Inflation in a scale-invariant universe

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A scale-invariant universe can have a period of accelerated expansion at early times: inflation. We use a frame-invariant approach to calculate inflationary observables in a scale-invariant theory of gravity involving two scalar fields: the spectral indices, the tensor-to-scalar ratio, the level of isocurvature modes, and non-Gaussianity. We show that scale symmetry leads to an exact cancellation of isocurvature modes and that, in the scale symmetry–broken phase, this theory is well described by a single scalar-field theory. We find the predictions of this theory strongly compatible with current observations.

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I. INTRODUCTION

The evolution of our Universe is simple to describe yet difficult to explain. Current observations allow a picture in which there were two periods of accelerated expansion, one at very early times (dubbed inflation) and another at late times (i.e., today), separated by periods of radiation and matter domination. The hierarchy between the energy scales of the two regimes of accelerated expansion is extreme and difficult to understand in terms of our current knowledge of the interplay between particles, fields, and gravity. Given this state of affairs, it is essential to find a consistent and simple explanation.

If one is to embrace inflation as an essential feature of the early Universe (although one should, of course, countenance alternatives), it makes sense to explore alternative ideas that may explain the hierarchy of scales one encounters. A tried and tested approach is to invoke new symmetries that can naturally lead to such a hierarchy. In this paper, we will explore one such symmetry—scale (or Weyl) invariance—that has been shown to lead to the type of behavior we are seeking to understand [1–20].

It has been shown that two scalar fields with a scale-invariant potential can be nonminimally coupled to gravity in such a way as to lead to a completely scale-invariant theory of the Universe. While there are no dimensionful coupling constants, scale symmetry is spontaneously broken and can generate a Planck mass, an effective cosmological constant, and particle masses. While in the symmetry-broken phase dimensionful quantities emerge, the only meaningful, measurable quantities are ratios of dimensionful quantities that are completely set by the dimensionless parameters of the underlying theory. A judicious choice of these parameters allows us to obtain two periods of accelerated expansion that are consistent with current observations.

In this paper, we will scrutinize the inflationary regime of the scale-invariant universe. Given that such a universe involves two scalar fields, one should expect a richer, more complex, phenomenology than a usual single-field model. In particular, one should inspect the possible presence of isocurvature modes [21] as well as non-negligible non-Gaussianity [22]. The conventional approach for studying such models is to transform them from the Jordan frame into the Einstein frame to work out the properties of the scalar-field evolution. In this paper, we will explore this phenomenology, using the frame-invariant approach of Ref. [23]. We will find that the mechanism of scale-symmetry breaking greatly simplifies the calculations and that the final answer can be understood in terms of an effective single-field model.

Our analysis extends previous related work in a number of ways:

(i) Using the analytic solutions for the scalar-field evolution found in Ref. [1], we analyze various primordial observables in detail, finding good agreement with previous results, e.g., from Refs. [1–3,23], where overlap exists. We prove that, at next-to-leading order in slow roll, isocurvature modes decouple completely in our scale-invariant setup, also away from the attractor solution (thus extending the related attractor solution result of Ref. [3]).

(ii) We explicitly derive the corresponding effective single-field theory and show it leads to the same predictions.

(iii) We extend previous results by computing predictions for the running of the tensorial spectral index and the non-Gaussian $f_{\rm NL}$ parameter(s).

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(iv) We work out predictions of the model in the recently developed frame-covariant setup of Ref. [23]. We use this, e.g., to investigate what the precise nature of the link between decoupling isocurvature mode(s) and scale invariance is and show that this is a consequence of working in a two-field scale-invariant model.

This paper is structured as follows. In Sec. II, we present the essential characteristics of the scale-invariant Universe with a particular emphasis on the inflationary regime; we recapitulate the analytic solutions of the field evolution, first found in Ref. [1]. In Sec. III, we summarize the frame-invariant approach of Ref. [23]. In Sec. IV, we explore the two-field dynamics and the isocurvature sector to assess how close this theory is to single-field dynamics. In Sec. V, we calculate the observables—the various spectral indices, the amplitude of tensor modes, and non-Gaussianity—and show that we can also derive these results from an effective single scalar-field theory. In Sec. VI, we discuss our findings.

II. MODEL

In this paper, we will work with a model with two scalar fields, $\phi^A \equiv (\phi_1, \phi_2)$,[1] coupled to gravity. In the Jordan frame (in which we will present the results of this section), the action is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M^2(\vec{\phi})R - \frac{1}{2}\sum_{A=1}^{2}\nabla_\mu\phi^A\nabla^\mu\phi^A - W(\vec{\phi})\right], \tag{1}$$

where $M^2 = -\frac{1}{6}\sum_{A=1}^{2}\alpha_A(\phi^A)^2$ and $W(\vec{\phi}) = \sum_{A,B=1}^{2}\lambda_{AB}(\phi^A)^2(\phi^B)^2$ and Einstein summation convention is *not* assumed. This theory has no input mass scales and is conformally invariant if $\alpha_A = 1$.

The equations of motion are given by

$$\sum_{B=1}^{2}\left[I_{AB} + \frac{\alpha_A\phi^A\alpha_B\phi^B}{6M^2}\right]\Box\phi_B = \mathcal{X}_A, \tag{2}$$

where

$$\mathcal{X}_A = \frac{\alpha_A\phi^A}{6M^2}\sum_{B=1}^{2}(\alpha_B - 1)(\dot{\phi}^B)^2 + \frac{4\alpha_A\phi^A}{6M^2}W + W_{,A} \tag{3}$$

and $A_{,X} = \partial A/\partial\phi^X$.

This system has a conserved Noether current, $\nabla_\mu K^\mu = 0$, where $K^\mu = \nabla^\mu K$ and

$$K = \frac{1}{2}\sum_{A=1}^{2}(1 - \alpha_A)(\phi^A)^2. \tag{4}$$

If we take ϕ^A to be functions of t only and consider a homogeneous and isotropic metric of the form $g_{\alpha\beta} = (-1, a^2\delta_{ij})$, we have that

$$\ddot{K} + 3\left(\frac{\dot{a}}{a}\right)\dot{K} = 0 \tag{5}$$

so that

$$K = c_1 + c_2\int\frac{dt}{a^3(t)}. \tag{6}$$

We see here one of the fundamental characteristics of this theory: scale invariance is spontaneously broken as K settles down to a constant value, corresponding to an ellipses in the $\vec{\phi}$ plane. The value of K is not set by the potential but by the initial value of $\vec{\phi}$, which makes this mechanism significantly different from the more conventional forms of spontaneous symmetry breaking—we have dubbed this particular mechanism "inertial symmetry breaking" [24]. Although $\vec{\phi}$ can still vary along the ellipse, it is confined to that trajectory, which is not invariant under scale transformations.

