphi^1_{\mathbf{0}}(\{TL, R\}) \geq \varepsilon)$,
- R and KT are positively interacting $(\varphi^1_{\mathbf{0}}(\{R, KT\}) \geq \varepsilon)$,
- TL and KT are positively interacting $(\varphi^1_{\mathbf{0}}(\{TL, KT\}) \geq \varepsilon)$,
- The interaction between R and KT is greater than the interaction between TL and KT $(\varphi^1_{\mathbf{0}}(\{R, KT\}) \geq \varphi^1_{\mathbf{0}}(\{TL, KT\}) + \varepsilon$ and $\varphi^1_{\mathbf{0}}(\{TL, KT\}) \geq \varepsilon)$,
- The interaction between R and TL is greater than the interaction between TL and KT $(\varphi^1_{\mathbf{0}}(\{R, TL\}) \geq \varphi^1_{\mathbf{0}}(\{TL, KT\}) + \varepsilon$ and $\varphi^1_{\mathbf{0}}(\{TL, KT\}) \geq \varepsilon)$,
- With respect to R, NRP and PTCP are positively interacting $(\varphi^2_{\mathbf{2}}(\{NRP, PTCP\}) \geq \varepsilon)$,
- CR and PTCP are negatively interacting $(\varphi^2_{\mathbf{0}}(\{CR, PTCP\}) \leq -\varepsilon)$,
- NRP and RKTR are positively interacting $(\varphi^2_{\mathbf{0}}(\{NRP, RKTR\}) \geq \varepsilon)$,
- NPA and NSO are negatively interacting $(\varphi^2_{\mathbf{0}}(\{NPA, NSO\}) \leq -\varepsilon)$,
- MGOT and NRP are positively interacting $(\varphi^2_{\mathbf{0}}(\{MGOT, NRP\}) \geq \varepsilon)$,

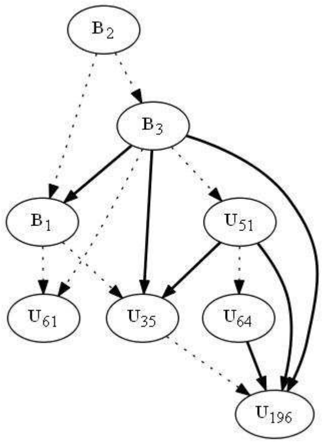
Applying NAROR at the comprehensive level, as well as on the three macro-criteria, we get the necessary preference relations shown in Figures 4(a)-4(d). Let us observe that the blocks $B_1, \ldots, B_7$ in Figures 4(b)-4(d) are composed of universities having exactly the same evaluations on the elementary criteria descending from the considered macro-criterion. Therefore, for example, $B_1$ is composed of $U_{25}$ and $U_{170}$ since they have exactly the same evaluations (5 and 4) on MGR and MGOT, being the two elementary criteria descending from macro-criterion TL.

Looking at Figures 4(a)-4(d) it seems that $U_{61}$ can be seen as the best university. Indeed, while it is evident that on R and KT this university dominates all the others, at the comprehensive level it is necessarily preferred to six out of the eleven universities. Analyzing more in detail the results of NAROR at the intermediate level, one can observe that the preference information provided by the DM results in many bold arrows, i.e., necessary preference relations which are not dominance relations. For example, on TL, $U_{51}$ is necessarily preferred to $U_{35}$ and $U_{196}$, while on R, the universities belonging to $B_4$ are necessarily preferred to the universities belonging to $B_5$. Moreover, on KT, $U_{117}$ is necessarily preferred to $U_{196}$. Let us remind that the results we are showing concern the universities belonging to the first nondominated front only but they are enough to observe that the application of NAROR puts many new couples in the necessary preference relations, both at the comprehensive level, and on particular macro-criteria, contributing in this way to a better understanding of the decision problem by the DM.

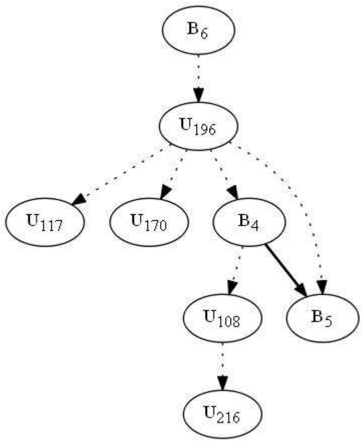
After applying NAROR, we applied the SMAA methodology on the set of compatible value functions at the comprehensive level and at the level of macro-criteria. At first, for each considered university, we looked at the best and at the worst position the university could get considering the



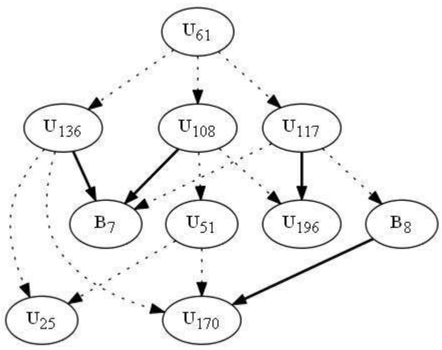

(a) Comprehensive level




(b) Teaching and Learning (TL); $B_1 = \{U_{25}, U_{170}\}$, $B_2 = \{U_{117}, U_{216}\}$, $B_3 = \{U_{108}, U_{129}, U_{136}\}$




(c) Research (R); $B_4 = \{U_{25}, U_{64}, U_{129}\}$, $B_5 = \{U_{35}, U_{51}\}$, $B_6 = \{U_{61}, U_{136}\}$




(d) Knowledge Transfer (KT); $B_7 = \{U_{35}, U_{64}\}$, $B_8 = \{U_{129}, U_{216}\}$

Figure 4: Necessary preference relation at the comprehensive level, as well as with respect to macro-criteria TL, R and KT. Dotted arrows represent the dominance relation, while bold arrows represent necessary preference relations obtained by NAROR.

whole set of capacities compatible with the preferences provided by the DM. We also looked at as well as the three highest rank acceptability indices showing, therefore, which are the most likely positions for a university at hand.

Table 6: Rank Acceptability Indices. For each university, we reported the best and the worst possible positions, as well as the three highest rank acceptability indices. The blocks $B_1, \ldots, B_8$ are composed of universities having exactly the same evaluations. Let us note that with respect to all elementary criteria there are 220 different performance vectors of the universities, so at the comprehensive level the rank acceptability indices are computed for the positions going from the 1 to 220. Analogously, the rank acceptability indices with respect to TL are computed for the positions from the 1 to 16, with respect to R from 1 to 29, while on KT, from the 1 to 55.

