Preference disaggregation method for value-based multi-decision sorting problems with a real-world application in nanotechnology

Miłosz Kadziński [a,*], Krzysztof Martyn [a,b], Marco Cinelli [a], Roman Słowiński [a,c], Salvatore Corrente [d], Salvatore Greco [d,e]

[a] *Institute of Computing Science, Poznań University of Technology, Poznań, Poland*
[b] *Poznan Supercomputing and Networking Center, Institute of Bioorganic Chemistry PAS, Poznań, Poland*
[c] *Systems Research Institute, Polish Academy of Sciences, Warsaw, Poland*
[d] *Department of Economics and Business, University of Catania, Catania, Italy*
[e] *CORL, Portsmouth Business School, University of Portsmouth, Portsmouth, United Kingdom*

ARTICLE INFO

ABSTRACT

We consider a problem of multi-decision sorting subject to multiple criteria. In the newly formulated decision problem, besides performances on multiple criteria, alternatives get evaluations on multiple interrelated decision attributes involving preference-ordered classes. We propose a dedicated method for dealing with such a problem, incorporating a threshold-based value-driven sorting procedure. The Decision Maker (DM) is expected to holistically evaluate a subset of reference alternatives by indicating the quality or risk level on a pre-defined scale of each decision attribute. Based on these evaluations, we construct a set of interrelated preference models, one for each decision attribute, compatible with intra- and inter-decision constraints imposed by such indirect preference information. We also formulate a new way of dealing with potentially non-monotonic criteria by discovering local monotonicity changes in different performance scale regions. The marginal value functions for criteria with unknown monotonicity are represented as a sum of two value functions assuming opposing preference directions, one non-decreasing and the other non-increasing. This permits to obtain an aggregated marginal value function with an arbitrary non-monotonic shape. The practical usefulness of the approach is demonstrated on a case study concerning risk management related to handling (i.e., production, use, manipulation, and processing) nanomaterials in different conditions. We analyze the expert judgments and discuss the inferred preference models, which can be applied to support health and safety managers in reducing the possible risk associated with the respective exposure scenario.

1. Introduction

Multiple Criteria Decision Analysis (MCDA) concerns decision problems where a set of alternatives are evaluated on a family of criteria, which represent all relevant, heterogeneous viewpoints on the quality of alternatives [1,2]. Many such decision problems fall into the general category of classification, where the alternatives need to be assigned to distinct classes [3]. If the classes are completely ordered, one deals with ordinal classification or, equivalently, sorting problems [4]. They are considered, e.g., in the ABC analysis, which is a type of inventory categorization method where high-, mid-, and low-value alternatives need to be identified [5], in medical diagnosis, where high-risk patients need to be distinguished from the low-risk ones [6], in business

failure prediction, where firms are sorted into healthy, uncertain, and close to bankruptcy [7], or in nanomanufacturing, where synthesis processes of nanomaterials can be sorted according to their greenness level [8].

Decision aiding sorting methods aim to provide recommendations to the Decision Maker (DM) regarding the assignment of alternatives to pre-defined and ordered classes [9]. There are various approaches serving this purpose though differing with respect to the underlying assumptions and characteristics of delivered results. In particular, various methods expect the DM to provide different types of preference information through a co-constructive elicitation process led by a decision analyst. On the one hand, some methods assume that the DM would directly specify values for a set of parameters of an assumed sorting model [10,11]. However, this is a bit unrealistic because (s)he usually has difficulties with keeping consistency between the supplied values and the model output [4]. On the other hand, preference disaggregation procedures have been proposed to prevent such difficulties [12]. They aim at deriving compatible model

---
* Correspondence to: Institute of Computing Science, Poznań University of Technology, Piotrowo 2, 60-965 Poznań, Poland.
E-mail address: milosz.kadzinski@cs.put.poznan.pl (M. Kadziński).

parameters from the analysis of the DM's comprehensive judgments (assignment examples) concerning a subset of reference alternatives. This allows generalizing the DM's policy to an entire set of alternatives through the use of a suitably parametrized sorting model. Specifically, UTADIS disaggregates the DM's assignment examples into marginal value functions and class thresholds separating the consecutive decision classes on a scale of comprehensive value [13,14]. This idea was found appealing in such various fields as finance [15], energy management [16], or stock portfolio analysis [17].

Motivated by the complexity of real-world sorting problems, UTADIS has been extended in various ways. First, different procedures for dealing with inconsistency of assignment examples with an assumed model have been proposed [14,18]. Second, a hierarchical classification approach, called MHDIS, has been introduced in [19]. Third, the Multiple Criteria Hierarchy Process has been adapted to UTADIS to allow handling preference information and deriving recommendations at both comprehensive and intermediate levels of the hierarchy of criteria [20]. Moreover, novel preference modeling procedures have been designed to admit the specification of desired class cardinalities, assignment-based pairwise comparisons [21], or valued assignment examples [22]. Furthermore, the frameworks for robustness analysis have been proposed [4,23,24] to exploit infinitely many instances of the preference model (e.g., value functions and class thresholds) compatible with the DM's holistic decisions. While all approaches mentioned above considered a single DM, [25] introduced a group decision framework, called UTADIS^{GMS}-GROUP, investigating the spaces of agreement and disagreement between sorting recommendations obtained for different DMs. Some other recent methodological advancements of the UTADIS method concern the form of an employed value-based model. In this regard, an additive value function has been extended in [26] to account for positively and negatively interacting pairs of criteria. The other stream concerned dealing with the non-monotonicity of preferences on a per-criterion level. In particular, [27] defined a broad spectrum of non-monotonic shapes that could be considered along with the gain- and cost-type criteria. Moreover, [28] and [29] introduced the models admitting non-monotonicity of marginal value functions while not restraining their complexity. In turn, [29,30] and [31] minimized, respectively, the variation in the slope or the number of changes of non-monotonicity in the shape of marginal value functions to ensure the most interpretable sorting model.

The contribution of this paper is three-fold. First, we introduce a new problem of multi-decision sorting in MCDA and propose a dedicated method for dealing with it. In this problem, each alternative is evaluated in terms of multiple decision attributes involving preference-ordered classes. We expect the DM to assign a subset of reference alternatives to classes of each decision attribute by indicating a quality or risk level on a scale pre-defined for all decision attributes. A similar setting has been considered in [32] and [33] in the context of credit rating problems. On the one hand, [32] adapted the UTADIS method to infer a single threshold-based value-driven sorting model compatible with the ratings provided by Moody's and Standard & Poor's, hence providing a precise recommendation based on potentially conflicting inputs for the same alternative. On the other hand, [33] used the three credit rating agencies' recommendations to form an interval rating that was subsequently used as a potentially imprecise reference benchmark to be reproduced by the ELECTRE TRI-nC method [34].

The multi-decision sorting problem and dedicated approach introduced in this paper are original in the sense of requiring construction of a set of interrelated preference models, one for each decision attribute. Such a requirement contrasts with the inference of a single sorting model that would align with multiple classifications at the same time [32] or an imprecise assignment built on multiple ratings for the same alternative [33]. Specifically, we propose a threshold-based value-driven sorting method. It involves a set of intra- and inter-decision constraints. The former ones ensure appropriate relations between comprehensive values of different alternatives for an individual value function used to derive the assignments for a single decision attribute. The latter correspond to the relations between comprehensive values of the same alternative for multiple value functions employed for classifying this alternative given various decision attributes. This makes sense when the classes of various decision attributes correspond to the same default categories, having the same scope and interpretation.

This paper's second contribution derives from presenting the results of a case study concerning risk management related to handling nanomaterials in different conditions [35]. The production, processing, and use of nanomaterials may lead to health or life exposure. Depending on the particular exposure scenario, different types of precautions or safety measures can be used to counteract the respective risk [36]. Also, some precautions meet general hazards, whereas others are dedicated to deal with some specific dangers. Each of the precaution types (e.g., incorporation of some personal protective equipment, engineering controls, or work practices) can be interpreted as a decision attribute with pre-defined preference-ordered classes representing different levels of risk [37]. When facing hazards that particular nanomaterials carry with them, some precautions are required, and others are optional or unnecessary [31]. However, different precautions are not independent, being defined on the same set of criteria and related in terms of their interpretation.

There is a need for a method dealing with multiple interrelated preference-ordered decision attributes to tackle such a problem. In this regard, MCDA has little to offer. This, in turn, implies that such complex problems would typically be decomposed into independent ones. This would allow for modeling intra-decision dependencies, neglecting the inter-decision relations that could negatively affect results' usefulness. Other ideas are solutions derived from multi-label classification, such as label powerset [38] or probabilistic classifier chains [39]. The label powerset generates a vast number of classes and requires many examples so that each class has a sufficient number of its representatives. The latter is difficult to satisfy for the case study. Probabilistic classifier chains offer different solutions depending on the order of the decisions under consideration and require repeated solving of the same problem. The limitations of the existing approaches motivated the development of a dedicated multiple criteria sorting method. In the context of the considered case study, the information coming from the proposed approach will help the DMs in assessing the risk related to the treatment of nanomaterials in different conditions. Specifically, it can be used for recommending the level of need for the use of specific personal protective equipment, engineering controls, or work practices.

Our third contribution consists of proposing a new way of dealing with potentially non-monotonic criteria. Non-monotonic criteria appear in the MCDA problem when, for some attributes, neither the univocal preference direction could be specified, nor the non-monotonic shape of respective marginal value function could be defined a priori. This happens in our case study. Then, such a shape needs to be inferred from data describing the multiple criteria problem and the DM's holistic judgments. In particular, the method should verify whether a monotonic relationship exists, if it is of gain- or cost-type, or if the monotonicity is not global. The latter scenario could reveal some local positive or negative relationships in different parts of the investigated performance scale [40]. To perform this task, we propose a new

approach that attempts to discover local monotonicity changes without requiring the DM to fix the preference directions for all criteria. Specifically, we represent the marginal value functions of potentially non-monotonic criteria as a sum of marginal value functions assuming opposing preference directions, one non-decreasing and the other non-increasing. This permits to obtain an aggregated marginal value function with an arbitrary non-monotonic shape. However, the two monotonic components remain easy to interpret.

The remainder of the paper is organized in the following way. Section 2 is devoted to the new method dealing with multi-decision sorting problems and handling potentially non-monotonic criteria. Section 3 illustrates the use of the proposed method on a didactic instance. In Section 4, we report the results of a case study concerning the analysis of exposure scenarios related to the treatment of nanomaterials in various conditions. The last section concludes and outlines the ideas for a future work.

2. Notation and problem statement

We use the following notation:

- $A = \{a_1, a_2, \ldots, a_i, \ldots, a_n\}$ – a finite set of n alternatives;
- $A^R = \{a^*, b^*, \ldots\}$ – a finite set of reference alternatives, which are holistically judged by the DM; we assume that $A^R \subseteq A$;
- $G = \{g_1, g_2, \ldots, g_j, \ldots, g_m\}$ – a finite set of m criteria, $g_j : A \to \mathbb{R}$ for all $j \in J = \{1, 2, \ldots, m\}$;
- $X_j = \{x_j \in \mathbb{R} : g_j(a_i) = x_j, a_i \in A\}$ – a set of all different performances on $g_j, j \in J$;
- $x_j^1, x_j^2, \ldots, x_j^{n_j(A)}$ – increasingly ordered values of X_j, $x_j^k < x_j^{k+1}, k = 1, 2, \ldots, n_j(A) - 1$, where $n_j(A) = |X_j|$ and $n_j(A) \le n$; consequently, $X = \prod_{j=1}^{m} X_j$ is the performance space;
- $\mathcal{D} = \{D_1, D_2, \ldots, D_l\}$ – a finite set of l decision attributes;
- $C_1^{D_s}, C_2^{D_s}, \ldots, C_p^{D_s}$ – p pre-defined preference-ordered classes defined for each decision attribute $D_s, s = 1, \ldots, l$, where $C_{h+1}^{D_s}$ is preferred to $C_h^{D_s}, h = 1, \ldots, p - 1$; moreover, $H = \{1, \ldots, p\}$. Remark that the number of pre-defined preference-ordered classes for all l decision attributes is the same and equal to p. Qualitative meaning of class C_h is also the same for all decision attributes $D_s \in \mathcal{D}$.
- $b_0^{D_s}, b_1^{D_s}, \ldots, b_p^{D_s}$ – thresholds separating the classes on decision attribute $D_s, s = 1, \ldots, l$, such that $b_{h-1}^{D_s}$ and $b_h^{D_s}$ are, respectively, the lower and upper comprehensive values admissible for alternatives assigned to $C_h^{D_s}, h = 1, \ldots, p$.

In what follows, we discuss the employed preference model and preference information. We present the mathematical constraints that allow dealing with multi-decision sorting problems while originally handling potentially non-monotonic criteria. The latter ones are interpreted as criteria with unknown monotonicity. This means that a decision analyst and the DM cannot specify a preference direction for them, and they admit that such a direction may not exist. Moreover, they accept that the shapes of marginal value functions for these criteria will be inferred through disaggregating the DM's holistic preferences. The resulting shape will determine if the monotonic relation can be imposed in the entire performance space of a given criterion, and, if not, what are the local relationships of monotonicity in different regions of this space.

2.1. Preference model

For each decision attribute $D_s \in \mathcal{D}$, a comprehensive quality of each alternative $a_i \in A$ is quantified using an additive value function defined as the sum of marginal values $u_j^{D_s}(a_i)$ on all criteria $g_j, j = 1, \ldots, m$:

$$U^{D_s}(a_i) = \sum_{j=1}^{m} u_j^{D_s}(a_i) \in [0, 1]. \tag{1}$$

Alternatives are evaluated in terms of three types of criteria: gain $g_j \in G_g$, cost $g_j \in G_c$, and potentially non-monotonic $g_j \in G_n$ ($G_g \cup G_c \cup G_n = G$). For the gain-type criteria, greater performances are more preferred than smaller performances. This implies the following monotonicity and normalization constraints:

$$u_j^{D_s}\left(x_j^k\right) \ge u_j^{D_s}\left(x_j^{k-1}\right), \; k = 2, \ldots, n_j(A), \; \text{and} \; u_j^{D_s}\left(x_j^1\right) = 0. \tag{2}$$

Analogously, for the cost-type criteria, smaller performances are more preferred:

$$u_j^{D_s}\left(x_j^k\right) \le u_j^{D_s}\left(x_j^{k-1}\right), \; k = 2, \ldots, n_j(A), \; \text{and} \; u_j^{D_s}\left(x_j^{n(A)}\right) = 0. \tag{3}$$

Example marginal value functions for the gain- and cost-type criteria are presented in Fig. 1. Note that these functions are, respectively, non-decreasing and non-increasing.

The marginal value function for the potentially non-monotonic criterion $g_j \in G_n$ is modeled as the sum of marginal values derived from the non-decreasing and non-increasing components contributing to the comprehensive assessment of alternatives from this particular viewpoint:

$$u_j^{D_s}\left(x_j^k\right) = u_{j,nd}^{D_s}\left(x_j^k\right) + u_{j,ni}^{D_s}\left(x_j^k\right), \; k = 1, \ldots, n_j(A),$$

where the monotonicity of $u_{j,nd}^{D_s}$ and $u_{j,ni}^{D_s}$ is modeled in a standard way:

$$u_{j,nd}^{D_s}\left(x_j^k\right) \ge u_{j,nd}^{D_s}\left(x_j^{k-1}\right), \; k = 2, \ldots, n_j(A), \; \text{and} \; u_{j,nd}^{D_s}\left(x_j^1\right) = 0, \tag{4}$$

$$u_{j,ni}^{D_s}\left(x_j^k\right) \le u_{j,ni}^{D_s}\left(x_j^{k-1}\right), \; k = 2, \ldots, n_j(A), \; \text{and} \; u_{j,ni}^{D_s}\left(x_j^{n(A)}\right) = 0. \tag{5}$$

In case the method discovers that a monotonic relationship exists, either $u_{j,nd}^{D_s}\left(x_j^k\right)$ or $u_{j,ni}^{D_s}\left(x_j^k\right)$ takes non-negative values for $k = 1, \ldots, n_j(A)$ and the other component is zeroed for all performances. Then, the resulting marginal value function $u_j^{D_s}$ is also monotonic. In Figs. 2a and b, we present the examples of such non-decreasing and non-increasing functions along with the two components.