At late times, there is a fixed point on the ellipse, when $\dot{\phi}^A = 0$ and

$$\frac{4\alpha_A\phi^A}{6M^2}W + W_{,A} = 0. \tag{7}$$

An explicit solution is

$$\left(\frac{\phi_2}{\phi_1}\right)^2 = \frac{\lambda_{11}\alpha_2 - \lambda_{12}\alpha_1}{\lambda_{22}\alpha_1 - \lambda_{21}\alpha_2}. \tag{8}$$

We can see that the final, fixed-point, end state is set by the ratio of the coupling constants; any dimensionful constants, such as the effective Planck mass, M^2, will depend on an arbitrary (or accidental) scale arising from the spontaneous breaking of scale symmetry.

A remarkable feature of this model is that the degree of freedom (d.o.f.) orthogonal to the constraint surface given by Eq. (4)—the dilaton—completely decouples from the other d.o.f. [25]. To slightly belabor this point, given that the dilaton is the Goldstone boson of the broken symmetry, one might expect it to be derivatively coupled. In fact, it can be shown that the scale invariance of the theory ensures that the dilaton—the putative mediator of a fifth force—decouples from the matter sector, has only a kinetic term, and is thus unconstrained by laboratory or astrophysical effects [25]. We will see that this fact will play a role when

[1] Capital latin letters are therefore field-space indices that run from 1 to 2 (e.g., $A = 1, 2$). They are raised and lowered with a field-space metric, which we will introduce in the following section.

we study the evolution of perturbations in the inflationary regime.

Our focus, in this paper, will not be on the end state but on a putative period of slow roll on the ellipse, before $\vec{\phi}$ settles down on the final fixed point. The equations of motion in this slow-roll regime are given by

$$\sum_{B=1}^{2}\left[I_{AB}+\frac{\alpha_A\phi^A\alpha_B\phi^B}{6M^2}\right][-3H\phi^B]=\frac{4\alpha_A\phi^A}{6M^2}W+W_{,A}. \quad (9)$$

If we assume that $W\simeq\lambda_{22}(\phi_2)^4$, we have

$$\begin{pmatrix}\frac{4\alpha_1\phi_1}{6M^2}W+W_{,1}\\ \frac{4\alpha_2\phi_2}{6M^2}W+W_{,2}\end{pmatrix}=\frac{4\lambda_{22}\alpha_1\phi_1\phi_2^4}{6M^2}\begin{pmatrix}1\\ -\frac{\phi_1}{\phi_2}\end{pmatrix}. \quad (10)$$

In this regime, we can solve the equations of motion exactly [1]. Defining $M_A^2=-\frac{\alpha_A}{6}(\phi^A)^2$, we have

$$M_1^2=M_E^2e^{-\nu N_J}$$
$$M_2^2=M_E^2[1+\gamma(1-e^{-\nu N_J})], \quad (11)$$

where $\nu=-\frac{4}{3}\alpha_1$, $\gamma=\frac{\alpha_2(1-\alpha_1)}{\alpha_1(1-\alpha_2)}$, and N_J is the number of e-foldings until the end of inflation in Jordan frame.[2] We have shown that these analytical solutions are an exquisite approximation to the full equations of motion in the slow-roll regime. We will work with this solution in all that follows in this paper (although we will at some point compare with numerical solutions).

We can obtain the dynamics of the Einstein-frame scale factor a_E and the corresponding Hubble rate H_E through a conformal transformation of the form

$$a_E=M(\phi_1,\phi_2)a$$
$$H_E=H+\frac{\dot{M}}{M}. \quad (12)$$

Thus, we can reconstruct the Einstein-frame quantities. The final piece in the dictionary is the transformation between Einstein- and Jordan-frame e-foldings, which is given by

$$N_E=N_J+\ln\left(\frac{M_f}{M_i}\right),$$
$$=N_J+\frac{1}{2}\ln\left(\frac{2\alpha_1(1-\alpha_2)}{\alpha_1+\alpha_2-2\alpha_1\alpha_2+(\alpha_1-\alpha_2)e^{-\nu N_J}}\right), \quad (13)$$

where M_f and M_i are the final and initial values of M (at the end and start of inflation), respectively. One can implicitly solve to find $N_J(N_E)$ and in this way consistently map the solution (11) into the Einstein frame. Note that one can always uniquely relate the scalar-field values at any given time to the corresponding N_E and N_J.

Scale-invariant theories are particularly interesting because they provide a possible explanation for the hierarchical difference between the Planck scale and the electroweak scale, the scale invariance requiring vanishing masses until spontaneously broken. As originally constructed [3–5], ϕ_2 was taken to model the Higgs with an hierarchy of vacuum expectation values $\frac{\phi_2}{\phi_1}\ll 1$. Thus, ϕ_1 is dominantly responsible for setting the Planck mass, and ϕ_2 sets the electroweak scale with the "Higgs" self-coupling $\lambda_{22}=O(1)$. In this limit, one gets "Higgs inflation" with $|\alpha_2|\gg 1$ needed to have an acceptable scale of inflation.[3]

The other scalar couplings λ_{11} and λ_{12} must be hierarchically small to allow for a small cosmological constant and to keep the Higgs mass at the electroweak scale. In the absence of gravity, this ordering of couplings is natural due to the underlying shift symmetry of the Weyl-invariant scalar potential. This shift symmetry is broken by the Higgs coupling to the Ricci scalar, and to determine whether the hierarchy survives requires a calculation of gravitational radiative corrections—an issue in need of further elaboration.

It is possible to generalize the scale-invariant model to one with many scalar fields. The dynamics will be qualitatively similar: inertial symmetry breaking will occur, but now the symmetry-broken phase will lie on a (hyper)ellipsoid, and there will be richer dynamics to deal with. In Appendix B, we briefly touch on one such case to discuss a particular aspect related to the perturbations.

III. FRAME-INVARIANT SLOW-ROLL PARAMETERS

There is substantial literature on multifield, inflationary perturbations in the slow-roll regime [21,26–29]. When the dynamics involves more than one field, the trajectory in field space will play a crucial role in how perturbations evolve and, in particular, whether the curvature perturbation is preserved on superhorizon scales or whether it varies, sourcing isocurvature perturbations. As shown in Ref. [21], the curvature of the field trajectories plays a crucial role in the quantifying how isocurvature perturbations are sourced.