(a) Comprehensive level

| University | Best $\left(b_{k,\mathbf{0}}^{Best}\right)$ | Worst $\left(b_{k,\mathbf{0}}^{Worst}\right)$ | $high_1\left(b_{k,\mathbf{0}}^{high_1}\right)$ | $high_2\left(b_{k,\mathbf{0}}^{high_2}\right)$ | $high_3\left(b_{k,\mathbf{0}}^{high_3}\right)$ |
|---|---|---|---|---|---|
| $U_{25}$ | 18 (0.11%) | 103 (0.01%) | 52 (3.65%) | 55 (3.45%) | 58 (3.39%) |
| $U_{35}$ | 80 (0.01%) | 132 (0.05%) | 106 (4.68%) | 108 (4.04%) | 105 (3.87%) |
| $U_{51}$ | 47 (0.1%) | 83 (0.01%) | 65 (6.18%) | 60 (6%) | 70 (5.71%) |
| $U_{61}$ | 1 (93.29%) | 2 (6.71%) | | | |
| $U_{64}$ | 47 (0.02%) | 133 (0.05%) | 90 (2.82%) | 87 (2.72%) | 88 (2.66%) |
| $U_{108}$ | 14 (0.16%) | 51 (0.01%) | 28 (8.55%) | 34 (6.19%) | 33 (5.8%) |
| $U_{117}$ | 23 (0.15%) | 90 (0.06%) | 63 (3.22%) | 67 (3.15%) | 64 (2.82%) |
| $U_{129}$ | 6 (0.2%) | 68 (0.11%) | 23 (4.8%) | 32 (4.24%) | 37 (4.12%) |
| $U_{136}$ | 1 (6.71%) | 12 (0.12%) | 2 (60.18%) | 3 (14.36%) | 5 (8.88%) |
| $U_{170}$ | 58 (0.04%) | 99 (0.04%) | 80 (6.04%) | 82 (6.12%) | 83 (7.02%) |
| $U_{196}$ | 40 (0.01%) | 77 (0.33%) | 58 (5.88%) | 57 (5.64%) | 55 (5.6%) |
| $U_{216}$ | 12 (0.04%) | 81 (0.01%) | 43 (4.59%) | 46 (4.04%) | 42 (3.65%) |

(b) Teaching and Learning (TL)

| University | Best $\left(b_{k,\mathbf{1}}^{Best}\right)$ | Worst $\left(b_{k,\mathbf{1}}^{Worst}\right)$ | $high_1\left(b_{k,\mathbf{1}}^{high_1}\right)$ | $high_2\left(b_{k,\mathbf{1}}^{high_2}\right)$ | $high_3\left(b_{k,\mathbf{1}}^{high_3}\right)$ |
|---|---|---|---|---|---|
| $B_1$ | 3 (61.91%) | 5 (18.47%) | 3 (61.91%) | 4 (19.62%) | 5 (18.47%) |
| $B_2$ | 1 (100.00%) | 1 (100.00%) | | | |
| $B_3$ | 2 (100.00%) | 2 (100.00%) | | | |
| $U_{35}$ | 5 (5.93%) | 9 (19.37%) | 6 (32.95%) | 8 (28.3%) | 9 (19.37%) |
| $U_{51}$ | 3 (38.09%) | 5 (21.64%) | 4 (40.27%) | 3 (38.09%) | 5 (21.64%) |
| $U_{61}$ | 4 (21.64%) | 7 (4.04%) | 6 (37.63%) | 5 (36.69%) | 4 (21.64%) |
| $U_{64}$ | 4 (18.47%) | 10 (0.05%) | 7 (23.9%) | 6 (23.52%) | 4 (18.47%) |
| $U_{196}$ | 8 (4.48%) | 13 (28.09%) | 13 (28.09%) | 12 (22.16%) | 10 (21.72%) |

(c) Research (R)

| University | Best $\left(b_{k,\mathbf{1}}^{Best}\right)$ | Worst $\left(b_{k,\mathbf{1}}^{Worst}\right)$ | $high_1\left(b_{k,\mathbf{1}}^{high_1}\right)$ | $high_2\left(b_{k,\mathbf{1}}^{high_2}\right)$ | $high_3\left(b_{k,\mathbf{1}}^{high_3}\right)$ |
|---|---|---|---|---|---|
| $B_4$ | 5 (6.9%) | 16 (1.27%) | 8 (22.16%) | 10 (13.81%) | 11 (13.72%) |
| $B_5$ | 15 (1.28%) | 23 (0.14%) | 21 (24.59%) | 18 (20.95%) | 20 (20.65%) |
| $B_6$ | 1 (100.00%) | 1 (100.00%) | | | |
| $U_{108}$ | 6 (0.32%) | 19 (4.23%) | 13 (16.05%) | 14 (15.25%) | 12 (11.63%) |
| $U_{117}$ | 20 (1.71%) | 26 (29.18%) | 25 (48.64%) | 26 (29.18%) | 24 (14.85%) |
| $U_{170}$ | 12 (3.03%) | 21 (1.34%) | 18 (27.28%) | 17 (21.32%) | 19 (16.49%) |
| $U_{196}$ | 4 (43.99%) | 10 (0.01%) | 4 (43.99%) | 5 (32.95%) | 6 (10.96%) |
| $U_{216}$ | 14 (4.66%) | 21 (1.71%) | 16 (23.79%) | 15 (18.16%) | 19 (17.19%) |

(d) Knowledge Transfer (KT)

| University | Best $\left(b_{k,\mathbf{1}}^{Best}\right)$ | Worst $\left(b_{k,\mathbf{1}}^{Worst}\right)$ | $high_1\left(b_{k,\mathbf{1}}^{high_1}\right)$ | $high_2\left(b_{k,\mathbf{1}}^{high_2}\right)$ | $high_3\left(b_{k,\mathbf{1}}^{high_3}\right)$ |
|---|---|---|---|---|---|
| $B_7$ | 30 (0.18%) | 42 (0.3%) | 36 (21.38%) | 35 (18.18%) | 34 (15.78%) |
| $B_8$ | 15 (1.28%) | 23 (0.14%) | 21 (24.59%) | 18 (20.95%) | 20 (20.65%) |
| $U_{25}$ | 21 (0.09%) | 40 (0.09%) | 30 (25.86%) | 29 (11.91%) | 31 (10.55%) |
| $U_{51}$ | 13 (1.04%) | 20 (0.38%) | 16 (30.46%) | 17 (22.81%) | 18 (21.49%) |
| $U_{61}$ | 1 (100.00%) | 1 (100.00%) | | | |
| $U_{108}$ | 7 (4.92%) | 14 (0.34%) | 8 (20.09%) | 10 (19.71%) | 11 (17.92%) |
| $U_{117}$ | 4 (20.81%) | 14 (0.32%) | 7 (22.22%) | 4 (20.81%) | 5 (19.14%) |
| $U_{136}$ | 4 (5.2%) | 20 (3.17%) | 7 (11.48%) | 10 (10.91%) | 5 (9.05%) |
| $U_{170}$ | 25 (0.3%) | 36 (0.5%) | 31 (34.4%) | 32 (29.06%) | 30 (9.87%) |
| $U_{196}$ | 26 (0.01%) | 39 (0.77%) | 36 (17.57%) | 33 (16.3%) | 35 (15.43%) |

Looking at Tables 6(a)-6(d) the following observations can be made.