When both components $u_{j,nd}^{D_s}$ and $u_{j,ni}^{D_s}$ take some positive values over the range from x_j^1 to $x_j^{n_j(A)}$, then any non-monotonic shape of the marginal value function $u_j^{D_s}$ can be obtained. However, this may yield a comprehensive marginal value function, which is not equal to zero for the worst performance on the non-monotonic criterion. Such a situation is undesired because it is hard to interpret such a model, and, moreover, the scale of values attained by the comprehensive model gets reduced. To prevent such a scenario, the marginal value function should be normalized so that $\exists x_j^k \in X_j$ such that $u_j^{D_s}\left(x_j^k\right) = 0$. This can be obtained by subtracting a value of bias $t_j^{D_s} \ge 0$ from the marginal values of g_j, and adding a constraint that $u_j^{D_s}\left(x_j^k\right)$ should be non-negative for all performances $x_j^k, k = 1, \ldots, n_j(A)$:

$$u_j^{D_s}\left(x_j^k\right) = u_{j,nd}^{D_s}\left(x_j^k\right) + u_{j,ni}^{D_s}\left(x_j^k\right) - t_j^{D_s}, \; k = 1, \ldots, n_j(A), \tag{6}$$

$$1 \ge t_j^{D_s} \ge 0, \tag{7}$$

$$u_j^{D_s}\left(x_j^k\right) \ge 0, \; k = 1, \ldots, n_j(A). \tag{8}$$

In Figs. 2c–f, we present the two components, a value of bias, and the resulting non-monotonic functions of different types: A-, V-, W-, and M-type functions. The elementary components are monotonic, but the marginal functions which aggregate them
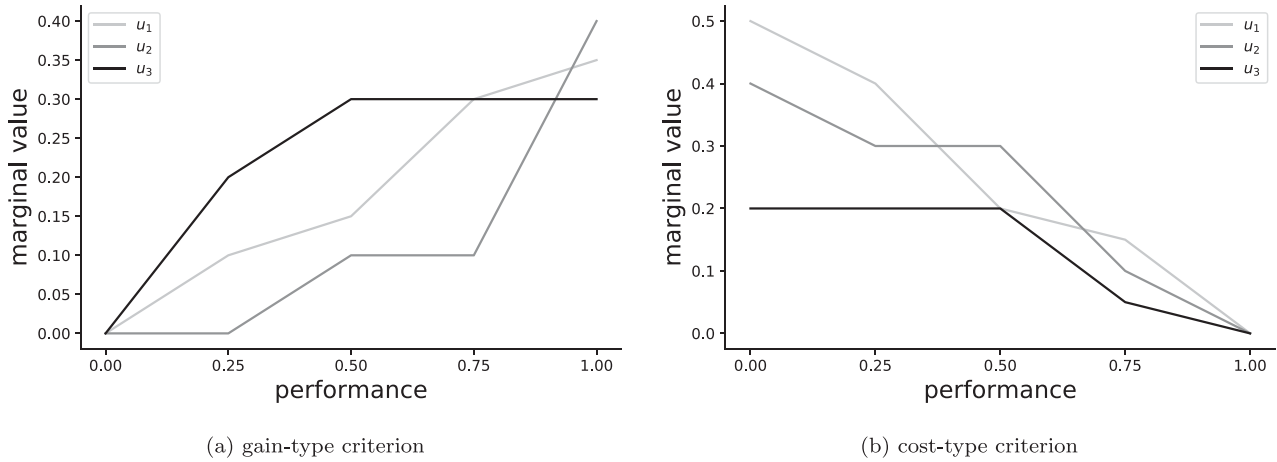
(a) gain-type criterion

(b) cost-type criterion

Fig. 1. Example monotonic marginal value functions.

with a bias are arbitrarily non-monotonic. Such functions can capture the local relationships of monotonicity that can be positive in some of the considered performance space and negative in the other part of the same space. Since in the proposed approach, these functions are inferred from the assignment examples, their complexity (i.e., the non-monotonic character, the number of monotonicity changes, or differences in slopes in various regions of the performance space) depends on the dependencies observed in the preferences of alternatives and input preference information.

To assign the alternatives to pre-defined and ordered classes for decision attribute $D_s \in \mathcal{D}$, we will apply a threshold-based sorting procedure. It derives the assignment of alternative $a_i \in A$ from the comparison of $U^{D_s}(a_i)$ with a set of thresholds $b_h^{D_s}$, $h = 0, \ldots, p$, such that for $D_s \in \mathcal{D}$:

$$b_0^{D_s} = 0, \ b_{p-1}^{D_s} \leq 1 - \varepsilon, \ \text{and} \ b_p^{D_s} = 1 + \varepsilon, \tag{9}$$

$$b_h^{D_s} \geq b_{h-1}^{D_s} + \varepsilon, \ h = 2, \ldots, p - 1, \tag{10}$$

where ε is an arbitrarily small positive value. In this way, the values of the worst and the best thresholds are set to, respectively, zero and greater than one. Moreover, there is a difference between the extreme thresholds delimiting each class so that it could accommodate some alternatives. Then, a_i is assigned to class $C_h^{D_s}$ iff $b_{h-1}^{D_s} \leq U^{D_s}(a_i) < b_h^{D_s}$, i.e., if a_i is at least as good as the respective lower threshold and strictly worse than the corresponding upper threshold. Such a threshold-based sorting procedure is illustrated in Fig. 3. Eqs. (1)–(10) form a core component of a larger set of linear programming constraints defining a set of instances of an assumed sorting model that are compatible with the DM's preference information. We will refer to it as E^{BASE}.

2.2. Preference information

We expect the DM to specify the desired assignments for a subset of reference alternatives $a^* \in A^R \subseteq A$ on each decision attribute $D_s \in \mathcal{D}$:

$$a^* \in A^R \rightarrow C_{DM}^{D_s}(a^*). \tag{11}$$

Note that the classes provided by the DM for different decision attributes $D_s \in \mathcal{D}$ can be different for the same reference alternative. Such holistic preference information is used in a twofold way. On the one hand, we need to reproduce the desired

assignments on each decision attribute, i.e.:

$$\left. \begin{array}{l} \text{for all } a^* \in A^R : \\ \quad v(a^*) \in \{0, 1\}, \\ \quad \text{for all } D_s \in \mathcal{D} : \\ \qquad U^{D_s}(a^*) \geq b_{C_{DM}^{D_s}(a^*)-1}^{D_s} - v(a^*), \\ \qquad U^{D_s}(a^*) + \varepsilon \leq b_{C_{DM}^{D_s}(a^*)}^{D_s} + v(a^*). \end{array} \right\} \quad E^R(intra - \mathcal{D}) \tag{12}$$

In case $v(a^*) = 0$, $U^{D_s}(a^*)$ falls in the range corresponding to class $C_{DM}^{D_s}(a^*)$, i.e., $[b_{C_{DM}^{D_s}(a^*)-1}^{D_s}, b_{C_{DM}^{D_s}(a^*)}^{D_s})$. Then, the assignment provided for a^* on D_s is reproduced. If $v(a^*) = 1$, the respective constraints are always satisfied, being relaxed. The binary variables $v(a^*)$, $a^* \in A^R$, will be subsequently used to minimize the prediction distance of the inferred model from the reference data in case the sorting model would not be able to align with all assignment examples.

On the other hand, in line with the specificity of the multi-decision sorting problem, we will compare the desired assignments for each reference alternative $a^* \in A^R$ for different pairs of decision attributes. Let us remind that both the number and interpretation of classes are the same for all decision attributes. In this way, the classes specified by the DM determine an order of labels associated with each reference alternative. If $C_{DM}^{D_s}(a^*)$ is more preferred than $C_{DM}^{D_t}(a^*)$, this can be interpreted as the label D_s being more desired for a^* than label D_t. Consequently, a comprehensive value of a^* associated with D_s should be greater than its respective value associated with D_t, i.e.:

$$\left. \begin{array}{l} \text{for all } a^* \in A^R : \\ \quad \text{if } C_{DM}^{D_s}(a^*) > C_{DM}^{D_t}(a^*) : \\ \qquad U^{D_s}(a^*) \geq U^{D_t}(a^*) + \varepsilon - v(a^*). \end{array} \right\} \quad E^R(inter - \mathcal{D}) \tag{13}$$

Analogously as in $E^R(intra - D_s)$, binary variable $v(a^*)$ implies that the respective constraints associated with the assignments of a^* are either instantiated (when $v(a^*) = 0$) or relaxed (when $v(a^*) = 1$).

2.3. Compatible sorting model

We aim to infer a sorting model that would be compatible with the provided assignment examples while respecting the assumptions on additivity, monotonicity, and normalization, as well as intra- and inter-decision constraints. The model is composed of a set of interrelated additive value functions and vectors of class thresholds such that a single function is associated with a single
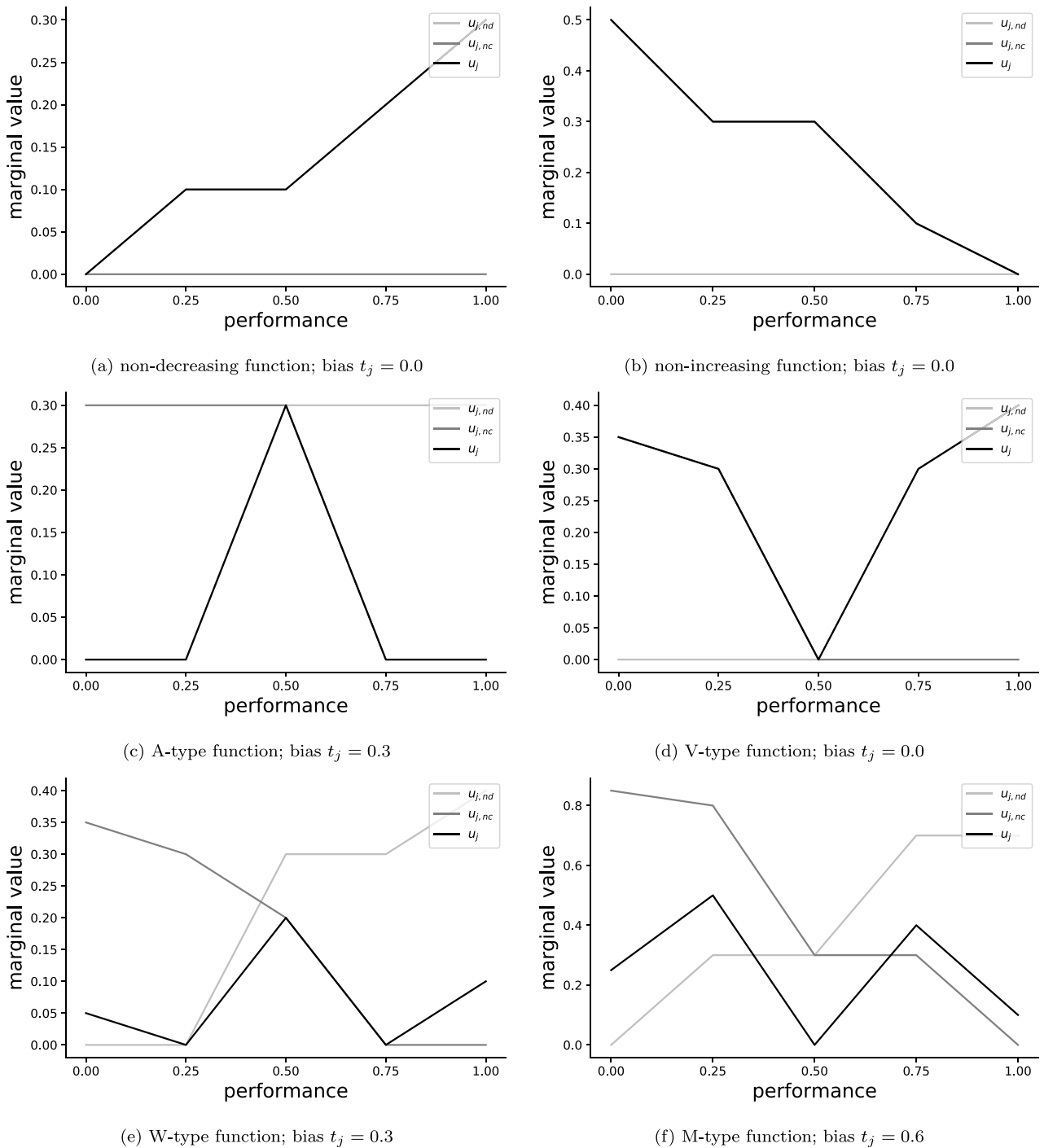
(a) non-decreasing function; bias $t_j = 0.0$

(b) non-increasing function; bias $t_j = 0.0$

(c) A-type function; bias $t_j = 0.3$

(d) V-type function; bias $t_j = 0.0$

(e) W-type function; bias $t_j = 0.3$

(f) M-type function; bias $t_j = 0.6$

Fig. 2. Example non-decreasing ($u_{j,nd}$) and non-increasing ($u_{j,ni}$) components along with resulting marginal value functions (u_j) of different types for the potentially non-monotonic criteria.

vector of thresholds corresponding to each decision attribute. We admit that the reference assignments are burdened with some error, though we would like the model to be compatible with as many assignments of reference alternatives as possible. For this purpose, we solve the following Mixed-Integer Linear Programming (MILP) model:

$$\text{Minimize } f_w = (r \cdot l + 1) \sum_{a^* \in A^R} v(a^*) + \sum_{j \in G_n, D_s \in \mathcal{D}} t_j^{D_s},$$

$$\text{subject to } E^{BASE} \cup E^R(intra - \mathcal{D}) \cup E^R(inter - \mathcal{D}). \quad (14)$$

The primary goal of the above objective function f_w is to minimize the number of reference alternatives for which the desired assignments are inconsistent with an assumed preference model, i.e., $\sum_{a^* \in A^R} v(a^*)$. The secondary goal is to minimize a sum of bias values for all decision attributes and all potentially non-monotonic criteria, i.e., $\sum_{j \in G_n, D_s \in \mathcal{D}} t_j^{D_s}$. To ensure a lexicographic optimization of these two targets, we multiply the first component by $(r \cdot l + 1)$ and the second by 1. Note that $(r \cdot l + 1)$ is greater than a maximal possible sum of all bias values. Based on Eqs. (6)
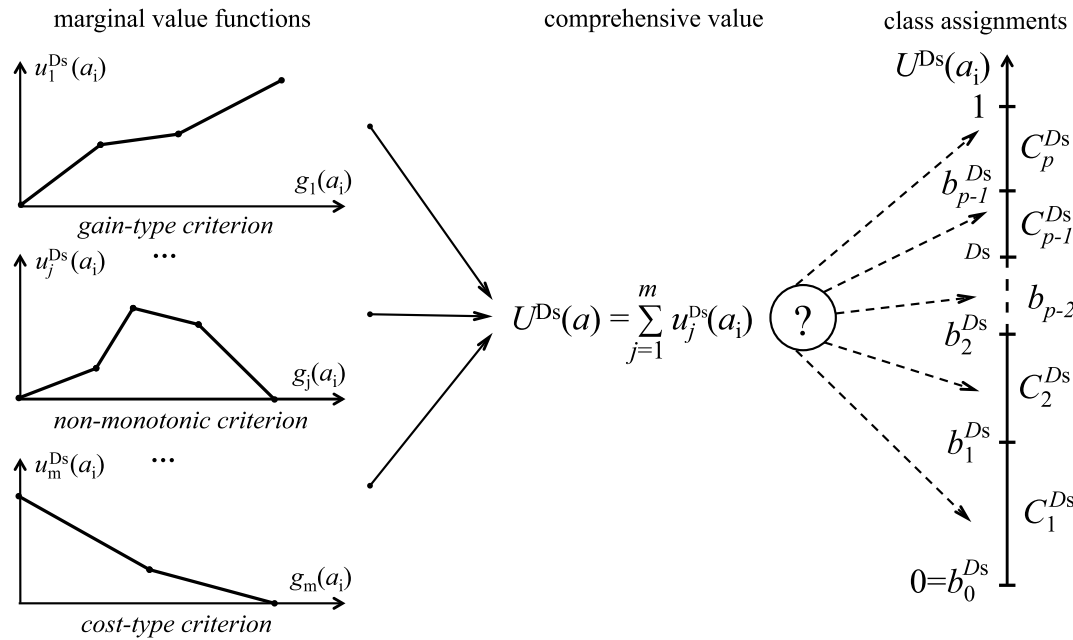
marginal value functions   comprehensive value   class assignments

Fig. 3. Threshold-based sorting procedure driven by an additive value function aggregating marginal values corresponding to criteria of different types.

and (7), such a sum is constrained by $r \cdot l$, i.e.:

$$\sum_{j \in G_n, \, D_s \in \mathcal{D}} t_j^{D_s} \leq r \cdot l, \tag{15}$$

where $r = |G_n|$ is the number of potentially non-monotonic criteria, and l is the number of decision attributes. As a result, the above model always favors a lesser number of reference alternatives $a^* \in A^R$ that need to be removed to restore consistency. Specifically, among the models for which such a number is minimal, we favor the one for which the sum of biases on all criteria with unknown monotonicity is as small as possible.