Over the past couple of decades, a more geometric approach—in which the geometry of field space, through the field-space metric that enters the definition of the kinetic term of the scalar-field action, can be used to determine the evolution of perturbation in the case of multifield inflation—has emerged. In the case of nonminimal coupling, the favored approach is to conformally transform to the Einstein frame and apply the standard slow-roll formalism. A battery of readily available algorithms, which numerically solve the

[2] In other words, we have implicitly defined $N_J=0$ at the end of inflation.

[3] If ϕ_2 does not model the Higgs, it is possible for λ_{22} to be small, and in this case, α_2 need not be large [1].

transport equations and can be solved in the case of generic potentials and actions that lead to scalar-field evolution that is sufficiently close to the slow-roll regime, has been made available by a number of authors. In this paper, we will follow a slightly different approach proposed in Ref. [23] (and foreshadowed by Ref. [30]) and consider a *frame-invariant* formalism for calculating the inflationary observables (we have checked that we obtain the same results if we use the standard, Einstein-frame, approach and illustrate that in subsequent sections).

The fundamental quantities that one needs to consider are the frame-invariant metric[4]

$$G_{AB} = \frac{\delta_{AB}}{M^2} + \frac{3}{2}\frac{M^2_{,A}M^2_{,B}}{M^4} \tag{14}$$

and potential

$$U = \frac{W}{M^4}. \tag{15}$$

Note that the frame-invariant metric is simply the field-space metric one obtains when transforming to the Einstein frame.

One can construct a covariant vector on field space, X_A, given by

$$X_A = (\ln U)_{,A}, \tag{16}$$

which is the frame field of the curvature perturbation or the tangent to the geodesics *in field space* traced out by the scalar-field evolution. We can then construct the corresponding contravariant vector by raising indices with G^{AB}. Furthermore, we can use G_{AB} to construct the connection coefficients, $\Gamma^A{}_{BC}$, which will go into the definition of a bona fide covariant derivative; for example, we have that

$$\nabla_A X^B = X^B_{,\phi_A} + \Gamma^B{}_{AC}X^C. \tag{17}$$

We can then define the frame-invariant potential slow-roll parameters [23]

$$\bar{\epsilon}_U = \frac{1}{2}X^A X_A \tag{18}$$

associated to the norm of the flow vector in field space and its directed derivatives along the flow:

$$\bar{\eta}_U = -X^A(\ln \bar{\epsilon}_U)_{,\phi_A}$$
$$\bar{\xi}_U = -X^A(\ln \bar{\eta}_U)_{,\phi_A}. \tag{19}$$

A defining feature of these slow-roll parameters as defined above is that they reduce to the standard Hubble slow-roll parameters in the slow-roll approximation.

In this regime, it is also important to define a set of parameters that are crucial for evaluating the strength of the isocurvature perturbations. A leading parameter is the acceleration vector between paths in the geodesic flow

$$\omega^A = X^B \nabla_B \left[\frac{X^A}{\sqrt{2\bar{\epsilon}_U}}\right] \tag{20}$$

given here up to second order in the slow-roll parameters and that should be complemented by two additional parameters,

$$\bar{\eta}_{ss} = \frac{\omega^A \omega^B}{\omega^2}[\nabla_A X_B + X_A X_B] + \frac{2}{3}\bar{\epsilon}_U R^A{}_A$$
$$\bar{\eta}_{\sigma\sigma} = X^A X^B[\nabla_A X_B + X_A X_B], \tag{21}$$

where we have used the Ricci tensor R_{AB} of our curved field space. From these parameters (and especially from ω^A), we can reconstruct how curved the trajectories are in field space and, in particular, what the transfer function that converts curvature perturbations into isocurvature perturbations is.

To do so, we finally also need to promote the implicit definition of the number of e-foldings in (11) to a frame-covariant one. Making use of the frame-covariant time derivative $\mathcal{D}_t T \equiv \frac{d\phi^C}{dt}\nabla_C T$ (for any tensor T—see Ref. [23] for details), we can use Eq. (12) to get

$$\mathcal{H} \equiv \mathcal{D}_t a/a = H_E, \tag{22}$$

where t is the physical time and H_E satisfies (12). Analogously, we can then define a frame-covariant e-folding number, $dN = -\mathcal{H}dt$. Solving this equation, we have that the frame-covariant e-folding number N is given by

$$N = N_E. \tag{23}$$

From Eq. (13), this gives us N as a function of N_J or, inverting the relation, lets us express N_J as a function of N and as such yields an explicitly frame-covariant version of (11).

IV. ISOCURVATURE MODES AND THE ATTRACTOR

Multifield models generically produce entropy transfer between modes, leading to isocurvature effects on top of the standard adiabatic evolution [21]. This is particularly important on superhorizon scales, in which the comoving curvature perturbation is conserved during adiabatic evolution but evolves in the presence of isocurvature

[4]Here, we have assumed canonical kinetic interactions for the scalar fields of the form $-\frac{1}{2}\delta_{AB}\nabla_\mu\phi^A\nabla^\mu\phi^B$. If the kinetic structure is nontrivial in field space, i.e., we have kinetic interactions of the form $-\frac{1}{2}k_{AB}(\vec{\phi})\nabla_\mu\phi^A\nabla^\mu\phi^B$, then the frame-invariant metric becomes $G_{AB} = \frac{k_{AB}}{M^2} + \frac{3}{2}\frac{M^2_{,A}M^2_{,B}}{M^4}$.

perturbations [31]. Before computing observables, it is therefore important to investigate whether isocurvature modes are present and impact the evolution of modes.