- At comprehensive level, $U_{61}$ is confirmed to be the best among the considered universities since it has rank acceptability index for the 1st position equal to 93.29%, while the remaining 6.71% is its rank acceptability index for the 2nd position; analogously, $U_{136}$ could be considered a pretty good university since it takes always a position between the 1st and the 12th and its highest rank acceptability indices correspond to the 2nd and to the 3rd position. At the same time, even if $U_{35}$ and $U_{64}$ belong to the first nondominated front, they do not take very high positions in the complete rankings obtained at the comprehensive level. Indeed, on one hand, the highest position reached by $U_{35}$ is the 80th, while its highest rank acceptability index corresponds to position 106. On the other hand, $U_{64}$ reaches positions between the 47th and the 133th, and its highest rank acceptability index corresponds to position 90.

- With respect to TL, the complete ranking is almost deterministic. Indeed, the universities belonging to block $B_1$, that are $U_{25}$ and $U_{170}$, are always in the 1st position, while the universities belonging to block $B_2$, that are $U_{117}$ and $U_{216}$, are always in the 2nd position. Considering that on this macro-criterion there are only 16 possible positions, $U_{196}$ is bad on this macro-criterion since it takes always a position between the 8th and the 13th and its highest rank acceptability index corresponds to position 13.

- With respect to R, the universities belonging to block $B_4$, that are $U_{25}$, $U_{64}$ and $U_{129}$, are the best since they take always the 1st position. Good results are also obtained by university $U_{196}$ which takes always positions between the 4th and the 10th, and it has the highest rank acceptability index for position 4. $U_{117}$ is instead a bad university on this macro-criterion since it takes positions between the 20th and the 26th, and its highest rank acceptability index corresponds to position 25.

- On KT, $U_{61}$ is always the 1st while $U_{117}$ and $U_{136}$ are quite good since the highest position got by both of them is the 4th, and their highest rank acceptability index corresponds to position 7. At the same time, the universities belonging to block $B_7$, that are $U_{35}$ and $U_{64}$, are not very good on KT since they take always a position between the 30th and the 42th, and their highest rank acceptability index corresponds to position 36.

To compare the universities pairwise, we computed also the pairwise winning indices $p(U_h, U_k)$ providing the frequency with which university $U_h$ is preferred to university $U_k$ considering all criteria simultaneously, that is at the comprehensive level, as well as considering the three macro-criteria one by one.

Table 7: Pairwise Winning Indices

(a) Comprehensive level

| $p_{\mathbf{0}}(\cdot,\cdot)$ | $U_{25}$ | $U_{35}$ | $U_{51}$ | $U_{61}$ | $U_{64}$ | $U_{108}$ | $U_{117}$ | $U_{129}$ | $U_{136}$ | $U_{170}$ | $U_{196}$ | $U_{216}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $U_{25}$ | 0 | 99.87 | 77.21 | 0 | 94.75 | 2.69 | 55.06 | 3.36 | 0 | 94.93 | 54.78 | 19.78 |
| $U_{35}$ | 0.13 | 0 | 0 | 0 | 28.34 | 0 | 0 | 0 | 0 | 1.13 | 0 | 0 |
| $U_{51}$ | 22.79 | 100 | 0 | 0 | 88.08 | 0 | 32.1 | 0.82 | 0 | 96.44 | 23.16 | 4.65 |
| $U_{61}$ | 100 | 100 | 100 | 0 | 100 | 100 | 100 | 100 | 93.29 | 100 | 100 | 100 |
| $U_{64}$ | 5.25 | 71.66 | 11.92 | 0 | 0 | 0 | 9.85 | 0 | 0 | 26.51 | 2.25 | 2.67 |
| $U_{108}$ | 97.31 | 100 | 100 | 0 | 100 | 0 | 96.16 | 47.92 | 0 | 100 | 99.67 | 86.38 |
| $U_{117}$ | 44.94 | 100 | 67.9 | 0 | 90.15 | 3.84 | 0 | 8.82 | 0 | 96.72 | 45.94 | 21.63 |
| $U_{129}$ | 96.64 | 100 | 99.18 | 0 | 100 | 52.08 | 91.18 | 0 | 0.24 | 100 | 90.22 | 99.95 |
| $U_{136}$ | 100 | 100 | 100 | 6.71 | 100 | 100 | 100 | 99.76 | 0 | 100 | 100 | 100 |
| $U_{170}$ | 5.07 | 98.87 | 3.56 | 0 | 73.49 | 0 | 3.28 | 0 | 0 | 0 | 1.33 | 0.3 |
| $U_{196}$ | 45.22 | 100 | 76.84 | 0 | 97.75 | 0.33 | 54.06 | 9.78 | 0 | 98.67 | 0 | 18.88 |
| $U_{216}$ | 80.22 | 100 | 95.35 | 0 | 97.33 | 13.62 | 78.37 | 0.05 | 0 | 99.7 | 81.12 | 0 |