3. Illustrative example

In this section, we illustrate the use of the proposed method on a simple didactic example composed of a pair of scenarios, denoted as Scenarios 1 and 2. The considered problem involves ten alternatives, which are evaluated on the following three criteria: g_1 of gain type, g_2 of cost type, and g_3 being potentially non-monotonic. For the performance matrix, see Table 1. The alternatives are comprehensively evaluated using two decision attributes (D_1 and D_2) with five preference ordered classes $C_1 - C_5$ such that C_5 and C_1 are, respectively, the most and the least preferred ones.

Let us first consider Scenario 1 for which the reference assignments are provided in Table 1. For example, a_1 is assigned to C_4 on D_1 and to C_3 on D_2, whereas the order of classes for a_5 is inverse. The inferred model is able to reconstruct the assignments for eight alternatives (see column $v(a^*)$ (Scenario 1) of Table 2). The comprehensive judgments for a_6 and a_{10} could not be reproduced. The assignments of a_6 were contradictory with those of a_7. Specifically, a_7 is more preferred than a_6 on g_1 and g_2, while attaining the same performance on g_3. However, the classes of a_7 are worse than the respective classes of a_6, which contradicts the dominance principle. Similarly, a_{10} dominates a_1 while being at least as good on all monotonic criteria and having the same performance on the non-monotonic criterion. However, the desired class of a_{10} on D_1 is worse than that of a_1, while they

Table 1
Performance matrix and the reference assignments considered in the two scenarios in the illustrative example.

Alternative	Criteria			Scenario 1		Scenario 2	
a_i	g_1	g_2	g_3	D_1	D_2	D_1	D_2
a_1	2	1	2	C_4	C_3	C_4	C_3
a_2	0	2	0	C_2	C_1	C_2	C_1
a_3	1	3	4	C_5	C_2	C_5	C_2
a_4	3	2	3	C_5	C_4	C_5	C_4
a_5	3	3	3	C_3	C_4	C_3	C_4
a_6	0	4	1	C_2	C_3	C_1	C_3
a_7	1	3	1	C_1	C_2	C_1	C_3
a_8	3	0	0	C_4	C_5	C_4	C_5
a_9	0	3	4	C_3	C_1	C_3	C_1
a_{10}	2	0	2	C_1	C_3	C_1	C_3

are both assigned to the same class on D_2. Also in this case, the assignments of a_{10} and a_1 could not be reproduced jointly.

The comprehensive values of all ten alternatives and the assignments generated with the inferred sorting model are presented in Table 2. For the separating class thresholds, see Table 3. Let us first discuss the assignments of reference alternatives which agree with the one specified by the DM. For example on D_1, a_1 is assigned to C_4 and a_3 is assigned to C_5. Not only a_3 attains a greater comprehensive value than a_1, but also the comprehensive values of these alternatives fall in the ranges delimited by the respective class thresholds (compare Tables 2 and 3). Similarly, on D_2, a_1 is assigned to a more preferred class than a_3, which is reflected in the relationship between their comprehensive values ($U^{D_2}(a_1) = 0.4762 > U^{D_2}(a_3) = 0.2886$). However, the inferred comprehensive values respect also the desired inter-decision relationships. For example, a_3 was assigned to C_5 on D_1 and to C_2 on D_2. As a result, its comprehensive value on D_1 ($U^{D_1}(a_3) = 0.6162$) is greater than that on D_2 ($U^{D_2}(a_3) = 0.2886$). In the same spirit, since a_8 was assigned to C_4 and C_5 on, respectively, D_1 and D_2, we have $U^{D_1}(a_8) = 0.6078 < U^{D_2}(a_8) = 0.6162$. When it comes to the reference alternatives for which the desired assignments were not fully reproduced, the resulting class of a_6 on D_2 and a_{10}

Table 2

Comprehensive values $U(a^*)$, respective class assignments on decision attributes D_1 and D_2, and values of binary variables $v(a^*)$ for the two scenarios in the illustrative example.

Alternative	Scenario 1					Scenario 2				
a_i^*	$U^{D_1}(a^*)$	$U^{D_2}(a^*)$	D_1	D_2	$v(a^*)$	$U^{D_1}(a^*)$	$U^{D_2}(a^*)$	D_1	D_2	$v(a^*)$
a_1	0.4846	0.4762	C_4	C_3	0	0.1223	0.0057	C_1	C_2	1
a_2	0.2800	0.1402	C_2	C_1	0	0.1279	0.0001	C_2	C_1	0
a_3	0.6162	0.2886	C_5	C_2	0	0.5556	0.0056	C_5	C_2	0
a_4	0.6722	0.5408	C_5	C_4	0	0.5556	0.5001	C_5	C_4	0
a_5	0.4762	0.4846	C_3	C_4	0	0.4944	0.5000	C_3	C_4	0
a_6	0.0002	0.0002	C_1	C_1	1	0.0613	0.4444	C_1	C_3	0
a_7	0.1402	0.1486	C_1	C_2	0	0.1224	0.4500	C_1	C_3	0
a_8	0.6078	0.6162	C_4	C_5	0	0.5500	0.5556	C_4	C_5	0
a_9	0.4762	0.1402	C_3	C_1	0	0.4944	0.0001	C_3	C_1	0
a_{10}	0.6722	0.4763	C_5	C_3	1	0.1224	0.4444	C_1	C_3	0

Table 3

The class thresholds for the two scenarios in the illustrative example.

Scenario	Decision attribute	b_1	b_2	b_3	b_4
Scenario 1	D_1	0.1445	0.3585	0.4800	0.6100
	D_2	0.1445	0.3585	0.4800	0.6100
Scenario 2	D_1	0.1251	0.1807	0.4972	0.5527
	D_2	0.0028	0.4416	0.4972	0.5527

on D_1 was C_1 as compared to, respectively, C_2 and C_5 in the DM's judgments.

The marginal value functions inferred for Scenario 1 are presented in Fig. 4. The imposed monotonicity constraints are respected for the gain (g_1) and cost (g_2) criteria and the monotonic components of g_3. The shapes of marginal value functions on the different decisions are similar. The differences concern slightly greater marginal value assigned to performance 3 on g_1 for D_2 and to performances 0–2 on g_2 and 3–4 on g_3 for D_1. The non-increasing component for g_3 was the same for both decision attributes. The marginal value function's overall course for g_3 took the V-shape with 1 being the least preferred performance.

As the other scenario (Scenario 2), let us consider slightly modified desired assignments (see Table 1). When compared to Scenario 1, the assignments of a_7 on D_2 and a_6 on D_1 were changed to, respectively, C_3 and C_1. Now, only assignments of alternative a_1 could not be reproduced with an assumed model (see Table 2, column $v(a^*)$) for Scenario 2. The comprehensive values and the respective assignments are presented in Table 2 and the class thresholds are given in Table 3. Similarly as for Scenario 1, we could observe that the classifications on the two decision attributes and the relationships between comprehensive values attained by each alternative on D_1 and D_2 are preserved. For example, a_2 is assigned to C_2 on D_1 and to C_1 on D_2, because $b_2^{D_1} = 0.1807 > U^{D_1}(a_2) = 0.1279 \geq b_1^{D_1} = 0.1251$, $b_2^{D_2} = 0.0028 > U^{D_2}(a_2) = 0.0001$, and $U^{D_1}(a_2) > U^{D_2}(a_2)$. However, the primary motivation for considering Scenario 2 is to show the impact of eliminating a bias for a non-monotonic criterion (for the marginal value functions, see Fig. 5). Indeed, when summing up the non-decreasing and non-increasing components for g_3 for decision D_2, all performances would be assigned positive marginal values. To ensure that the worst performance on g_3 (for this scenario – $g_3^{D_2}(a) = 2$) was assigned zero, the constructed model subtracted a bias $t_3^{D_2} = 0.4444$. In this way, a comprehensive value of the anti-ideal alternative was also zeroed, while not affecting the relative comparison of existing alternatives.

To support the comprehension of different types of constraints defining a set of inter-related sorting models, in Table 4, we illustrate the use of these constraints in the context of an example presented in this section. Specifically, for nine different constraint types, we provide their general form, an example constraint for a specific decision attribute, alternative, criterion, performance, or class, and the values assigned in this example constraint to the variables by the sorting models inferred for Scenario 2.

4. Multi-decision sorting in the context of exposure management of nanomaterials

Nanomaterials are particles with a size of several dozen nanometers and physicochemical properties being significantly different from the materials of larger sizes composed of the same atoms [35]. Due to these specific properties, they can improve the performance of products in several application areas, including energy production and storage [41], water treatment [42], healthcare [43], and food preservation [44], to name a few. The production of nanomaterials is based on the manipulation of materials at the nanoscale (1–100 nanometers), which requires caution and protective measures to guarantee their safe handling.

Since nanotechnology is a relatively new scientific field, all the potentially harmful effects of individual nanomaterials and threats resulting from their production and employment are not yet precisely known [45,46]. The research on this subject is still ongoing, but the safety standards used in nanomaterials production are currently mainly adopted from similar chemical production processes [37,47,48]. Nevertheless, the safety of nanomaterials production processes is a pressing issue in the area of nanotechnology [48,49]. In this perspective, the development of guidelines for the appropriate selection of precautions for nanomanufacturing would be a beneficial contribution.

4.1. Problem definition

4.1.1. Criteria

When evaluating nanomanufacturing exposure scenarios, there are several characteristics of the nanomaterials and operating conditions that need to be accounted for. In this case study, we will consider the following ten criteria, which are common descriptors for this type of scenarios [31,35,48]:

- *Particle size* (g_1) – in general, the smaller the size, the easier the nanomaterial gets through any filter. Nevertheless, since nanomaterials have different toxicological profiles according to their size, it is not yet possible to generalize a monotonic dependency between size and harmfulness [50].
- *Toxicity* (g_2) determines type of effect the nanomaterial has on human health [51].
- *Airborne capacity* (g_3) characterizes the engineered nanomaterials' capacity to spread in the workplace through the air stream. It is scored on 4-point scale from none to high, with none and high being, respectively, the most and the least preferred performances [52].
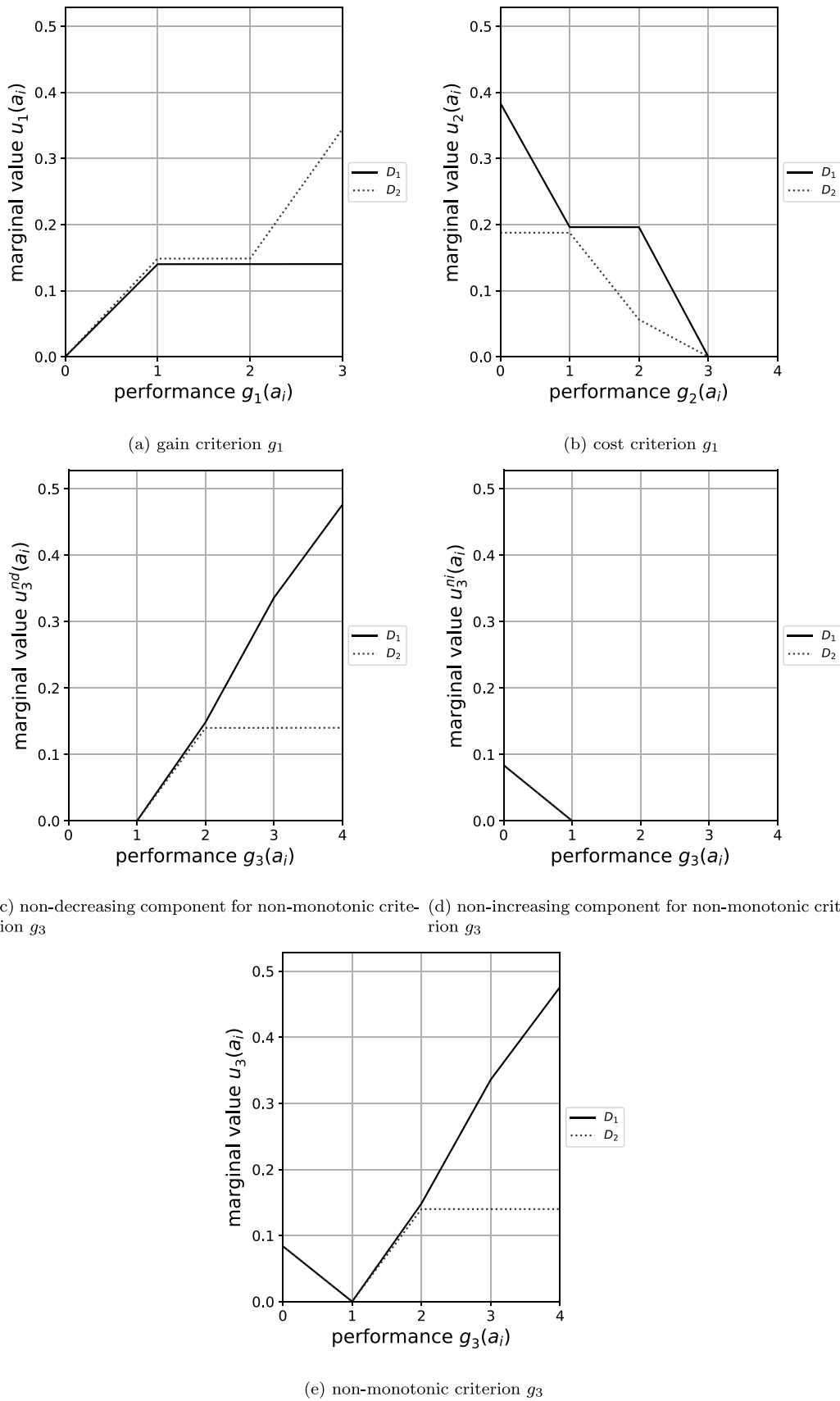
(a) gain criterion g_1

(b) cost criterion g_1

(c) non-decreasing component for non-monotonic criterion g_3

(d) non-increasing component for non-monotonic criterion g_3

(e) non-monotonic criterion g_3

Fig. 4. Marginal value functions for all criteria and decision D_1 and D_2 for Scenario 1 of the illustrative example.

(a) gain criterion g_1

(b) cost criterion g_2

(c) non-decreasing component for non-monotonic criterion g_3

(d) non-increasing component for non-monotonic criterion g_3
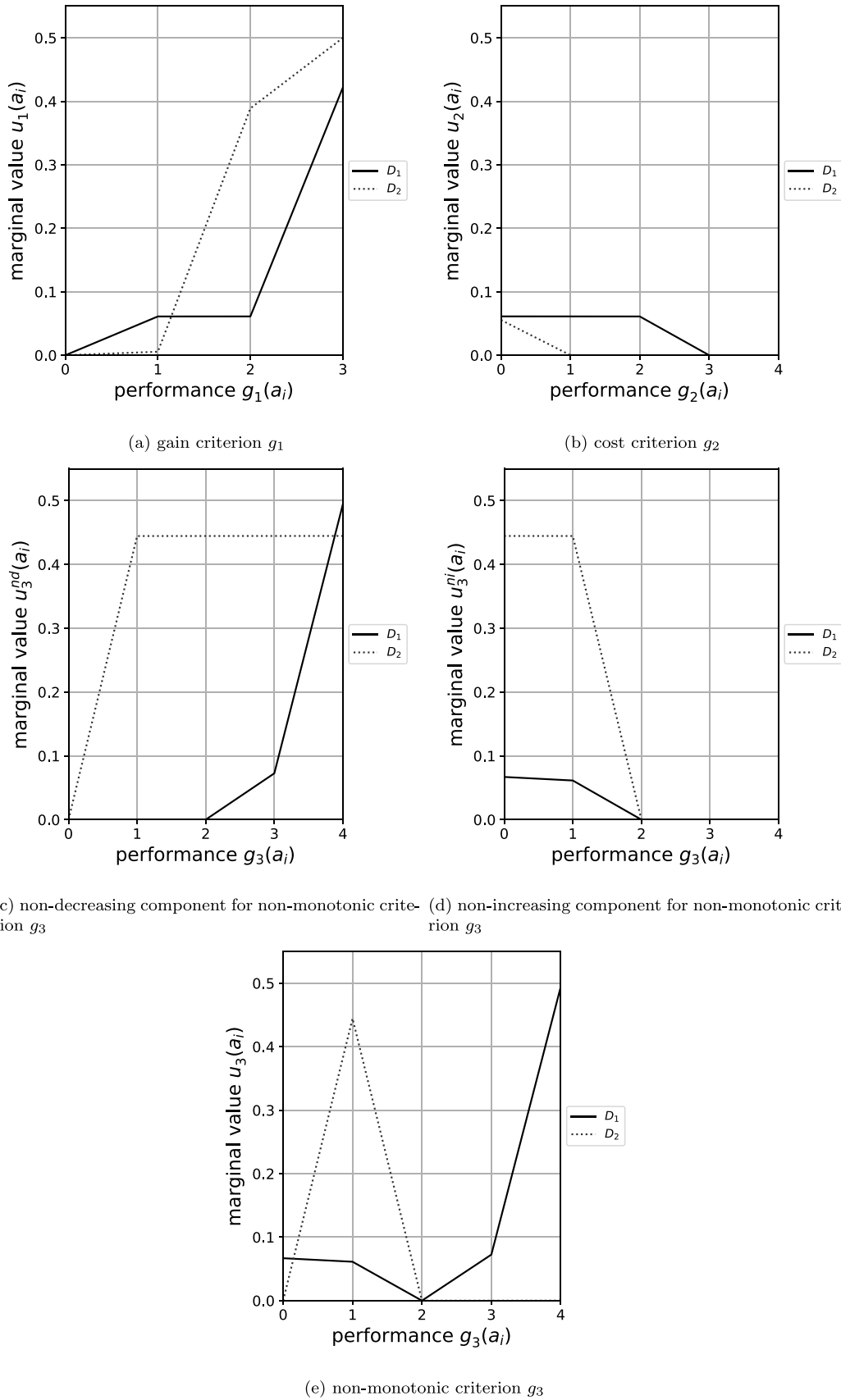
(e) non-monotonic criterion g_3

Fig. 5. Marginal value functions for all criteria and decision D_1 and D_2 for Scenario 2 of the illustrative example.