Isocurvature effects in two-field models of the type considered here can be parametrized by and encoded via the transfer functions $T_{\mathcal{RS}}$ and $T_{\mathcal{SS}}$, which are defined via

$$\begin{pmatrix} \mathcal{R} \\ \mathcal{S} \end{pmatrix} = \begin{pmatrix} 1 & T_{\mathcal{RS}} \\ 0 & T_{\mathcal{SS}} \end{pmatrix} \begin{pmatrix} \mathcal{R}_* \\ \mathcal{S}_* \end{pmatrix}, \tag{24}$$

where $\mathcal{R}$ and $\mathcal{S}$ are the curvature and entropy perturbations and $*$ denotes the horizon exit at N_*. In multifield models with ≥ 3 fields, additional isocurvature modes are present, and the above transfer functions get complemented by additional ones linking all neighboring modes (i.e., each $\mathcal{S}_{(n)}$ and $\mathcal{S}_{(n+1)}$)—see Ref. [23] for details. Going back to the two-field context, a derived transfer angle Θ is defined by

$$\cos\Theta = \frac{1}{\sqrt{1+T_{\mathcal{RS}}^2}}. \tag{25}$$

In integral form, the transfer functions can then be written as

$$T_{\mathcal{RS}}(N_*, N) = -\int_{N_*}^{N} dN' A(N') T_{\mathcal{SS}}(N_*, N'),$$
$$T_{\mathcal{SS}}(N_*, N) = \exp\left[-\int_{N_*}^{N} dN' B(N')\right]. \tag{26}$$

Note that the e-folding number used here is the frame-covariant one and $N_\star$ is defined to be positive. For two-field models, A and B satisfy [32]

$$A = 2\omega, \qquad B = -2\bar{\varepsilon}_U - \bar{\eta}_{ss} + \bar{\eta}_{\sigma\sigma} - \frac{4}{3}\omega^2, \tag{27}$$

where we have defined $\omega^2 = |\omega_A \omega^A|$, with indices raised and lowered with G_{AB}. In evaluating the isocurvature effects, let us first note that ε_U and $\bar{\eta}_{\sigma\sigma}$ and $R^A{}_A$ are well defined, finite, and generically nonzero expressions for our model. We have derived explicit expressions for these quantities but will not require these here. The important quantity is ω.

To proceed, we should first note that scale invariance imposes a set of consistency conditions on the quantities at play in the expressions of Sec. III. For example, we have that

$$\ln U(\lambda\vec{\phi}) = \ln U(\vec{\phi}) \tag{28}$$

for arbitrary λ, which in turn leads to the constraint

$$\left.\frac{d\ln U(\lambda\vec{\phi})}{d\lambda}\right|_{\lambda=1} = \phi^A X_A = 0. \tag{29}$$

This immediately allows us to explicitly write out X_A as

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \frac{X_1}{\phi_2} \begin{pmatrix} \phi_2 \\ -\phi_1 \end{pmatrix}. \tag{30}$$

When indices are raised with G^{AB}, we get

$$\begin{pmatrix} X^1 \\ X^2 \end{pmatrix} \propto \begin{pmatrix} (\alpha_2 - 1)\phi_2 \\ -(\alpha_1 - 1)\phi_1 \end{pmatrix}. \tag{31}$$

Interestingly, this is the orthogonal, contravariant vector to $\partial_A K$, i.e., to the ellipse from Eq. (4). We can define a unit vector $\hat{X}^A = X^2/\sqrt{2\bar{\varepsilon}_U}$ and rescale ω^A such that

$$\hat{\omega}^A \equiv \frac{\omega^A}{\sqrt{2\bar{\varepsilon}_U}} = \hat{X}^B \nabla_B \hat{X}^A. \tag{32}$$

There are a few properties to note about this expression. First of all, because of the structure in X^A arising from scale invariance, there is no λ_{AB} dependence in $\hat{X}_A$. Furthermore, we generally (and independently of scale invariance) have that $\hat{X}_A \hat{\omega}^A = 0$, which means that $\hat{\omega}^A \propto \phi^A$ for a scale-invariant setup like ours. Putting everything together, one can evaluate the proportionality constant and in fact explicitly show that

$$\hat{\omega}^A = 0, \tag{33}$$

which means that $\hat{X}^A$ is a geodesic flow associated with the metric G_{AB}. This result can be seen as an extension of the result of Ref. [3], in which an analogous turn rate was shown to vanish for a subset of (1) on the attractor solution. Specifically, there, it was shown that (on the attractor solution) the turn rate vanishes for a potential $W = \frac{\lambda}{4}(\phi_2^2 - \frac{\alpha}{\lambda}\phi_1^2)^2$, which [in the context of our (1)] is equivalent to assuming a specific choice of λ_{11}. We therefore emphasize that (33) here holds for arbitrary λ_{ij} and without assuming any specific solution (attractor or otherwise)—it follows directly from (1) and (20). The result (33) can also be neatly interpreted in terms of the equation of motion for ϕ^A, which can be written as

$$\mathcal{D}_t\mathcal{D}_t\phi^A + 3\mathcal{H}(\mathcal{D}_t\phi^A) + fU^{,\phi_A} = 0, \tag{34}$$

where we recall the definition $\mathcal{D}_t T \equiv \frac{d\phi^C}{dt}\nabla_C T$ for any tensor T. Since $\hat{X}^A \propto U^{,A}$, the statement that $\hat{X}^A$ is a geodesic flow associated with the metric G_{AB} becomes equivalent to the observation that the drag-term $-3\mathcal{H}\mathcal{D}_t\phi^A$ is aligned with the force term $fU^{,A}$. We reiterate that our starting expression for ω^A (20) was accurate up to second order in slow roll, so the same is true for the above derivation. This is crucial, since both $\hat{\omega}^A = 0$ and the alignment of drag term and force term in the equations of motion are trivially true at first order in slow-roll parameters but highly nontrivial at higher orders.

While we can understand the cancellation of $\hat{\omega}^A = 0$ in a geometric way via the above reasoning, one may wonder whether this cancellation can also be related to another underlying feature. In fact, we have checked that one can add further dimensionless coefficients to the model, e.g., via $\lambda_{1112}\phi_1^3\phi_2$ and/or $\lambda_{1222}\phi_1\phi_2^3$ terms in the potential, and the conclusion remains unchanged. This strongly suggests that dimensionless coefficients (by themselves) never contribute to ω^A, i.e., that $\omega^A = 0$ is intimately tied to the scale-free nature of our model (at least for a two-dimensional field space—see the discussion below). Note that this changes as soon as any dimensionful coefficient is added. We have explicitly checked, that as soon as, e.g., a constant (and of course dimensionful) Planck mass $M_{\rm Pl}$ is added to the terms multiplying the Ricci scalar or a quadratic mass term is added (controlled by a new parameter m^2) or a sixth-order interaction such as ϕ_i^6/m^2 is added ω^A picks up nonzero contributions. Crucially, all dimensionless parameters of the model then also enter the expression and affect ω^A's value, but in order to have a nonzero ω^A in the first place, the presence of at least one such dimensionful parameter is required.

What does this mean for the total isocurvature contributions for our model? From (26) and (27), we obtain that $T_{\mathcal{RS}} = 0$ and $\Theta = 0$ as a direct consequence of $\omega = 0$. In other words, no isocurvature effects affect observables related to the curvature mode (within the approximations we have used throughout, i.e., up to second order in slow roll). Second, note that B is finite and generically nonzero,[5] meaning that an initially present isocurvature mode can still undergo a nontrivial evolution due to $T_{\mathcal{SS}}$. However, this is of course decoupled from the curvature mode, given that $T_{\mathcal{RS}} = 0$, and so the isocurvature mode can never be sourced by the curvature mode.