(b) Teaching and Learning (TL)

| $p_{\mathbf{1}}(\cdot,\cdot)$ | $B_1$ | $B_2$ | $B_3$ | $U_{35}$ | $U_{51}$ | $U_{61}$ | $U_{64}$ | $U_{196}$ |
|---|---|---|---|---|---|---|---|---|
| $B_1$ | 0 | 0 | 0 | 100 | 61.91 | 100 | 81.53 | 100 |
| $B_2$ | 100 | 0 | 100 | 100 | 100 | 100 | 100 | 100 |
| $B_3$ | 100 | 0 | 0 | 100 | 100 | 100 | 100 | 100 |
| $U_{35}$ | 0 | 0 | 0 | 0 | 0 | 8.97 | 36.94 | 100 |
| $U_{51}$ | 38.09 | 0 | 0 | 100 | 0 | 78.36 | 100 | 100 |
| $U_{61}$ | 0 | 0 | 0 | 91.03 | 21.64 | 0 | 63.26 | 100 |
| $U_{64}$ | 18.47 | 0 | 0 | 63.06 | 0 | 36.74 | 0 | 100 |
| $U_{196}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(c) Research (R)

| $p_{\mathbf{2}}(\cdot,\cdot)$ | $B_4$ | $B_5$ | $B_6$ | $U_{108}$ | $U_{117}$ | $U_{170}$ | $U_{196}$ | $U_{216}$ |
|---|---|---|---|---|---|---|---|---|
| $B_4$ | 0 | 98.73 | 0 | 100 | 100 | 94.37 | 14.74 | 100 |
| $B_5$ | 1.27 | 0 | 0 | 7.13 | 100 | 0 | 0 | 29.51 |
| $B_6$ | 100 | 100 | 0 | 100 | 100 | 100 | 100 | 100 |
| $U_{108}$ | 0 | 92.87 | 0 | 0 | 100 | 80.23 | 0.37 | 100 |
| $U_{117}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.71 |
| $U_{170}$ | 5.63 | 100 | 0 | 19.77 | 100 | 0 | 0 | 43.99 |
| $U_{196}$ | 85.26 | 100 | 0 | 99.63 | 100 | 100 | 0 | 100 |
| $U_{216}$ | 0 | 70.49 | 0 | 0 | 98.29 | 56.01 | 0 | 0 |

(d) Knowledge Transfer (KT)

| $p_{\mathbf{3}}(\cdot,\cdot)$ | $B_7$ | $B_8$ | $U_{25}$ | $U_{51}$ | $U_{61}$ | $U_{108}$ | $U_{117}$ | $U_{136}$ | $U_{170}$ | $U_{196}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $B_7$ | 0 | 0 | 5.83 | 0 | 0 | 0 | 0 | 0 | 6.99 | 36.92 |
| $B_8$ | 100 | 0 | 100 | 97.27 | 0 | 52.58 | 0 | 52.26 | 100 | 100 |
| $U_{25}$ | 94.17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 89.29 | 89.29 |
| $U_{51}$ | 100 | 2.73 | 100 | 0 | 0 | 0 | 0 | 10.75 | 100 | 100 |
| $U_{61}$ | 100 | 100 | 100 | 100 | 0 | 100 | 100 | 100 | 100 | 100 |
| $U_{108}$ | 100 | 47.42 | 100 | 100 | 0 | 0 | 25.01 | 52.24 | 100 | 100 |
| $U_{117}$ | 100 | 100 | 100 | 100 | 0 | 74.99 | 0 | 68.77 | 100 | 100 |
| $U_{136}$ | 100 | 47.74 | 100 | 89.25 | 0 | 47.76 | 31.23 | 0 | 100 | 100 |
| $U_{170}$ | 93.01 | 0 | 10.71 | 0 | 0 | 0 | 0 | 0 | 0 | 89.29 |
| $U_{196}$ | 63.08 | 0 | 10.71 | 0 | 0 | 0 | 0 | 0 | 10.71 | 0 |

Further information can be obtained looking at the pairwise winning indices in Tables 7(a)-7(d):

- At the comprehensive level, $U_{61}$ is preferred to all but one university in the first nondominated front with a frequency equal to 100% while it is preferred to $U_{136}$ with a frequency of 93.29%. Analogously, $U_{136}$ is preferred to all but two other universities in the first nondominated front with a frequency equal to 100%, while it is preferred to $U_{61}$ with a frequency of 6.71%, while almost always, it is preferred to $U_{129}$ ($p_0(U_{136}, U_{129}) = 99.76\%$). Looking at the worst universities in the first nondominated front, $U_{35}$ could be considered as a bad university since it is never preferred to majority of the universities in this front apart from $U_{25}$, $U_{64}$ and $U_{170}$, to which it is sometimes preferred.

- With respect to TL, universities belonging to block $B_2$, that are $U_{117}$ and $U_{216}$ are obviously always preferred to all other universities, while $U_{196}$ is really bad since it is never preferred to any other university belonging to the first nondominated front.

- With respect to R, the universities belonging to block $B_6$, that are $U_{61}$ and $U_{136}$, are always preferred to all the other universities, while $U_{117}$ could be considered the worst among the twelve universities at hand since it is only preferred to $U_{216}$ with a frequency equal to 1.71%.

- With respect to KT, $U_{61}$ is preferred to all other universities since it gets the best performances on all elementary criteria descending from this macro-criterion, while $U_{117}$ could be considered a pretty good university with respect to KT since it is preferred to all other universities (apart from $U_{61}$) with a frequency at least 68.77%. Analogously, universities belonging to block $B_7$, that are $U_{35}$ and $U_{64}$, could be considered really bad since all other universities are almost always preferred to them.

Table 8: Barycenter values of the Möbius representation of compatible capacities

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $m(\{MGR\})$ | $m(\{MGOT\})$ | $m(\{NRP\})$ | $m(\{CR\})$ | $m(\{PTCP\})$ | $m(\{NPA\})$ | $m(\{NSO\})$ | $m(\{RKTR\})$ | $m(\{MGR, MGOT\})$ | $m(\{MGR, NRP\})$ | $m(\{MGR, CR\})$ | $m(\{MGR, PTCP\})$ |
| 0.0406 | 0.0841 | 0.0504 | 0.1119 | 0.1485 | 0.0636 | 0.1277 | 0.1646 | 0.0139 | 0.0077 | 0.0506 | 0.0586 |
| $m(\{MGR, NPA\})$ | $m(\{MGR, NSO\})$ | $m(\{MGR, RKTR\})$ | $m(\{MGOT, NRP\})$ | $m(\{MGOT, CR\})$ | $m(\{MGOT, PTCP\})$ | $m(\{MGOT, NPA\})$ | $m(\{MGOT, NSO\})$ | $m(\{MGOT, RKTR\})$ | $m(\{NRP, CR\})$ | $m(\{NRP, PTCP\})$ | $m(\{NRP, NPA\})$ |
| 0.0085 | 0.0005 | 0.0058 | 0.0238 | -0.0027 | -0.0249 | 0.0067 | 0.0103 | 0.0009 | 0.0479 | 0.0581 | 0.0117 |
| $m(\{NRP, NSO\})$ | $m(\{NRP, RKTR\})$ | $m(\{CR, PTCP\})$ | $m(\{CR, NPA\})$ | $m(\{CR, NSO\})$ | $m(\{CR, RKTR\})$ | $m(\{PTCP, NPA\})$ | $m(\{PTCP, NSO\})$ | $m(\{PTCP, RKTR\})$ | $m(\{NPA, NSO\})$ | $m(\{NPA, RKTR\})$ | $m(\{NSO, RKTR\})$ |
| 0.0100 | 0.0147 | -0.0464 | 0.0205 | 0.0163 | -0.0082 | 0.0113 | -0.0053 | 0.0024 | -0.0147 | -0.0119 | -0.0573 |