Table 4

Example constraints defining two inter-related sorting models for Scenario 2.

Type	Constraint
Comprehensive value of an alternative defined using an additive value function	
General constraint form	$U^{D_s}(a_i) = \sum_{j=1}^{m} u_j^{D_s}(a_i)$
Example constraint for D_2 and a_7	$U^{D_2}(a_7) = u_1^{D_2}(a_7) + u_2^{D_2}(a_7) + u_3^{D_2}(a_7)$
Example values assigned to the model variables	$0.4500 = 0.0056 + 0.0 + 0.4444$
Constraints on a comprehensive value of an alternative	
General constraint form	$1 \geq U^{D_s}(a_i) \geq 0$
Example constraint for D_2 and a_7	$1 \geq U^{D_2}(a_7) \geq 0$
Example values assigned to the model variables	$1 \geq 0.4500 \geq 0$
The least preferred marginal value for a monotonic criterion of a gain-type	
General constraint form	$u_j^{D_s}(x_j^1) = 0$
Example constraint for D_1, g_1, and $x_1^1 = 0$	$u_1^{D_1}(0) = 0$
Example values assigned to the model variables	$0 = 0$
Monotonicity constraint for a gain-type criterion	
General constraint form	$u_j^{D_s}(x_j^k) \geq u_j^{D_s}(x_j^{k-1})$
Example constraint for D_2, g_1, $x_1^3 = 2$, and $x_1^2 = 1$	$u_1^{D_2}(2) \geq u_1^{D_2}(1)$
Example values assigned to the model variables	$0.3888 \geq 0.0056$
Monotonicity constraint for a cost-type criterion	
General constraint form	$u_j^{D_s}(x_j^k) \leq u_j^{D_s}(x_j^{k-1})$
Example constraint for D_2, g_2, $x_2^2 = 1$, and $x_2^1 = 0$	$u_2^{D_2}(1) \leq u_2^{D_2}(0)$
Example values assigned to the model variables	$0.0001 \leq 0.0556$
Marginal value for a potentially non-monotonic criterion	
General constraint form	$u_j^{D_s}(x_j^k) = u_{j,nd}^{D_s}(x_j^k) + u_{j,ni}^{D_s}(x_j^k) - t_j^{D_s}$
Example constraint for D_2, g_3, and $x_3^2 = 1$	$u_3^{D_2}(1) = u_{3,nd}^{D_2}(1) + u_{3,ni}^{D_2}(1) - t_3^{D_2}$
Example values assigned to the model variables	$0.4444 = 0.4445 + 0.4445 - 0.4446$
Constraints between the neighboring thresholds separating decision classes	
General constraint form	$b_h^{D_s} \geq b_{h-1}^{D_s} + \varepsilon$
Example constraint for D_1 and $h = 3$	$b_3^{D_1} \geq b_2^{D_1} + \varepsilon$
Example values assigned to the model variables	$0.4972 \geq 0.1807 + 0.0001$
Intra-decision constraints imposed by an assignment example	
General constraint form	$U^{D_s}(a^*) \geq b_{C_{DM}^{D_s}(a^*)-1}^{D_s} - v(a^*)$
Example constraint for $a_5 \rightarrow C_3^{D_1}$	$U^{D_1}(a_5) \geq b_2^{D_1} - v(a_5)$
Example values assigned to the model variables	$0.4944 \geq 0.4416 - 0$
General constraint form	$U^{D_s}(a^*) + \varepsilon \leq b_{C_{DM}^{D_s}(a^*)}^{D_s} + v(a^*)$
Example constraint for $a_5 \rightarrow C_3^{D_1}$	$U^{D_1}(a_5) + \varepsilon \leq b_3^{D_1} + v(a_5)$
Example values assigned to the model variables	$0.4944 + 0.0001 \leq 0.4972 + 0$
Inter-decision constraint imposed by assignment examples	
General constraint form	$U^{D_s}(a^*) \geq U^{D_t}(a^*) + \varepsilon - v(a^*)$
Example constraint for $a_7 \rightarrow C_3^{D_2}$ and $a_7 \rightarrow C_1^{D_1}$	$U^{D_2}(a_7) \geq U^{D_1}(a_7) + \varepsilon - v(a_7)$
Example values assigned to the model variables	$0.4500 \geq 0.1224 + 0.0001 - 0$

- *Detection limit* (g_4) specifies the capacity of the instruments used for exposure assessment to detect the nanomaterials. The better the detection limit is, the safer the exposure scenario is assumed to be [35].
- *Exposure limit* (g_5) indicates the limit of exposure, expressed on five ranges, for a given exposure scenario. The lower this limit, the less risk concerning a given exposure is [35].
- *Quantity* (g_6) of a nanomaterial (in kg) handled in a given scenario. Smaller quantities are preferred as they imply a smaller chance of exposure [53].
- *Engineering controls* (g_7) is a potentially non-monotonic attribute, specifying a setting in which the nanomanufacturing tasks are performed. It refers to the combinations of open (O) or closed (C) system and positive (PP) or negative (NP) pressure.
- *Number of employees* (g_8) indicates the number of people required to handle a given exposure scenario. One cannot define a priori how this attribute is associated with the risk of an exposure scenario [35].
- *Duration of exposure* (g_9) is negatively associated with the risk, i.e., the shorter duration is deemed to be less risky [53].

- *Multiple exposures* (g_{10}) is related to the frequency of exposure (a scenario is safer in case the number of exposures is lesser) [54].

The measurement units, preference directions, performance scales, and encoding of performances for all criteria are provided in Table 5. In general, there are six criteria of cost type, a single gain criterion, and three criteria for which the preference direction is unknown.

4.1.2. Alternatives

The considered set of alternatives is composed of exposure scenarios for nanomanufacturing generated by the JMP software [35]. They correspond to the existing and future types of nanomaterials and manufacturing processes. To demonstrate the proposed method's applicability, we consider a set of 45 exposure scenarios, denoted by $a_1 - a_{45}$. For their performances, see Tables 6 and 7.

4.1.3. Multi-decision sorting

The alternatives are holistically evaluated in terms of four decision attributes that could be considered individually. However,

Table 5
A set of criteria considered in the risk management of exposure scenarios for nanomanufacturing.

g_j	Criterion	Preference	Performance	Code
g_1	Particle size (nm)	none	< 2	1
			2–10	2
			10–100	3
			100–500	4
			500–1000	5
			> 1000	6
g_2	Toxicity	cost	Low	1
			Moderate	2
			High	3
g_3	Airborne capacity	cost	None	0
			Low	1
			Moderate	2
			High	3
g_4	Detection limit	gain	None	0
			Poor	1
			Moderate	2
			Good	3
g_5	Exposure limit (fiber/cc)	cost	< 0.1	1
			0.1–0.2	2
			0.2–0.5	3
			0.5–1.0	4
			> 1.0	5
g_6	Quantity (kg)	cost	< 1	1
			1–100	2
			100–1000	3
			1000–10000	4
			> 10000	5
g_7	Engineering controls	none	O-PP	1
			O-NP	2
			C-PP	3
			O-NP	4
g_8	Number of employees	none	1–3	1
			3–10	2
			11–50	3
			51–100	4
			101–500	5
g_9	Duration of exposure (h)	cost	incidental	1
			< 0.25	2
			< 1	3
			1–5	4
			5–8	5
g_{10}	Multiple exposure (number)	cost	none	0
			1–3	1
			> 3	2
			unknown	3

the multiplicity of these attributes, the reference to the same aspect of production, i.e., safety, and the resulting inter-relations between the examined decisions form the basis for multi-decision sorting. Specifically, the decision attributes correspond to four types of precautions that can be used to reduce the risk. They concern three main aspects of safety: *respirator* (D_1) represents a personal protective equipment, *fume hood* (D_2) and *fume hood with HEPA filter* (D_3) stand for the engineering controls, and *HEPA vacuum cleaner* (D_4) corresponds to the work practices. Let us note that a respirator is a form of a mask with a filter that protects against dangerous substances in the air. A fume hood is a type of ventilation system protecting against harmful gases and toxins. Finally, High Efficiency Particulate Air (HEPA) filter is a filter that has a very high capacity of retaining particles in the range of several micrometers and above, as well as below.

For each precaution type, we make decisions about its requirement during the nanomanufacturing process. The holistic

preference on each decision attribute includes five preference-ordered classes corresponding to the levels of need for the specific precaution: required (C_1), might be required (C_2), optional (C_3), might be optional (C_4), and not required (C_5). The reasoning on the decision attributes is the following: if the exposure scenario is deemed as risky, then a given precaution will be indicated as required. If it is not, then the expert would indicate no need for the precaution. A non-risky scenario is preferred to the risky one.

4.2. Preference information

For forty exposure scenarios ($a_1 - a_{40}$), we consider input provided by the health and safety managers in the form of class assignments on four decision attributes [35]. The experts were asked in a survey what precautions should be taken and in what intensity they should be used given a set of production parameters and features of the nanomaterials based on those presented in Table 5. For the scenarios deemed as risky and dangerous by the specialists, a given precaution is required. In the case of safer scenarios, the requirements are lower, and the necessity of some precaution types is not required. For the answers of the experts, see Table 9. The numbers of reference alternatives assigned to each class for the four decisions are presented in Table 8. The most common decisions are "required" (C_1), "optional" (C_3) and "not required" (C_5), whereas the least chosen classes in the survey classes are "might be required" (C_2) and "might be optional" (C_4).

Let us discuss the performances and assignments for the three example reference alternatives (a_{26}, a_5, and a_1). Alternative a_{26} attains very favorable performances on four criteria of cost type g_2, g_6, g_9 and g_{10} and the best performance on gain criterion g_4. As a result, the assigned classes for *respirator*, *fume hood with HEPA filter* and *HEPA vacuum cleaner* are "not required" and for *fume hood* – "required". Consequently, the most risky evaluation in terms of *fume hood* is linked to the performances on g_3, g_5, and the potentially non-monotonic criteria. Furthermore, a_5 performs poorly on g_2, g_3, g_4, g_5, g_9 and g_{10}, which was an important reason to classify this scenario to C_1 ("precaution is required") for all precaution types. Finally, a_1 attains favorable performances on criteria g_2, g_3, g_4 and g_{10}, while being less advantageous on criteria g_5, g_6 and g_9. Therefore, its classification for all decisions is between "might be required" (C_2) and "optional" (C_3).

4.3. Research questions

The research goal consists of understanding under which operational conditions and according to which characteristics of the nanomaterials, different types of precautions can be required, might be required, are optional, might be optional, or not be required. This contribution results in a sorting model capable of providing decision recommendations on multiple risk management measures, corresponding to various precautions, for the same exposure scenario.

We wish to find a set of additive value functions and class thresholds that will describe the allocation to a particular class for each decision attribute based on the scenario described in terms of a set of ten criteria/attributes. Such a preference model is expected to capture the patterns from experts' choices. Then, we will demonstrate that these discovered regularities can be used to support decision making. Thus, the inferred preference model involving intra- and inter-decision relations will be used to classify a set of non-reference exposure scenarios.

Table 6
Performance matrix of the reference exposure scenarios $a_1 - a_{40}$ (↑ and ↓ indicate gain and cost criteria, respectively; – denotes criteria without a pre-defined preference direction).

	g_1 –	g_2 ↓	g_3 ↓	g_4 ↑	g_5 ↓	g_6 ↓	g_7 –	g_8 –	g_9 ↓	g_{10} ↓		g_1 –	g_2 ↓	g_3 ↓	g_4 ↑	g_5 ↓	g_6 ↓	g_7 –	g_8 –	g_9 ↓	g_{10} ↓
a_1	4	1	0	3	3	4	3	5	4	1	a_{21}	6	1	2	2	2	3	1	4	4	1
a_2	5	2	0	0	2	4	1	2	3	1	a_{22}	6	1	3	0	5	3	3	1	2	1
a_3	5	2	3	2	4	1	2	5	5	0	a_{23}	4	1	0	3	4	2	1	1	3	2
a_4	4	1	3	2	3	4	2	4	3	2	a_{24}	6	3	2	1	5	5	2	3	5	0
a_5	1	3	3	0	3	1	2	2	4	3	a_{25}	2	1	2	0	1	4	4	4	2	0
a_6	3	1	1	0	4	5	1	4	1	3	a_{26}	1	1	2	3	2	1	3	3	1	0
a_7	3	1	1	1	2	3	2	3	3	3	a_{27}	1	1	2	1	1	2	2	2	3	1
a_8	4	3	3	1	1	1	4	3	3	0	a_{28}	6	2	2	2	5	4	1	1	1	2
a_9	4	2	0	1	5	5	1	1	5	1	a_{29}	5	1	1	3	4	3	1	1	1	1
a_{10}	2	1	3	2	1	5	1	3	4	3	a_{30}	2	3	1	2	5	2	2	1	1	1
a_{11}	2	2	3	0	1	3	2	3	5	2	a_{31}	6	2	2	2	3	3	4	2	2	3
a_{12}	4	2	1	1	1	3	4	1	4	3	a_{32}	6	2	0	0	1	2	2	1	3	0
a_{13}	1	1	3	3	4	5	1	2	2	0	a_{33}	5	1	0	1	3	5	2	4	2	2
a_{14}	6	1	0	2	4	1	3	3	4	3	a_{34}	1	3	1	2	3	3	2	1	4	0
a_{15}	2	3	0	1	3	4	3	1	4	0	a_{35}	6	1	2	3	4	2	2	3	3	2
a_{16}	6	3	1	3	3	1	1	4	5	0	a_{36}	5	3	2	1	2	4	4	5	5	3
a_{17}	6	1	0	2	2	2	3	5	4	0	a_{37}	5	3	2	3	1	2	3	1	2	3
a_{18}	1	1	3	2	1	5	1	5	5	1	a_{38}	4	2	3	3	2	1	2	2	2	1
a_{19}	1	1	1	2	2	3	2	2	1	0	a_{39}	2	1	0	0	5	4	2	5	1	3
a_{20}	6	1	1	0	1	1	4	5	2	0	a_{40}	6	2	1	0	2	4	1	3	4	2

Table 7
Performance matrix of the non-reference exposure scenarios $a_{41} - a_{45}$ (↑ and ↓ indicate gain and cost criteria, respectively; – denotes criteria without a pre-defined preference direction).

	g_1 –	g_2 ↓	g_3 ↓	g_4 ↑	g_5 ↓	g_6 ↓	g_7 –	g_8 –	g_9 ↓	g_{10} ↓
a_{41}	5	2	3	2	3	3	3	3	1	2
a_{42}	2	1	0	2	2	2	2	2	3	3
a_{43}	5	1	0	1	0	4	1	1	2	3
a_{44}	0	2	2	0	3	3	0	3	2	1
a_{45}	1	1	2	2	3	4	2	0	4	2

4.4. Results

4.4.1. Marginal value functions

The marginal value functions for the ten criteria and four decision attributes are presented in Figs. 6 and 7. They preserve the imposed monotonicity constraints. In particular, the marginal value function u_2 for the cost-type criterion *toxicity* is non-increasing, i.e., a value assigned to the "moderate" performance is always greater than to the "high" performance and equal or lesser (depending on the decision attribute) than the value corresponding to the "low" toxicity (see Fig. 6). Similarly, the marginal value function u_4 for the gain-type criterion *detection limit* is non-decreasing. It assigns a strictly greater value to "poor" than to "null" performance, and exhibits a stable or a slightly increasing trend from "poor" through "moderate" to "good" detection limit (see Fig. 6). On the contrary, the marginal value functions for the criteria with unknown monotonicity exhibit a non-monotonic trend. For example, the least marginal value on u_1 does not correspond to either of the extreme performances (see Fig. 6). However, the corresponding non-decreasing and non-increasing components adhere to the monotonicity constraints.