While the focus of this paper is the scale-invariant model with two scalar fields, one has to consider the fact that this is a special case; the constraint is a one-dimensional curve—the ellipse (4)—on which the inflationary trajectory lies. Fluctuations *along* the ellipse correspond to adiabatic perturbations, and fluctuations *orthogonal* to the ellipse correspond to isocurvature fluctuations. That orthogonal d.o.f. corresponds to the dilaton, which, as we have shown in Ref. [25], completely decouples. This means that we do not expect that particular isocurvature mode to be seeded or to interact with the adiabatic mode. Given that it is the only isocurvature mode in this theory, we recover what we found.

To confirm our intuition, we can generalize our analysis to the case of multiscalar fields, in which the situation is more complex. There, the constraint surface is a hyper-ellipsoid in which the inflationary trajectory is embedded. Again, there will be an isocurvature mode associated to the dilaton, i.e., orthogonal to the surface, but now there will also be isocurvature modes lying on the constraint surface. These will not decouple from the adiabatic mode and can be seeded during inflation. The hallmark for this is that ω^A will not be zero in this case. For an example, we have considered the case of three scalar fields with a setup that is essentially equivalent to our model: $\alpha_1 < \alpha_2$, α_3 and the potential (which now consists of all quartic combinations of ϕ_1^2, ϕ_2^2, and ϕ_3^2) is dominated by $\lambda_{22}\phi_2^4$. In Appendix B, we discuss this case in more detail, explicitly showing that ω^A and its norm are nonzero, which means isocurvature perturbations are clearly present in the case with more than two fields.

V. OBSERVABLES AND SINGLE-FIELD DYNAMICS

We are now ready to compute the observable predictions of our model. Let us quickly summarize the dynamical regime we are exploring. We are assuming that $W \simeq \lambda_{22}\phi_2^4$ during the inflationary regime. In Ref. [1], we showed that this was a well-defined slow-roll regime that allowed us to find the analytical solutions of Sec. II. Furthermore, we have that $|\alpha_1| \ll 1$, while α_2 is unconstrained.

If we now turn to two-point functions of scalar and tensor perturbations, we are interested in the spectral index of scalar perturbations n_S, its running α_S, the spectral index of tensor perturbations n_T, its running α_T, and finally the tensor-to-scalar ratio r. Their frame-invariant definitions are [23]

$$\begin{aligned} n_S &= 1 - 2\bar{\epsilon}_U - \bar{\eta}_U - \mathcal{D}_N(1 + T_{RS}^2) \\ \alpha_S &= -2\bar{\epsilon}_U\bar{\eta}_U - \bar{\eta}_U\bar{\xi}_U + \mathcal{D}_N\mathcal{D}_N(1 + T_{RS}^2) \\ n_T &= -2\bar{\epsilon}_U \\ \alpha_T &= -2\bar{\epsilon}_U\bar{\eta}_U \\ r &= 16\bar{\epsilon}_U\cos^2\Theta, \end{aligned} \tag{35}$$

where $\mathcal{D}_N$ is the frame-covariant derivative with respect to N, but since we have already seen that the transfer function T_{RS} vanishes in our setup, all terms involving $\mathcal{D}_N$ drop out trivially, and $\cos^2\Theta = 1$. Note that we therefore trivially obtain the consistency relation $r = -8n_T$.

Making use of (11), we accordingly obtain exact expressions for all these observables. Expanding up to leading order in α_1 for each parameter, we find[6]

[5] A calculation analogous to that above shows that $\bar{\eta}_{ss}$ is a finite, nondivergent quantity (the vanishing ω^2 in the denominator is compensated for by factors in the numerator).

[6] Note that in this small α_1 expansion we have not expanded the exponential $e^{-\nu N_J}$, since it can be order 1 even if $|\alpha_1| \ll 1$.

$$\begin{aligned} n_S &= 1 + \frac{4\alpha_1(e^{-\nu N_J}+1)}{3(1-e^{-\nu N_J})} + \mathcal{O}(\alpha_1^2), \\ r &= \frac{64\alpha_1^2(\alpha_2-1)e^{-\nu N_J}}{3\alpha_2(e^{-\nu N_J}-1)^2} + \mathcal{O}(\alpha_1^3), \\ \alpha_S &= -\frac{32\alpha_1^2 e^{-\nu N_J}}{9(e^{-\nu N_J}-1)^2} + \mathcal{O}(\alpha_1^3), \\ n_T &= -\frac{8\alpha_1^2(\alpha_2-1)e^{-\nu N_J}}{3\alpha_2(e^{-\nu N_J}-1)^2} + \mathcal{O}(\alpha_1^3), \\ \alpha_T &= -\frac{32\alpha_1^3(\alpha_2-1)e^{-\nu N_J}(1+e^{-\nu N_J})}{9\alpha_2(e^{-\nu N_J}-1)^3} + \mathcal{O}(\alpha_1^3). \end{aligned} \quad (36)$$

We have checked that, with the fiducial parameter values of Ref. [2], these expressions are accurate at roughly percent level (when compared with the full expressions). Note that N_J here (in the spirit of frame covariance) should be seen as a function of N. This can be obtained by inverting (23), which at leading order in α_1 becomes

$$N = N_J + \frac{1}{2}\ln\left(\frac{2\alpha_1(\alpha_2-1)}{\alpha_2(e^{-\nu N_J}-1)}\right) + \mathcal{O}(\alpha_1). \quad (37)$$

Also, here and in what follows, we are focusing on the modes relevant for observables today, by picking a fiducial $N_J \sim 60$. Using (37), one can show this corresponds to $N \sim 58.5$.[7]

These results extend, and are also completely consistent with, those found in Ref. [1], in which calculations were done using the $H(N)$ formalism, in the Einstein frame (see also Ref. [3]). It is instructive to pursue this further. As we saw in Sec. IV, isocurvature perturbations are zero up to, at least, second order, which means that the there are no perturbations orthogonal to the field trajectory. One might have guessed that would be the case, given that the field is evolving along the scale symmetry–broken locus of field space, i.e., the ellipse of Eq. (4), but this does not immediately follow; the trajectory along the ellipse has curvature that one might naively associate with normal forces and thus isocurvature perturbations. Given that this is not the case (due to the way in which the ϕ^A map onto curvature and isocurvature modes) and $\omega^A = 0$, we can simplify the analysis considerably by reducing the theory to a single-field model.