In order to get a ranking of the considered universities with respect to TL, R, KT and at the comprehensive level, we computed the barycenter of the Möbius representation of capacities compatible with the preferences provided by the DM. Their values are shown in Table 8. From this table one can conclude that, considered alone, the most important criterion is RKTR ($m(\{RKTR\}) = 0.1646$), followed by PTCP ($m(\{PTCP\}) = 0.1485$) and NSO ($m(\{NSO\}) = 0.1277$), while MGR is the least important one ($m(\{MGR\}) = 0.0406$). Moreover, apart from information provided by the DM about interactions between some elementary criteria, Table 8 shows other interactions, like positive interaction between MGR and MGOT or negative interaction between NSO and RKTR.

Using the barycenter of the Möbius representation to compute the Choquet integral value for each university, we get four complete rankings of universities at the comprehensive level and at the levels of macro-criteria. In Tables 9(a)-9(d) we show the complete rankings of the twelve universities from the first nondominated front, indicating their positions in complete ranking of all 220 universities.

One can observe that the Tech University Denmark is the best among the considered universities at the comprehensive level, as well as on R and KT, while it takes the 5th position with respect to TL. It is interesting to note that university of Trieste has a high position with respect to R ($5th$) while it has a bad position with respect to KT ($35th$). Lomonosow Moscow State University behaves exactly in the opposite way, getting a bad position with respect to R ($25th$) and a good position with respect to KT ($6th$). These observations shed light on the usefulness of the MCHP in providing a valuable insight into the problem at hand at different nodes of the hierarchy of criteria.
Even if we performed the analysis of the results for the alternatives belonging to the first nondominated front, for the sake of completeness, in Table 10 we list the first ten universities in the ranking at the comprehensive level obtained considering the barycenter of the Möbius representation of the capacities compatible with the preference information provided by the DM. Moreover, we reported also the rank acceptability indices of the same universities with respect to the first five positions in the ranking.

Looking at Table 10, one can argue that something is wrong in the presented results since only two of the universities in the first ten positions in the comprehensive ranking belong to the first nondominated front, that are the Tech University Denmark and the Newcastle University, even if it

Table 9: Rankings of universities from the first nondominated front, using Möbius representation shown in Table 8

(a) Comprehensive level

| Position in the complete ranking | University | Country |
|---|---|---|
| *1st* | Tech U Denmark | Denmark |
| *2nd* | Newcastle U | United Kingdom |
| *31th* | U Limerick | Ireland |
| *32th* | Mondragon U | Spain |
| *41th* | WHU School of Management | Germany |
| *53th* | Bocconi University | Italy |
| *54th* | U Trieste | Italy |
| *58th* | Lomonosow Moscow State U | Russia |
| *67th* | U Cordoba | Spain |
| *78th* | U Salamanca | Spain |
| *91th* | Dublin Inst. Tech | Ireland |
| *105th* | Budapest U Tech & Economics | Hungary |

(b) Teaching and Learning (TL)

| Position in the complete ranking | University | Country |
|---|---|---|
| *1st* | Lomonosow Moscow State U | Russia |
| | WHU School of Management | Germany |
| *2nd* | U Limerick | Ireland |
| | Mondragon U | Spain |
| | Newcastle U | United Kingdom |
| *3rd* | Bocconi University | Italy |
| | U Salamanca | Spain |
| *4th* | U Cordoba | Spain |
| *5th* | Tech U Denmark | Denmark |
| *6th* | Dublin Inst. Tech | Ireland |
| *8th* | Budapest U Tech & Economics | Hungary |
| *12th* | U Trieste | Italy |

(c) Research (R)

| Position in the complete ranking | University | Country |
|---|---|---|
| *1st* | Tech U Denmark | Denmark |
| | Newcastle U | United Kingdom |
| *5th* | U Trieste | Italy |
| *9th* | Bocconi University | Italy |
| | Dublin Inst. Tech | Ireland |
| | Mondragon U | Spain |
| *13th* | U Limerick | Ireland |
| *17th* | WHU School of Management | Germany |
| *18th* | U Salamanca | Spain |
| *19th* | Budapest U Tech & Economics | Hungary |
| | U Cordoba | Spain |
| *25th* | Lomonosow Moscow State U | Russia |

(d) Knowledge Transfer (KT)

| Position in the complete ranking | University | Country |
|---|---|---|
| *1st* | Tech U Denmark | Denmark |
| *6th* | Lomonosow Moscow State U | Russia |
| *9th* | Mondragon U | Spain |
| | WHU School of Management | Germany |
| *10th* | U Limerick | Ireland |
| *11th* | Newcastle U | United Kingdom |
| *16th* | U Cordoba | Spain |
| *28th* | Bocconi University | Italy |
| *31th* | U Salamanca | Spain |
| *35th* | U Trieste | Italy |
| *36th* | Budapest U Tech & Economics | Hungary |
| | Dublin Inst. Tech | Ireland |

Table 10: First ten universities in the ranking at the comprehensive level obtained by considering the barycenter of the Möbius representations of the capacities compatible with the preferences provided by the DMs. Moreover, we provide the rank acceptability indices of the same universities for the first five positions.

| **Position** | **University** | **Country** | $b_k^1$ | $b_k^2$ | $b_k^3$ | $b_k^4$ | $b_k^5$ | $b_k^6$ | $b_k^7$ | $b_k^8$ | $b_k^9$ | $b_k^{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1st | Tech U Denmark | Denmark | 93.29 | 6.71 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2nd | Newcastle U | United Kingdom | 6.71 | 60.18 | 14.36 | 6.99 | 8.88 | 0.61 | 1.08 | 0.32 | 0.3 | 0.31 |
| 3th | Eindhoven U Tech | The Netherlands | 0 | 33.11 | 16.54 | 34.88 | 10.72 | 4.75 | 0 | 0 | 0 | 0 |
| 4th | U Liverpool | United Kingdom | 0 | 0 | 50.12 | 20.91 | 6.44 | 16.13 | 1.46 | 1.58 | 0.84 | 1.07 |
| 5th | U Bern | Switzerland | 0 | 0 | 0 | 11 | 38.28 | 16.13 | 21 | 4.24 | 1.39 | 0.85 |
| 6th | Karlsruhe Inst. Tech (Kinst. Tech) | Denmark | 0 | 0 | 7.49 | 12.43 | 17.79 | 26.64 | 12.56 | 18.71 | 3.92 | 0.41 |
| 7th | Tech U München | Germany | 0 | 0 | 11.26 | 8.47 | 9.22 | 14.52 | 26.74 | 10.58 | 9.3 | 3.23 |
| 8th | U Liege | Belgium | 0 | 0 | 0 | 0.19 | 0.51 | 1.29 | 3.91 | 8.51 | 16.29 | 21.56 |
| 9th | U Stuttgart | Germany | 0 | 0 | 0 | 0.89 | 2.46 | 2.03 | 7.37 | 13.49 | 10.67 | 10.63 |
| 10th | U Groningen | The Netherlands | 0 | 0 | 0 | 0 | 0 | 4.52 | 5.53 | 10.71 | 18.63 | 10.21 |