The impact of each criterion on the recommended decision can be estimated with the maximal share of each criterion in the comprehensive value (see Table 10). The greatest shares correspond to: for *respirator* – *airborne capacity* and *detection limit*, for *fume hood* – *particle size* and *airborne capacity*, for *fume hood with HEPA filter* – *airborne capacity*, and for *HEPA vacuum cleaner* – *exposure limit* and *airborne capacity*. The values of bias for all non-monotonic criteria are given in Table 11. They allowed normalizing the performance of an anti-ideal alternative to zero, as described in Section 2.

For the marginal value function u_1 for *particle size*, the greatest value is assigned to the size greater than 1000 nm for all decisions but *fume hood with HEPA filter*, for which the greatest value is attained for the size of lesser than 2 nm. The function is of "W" shape for *respirator*, *fume hood with HEPA filter* and *HEPA vacuum cleaner* with a significant peak corresponding to sizes of 10 − 100 nm or 100 − 500 nm. Such a shape is implied by the largest decrease of value for the non-increasing component observed between sizes 10–100 nm, 100–500 nm, and 500–1000 nm, and the largest increase of value for the non-decreasing component observed for sizes between 2–10 nm, 10–100 nm, and 100–500 nm. For *fume hood*, the shape of u_1 is similar to "V", and the zero value is assigned to the intermediate size.

The value function u_2 for *toxicity* indicates a negligible difference between the low and moderate performances. Such a difference is slightly greater only for *HEPA vacuum cleaner*. Intuitively, the precautions are less required with low toxicity. This criterion has a very low impact on the comprehensive value when considering *respirator* and *fume hood*. This means that this precaution type is needed even with low toxicity.

Airborne capacity has a very significant impact on the alternatives' assignments. The "null" performance vastly contributes to reducing the requirement of a given precaution type. In addition, for *fume hood* the value differences between performances "null" and "low" or "moderate" and "high" are very marginal or non-existing. Thus, in this case, only the difference between "low" and "moderate" matters. For the remaining decision attributes, the difference between "moderate" and "high" and "low" and "moderate" are huge.

The shapes of marginal value functions for *detection limit* (u_4) reveal high discrepancy between the decisions, even if they are similar in terms of a general trend. For *fume hood*, this criterion has almost no effect on the comprehensive value, whereas for *respirator* – the impact of g_4 is significant. The main difference in terms of a trend is that for *fume hood with HEPA filter*, the difference between values assigned to "poor" and "moderate" or "good'" detection limits is negligible, whereas for the *respirator* and *HEPA vacuum cleaner* it is around 0.025.

Analogously, the slight differences in the shapes of value functions for various decision attributes can be observed for the *exposure limit* (u_5). For *respirator*, *fume hood with HEPA filter*, and *HEPA vacuum cleaner*, the greatest value difference is between the performances of < 0.1 and 0.1−0.2, whereas for *fume hood* – the
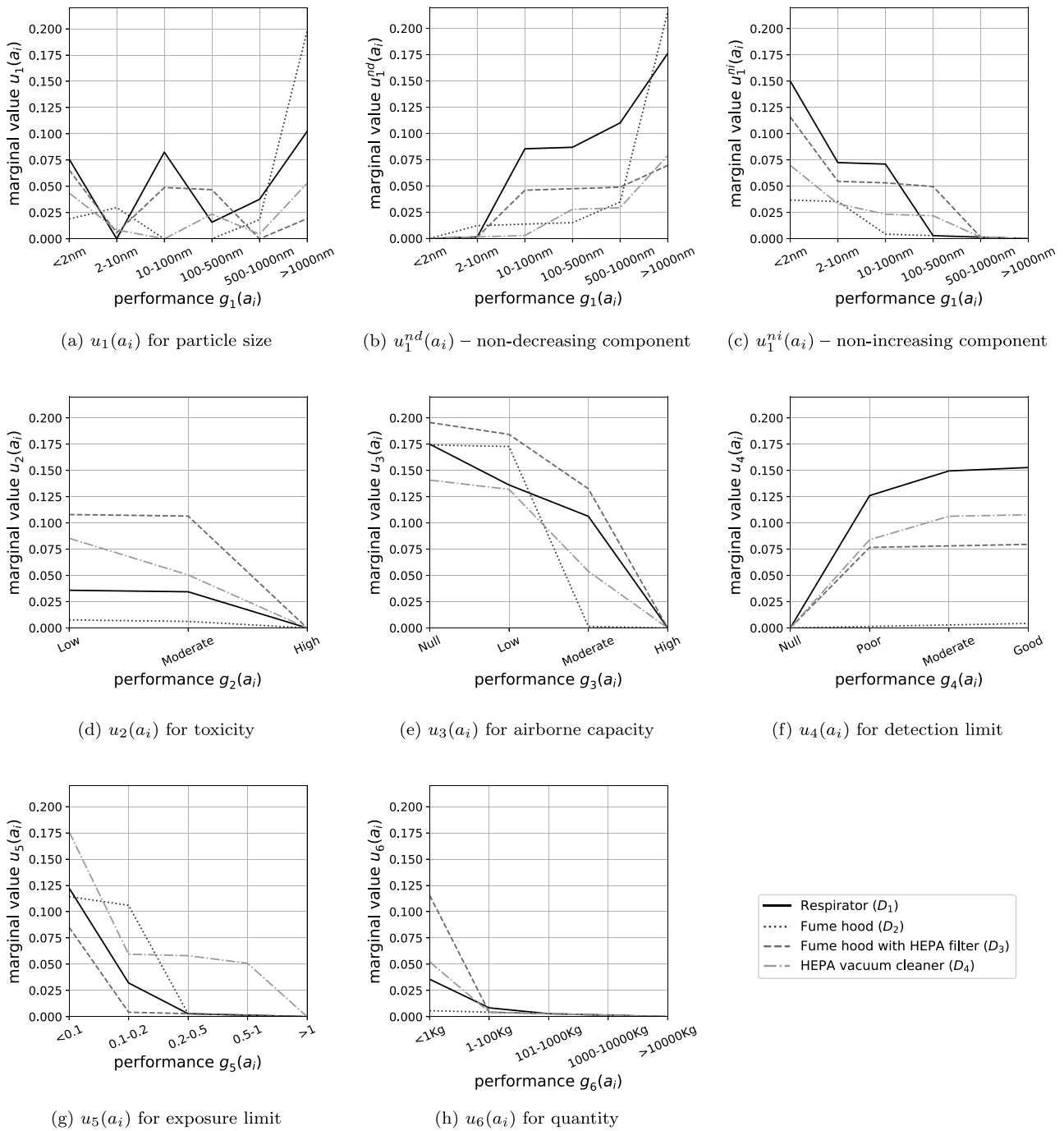
(a) $u_1(a_i)$ for particle size

(b) $u_1^{nd}(a_i)$ − non-decreasing component

(c) $u_1^{ni}(a_i)$ − non-increasing component

(d) $u_2(a_i)$ for toxicity

(e) $u_3(a_i)$ for airborne capacity

(f) $u_4(a_i)$ for detection limit

(g) $u_5(a_i)$ for exposure limit

(h) $u_6(a_i)$ for quantity

Fig. 6. Marginal value functions for criteria g_1 – g_6 for four decision attributes.

Table 8
The number of reference alternatives assigned to a given class on four decision attributes.

Decision	Respirator (D_1)	Fume hood (D_2)	Fume hood with HEPA filter (D_3)	HEPA vacuum cleaner (D_4)
"Required" (C_1)	8	27	10	6
"Might be required" (C_2)	3	3	4	6
"Optional" (C_3)	14	5	14	17
"Might be optional" (C_4)	3	2	2	2
"Not required" (C_5)	12	3	10	9

most significant difference is between the limits of $0.1 - 0.2$ and $0.2 - 0.5$. In addition, for all types of precautions but *HEPA vacuum cleaner*, the marginal value assigned to performances at least 0.2 is close to zero.

The marginal value function for *quantity* (u_6) indicates that the production of small quantities of nanomaterial is considered less risky. In contrast, for the mass production above 1kg — the marginal values are close to zero. The production of a
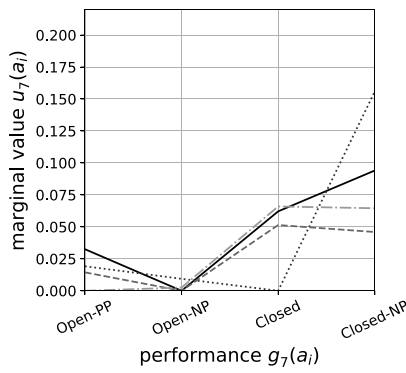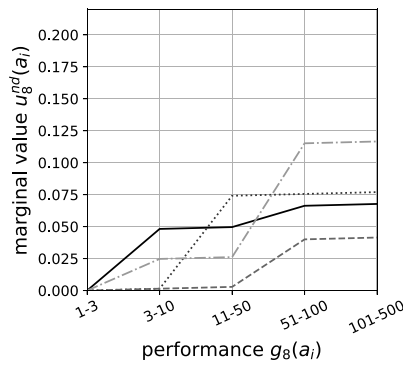
(a) $u_7(a_i)$ for engineering controls
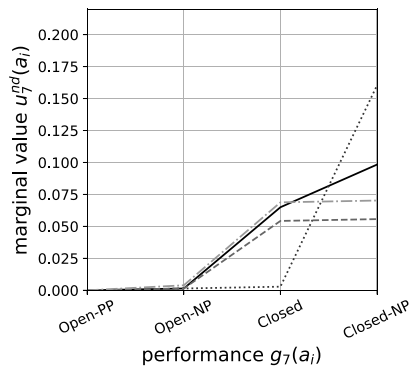
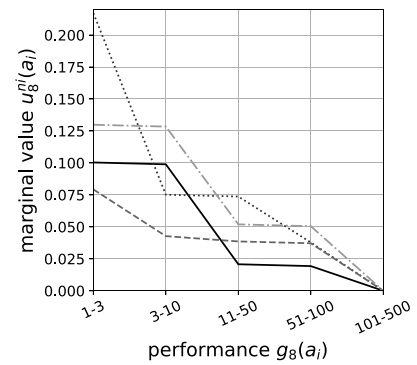(b) $u_7^{nd}(a_i)$ for engineering controls
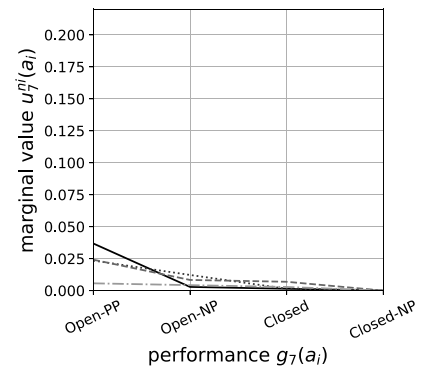
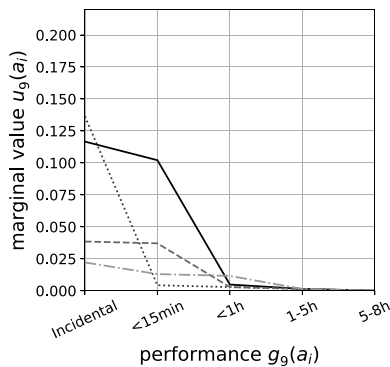(c) $u_7^{ni}(a_i)$ for engineering controls
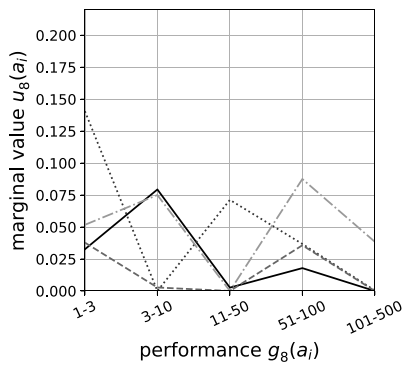
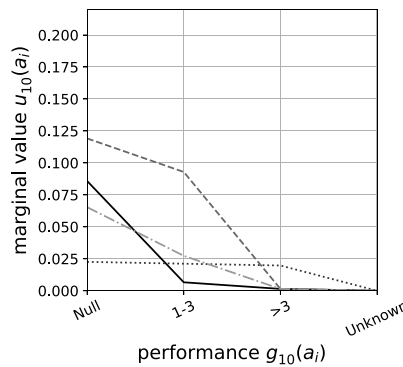(d) $u_8(a_i)$ for the number of employees

(e) $u_8^{nd}(a_i)$ for the number of employees

(f) $u_8^{ni}(a_i)$ for the number of employees

(g) $u_9(a_i)$ for duration of exposure
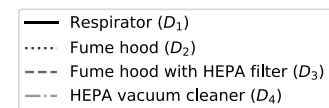
(h) $u_{10}(a_i)$ for multiple exposures

Fig. 7. Marginal value functions for criteria $g_7 - g_{10}$ for four decision attributes.

small amount of nanomaterial reduces the chance of exposure. At higher manufacturing loads, the chances of accidental or undesired contact are higher. In the context of *fume hood*, g_6 has no significant impact on the comprehensive value, and for each quantity produced, this precaution is required.

The marginal value functions for the *engineering controls* (u_7) indicate that the closed systems are safer than open ones, particularly those with the negative pressure. The most desired configuration depends on the decision attribute. *Fume hood* is more required in the closed system, and for the remaining types of precautions, an open system with negative pressure is the most needed. The non-decreasing component (u_7^{nd}) is prevailing, implying high marginal values for the closed systems, whereas

the non-increasing component (u_7^{ni}) assigns low values to all possible configurations of the *engineering controls*.

The marginal functions for the *number of employees* (u_8) reveal a slightly different shape for each decision attribute. We can observe two main peaks corresponding to 3–10 employees for *respirator* and *HEPA vacuum cleaner* or 51–100-employees for *respirator*, *HEPA vacuum cleaner* and *fume hood with HEPA filter*. The performances with the least assigned marginal value are 11–50 employees for *respirator*, *HEPA vacuum cleaner*, and *fume hood with HEPA filter* or 101–500 employees for *respirator*, *fume hood*, and *fume hood with HEPA filter*. For all decision attributes, the values assigned to 101–500 employees are

Table 9
Class assignments provided by the experts for reference alternatives and their comprehensive values for four decision attributes.

$i \, / \, j$	$C_{DM}^{D_j}(a_i)$				$U^{D_j}(a_i)$				$i \, / \, j$	$C_{DM}^{D_j}(a_i)$				$U^{D_j}(a_i)$			
	1	2	3	4	1	2	3	4		1	2	3	4	1	2	3	4
a_1	3	2	2	3	0.4539	0.2134	0.5800	0.5502	a_{21}	3	2	3	3	0.4874	0.3970	0.4894	0.4773
a_2	2	1	2	2	0.4039	0.3493	0.4207	0.3707	a_{22}	1	1	1	1	0.3445	0.3742	0.3493	0.2993
a_3	1	1	2	2	0.3445	0.0668	0.4207	0.3707	a_{23}	3	1	3	3	0.4607	0.3742	0.4922	0.4773
a_4	1	1	1	3	0.2296	0.0835	0.2772	0.3779	a_{24}	3	1	1	1	0.4231	0.3037	0.3476	0.2588
a_5	1	1	1	1	0.1954	0.0382	0.1881	0.2330	a_{25}	5	1	5	5	0.5659	0.3742	0.5708	0.5559
a_6	3	1	3	3	0.4231	0.3742	0.4312	0.3779	a_{26}	5	1	5	5	0.7060	0.3742	0.7145	0.5559
a_7	3	1	3	3	0.4231	0.3742	0.4279	0.3779	a_{27}	5	1	5	5	0.5659	0.1814	0.5708	0.5639
a_8	3	1	3	3	0.4874	0.3742	0.4922	0.4773	a_{28}	4	4	4	4	0.5772	0.5255	0.4300	0.3405
a_9	3	1	5	3	0.4231	0.3628	0.5708	0.3779	a_{29}	4	4	4	4	0.5551	0.5245	0.5598	0.4845
a_{10}	1	1	1	3	0.3445	0.2464	0.2929	0.3779	a_{30}	3	3	3	2	0.4500	0.5171	0.4417	0.3548
a_{11}	1	1	1	1	0.1639	0.2534	0.2019	0.2420	a_{31}	5	1	3	3	0.6738	0.3742	0.4279	0.4773
a_{12}	5	5	5	5	0.5659	0.5957	0.5877	0.5867	a_{32}	5	5	5	5	0.5659	0.6721	0.5708	0.5559
a_{13}	5	1	3	3	0.5659	0.0783	0.4279	0.4411	a_{33}	4	1	3	3	0.4988	0.2745	0.4574	0.4773
a_{14}	5	3	5	5	0.5659	0.4622	0.5708	0.5559	a_{34}	3	1	3	3	0.4874	0.3742	0.4922	0.4640
a_{15}	3	1	3	3	0.4874	0.3742	0.4922	0.4773	a_{35}	2	1	1	2	0.4159	0.3199	0.3493	0.3707
a_{16}	5	3	5	5	0.5659	0.4619	0.5708	0.5559	a_{36}	2	1	1	2	0.3975	0.2850	0.2609	0.3064
a_{17}	5	3	5	5	0.6526	0.5171	0.5815	0.6206	a_{37}	5	1	3	3	0.6244	0.2875	0.4279	0.4773
a_{18}	3	1	3	3	0.4231	0.1845	0.4442	0.4773	a_{38}	3	1	3	3	0.4589	0.1570	0.4854	0.4114
a_{19}	5	3	5	5	0.7145	0.4794	0.6035	0.5948	a_{39}	1	1	1	1	0.3290	0.3591	0.3493	0.2993
a_{20}	3	2	2	2	0.7145	0.6815	0.7145	0.6801	a_{40}	1	5	1	1	0.3445	0.5957	0.3331	0.2993

Table 10
The maximal shares of the individual criteria in the comprehensive values (in %) for four decision attributes.