Substituting the solutions for ϕ^A (11) into the ellipse equation (4) and (without loss of generality) setting M_E to unity in what follows, we find that

$$K = 6 - \frac{3}{\alpha_1} - \frac{3}{\alpha_2}. \quad (38)$$

Solving for the ellipse, we can therefore express the whole theory in terms of a single d.o.f., which we choose to be $\phi \equiv \phi_2$,

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\hat{M}^2(\phi)R - \frac{\hat{k}(\phi)}{2}\nabla_\mu\phi\nabla^\mu\phi - \hat{W}(\phi)\right], \quad (39)$$

where we see explicitly that the single-field formulation comes at the expense of introducing a noncanonical kinetic term. The model functions are given by

$$\begin{aligned} \hat{M}^2 &= \frac{\alpha_2\phi^2 + \alpha_1(2K-\phi^2)}{6(\alpha_1-1)}, \\ \hat{k} &= -\frac{2K(1-\alpha_1) + (\alpha_2-1)(\alpha_2-\alpha_1)\phi^2}{(\alpha_1-1)(2K+(\alpha_2-1)\phi^2)} \\ \hat{U} &= \frac{\hat{W}}{\hat{M}^4} = \frac{36(\alpha_1-1)^2\lambda_{22}\phi^4}{(\alpha_2\phi^2+\alpha_1(2K-\phi^2))^2}. \end{aligned} \quad (40)$$

Recalling the definition of the frame-invariant metric in the presence of nontrivial kinetic terms for the scalar(s) (14) and noting that the field-space metric is a simple scalar function in the case of a one-dimensional field space as we are considering here, we have

$$\hat{G} = \frac{12(\alpha_1-1)K(2\alpha_1K + (\alpha_1-\alpha_2)(\alpha_2-1)\phi^2)}{(2K+(\alpha_2-1)\phi^2)(2\alpha_1K+(\alpha_2-\alpha_1)\phi^2)^2}. \quad (41)$$

Expressed in this way, we have $\hat{G} = G_{AB}$ and consequently $G^{AB} = \hat{G}^{-1}$ and can express the first two slow-roll parameters as

$$\hat{\varepsilon}_{\hat{U}} = \frac{\hat{U}_{,\phi}^2}{2\hat{G}\hat{U}^2}, \qquad \hat{\eta}_{\hat{U}} = -\frac{\hat{\varepsilon}_{\hat{U},\phi}\hat{U}_{,\phi}}{\bar{\varepsilon}_{\hat{U}}G\hat{U}}. \quad (42)$$

Evaluating this and expanding in α_1, we obtain precisely the same expressions for n_S and r as in (36). In fact, we have explicitly checked that the two approaches yield identical predictions up to eighth order in α_1.

We can now focus on the actual values of the observables. The main observables, i.e., the ones for which we have the tightest constraints, are n_S and r. In Fig. 1 we can see that, for sufficiently small values of α_1, $n_S \simeq 0.96$; i.e., it lies comfortably within the observational constraints from the Planck data [33]. In fact, given that n_S is solely dependent on α_1, we can immediately convert current constraints on n_S (e.g., $n_S = 0.9652 \pm 0.0047$) into constraints on α_1:

$$|\alpha_1| < 0.019. \quad (43)$$

Note that there is an upper bound on n_S for $\alpha_1 \to 0$ such that $n_S < 1$–$2/N \simeq 0.97$.

In Fig. 2, we can see that we naturally obtain a small value of r, well within current constraints. A conservative expression comes from taking $\alpha_1 \to 0$:

[7]Incidentally, this is precisely in the parameter range explored by Ref. [3], which corresponds to $57 \lesssim N \lesssim 59$.

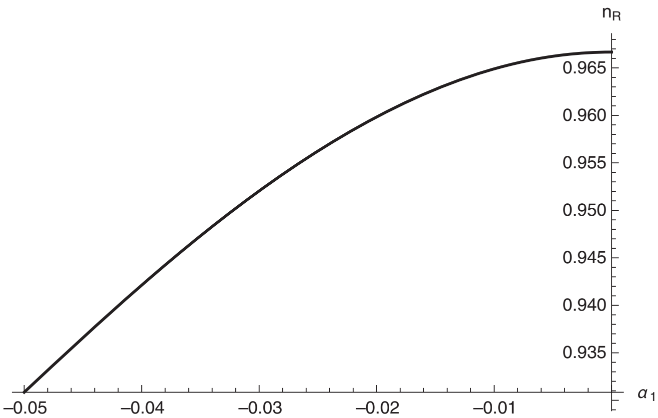

FIG. 1. Plot of n_S vs α_1. Note that the spectral index of scalar perturbations does not depend on α_2 at leading order in α_1, which is unlike the result for r above.

$$r \simeq \frac{12}{N^2}\frac{(\alpha_2 - 1)}{\alpha_2} \simeq \frac{1}{300}\frac{(\alpha_2 - 1)}{\alpha_2}. \tag{44}$$

Current constraints on $r < 0.07$ lead to a conservative bound on α_2 such that

$$\alpha_2 < -0.048. \tag{45}$$

Given the constraint on α_1 (43), there is interestingly also a lowest value for r in our model, namely, $r > 0.0026$. The other observables are, currently, unconstrained but could in principle be measured with future cosmic microwave background measurements (in the case of n_T and α_T) and high-redshift 21 cm missions that can probe small wavelengths in the linear regime (in the case of α_S). The numerical predictions our model makes for these parameters are $\alpha_S \sim -5 \times 10^{-4} \pm 10\%$, depending on the precise value of α_1. Note that, at leading order in α_1, α_S is independent of α_2, just like n_S. n_T and α_T both do have explicit dependence on α_2 and α_1. However, within the allowed range for α_1 (43), the value for n_T is recovered well by the limiting expression as $\alpha_1 \to 0$, and we then find