is not the case. Indeed, the fact that a university belongs to the first nondominated front means only that there is not any other university dominating it in consequence of its excellence in one or more of the elementary criteria. This does not mean that at the comprehensive level, that is considering all elementary criteria simultaneously, a university having not any excellence in some elementary criteria but having in average good performance could not be a good university. For example, the Liverpool University belongs to the 2nd nondominated front but its performances are such that it takes the 3rd position in the final ranking with a frequency of 50.12%. Even more, we could observe that three of the first ten universities belong to the 2nd nondominated front (Eindhoven University Tech and Liverpool University), three at the 3rd nondominated front (Bern University, Karlsruhe Inst. Tech and Tech University of München), while two belong to the 4th nondominated front (Stuttgart University and Groningen University). Once more, we would like to underline that, even if we performed the analysis of the results for the universities belonging to the first nondominated front, the DM could make a similar analysis on every other subset of universities (s)he is interested in, therefore universities in another nondominated front or universities belonging to the same country and so on.

An interested reader can download the file containing complete results of the application of NAROR and SMAA on the full set of 220 universities from data-MCHP-NAROR-SMAA.

# 5 Conclusions

In this paper, we presented a methodology of handling a hierarchical structure of interacting criteria in the multiple criteria ranking problem. To this end, we applied the Multiple Criteria Hierarchy Process with the Choquet integral preference model. The preference information provided by the user in the course of the decision aiding process has the form of pairwise comparisons of some alternatives and some criteria at different levels of the hierarchy of criteria. The set of instances of the Choquet integral compatible with this preference information is identified using the Robust Ordinal Regression (ROR). Then, Stochastic Multiobjective Acceptability Analysis (SMAA) is applied on this set of compatible instances, leading to recommendations in the form of complete rankings of alternatives at the comprehensive level of the hierarchy of criteria and with respect to all subcriteria excluding the elementary ones. SMAA provides, moreover, many useful indices permitting to assess the relative quality of particular alternatives in different nodes of the hierarchy tree, i.e., with respect to different macro-criteria..

The presented methodology performs a constructive learning process in which the user learns from the results supplied by ROR and SMAA indices, and the method learns from the preference information supplied incrementally by the user in successive iterations. This process ceases when the obtained recommendations and indices are conclusive enough for the user.

We envisage to apply the hierarchical Choquet integral preference model in conjunction with ROR and SMAA in case of criteria involving different evaluation scales. In this case, the method presented recently in [4] can be used to construct a common scale without the need of normalizing the evaluations.

# Acknowledgments

The second and the third authors wish to acknowledge funding by the "Programma Operativo Nazionale" Ricerca & Competitivitá "2007-2013" within the project "PON04a2 E SINERGREEN-RES-NOVAE".
The fourth author wishes to acknowledge financial support from the Polish National Science Center, DEC-2015/18/A/ST6/00171.

# Appendix

**Proof of Proposition 2.1**
We shall prove Proposition 2.1 by induction over $\alpha$.

- First, let us prove the thesis for $\alpha = 1$. In this case, considering criterion $G_{(\mathbf{r},w_1)}$ as subcriterion of criterion $G_{\mathbf{r}}$ at the level $k$, we have

$$m^k_{\mathbf{r}}(\{G_{(\mathbf{r},w_1)}\}) = \mu^k_{\mathbf{r}}(\{G_{(\mathbf{r},w_1)}\}) = \frac{\mu(E(\{G_{(\mathbf{r},w_1)}\}))}{\mu(E(G_{\mathbf{r}}))} = \frac{\displaystyle\sum_{T \subseteq E(G_{(\mathbf{r},w_1)})} m(T)}{\mu(E(G_{\mathbf{r}}))}.$$

The first equality is obtained by eq. (14) defining the Möbius transformation $m_{\mathbf{r}}^k$ of the capacity $\mu_{\mathbf{r}}^k$; the second equality is obtained by equation (10) defining the capacity $\mu_{\mathbf{r}}^k$ in terms of the capacity $\mu$ while the third one is obtained by equation (1) defining the Möbius transformation $m$ of the capacity $\mu$.

- Let us suppose that the thesis is true for $\alpha = n-1$, that is, for all $\left\{G_{(\mathbf{r},w_1)}, \ldots, G_{(\mathbf{r},w_{n-1})}\right\} \subseteq \mathcal{G}_{\mathbf{r}}^k$,

$$m_{\mathbf{r}}^k(\left\{G_{(\mathbf{r},w_1)}, \ldots, G_{(\mathbf{r},w_{n-1})}\right\}) = \frac{\displaystyle\sum_{\substack{T_1 \subseteq E(G_{(\mathbf{r},w_1)}),\ T_1 \neq \emptyset, \\ \ldots \\ T_\alpha \subseteq E(G_{(\mathbf{r},w_{n-1})}),\ T_{n-1} \neq \emptyset,}} m(\{T_1, \ldots, T_{n-1}\})}{\mu(E(G_{\mathbf{r}}))}.$$

- Now, let us prove that the thesis is true for $\alpha = n$.
  Let $\left\{G_{(\mathbf{r},w_1)}, \ldots, G_{(\mathbf{r},w_n)}\right\} \subseteq \mathcal{G}_{\mathbf{r}}^k$ and let us compute $\mu_{\mathbf{r}}^k\left(\left\{G_{(\mathbf{r},w_1)}, \ldots, G_{(\mathbf{r},w_n)}\right\}\right)$.