Criterion	Respirator (D_1)	Fume hood (D_2)	Fume hood with HEPA filter (D_3)	HEPA vacuum cleaner (D_4)
Particle size (g_1)	10.22%	20.6%	7.32%	6.18%
Toxicity (g_2)	3.58%	0.79%	12.04%	9.97%
Airborne capacity (g_3)	17.5%	18.15%	21.82%	16.44%
Detection limit (g_4)	15.27%	0.44%	8.87%	12.58%
Exposure limit (g_5)	12.24%	11.91%	9.5%	20.57%
Quantity (g_6)	3.56%	0.59%	12.88%	6.09%
Engineering controls (g_7)	9.4%	16.26%	5.73%	7.7%
Number of employees (g_8)	7.93%	14.65%	4.22%	10.22%
Duration of exposure (g_9)	11.66%	14.21%	4.29%	2.58%
Multiple exposure (g_{10})	8.58%	2.35%	13.28%	7.63%

Table 11
Values of bias for each non-monotonic criterion for all decision attributes.

Criterion	Respirator (D_1)	Fume hood (D_2)	Fume hood with HEPA filter (D_3)	HEPA vacuum cleaner (D_4)
Particle size (g_1)	0.10	0.03	0.07	0.04
Engineering controls (g_7)	0.01	0.01	0.01	0.01
Number of employees (g_8)	0.09	0.11	0.06	0.11

smaller than those associated with 51–100 employees. The non-increasing (u_8^{ni}) and non-decreasing (u_8^{nd}) components explain why the resulting marginal functions differ across various decisions. For *respirator*, the non-decreasing component increases only between 3–10 and 11–50 employees, whereas the functions for the remaining decisions in this performance area are stable. In turn, they increase for the number of employees between 1–3 and 3–10 and between 11–50 and 51–100.

The *duration of exposure* (u_9) is particularly important in the context of *respirator* and *fume hood*. When the time exceeds one hour, there is a greater safety concern, and thus all precautions are more required. A short duration of exposure can motivate the reduction of safety requirements.

The analysis of marginal functions for the *number of exposures* (u_{10}) indicates that if the exposures are non-existing, the marginal value is high, hence leading to the assignment to a less risky class for all decision attributes. In case there is at least one exposure, the *respirator* is more required. The precautions involving the HEPA filter are required when three exposures are exceeded. In turn, *fume hood* is equally necessary for all values when the *number of exposures* is known, and the marginal functions attain zero when the value of this criterion cannot be determined.

In general, the marginal value functions for the decision attributes concerning the use of the HEPA filter are similar for the *airborne capacity*, *detection limit*, *quantity*, *engineering controls*, *duration of exposure*, and *number of exposures*. In turn, the marginal functions corresponding to the two *fume hoods* differ on all criteria. This may suggest that these two precautions are complementary, and depending on the conditions, one should choose the fume hood either with the filter or without it. Finally, the functions for the *respirator* are more similar to those derived from the analysis for the *HEPA vacuum cleaner* and *fume hood with HEPA filter* than for the *fume hood*.

4.4.2. Class assignments for the reference alternatives

The comprehensive values and class assignments for the forty reference alternatives with respect to the four decision attributes are provided in Table 9. The constructed model reproduced the desired assignments for all reference exposure scenarios but a_1, a_{20}, and a_{28}. These alternatives form the minimal subset that had to be removed to impose the consistency of the experts' judgments with an assumed preference model. The comparison of desired and resulting assignments for these three exposure scenarios is given in Table 12. For example, the inferred model

Table 12
Class assignments derived with the constructed preference model for the reference alternatives, not aligning with the ones desired by the experts.

i / j	$C_{DM}^{D_j}(a_i)$				$C^{D_j}(a_i)$			
	1	2	3	4	1	2	3	4
a_1	3	2	2	3	3	1	5	4
a_{20}	3	2	2	2	5	5	5	5
a_{28}	4	4	4	4	5	4	3	2

Table 13
Class thresholds separating the five preference-ordered classes for four decision attributes.

D_j	$b_1^{D_j}$	$b_2^{D_j}$	$b_3^{D_j}$	$b_4^{D_j}$
D_1	0.3480	0.4195	0.4909	0.5624
D_2	0.3778	0.4492	0.5207	0.5921
D_3	0.3529	0.4243	0.4958	0.5672
D_4	0.3028	0.3743	0.4809	0.5524

evaluated alternative a_{20} as "not required" (C_5) for all decision classes, while the experts indicated that it should be "optional" and "might be required". This was implied by the most preferred or nearly the best performances on the monotonic criteria g_2, g_3, g_5, g_6 and g_{10}, as well as the performances on the non-monotonic criteria g_1 and g_7 that were assigned the greatest marginal values according to the constructed model.

The thresholds separating the decision classes on a scale of a comprehensive value for all decisions are given in Table 13. Let us remind that the range delimited by these thresholds in which a comprehensive value of a given alternative falls determines its assignment to the respective class. For example, $U^{D_1}(a_2) = 0.4039$ is not lesser than $b_1^{D_1} = 0.3480$ and lesser than $b_2^{D_1} = 0.4195$, which allows to reproduce the assignment of a_2 to C_2 provided by the experts. Although these thresholds have similar values for various decision attributes, we can observe that, e.g., on D_4, they are by $0.04 - 0.07$ lower than on D_2.

4.4.3. Inter-decision relationships

Let us focus on the inter-decision relationships implied by the specificity of the considered multi-decision sorting problem. The impact of the individual criteria on the comprehensive values as well as the relations between the latter ones for different decision attributes are demonstrated in Fig. 8 for the four reference alternatives (a_9, a_3, a_{13}, and a_{33}).

For example, a_9 was assigned to C_5 for D_3, to C_3 for D_1 and D_4, and to C_1 for D_2. This information can be interpreted in such a way that when considering different types of precautions in the context of a_9, their ranking is the following: fume hood with HEPA filter (D_3), respirator (D_1) and HEPA vacuum cleaner (D_4), fume hood (D_2). Such a ranking is reflected in the comprehensive values on the respective decision attributes: $U^{D_3}(a_9) > U^{D_1}(a_9)$, $U^{D_4}(a_9) > U^{D_2}(a_9)$. The analysis of marginal values for a_9 indicates that, depending on the decision context, the same performance can yield very different contributions to the comprehensive values. This, in turn, may result in the extreme assignments for various decision attributes (e.g., the most preferred class on D_3 and the least preferred class on D_2). The comprehensive value of a_9 for *fume hood with HEPA filter* (D_3) is equal to 0.5707. Such a great value is implied mainly by the following high contributions from the individual criteria: $u_2^{D_3}(a_9) = 0.1065$, $u_3^{D_3}(a_9) = 0.1956$, $u_4^{D_3}(a_9) = 0.0767$, and $u_5^{D_3}(a_9) = 0.0928$. In turn, for *fume hood*, despite a slightly higher value for $u_8^{D_2}(a_9) = 0.1406$, the marginal value of a_9 derived from $u_5^{D_2}(a_9)$, $u_6^{D_2}(a_9)$, and $u_9^{D_2}(a_9)$ are nearly zero. As a result, comprehensive value is lower (0.3627) than for other decision attributes.

The desired assignments of a_3 were either C_2 on D_3 and D_4 or to C_1 on D_1 and D_2. Consequently, its comprehensive values on all decision attributes are relatively low, while being slightly higher for the *fume hood with HEPA filter* and *HEPA vacuum cleaner* than for the *respirator* or *fume hood*. The assignments to classes representing more risky scenarios are mainly due to the low marginal values from the following criteria: *airborne capacity* (u_3), *exposure limit* (u_5), *engineering controls* (u_7), *number of employees* (u_8), and *duration of exposure* (u_9). The differences in the assignments on D_1 and D_3 can be explained, e.g., with respect to *toxicity* ($u_2^{D_1} = 0.0344$ and $u_2^{D_3} = 0.1065$) and *quantity* ($u_6^{D_1} = 0.0356$ and $u_6^{D_3} = 0.1155$), making the *respirator* "required" with a comprehensive value of 0.3444 and *fume hood with HEPA filter* – "might be required" with a greater comprehensive value of 0.4207. When it comes to *fume hood*, a_3 attained very low values on all criteria, making it "required". In case of *HEPA vacuum cleaner*, a higher value justifying an assignment to C_2 is implied by the significant contributions from the following criteria: *toxicity*, *airborne capacity*, *detection limit*, *exposure limit*, *number of employees*, and *multiple exposure*.

The difference in marginal values assigned to the same performances for various decision attributes as well as the inter-decision relationships between comprehensive values implied by the experts' assignments can be also observed for a_{13} and a_{33} (see Fig. 8). On the one hand, the comprehensive values of a_{13} range from 0.0783 for D_2 to 0.5659 for D_1. On the other hand, the large differences in marginal values assigned to a_{33} on various decision attributes for *toxicity*, *detection limit*, *number of employees*, and *duration of exposure* imply that it can be assigned to classes ranging from C_1 for D_2 to C_4 for D_1. As a result, the ranking of precautions associated with a_{33} in terms of safety requirements (starting from the least required) reproduced by the constructed model is as follows: *respirator*, *fume with HEPA filter* and *HEPA vacuum cleaner*, *fume hood*.

4.4.4. Intra-decision relationships

To justify the assignments of alternatives to the respective classes, in Fig. 9, we demonstrate the comprehensive values of selected exposure scenarios and class thresholds. For each decision attribute, we depicted a single alternative assigned to each class. The comprehensive values of exposure scenarios assigned to better classes are greater than for the alternatives assigned to the classes associated with greater risk. For example, the following relations between comprehensive values on D_1: $U^{D_1}(a_{37}) > U^{D_1}(a_{33}) > U^{D_1}(a_7) > U^{D_1}(a_2) > U^{D_1}(a_4)$ reflect the expert judgments. Let us explain a few example assignments in terms of marginal values attained on the respective criteria and the comparison of comprehensive values with the class thresholds.

When it comes to the evaluation of a_2 in terms of *respirator* (D_1), it attains the greatest marginal values for *airborne capacity* ($u_3^{D_1}(a_2) = 0.1750$) and *number of employees* ($u_8^{D_1}(a_2) = 0.0793$). However, its negligible marginal values derived from *detection limit* ($u_4^{D_1}(a_2) = 0$), *quantity* ($u_6^{D_1}(a_2) = 0.0014$), *duration of exposure* ($u_9^{D_1}(a_2) = 0.0048$), and *multiple exposure* ($u_{10}^{D_1}(a_2) = 0.0065$) imply a relatively small comprehensive value. Since $b_1^{D_1} = 0.3480 \leq U^{D_1}(a_2) = 0.4039 < b_2^{D_1} = 0.4195$, a_2 is assigned to C_2 on D_1. As far as a_4 is concerned, it attained zero marginal values on a few criteria and very low values on other criteria ($u_3^{D_1}(a_4) = 0$, $u_5^{D_1}(a_4) = 0.0028$, $u_6^{D_1}(a_4) = 0.0014$, $u_7^{D_1}(a_4) = 0$, and $u_{10}^{D_1}(a_4) = 0.0014$). Therefore, despite high values for *toxicity* ($u_2^{D_1}(a_4) = 0.0358$) and *detection limit* ($u_4^{D_1}(a_4) = 0.1495$), it is assigned to C_1 due to $U^{D_1}(a_4) = 0.2296 < b_4^{D_1} = 0.3480$.

When comparing the assignments of a_{14} and a_{40} in terms of *fume hood* (D_2), these alternatives perform similarly on g_1, g_2,

(a) a_3
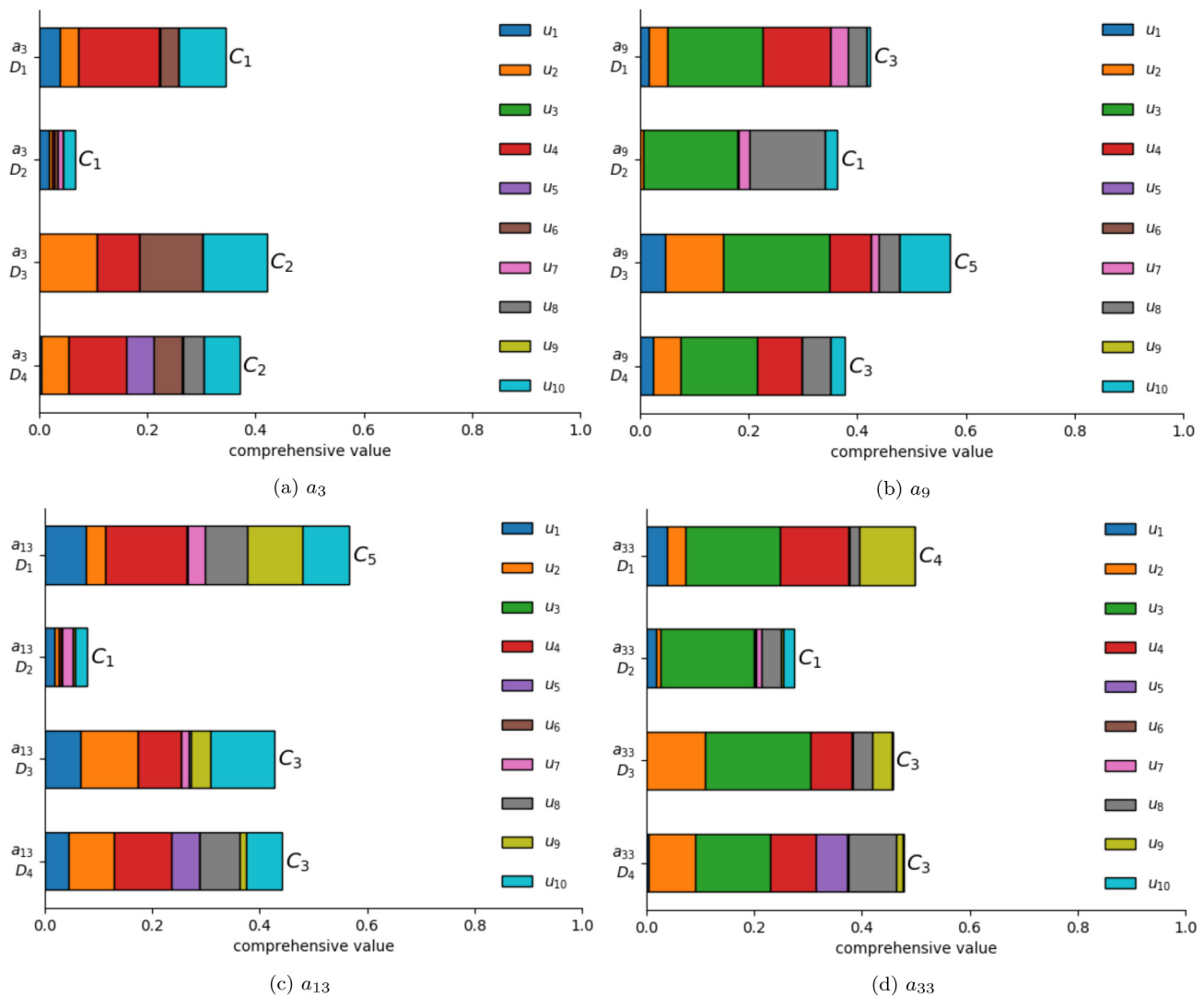

(b) a_9

(c) a_{13}

(d) a_{33}

Fig. 8. Marginal and comprehensive values and class assignments demonstrating the inter-decision relationships for four reference exposure scenarios in terms of four decision attributes (Respirator – D_1, Fume hood – D_2, Fume hood with HEPA filter – D_3, and HEPA vacuum cleaner – D_4).

g_3, and g_8 (e.g., $u_1^{D_2}(a_{14}) = 0.1976$ and $u_1^{D_2}(a_{40}) = 0.1976$ or $u_2^{D_2}(a_{14}) = 0.0076$ and $u_2^{D_2}(a_{40}) = 0.0062$). However, the more advantageous performances of a_{40} on g_5, g_7, and g_{10} imply that it is assigned to C_5 as compared to C_3 for a_{14}. Even though a_2 attains comparable marginal values to a_{40} on six criteria (g_2, g_3, g_4, g_5, g_6, and g_7), it is significantly less preferred on g_1 and g_8 (e.g., $u_1^{D_2}(a_2) = 0.0182 < u_1^{D_2}(a_{40}) = 0.1976$). As a result, a_2 has a very low comprehensive value ($U^{D_2}(a_2) = 0.3492$), justifying the assignment to the least preferred class C_1.