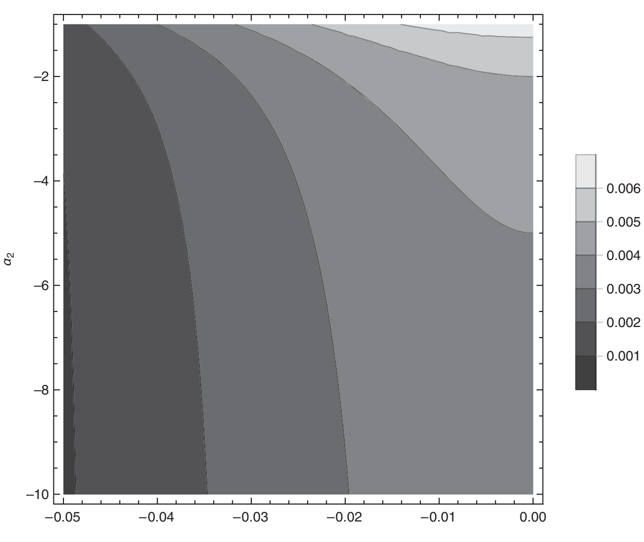

FIG. 2. Contour plot of r vs α_2 and α_1. Note that, as far as r is concerned and at least for the values shown here, all parameter values give observationally consistent predictions.

$$n_T \sim -4 \times 10^{-4} \cdot \frac{(\alpha_2 - 1)}{\alpha_2}. \tag{46}$$

For α_T, considering the limiting expression as $\alpha_1 \to 0$ gives even more accurate results [due to the extra α_1 suppression factor—see Eq. (36)], and we obtain there (36)

$$\alpha_T \sim -1.4 \times 10^{-5} \cdot \frac{(\alpha_2 - 1)}{\alpha_2}. \tag{47}$$

Finally, we can also investigate signatures of the scale-invariant model beyond the two-point function. While an exploration of the full bi- and trispectrum is beyond our scope, the local non-Gaussian parameter f_{NL} provides an observable of particular interest, since it is strongly suppressed in single-field models [34,35] and can therefore provide a smoking gun for multifield dynamics, if sizeable enough to be measured. Focusing on this local limit, one can then obtain the expression [36,37]

$$f_{NL}^{\rm local} \approx \frac{5}{6}\frac{N^{,A}N^{,B}(\nabla_A\nabla_B N)}{(N_{,C}N^{,C})^2}, \tag{48}$$

where N is the frame-covariant number of e-folds, as before. Note that this expression is essentially a (covariantized) version of the standard δN expression for f_{NL} [38], which there corresponds to a quasilocal configuration for the bispectrum (close, but not identical, to the local one—cf. the discussion in Ref. [39]). Taking (48) and noting that one can write $N_{,A} = UU_{,A}/(U_{,B}U^{,B})$ [23], after some algebra, we then find that $f_{NL}^{\rm local}$ can in fact succinctly be expressed as

$$f_{NL}^{\rm local} \approx -\frac{5}{6}\frac{X^A X^B \nabla_B X_A}{(X_C X^C)} = \frac{5}{12}\bar{\eta}_U. \tag{49}$$

Taking the same approach as for the other observables considered above, we can expand in α_1 and find the highly accurate expression

$$f_{NL}^{\rm local} \approx \frac{5\alpha_1(e^{-\nu N_J}+1)}{9(e^{-\nu N_J}-1)} + \mathcal{O}(\alpha_1^2) \simeq \frac{5}{12}(1-n_S)$$
$$\simeq \text{few} \times 10^{-2}, \tag{50}$$

where agreement between this expression to leading order in α_1 and the full expression (49) holds down to sub-1% level. Phrasing it in terms of n_s reproduces the (single-field) relation of Ref. [40], which is of course expected, given the existence of our effective single-field description (39). The $(n_S - 1)$ suppression in (50) then also follows from the well-known consistency relations for the three-point function [34,35]. Finally, note that we find $c_s = 1$ in the effective single-field picture (39), due to the independence of the model functions (40) on derivatives of ϕ. This immediately allows us to conclude that no sizeable equilateral non-Gaussianity is present in our model either, since for general single-field models $f_{NL}^{\rm equil} \lesssim 1/c_s^2$.

VI. DISCUSSION

In this paper, we have calculated the inflationary observables for inflation in a scale-invariant universe. While previous calculations had been undertaken in the Einstein frame under the assumption of single-field evolution, we chose to consider the full multifield model in a scale-invariant formalism. This allowed us to prove that, up to second order in slow roll, no isocurvature perturbations were generated in the inflationary regime. We showed that this was a particular feature of the two-field model we are considering in this paper and can be understood quite simply: the isocurvature mode is orthogonal to the constraint ellipse, and thus we can identify it with the dilaton. As we have shown before, the dilaton completely decouples from the other d.o.f. Our result reinforces the fact that the (effective) single-field approach is an excellent approximation.

Nevertheless, we persisted with the calculation, taking into account both fields, and found a set of analytic expressions for the inflationary observables: r, n_S, α_S, n_T, and α_T. These expressions are accurate at the subpercent level; r and n_S are in exact agreement with those found in Ref. [1]. As a final cross-check, we explicitly reduced the system to the dynamics of a single field by solving for the constraint in Eq. (4). Again, we recovered the same analytic results as we had determined in the multifield case, reinforcing the fact that isocurvature perturbations are completely absent. Finally, we assessed the level of non-Gaussianity in this model and found it to be small, of order $f_{NL}^{\rm local} \sim 10^{-2}$, and well within the current observationally allowed range.

Our calculations have confirmed that inflation in a scale-invariant universe is a completely viable model for the origin of structure, leading to acceptable observables. Furthermore, it is fundamentally well motivated; in future attempts at cosmological constraints, one is in a position to consider priors on the fundamental parameters as opposed to on the observables (such as r and n_S). Our results also reinforce the point made in Ref. [28]: if we are to accept inflation as the theory that explains the seeds for structure, then current data are strongly pushing us to have to accept nonminimal couplings. This is a striking statement about the fundamental structure of gravity and further incentive to consider theories such as the one discussed in this paper.

In this paper, we have not touched on other fundamental issues in inflation model building that need to be addressed: how did the inflationary regime begin, and how fine tuned are the initial conditions? In the scale-invariant model, these questions are intimately tied to the inertial symmetry breaking that occurs and leads the fields to lie on the constraint surface (the "ellipse"). The slow-roll conditions are naturally enforced on a large region of the ellipse, but whether, for a general set of initial conditions, the fields naturally end up in that region remains to be seen.