  - By equation (14), we have that

$$\mu_{\mathbf{r}}^k\left(\left\{G_{(\mathbf{r},w_1)}, \ldots, G_{(\mathbf{r},w_n)}\right\}\right) = \sum_{T \subseteq \left\{G_{(\mathbf{r},w_1)}, \ldots, G_{(\mathbf{r},w_n)}\right\}} m_{\mathbf{r}}^k(T) = \sum_{T \subset \left\{G_{(\mathbf{r},w_1)}, \ldots, G_{(\mathbf{r},w_n)}\right\}} m_{\mathbf{r}}^k(T) +$$

$$+ m_{\mathbf{r}}^k\left(\left\{G_{(\mathbf{r},w_1)}, \ldots, G_{(\mathbf{r},w_n)}\right\}\right) = \sum_{\beta=1}^{n} m_{\mathbf{r}}^k\left(\left\{G_{(\mathbf{r},w_\beta)}\right\}\right) + \sum_{\{\beta_1,\beta_2\} \subset \{w_1,\ldots,w_n\}} m_{\mathbf{r}}^k\left(\left\{G_{(\mathbf{r},\beta_1)}, G_{(\mathbf{r},\beta_2)}\right\}\right) +$$

$$+ \ldots + \sum_{\{\beta_1,\ldots,\beta_{n-1}\} \subset \{1,\ldots,n\}} m_{\mathbf{r}}^k\left(\left\{G_{(\mathbf{r},\beta_1)}, \ldots, G_{(\mathbf{r},\beta_{n-1})}\right\}\right) + m_{\mathbf{r}}^k\left(\left\{G_{(\mathbf{r},w_1)}, \ldots, G_{(\mathbf{r},w_n)}\right\}\right).$$

  For the inductive hypothesis, we have therefore that

$$\mu_{\mathbf{r}}^k\left(\left\{G_{(\mathbf{r},w_1)}, \ldots, G_{(\mathbf{r},w_n)}\right\}\right) = \sum_{\beta=1}^{n} \frac{\displaystyle\sum_{T_\beta \subseteq E\left(G_{(\mathbf{r},w_\beta)}\right)} m(T_\beta)}{\mu(E(G_{\mathbf{r}}))} +$$

$$+ \sum_{\{\beta_1,\beta_2\} \subset \{w_1,\ldots,w_n\}} \frac{\displaystyle\sum_{\substack{T_{\beta_1} \subseteq E\left(G_{(\mathbf{r},w_{\beta_1})}\right), T_{\beta_1} \neq \emptyset, \\ T_{\beta_2} \subseteq E\left(G_{(\mathbf{r},w_{\beta_2})}\right), T_{\beta_2} \neq \emptyset}} m\left(\{T_{\beta_1}, T_{\beta_2}\}\right)}{\mu(E(G_{\mathbf{r}}))} + \ldots + \tag{19}$$

$$+ \sum_{\{\beta_1,\ldots,\beta_{n-1}\} \subset \{1,\ldots,n\}} \frac{\displaystyle\sum_{\substack{T_{\beta_1} \subseteq E\left(G_{(\mathbf{r},w_{\beta_1})}\right), T_{\beta_1} \neq \emptyset, \\ \ldots \\ T_{\beta_{n-1}} \subseteq E\left(G_{(\mathbf{r},w_{\beta_{n-1}})}\right), T_{\beta_{n-1}} \neq \emptyset}} m\left(\left\{T_{\beta_1}, \ldots, T_{\beta_{n-1}}\right\}\right)}{\mu(E(G_{\mathbf{r}}))} + m_{\mathbf{r}}^k(\left\{G_{(\mathbf{r},w_1)}, \ldots, G_{(\mathbf{r},w_\alpha)}\right\}).$$

  - From equation (10) we have that

$$\mu_{\mathbf{r}}^k\left(\left\{G_{(\mathbf{r},w_1)}, \ldots, G_{(\mathbf{r},w_n)}\right\}\right) = \frac{\mu\left(E\left(\left\{G_{(\mathbf{r},w_1)}, \ldots, G_{(\mathbf{r},w_n)}\right\}\right)\right)}{\mu(E(G_{\mathbf{r}}))} = \frac{\displaystyle\sum_{T \subseteq E\left(\left\{G_{(\mathbf{r},w_1)}, \ldots, G_{(\mathbf{r},w_n)}\right\}\right)} m(T)}{\mu(E(G_{\mathbf{r}}))} =$$

$$= \sum_{\beta=1}^{n} \frac{\sum\limits_{T_\beta \subseteq E\left(G_{(\mathbf{r},w_\beta)}\right)} m(T_\beta)}{\mu(E(G_\mathbf{r}))} + \sum_{\{\beta_1,\beta_2\}\subset\{w_1,\ldots,w_\alpha\}} \frac{\sum\limits_{\substack{T_{\beta_1}\subseteq E\left(G_{(\mathbf{r},w_{\beta_1})}\right),\, T_{\beta_1}\neq\emptyset,\\ T_{\beta_2}\subseteq E\left(G_{(\mathbf{r},w_{\beta_2})}\right),\, T_{\beta_2}\neq\emptyset}} m\left(\{T_{\beta_1}, T_{\beta_2}\}\right)}{\mu(E(G_\mathbf{r}))} +$$

$$+ \ldots + \sum_{\{\beta_1,\ldots,\beta_{n-1}\}\subset\{1,\ldots,n\}} \frac{\sum\limits_{\substack{T_{\beta_1}\subseteq E\left(G_{(\mathbf{r},w_{\beta_1})}\right),\, T_{\beta_1}\neq\emptyset,\\ \cdots \\ T_{\beta_{\alpha-1}}\subseteq E\left(G_{(\mathbf{r},w_{\beta_{n-1}})}\right),\, T_{\beta_{n-1}}\neq\emptyset}} m\left(\left\{T_{\beta_1},\ldots,T_{\beta_{n-1}}\right\}\right)}{\mu(E(G_\mathbf{r}))} + \tag{20}$$

$$+ \frac{\sum\limits_{\substack{T_1\subseteq E\left(G_{(\mathbf{r},w_1)}\right),\, T_1\neq\emptyset,\\ \cdots \\ T_n\subseteq E(G_{(\mathbf{r},w_n)}),\, T_n\neq\emptyset}} m\left(\{T_1\cup\ldots\cup T_n\}\right)}{\mu(E(G_\mathbf{r}))}$$

From equations (19) and (20), we get the thesis.