4.5. Classification of the non-reference alternatives

The model inferred from the analysis of reference alternatives can be used to classify other exposure scenarios. Thus, we first used expert knowledge to build a preference model. The latter is subsequently applied to evaluate other alternatives in a way that is consistent with the experts' value system and hence could be accepted by them. In this way, the proposed method can support nanomaterials' exposure management, suggesting the reasons for concern regarding some nanomanufacturing tasks performed by the workers [47,55].

For this purpose, let us consider five non-reference alternatives presented in Table 7. Their comprehensive values and the

Table 14
Comprehensive values and class assignments for the non-references alternatives for the four decision attributes.

a_i / D_j	$C^{D_j}(a_i)$				$U^{D_j}(a_i)$			
	1	2	3	4	1	2	3	4
a_{41}	3	2	1	3	0.4701	0.4202	0.2204	0.3781
a_{42}	4	1	3	3	0.5168	0.2631	0.4892	0.4362
a_{43}	5	3	3	5	0.6445	0.5061	0.4890	0.5936
a_{44}	1	1	1	1	0.2470	0.1031	0.3468	0.2763
a_{45}	2	1	2	3	0.3878	0.2019	0.4150	0.3893

respective class assignments are given in Table 14. Since the comprehensive values attained by these alternatives differ vastly from one decision attribute to another, the suggested classes differ too. For example, a_{42} is assigned to C_1 on D_2, to C_3 on D_3 and D_4, and to C_4 on D_1, whereas the classes for a_{43} range from C_3 on D_2 and D_3 to C_5 on D_1 and D_4. Note, however, that although the comprehensive values of a_{44} differ with respect to various precaution types, they are all very low, justifying the assignment to C_1 on all decision attributes.

For the five non-reference alternatives, the contribution of the individual criteria in the comprehensive values, as well as the assignments derived from the comparison of comprehensive
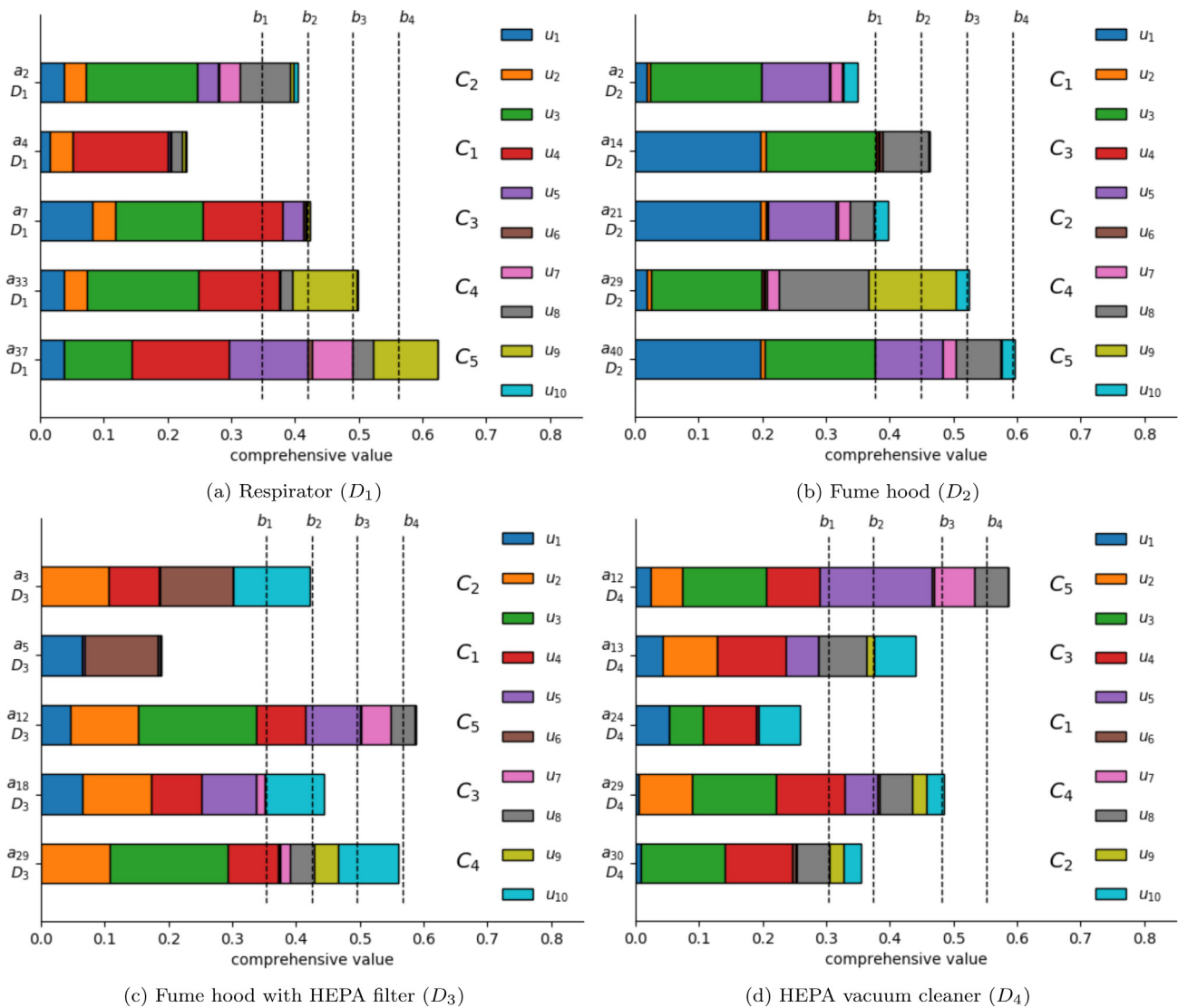
(a) Respirator (D_1)

(b) Fume hood (D_2)

(c) Fume hood with HEPA filter (D_3)

(d) HEPA vacuum cleaner (D_4)

Fig. 9. Marginal and comprehensive values and class assignments demonstrating the intra-decision relationships for five reference exposure scenarios in terms of four decision attributes.

values with class thresholds, are presented in Fig. 10. Let us justify the assignments obtained for two selected non-reference exposure scenarios (a_{43} and a_{45}).

When it comes to a_{43}, it is assigned to C_3 on D_2 and D_3 and to C_5 on D_1 and D_4. This alternative attains the extreme values on different criteria. However, it performs relatively well on the criteria with a significant impact on the classification, i.e., g_3, g_4 and g_5, which justifies its relatively high comprehensive values. They are slightly lower for *fume hood* (D_2) and *fume hood with HEPA filter* (D_3) mainly due to either zero ($u_8^{D_2}(a_{43}) = 0$) or negligible ($u_8^{D_3}(a_{43}) = 0.0027$) contribution of the *number of employees* (g_8). This criterion forms an example of the direction in which the health managers should work to verify if any of them can be improved to increase a comprehensive value and justify the assignment to a less risky class for all decision attributes.

As far as the evaluation of a_{45} is concerned, it attains high or moderate marginal values on g_2, g_3, g_4, g_7, and g_8 when assessed in terms of D_1, D_3, and D_4. This allows exceeding the lower threshold of class C_2 for these decision attributes. When it comes to D_2, the significant contribution to the overall quality of a_{45} is offered only by g_8, implying the assignment to the least preferred class C_1. A comprehensive evaluation of a_{45} as "optional" given

HEPA vacuum cleaner (D_4) is justified mainly by the value added by *quantity* (g_5). Nevertheless, the indication of class at most C_3 for all decision attributes can be perceived as a "safety warning flag", suggesting that this exposure scenario should be prioritized. In general, the greater risk associated with the respective class, the greater attention should be paid to its further investigation. The marginal value functions for which the alternative attains very low values should be analyzed to identify the performance modifications offering a significant increase of the comprehensive value.

In the e-Appendix (supplementary material available online), we compare the method introduced in this paper with the one we proposed in [31]. We also collate the outcomes obtained for the case study with both methods. This required a suitable methodological extension of the approach presented in [31] to a multi-decision setting considered in this work.

5. Conclusions and future work

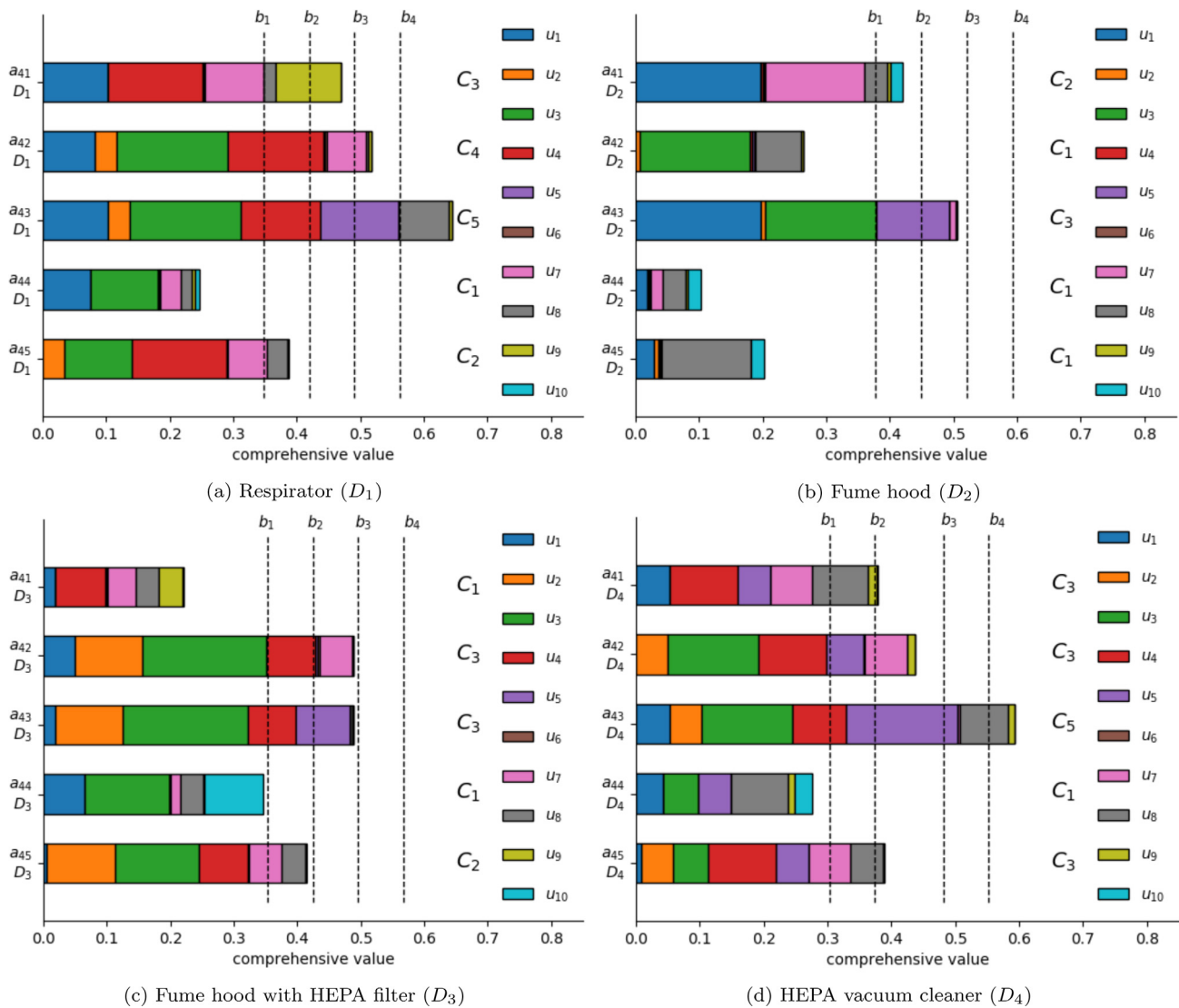We considered and formalized a new problem of multi-decision sorting in Multiple Criteria Decision Analysis. In this

Fig. 10. Marginal and comprehensive values for the five non-reference protocols in terms of four decision attributes.

problem, besides performances on multiple criteria, each alternative gets evaluated in terms of multiple decision attributes involving pre-defined preference-ordered classes. To solve such a problem, one needs to construct a set of individual sorting models, one for each decision attribute. They should reflect both intra-decision dependencies between the assignments of different alternatives and inter-decision dependencies between the classes desired for the same alternative on various decision attributes.

We have proposed a preference disaggregation method for dealing with the multi-decision sorting. In this approach, the DM is expected to assign a subset of reference alternatives to a single class for each decision attribute by indicating the quality or risk level on the pre-defined scale for all decision attributes. Such indirect preference information is used to learn a set of inter-related models composed of an additive value function and class thresholds separating the decision classes on a comprehensive value scale. The preference modeling involves intra- and inter-decision constraints intending to reproduce the assignments of as many reference alternatives as possible.

The proposed framework has been extended with a novel proposal for dealing with criteria for which the preference direction cannot be specified a priori. Explicitly, a marginal value function for each non-monotonic criterion is represented as a sum of

non-decreasing and non-increasing components. In this way, the resulting marginal function can take any arbitrary shape. Hence it can represent local monotonicity relationships in different regions of the performance scale. The interpretability of the model is enhanced by the monotonicity of non-decreasing and non-increasing components, as well as normalization imposed by the subtraction of biases, guaranteeing that an anti-ideal alternative would attain a zero comprehensive value.

The introduction of a new type of multiple criteria problem and the dedicated methodology have been motivated by the peculiarity of nanomaterials' exposure management. In this context, each exposure scenario is described in terms of various characteristics of a given nanomaterial and working conditions related to its production. However, it is also evaluated in terms of different safety measures corresponding to various types of precautions. Each precaution can be modeled as a decision attribute capturing the potential level of concern related to a nanomanufacturing scenario. We have considered four inter-related precautions representing personal protective equipment, engineering controls, and work practices.

The analysis of desired assignments provided by the health and safety managers for forty exposure scenarios allowed us to construct four inter-related classification models. These models

captured some patterns and regularities from experts' judgments for risk management in nanomanufacturing. In particular, the highest maximal share in the comprehensive values of alternatives was attributed to airborne capacity, detection limit, and exposure limit. Furthermore, the high variability of marginal values assigned to different performances on the same criterion indicated the directions for analysis of nanomanufacturing processes to reduce the risk level by vastly increasing the marginal value with a small modification of performance. For example, we can consider changing the toxicity from high to moderate, decreasing the airborne capacity from high to moderate or low, decreasing the exposure limit to less than 0.1 fiber/cc, reducing the quantity to less than 1kg, or nullifying multiple exposures. Even though the shapes of marginal value functions related to various decision attributes were similar for most criteria, some differences revealed the peculiarities of the risk management in the context of incorporating the respirator, fume hood with and without the HEPA filter, or HEPA vacuum cleaner. For example, the marginal functions for the precautions involving the HEPA filter were very similar, which is probably related to the notable reliance on this type of filter to reduce the potential concern during the production processes. On the contrary, the functions for fume hood with or without the HEPA filter were rather different, confirming their complementarity. We have also demonstrated that the constructed model can support decision-making by applying it to the classification of five non-reference exposure scenarios with unknown risk levels. These sorting models could thus be used to provide decision recommendations on multiple risk management measures – corresponding to various types of precautions – for nanomanufacturing processes, especially those where there is still high uncertainty in the operational conditions as well as the physicochemical and toxicological characteristics of the nanomaterials.