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APPENDIX A: GEOMETRY OF FIELD SPACE

For ease of notation, we define $f \equiv M^2 = -\frac{1}{6}\sum_A \alpha_A(\phi^A)^2$. We will need the following:

$$\begin{aligned} f &= -\frac{1}{6}\sum_A \alpha_A(\phi^A)^2 \\ f_{,B} &= -\frac{1}{3}\sum_A \alpha_A\phi^A\delta^A_B = -\frac{1}{3}\alpha_B\phi^B \\ f_{,B,C} &= -\frac{1}{3}\alpha_B\delta^B_C. \end{aligned} \tag{A1}$$

It is useful to define

$$F \equiv \sum_D f_{,D} f_{,D} = \frac{1}{9}\sum_D \alpha_D^2(\phi^D)^2. \tag{A2}$$

Inserting the above expressions into the definition of the field-space metric, we find

$$G_{AB} = \frac{1}{f}\left(\delta_{AB} + \frac{1}{6f}\alpha_A\alpha_B\phi^A\phi^B\right), \tag{A3}$$

and for the inverse field-space metric, we have

$$G^{AB} = f\delta^{AB} - \frac{3f^{,A}f^{,B}}{2(1+\frac{3F}{2f})} = f\delta^{AB} - \frac{\alpha_A\alpha_B\phi^A\phi^B}{6(1+\frac{3F}{2f})}. \tag{A4}$$

We can now also express the connection Γ as

$$\begin{aligned} \Gamma^A{}_{BC} &= \frac{2f\delta_{BC}f^{,A} - (3F+2f)(\delta^A_C f_{,B} + \delta^A_B f_{,C}) + 6ff^{,A}f_{,C,B}}{2f(3F+2f)} \\ &= \frac{(F+\frac{2}{3}f)(\delta^A_C\alpha_B\phi^B + \delta^A_B\alpha_C\phi^C) + \frac{2}{3}f\alpha_A\phi^A(\alpha_C-1)\delta_{BC}}{2f(3F+2f)}. \end{aligned} \tag{A5}$$

Finally, we have that

$$X_C = \frac{U_{,C}}{U} = \frac{2}{Uf^2}\sum_{A,B}\lambda_{AB}\phi^A(\phi^B)^2\delta^A_C + \frac{2}{3}\frac{\alpha_C\phi_C}{f}. \quad \text{(A6)}$$

APPENDIX B: ISOCURVATURE MODES FOR A SCALE-INVARIANT THREE-FIELD THEORY

Here, we briefly discuss a scale-invariant three-field model analogous to the two-field model presented in the main body of the paper. This will turn out to be instructive in understanding the origin of the decoupling of isocurvature from curvature modes in the two-field case. The action is still

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M^2(\vec{\phi})R - \frac{1}{2}\sum_{A=1}^{2}\nabla_\mu\phi^A\nabla^\mu\phi^A - W(\vec{\phi})\right], \quad \text{(B1)}$$

where we now have $M^2 = -\frac{1}{6}\sum_{A=1}^3\alpha_A(\phi^A)^2$ and $W(\vec{\phi}) = \sum_{A,B=1}^3\lambda_{AB}(\phi^A)^2(\phi^B)^2$. As before, a crucial quantity now is the turn rate ω^A; whenever $\omega^A = 0$, curvature and isocurvature modes decouple [32]. While we indeed found $\omega^A = 0$ in the two-field case, the three-field case is significantly different. One first new feature relevant to the computation of ω^A is that, while scale invariance still enforces $X_A\phi^A = 0$, this no longer in general eliminates all λ_{AB} dependence from $\hat{X}_A$. We therefore here, for simplicity, choose to set all parameters in the potential except for λ_{22} to zero, a choice that will be sufficient to show that generically $\omega^A \neq 0$ in multifield extensions of the two-field model considered in the main text. Explicitly calculating ω^A for the three-field model in question, we then obtain

$$\omega_1 = \frac{\alpha_1(1-\alpha_3){\phi_1}^2 + (1-\alpha_2)\alpha_3{\phi_2}^2 + (1-\alpha_3)\alpha_3{\phi_3}^2}{(\alpha_1-\alpha_3)\phi_1\phi_2\mathcal{A}},$$
$$\omega_2 = \frac{1}{\mathcal{A}},$$
$$\omega_3 = \frac{(\alpha_1-1)\alpha_1{\phi_1}^2 + \alpha_1(\alpha_2-1){\phi_2}^2 + (\alpha_1-1)\alpha_3{\phi_3}^2}{(\alpha_1-\alpha_3)\phi_2\phi_3\mathcal{A}}, \quad \text{(B2)}$$

where we have written $\omega^A \equiv \{\omega_1, \omega_2, \omega_3\}$ and have defined the shorthand notation

$$\mathcal{A}^2 \equiv -\frac{27(\sum_{A=1}^3\alpha_A(\alpha_A-1)\phi_A^2)^3\mathcal{B}^3}{2\alpha_1^2(\alpha_2-1)^4(\alpha_1-\alpha_3)^4\alpha_3^2{\phi_1}^4{\phi_2}^4{\phi_3}^4(-6M^2)^4},$$
$$\mathcal{B} \equiv \alpha_1^3{\phi_1}^4 + \alpha_1(\alpha_3-2)\alpha_3{\phi_1}^2{\phi_3}^2 + \alpha_3^2{\phi_3}^2((\alpha_2-1){\phi_2}^2 + (\alpha_3-1){\phi_3}^2) - \alpha_1^2{\phi_1}^2({\phi_1}^2 + {\phi_2}^2 - \alpha_2{\phi_2}^2 - \alpha_3{\phi_3}^2), \quad \text{(B3)}$$

using that $M^2 = -\frac{1}{6}\sum_{A=1}^3\alpha_A(\phi^A)^2$ as before. Given these expressions, we can then succinctly express the magnitude of the turn rate $\omega^2 = |\omega_A\omega^A|$ as

$$\omega^2 = \left|\frac{96\alpha_1^2(\alpha_2-1)^4(\alpha_1-\alpha_3)^2\alpha_3^2{\phi_1}^2{\phi_2}^2{\phi_3}^2KM^6}{\left[\sum_{A=1}^3\alpha_A(\alpha_A-1)\phi_A^2\right]^3\mathcal{B}^2}\right|, \quad \text{(B4)}$$

where K, in analogy to the constant from Eq. (4), satisfies

$$K = \frac{1}{2}\sum_{A=1}^3(1-\alpha_A)(\phi^A)^2 \quad \text{(B5)}$$

and describes the hyperellipsoid constraint surface in which the inflationary trajectory is embedded. Clearly, we therefore have a nonzero turn rate and associated mixing between curvature and isocurvature modes. This shows that the decoupling of these modes from one another cannot be a general consequence of scale invariance, irrespective of field-space dimension. In the three-dimensional case, we now have an isocurvature mode, orthogonal to the scalar-field trajectory, which lies on the constraint surface. Further work needs to be done to assess if this isocurvature mode is long lived.

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