**Proof of Proposition 2.2**

Let $m$ the Möbius representation of a $q$-additive capacity $\mu$, $\left\{G_{(\mathbf{r},w_1)},\ldots,G_{(\mathbf{r},w_\alpha)}\right\} \subseteq \mathcal{G}^k_\mathbf{r}$ with $\alpha > q$ and $m^k_\mathbf{r}$ the Möbius representation of the capacity $\mu^k_\mathbf{r}$. By Proposition (2.1), we have that

$$m^k_\mathbf{r}\left(\left\{G_{(\mathbf{r},w_1)},\ldots,G_{(\mathbf{r},w_\alpha)}\right\}\right) = \frac{\sum\limits_{\substack{T_1\subseteq E\left(G_{(\mathbf{r},w_1)}\right),\, T_1\neq\emptyset,\\ \cdots \\ T_\alpha\subseteq E\left(G_{(\mathbf{r},w_\alpha)}\right),\, T_\alpha\neq\emptyset}} m\left(\{T_1,\ldots,T_\alpha\}\right)}{\mu(E(G_\mathbf{r}))}.$$

Observing that the set $\{T_1,\ldots,T_\alpha\}$ will contain at least $q+1$ elements (since $\alpha > q$) and that the capacity $\mu$ is $q$-additive, we get that $m^k_\mathbf{r}\left(\left\{G_{(\mathbf{r},w_1)},\ldots,G_{(\mathbf{r},w_\alpha)}\right\}\right) = 0$ for all $\left\{G_{(\mathbf{r},w_1)},\ldots,G_{(\mathbf{r},w_\alpha)}\right\} \subseteq \mathcal{G}^k_\mathbf{r}$ with $\alpha > q$.

**Proof of Proposition 2.3**

1. Given $G_{(\mathbf{r},w)} \in \mathcal{G}^k_\mathbf{r}$, by equations (15) and (2.1) and, considering that the capacity $\mu$ is 2-additive, we have that

$$\varphi^k_\mathbf{r}\left(\left\{G_{(\mathbf{r},w)}\right\}\right) = \sum_{\mathcal{F}\subseteq\mathcal{G}^k_\mathbf{r}:\, G_{(\mathbf{r},w)}\in\mathcal{F}} \frac{m^k_\mathbf{r}(\mathcal{F})}{|\mathcal{F}|} = m^k_\mathbf{r}\left(\left\{G_{(\mathbf{r},w)}\right\}\right) + \sum_{G_{(\mathbf{r},w_1)}\subseteq\mathcal{G}^k_\mathbf{r}\setminus\left\{G_{(\mathbf{r},w)}\right\}} \frac{m^k_\mathbf{r}\left(\left\{G_{(\mathbf{r},w)},G_{(\mathbf{r},w_1)}\right\}\right)}{2} =$$

$$= \sum_{T\subseteq E\left(G_{(\mathbf{r},w)}\right)} \frac{m(T)}{\mu(E(G_\mathbf{r}))} + \frac{1}{2} \sum_{G_{(\mathbf{r},w_1)}\subseteq\mathcal{G}^k_\mathbf{r}\setminus\left\{G_{(\mathbf{r},w)}\right\}} \frac{\sum\limits_{\substack{T_1\subseteq E\left(G_{(\mathbf{r},w)}\right),\, T_1\neq\emptyset,\\ T_2\subseteq E\left(G_{(\mathbf{r},w_1)}\right),\, T_2\neq\emptyset}} m(\{T_1,T_2\})}{\mu(E(G_\mathbf{r}))} =$$

$$= \frac{1}{\mu(E(G_{\mathbf{r}}))}\left[\sum_{\mathbf{t}\in E\left(G_{(\mathbf{r},w)}\right)} m(g_{\mathbf{t}}) + \sum_{\mathbf{t}_1,\mathbf{t}_2\in E\left(G_{(\mathbf{r},w)}\right)} m\left(\{g_{\mathbf{t}_1}, g_{\mathbf{t}_2}\}\right)\right] + \frac{1}{\mu(E(G_{\mathbf{r}}))}\sum_{\substack{\mathbf{t}_1\in E\left(G_{(\mathbf{r},w)}\right),\\ \mathbf{t}_2\in E\left(\mathcal{G}^k_{\mathbf{r}}\setminus G_{(\mathbf{r},w)}\right)}} \frac{m\left(\{g_{\mathbf{t}_1}, g_{\mathbf{t}_2}\}\right)}{2} =$$

$$= \left[\sum_{\mathbf{t}\in E\left(G_{(\mathbf{r},w)}\right)} m(g_{\mathbf{t}}) + \sum_{\mathbf{t}_1,\mathbf{t}_2\in E\left(G_{(\mathbf{r},w)}\right)} m\left(\{g_{\mathbf{t}_1}, g_{\mathbf{t}_2}\}\right) + \sum_{\substack{\mathbf{t}_1\in E\left(G_{(\mathbf{r},w)}\right),\\ \mathbf{t}_2\in E\left(\mathcal{G}^k_{\mathbf{r}}\setminus G_{(\mathbf{r},w)}\right)}} \frac{m\left(\{g_{\mathbf{t}_1}, g_{\mathbf{t}_2}\}\right)}{2}\right]\frac{1}{\mu\left(E(G_{\mathbf{r}})\right)}.$$

2. Given $G_{(\mathbf{r},w_1)}, G_{(\mathbf{r},w_2)} \in \mathcal{G}^k_{\mathbf{r}}$, by equation (16) and Proposition 2.1 and, considering that the capacity $\mu$ is 2-additive, we have that

$$\varphi^k_{\mathbf{r}}\left(\left\{G_{(\mathbf{r},w_1)}, G_{(\mathbf{r},w_2)}\right\}\right) = \sum_{\mathcal{F}\subseteq\mathcal{G}^k_{\mathbf{r}}:\, G_{(\mathbf{r},w_1)},G_{(\mathbf{r},w_2)}\in\mathcal{F}} \frac{m^k_{\mathbf{r}}(\mathcal{F})}{|\mathcal{F}|-1} = m^k_{\mathbf{r}}\left(\left\{G_{(\mathbf{r},w_1)}, G_{(\mathbf{r},w_2)}\right\}\right) =$$

$$= \sum_{\substack{T_1\subseteq E\left(G_{(\mathbf{r},w_1)}\right),\, T_1\neq\emptyset,\\ T_2\subseteq E\left(G_{(\mathbf{r},w_2)}\right),\, T_2\neq\emptyset}} \frac{m\left(\{T_1,T_2\}\right)}{\mu(E(G_{\mathbf{r}}))} = \left[\sum_{\substack{\mathbf{t}_1\in E(G_{(\mathbf{r},w_1)}),\\ \mathbf{t}_2\in E(G_{(\mathbf{r},w_2)})}} m\left(\{g_{\mathbf{t}_1}, g_{\mathbf{t}_2}\}\right)\right]\frac{1}{\mu(E(G_{\mathbf{r}}))}$$

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