The potential extensions of the proposed method are fourfold. The motivation for the first development comes from a large number of constraints imposed by the intra- and inter-decision relationships and the use of binary variables that are needed to find the largest subset of reference alternatives for which an assumed model can reproduce the expert judgments. These factors imply that the proposed method, requiring a mathematical programming solver, cannot be applied in problems with thousands of alternatives and hundreds of decision attributes. An adaptation to such a setting would require the development of heuristic algorithms incorporating the machine learning techniques. As opposed to the proposed framework, they should not attempt to find an accurate, optimal solution, searching instead for a highly satisfactory model in an approximate way.

Secondly, the marginal functions for which the monotonicity direction cannot be pre-defined can be modeled differently without incorporating the non-decreasing and non-increasing components. In particular, the proposed methodology remains valid when the functions for potentially non-monotonic criteria are inferred to minimize either the number of changes in monotonicity [31] (see e-Appendix) or the sum of changes in slopes [30, 56]. Similarly, the framework remains valid with threshold-based sorting procedures incorporating other preference models to compute alternatives' scores than an additive value function (e.g., the Choquet integral [57,58]).

Thirdly, the proposed method can be extended with the robustness analysis framework [4,23,58]. In this approach, one should account for all compatible multi-decision sorting models instead of a single representative one. Then, one should capture the potential variability of assignments for the non-reference alternatives given the multiplicity of analyzed models consistent with the DM's judgments. Such an approach can be extended to analyze all maximal subsets of reference alternatives for which the provided assignments are consistent with an assumed model [59].

Finally, the idea of evaluating each alternative with a set of inter-related preference models can be adjusted to other problem types. For example, various value functions can be used to assess the suitability of a given alternative to be assigned to the groups of alternatives exhibiting different characteristics and being similar in terms of the DM's preferences. Then, it should be placed in a group for which the attained comprehensive value is the greatest. Such an approach could provide a novel way of dealing with multiple criteria nominal classification [60].

CRediT authorship contribution statement

Miłosz Kadziński: Conceptualization, Methodology, Validation, Formal analysis, Investigation, Data curation, Writing - original draft, Supervision, Project administration, Funding acquisition. **Krzysztof Martyn:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing - original draft, Visualization. **Marco Cinelli:** Conceptualization, Validation, Formal analysis, Investigation, Resources, Writing - review & editing. **Roman Słowiński:** Conceptualization, Methodology, Writing - review & editing. **Salvatore Corrente:** Conceptualization, Methodology, Writing - review & editing. **Salvatore Greco:** Conceptualization, Methodology, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.knosys.2021.106879.

References

[1] M. Cinelli, M. Kadziński, M. Gonzalez, R. Słowiński, How to support the application of multiple criteria decision analysis? Let us start with a comprehensive taxonomy, Omega 96 (2020) 102261.

[2] S. Greco, M. Ehrgott, J. Figueira, Multiple Criteria Decision Analysis – State of the Art Surveys, in: International Series in Operations Research & Management Science, Springer, New York, 2016.

[3] C. Zopounidis, M. Doumpos, Multicriteria classification and sorting methods: a literature review, European J. Oper. Res. 138 (2002) 229–246.

[4] S. Greco, V. Mousseau, R. Słowiński, Multiple criteria sorting with a set of additive value functions, European J. Oper. Res. 207 (3) (2010) 1455–1470.

[5] J. Liu, X. Liao, W. Zhao, N. Yang, A classification approach based on the outranking model for multiple criteria ABC analysis, Omega 61 (2016) 19–34.

[6] E. Saleh, J. Błaszczyński, A. Moreno, A. Valls, P. Romero-Aroca, S. de la Riva-Fernandez, R. Słowiński, Learning ensemble classifiers for diabetic retinopathy assessment, Artif. Intell. Med. 85 (2018) 50–63.

[7] R. Slowinski, C. Zopounidis, Application of the rough set approach to evaluation of bankruptcy risk, Intell. Syst. Account. Finance Manag. 4 (1) (1995) 27–41.

[8] M. Cinelli, S. Coles, M.N. Nadagouda, J. Błaszczyński, R. Słowiński, R.S. Varma, K. Kirwan, A green chemistry-based classification model for the synthesis of silver nanoparticles, Green Chem. 17 (2015) 2825–2839.

[9] C. Zopounidis, M. Doumpos, Multicriteria Sorting Methods, Springer, Boston, 2009, pp. 2379–2396.

[10] J. Almeida-Dias, J. Figueira, B. Roy, Electre Tri-C: A multiple criteria sorting method based on characteristic reference actions, European J. Oper. Res. 204 (3) (2010) 565–580.

[11] W. Yu, Aide multicritère à la décision dans le cadre de la problématique du tri: méthodes et applications (Ph.D. thesis), LAMSADE, Université Paris Dauphine, Paris, 1992.

[12] E. Jacquet-Lagrèze, Y. Siskos, Preference disaggregation: 20 years of MCDA experience, European J. Oper. Res. 130 (2) (2001) 233–245.

[13] J. Devaud, G. Groussaud, E. Jacquet-Lagreze, UTADIS: Une methode de construction de fonctions d'utilite additives rendant compte de jugements globaux, in: European Working Group on MCDA, Bochum, Germany, 1980.

[14] C. Zopounidis, M. Doumpos, PREFDIS: a multicriteria decision support system for sorting decision problems, Comput. & Oper. Res. 27 (7–8) (2000) 779–797.

[15] C. Zopounidis, M. Doumpos, A multicriteria decision aid methodology for sorting decision problems: The case of financial distress, Comput. Econ. 14 (3) (1999) 197–218.

[16] D. Diakoulaki, C. Zopounidis, G. Mavrotas, M. Doumpos, The use of a preference disaggregation method in energy analysis and policy making, Energy 24 (2) (1999) 157–166.

[17] K. Pendaraki, C. Zopounidis, M. Doumpos, On the construction of mutual fund portfolios: A multicriteria methodology and an application to the greek market of equity mutual funds, European J. Oper. Res. 163 (2) (2005) 462–481.

[18] V. Mousseau, L.C. Dias, J.R. Figueira, Dealing with inconsistent judgments in multiple criteria sorting models, 4OR 4 (2) (2006) 145–158.

[19] C. Zopounidis, M. Doumpos, Building additive utilities for multi-group hierarchical discrimination: the M.H.DIS method, Optim. Methods Softw. 14 (3) (2000) 219–240.

[20] S. Corrente, M. Doumpos, S. Greco, R. Słowiński, C. Zopounidis, Multiple criteria hierarchy process for sorting problems based on ordinal regression with additive value functions, Ann. Oper. Res. 251 (1) (2017) 117–139.

[21] M. Kadziński, K. Ciomek, R. Słowiński, Modeling assignment-based pairwise comparisons within integrated framework for value-driven multiple criteria sorting, European J. Oper. Res. 241 (3) (2015) 830–841.

[22] J. Liu, M. Kadziński, X. Liao, X. Mao, Y. Wang, A preference learning framework for multiple criteria sorting with diverse additive value models and valued assignment examples, European J. Oper. Res. 286 (3) (2020) 963–985.

[23] M. Kadziński, T. Tervonen, Stochastic ordinal regression for multiple criteria sorting problems, Decis. Support Syst. 55 (1) (2013) 55–66.

[24] M. Köksalan, S. Bilgin Özpeynirci, An interactive sorting method for additive utility functions, Comput. Oper. Res. 36 (9) (2009) 2565–2572.

[25] S. Greco, M. Kadziński, V. Mousseau, R. Słowiński, Robust ordinal regression for multiple criteria group decision: UTA-GMS-GROUP and UTADIS-GMS-GROUP, Decis. Support Syst. 52 (3) (2012) 549–561.

[26] S. Greco, V. Mousseau, R. Słowiński, Robust ordinal regression for value functions handling interacting criteria, European J. Oper. Res. 239 (3) (2014) 711–730.

[27] J. Rezaei, Piecewise linear value functions for multi-criteria decision-making, Expert Syst. Appl. 98 (2018) 43–56.

[28] M. Doumpos, Learning non-monotonic additive value functions for multicriteria decision making, OR Spectrum 34 (1) (2012) 89–106.

[29] M. Ghaderi, F. Ruiz, N. Agell, Understanding the impact of brand colour on brand image: A preference disaggregation approach, Pattern Recognit. Lett. 67 (2015) 11–18.

[30] J. Liu, X. Liao, M. Kadziński, R. Słowiński, Preference disaggregation within the regularization framework for sorting problems with multiple potentially non-monotonic criteria, European J. Oper. Res. 276 (3) (2019) 1071–1089.

[31] M. Kadziński, K. Martyn, M. Cinelli, R. Słowiński, S. Corrente, S. Greco, Preference disaggregation for multiple criteria sorting with partial monotonicity constraints: Application to exposure management of nanomaterials, Internat. J. Approx. Reason. 117 (2020) 60–80.

[32] A. Ulucan, K. Atici, A multiple criteria sorting methodology with multiple classification criteria and an application to country risk evaluation, Technol. Econ. Dev. Econ. 19 (1) (2013) 93–124.

[33] M. Doumpos, J. Figueira, A multicriteria outranking approach for modeling corporate credit ratings: An application of the Electre Tri-nC method, Omega 82 (2019) 166–180.

[34] J. Almeida-Dias, J. Figueira, B. Roy, A multiple criteria sorting method where each category is characterized by several reference actions: The electre Tri-nC method, European J. Oper. Res. 217 (3) (2012) 567–579.

[35] S. Naidu, Towards Sustainable Development of Nanomanufacturing (Ph.D. thesis), University of Tennessee, Knoxville, 2012.

[36] H. Goede, Y. Christopher-de Vries, E. Kuijpers, W. Fransman, A review of workplace risk management measures for nanomaterials to mitigate inhalation and dermal exposure, Ann. Work. Expo. Health 62 (8) (2018) 907–922.

[37] C. Oksel, N. Hunt, T. Wilkins, X. Wang, Risk management of nanomaterials - guidelines for the safe manufacture and use of nanomaterials, 2017, Sustainable Nanotechnologies Project, http://www.sun-fp7.eu.

[38] G. Tsoumakas, I. Katakis, Multi-label classification: An overview, Int. J. Data Warehous. Min. (IJDWM) 3 (3) (2007) 1–13.

[39] K. Dembcyzński, W. Cheng, E. Hüllermeier, Bayes optimal multilabel classification via probabilistic classifier chains, in: Proceedings of the 27th International Conference on International Conference on Machine Learning, ICML 10, (Madison, USA), Omnipress, 2010, pp. 279–286.

[40] J. Błaszczyński, S. Greco, R. Słowiński, Inductive discovery of laws using monotonic rules, Eng. Appl. Artif. Intell. 25 (2) (2012) 284–294.

[41] K. Zhang, K.O. Kirlikovali, Q.V. Le, Z. Jin, R.S. Varma, H.W. Jang, O.K. Farha, M. Shokouhimehr, Extended metal–organic frameworks on diverse supports as electrode nanomaterials for electrochemical energy storage, ACS Appl. Nano Mater. 3 (5) (2020) 3964–3990.

[42] M. Nasrollahzadeh, M. Sajjadi, S. Iravani, R.S. Varma, Green-synthesized nanocatalysts and nanomaterials for water treatment: Current challenges and future perspectives, J. Hazard. Mater. 401 (2021) 123401.

[43] E.O. Ogunsona, R. Muthuraj, E. Ojogbo, O. Valerio, T.H. Mekonnen, Engineered nanomaterials for antimicrobial applications: A review, Appl. Mater. Today 18 (2020) 100473.

[44] A.M. Abdelmonem, Application of carbon-based nanomaterials in food preservation area, in: K.A. Abd-Elsalam (Ed.), Carbon Nanomaterials for Agri-Food and Environmental Applications, Elsevier, Micro and Nano Technologies, 2020, pp. 583–593.

[45] M.M. Falinski, D.L. Plata, S.S. Chopra, T.L. Theis, L.M. Gilbertson, J.B. Zimmerman, A framework for sustainable nanomaterial selection and design based on performance, hazard, and economic considerations, Nat. Nanotechnol. 13 (8) (2018) 708–714.

[46] R.D. Klaper, The known and unknown about the environmental safety of nanomaterials in commerce, Small 16 (36) (2020) e2000690.

[47] P. Isigonis, D. Hristozov, C. Benighaus, E. Giubilato, K.G. Pizzol, E. Semenzin, I. Linkov, A. Zabeo, A. Marcomini, Risk governance of nanomaterials: Review of criteria and tools for risk communication, evaluation, and mitigation, Nanomaterials 9 (5) (2019) 696.

[48] M.L. Kirkegaard, P. Kines, K.C. Jeschke, K.A. Jensen, Risk perceptions and safety cultures in the handling of nanomaterials in academia and industry, Ann. Work. Expo. Health 64 (5) (2020) 479–489.

[49] V. Stone, M. Fuhr, P.H. Feindt, H. Bouwmeester, I. Linkov, S. Sabella, F. Murphy, K. Bizer, L. Tran, M. Agerstrand, C. Fito, T. Andersen, D. Anderson, E. Bergamaschi, J.W. Cherrie, S. Cowan, J.-F. Dalemcourt, M. Faure, S. Gabbert, A. Gajewicz, T.F. Fernandes, D. Hristozov, H.J. Johnston, T.C. Lansdown, S. Linder, H.J.P. Marvin, M. Mullins, K. Purnhagen, T. Puzyn, A. Sanchez Jimenez, J.J. Scott-Fordsmand, G. Streftaris, M. van Tongeren, N.H. Voelcker, G. Voyiatzis, S.N. Yannopoulos, P.M. Poortvliet, The essential elements of a risk governance framework for current and future nanotechnologies, Risk Anal. 38 (7) (2018) 1321–1331.

[50] A. Rahi, N. Sattarahmady, H. Heli, Toxicity of nanomaterials-physicochemical effects, Austin J. Nanomed. & Nanotechnol. 2 (2014) 1034.

[51] B. Fadeel, L. Farcal, B. Hardy, S. Vázquez-Campos, D. Hristozov, A. Marcomini, I. Lynch, E. Valsami-Jones, H. Alenius, K. Savolainen, Advanced tools for the safety assessment of nanomaterials, Nat. Nanotechnol. 13 (7) (2018) 537–543.

[52] S.F. Hansen, K.A. Jensen, A. Baun, Nanoriskcat: a conceptual tool for categorization and communication of exposure potentials and hazards of nanomaterials in consumer products, J. Nanopart. Res. 16 (1) (2013) 2195.

[53] D. Hristozov, S. Gottardo, M. Cinelli, P. Isigonis, A. Zabeo, A. Critto, M.V. Tongeren, L. Tran, A. Marcomini, Application of a quantitative weight of evidence approach for ranking and prioritising occupational exposure scenarios for titanium dioxide and carbon nanomaterials, Nanotoxicology 8 (2) (2014) 117–131.

[54] B.V. Duuren-Stuurman, S. Vink, K. Verbist, H. Heussen, D. Brouwer, D. Kroese, M.V. Niftrik, E. Tielemans, W. Fransman, Stoffenmanager nano version 1.0: a web-based tool for risk prioritization of airborne manufactured nano objects, Ann. Occup. Hyg. 56 (5) (2012) 525–541.

[55] F. Silva, P. Arezes, P. Swuste, Risk management: Controlling occupational exposure to nanoparticles in construction, in: F. Pacheco-Torgal, M.V. Diamanti, A. Nazari, C.G. Granqvist, A. Pruna, S. Amirkhanian (Eds.), Nanotechnology in Eco-Efficient Construction, second ed., Woodhead Publishing, 2019, pp. 755–784.

[56] M. Ghaderi, F. Ruiz, N. Agell, A linear programming approach for learning non-monotonic additive value functions in multiple criteria decision aiding, European J. Oper. Res. 259 (3) (2017) 1073–1084.

[57] S. Angilella, P. Catalfo, S. Corrente, A. Giarlotta, S. Greco, M. Rizzo, Robust sustainable development assessment with composite indices aggregating interacting dimensions: The hierarchical-SMAA-choquet integral approach, Knowl. Based Syst. 158 (2018) 136–153.

[58] G. Beliakov, J.-Z. Wu, D. Divakov, Towards sophisticated decision models: Nonadditive robust ordinal regression for preference modeling, Knowl. Based Syst. 190 (2020) 105351.

[59] M. Kadziński, M. Cinelli, K. Ciomek, S. Coles, M. Nadagouda, R. Varma, K. Kirwan, Co-constructive development of a green chemistry-based model for the assessment of nanoparticles synthesis, European J. Oper. Res. 264 (2) (2018) 472–490.

[60] A. Costa, J. Figueira, J. Borbinha, A multiple criteria nominal classification method based on the concepts of similarity and dissimilarity, European J. Oper. Res. 271 (1) (2018) 193–209.