Synthesizing Preference Information of Multiple Decision Makers in Terms of Collective Decision Rules

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Abstract. We propose an approach based on DRSA (Dominance-based Rough Set Approach) method for synthesizing preference information of multiple decision makers in a multicriteria classification problem. The proposed approach takes as input a common information table and generates a set of collective decision rules representing a generalized description of the preference information of the decision makers.

1 INTRODUCTION

DRSA (Dominance-based Rough Set Approach) [3] is an extension of rough sets theory [5] to deal with multicriteria classification problems. It takes as input a *decision table* describing the *decision objects* and generates as output a set of *decision rules*. DRSA is a single decision maker oriented method. However, there are some proposals to extend DRSA to group decision making [7][4][1]. But these proposals have several shortcomings as discussed in Section 7.

The objective of this paper is to introduce a DRSA-based approach for synthesizing preference information of multiple decision makers in a multicriteria classification problem. The proposed approach takes as input a common information table and generates a set of collective decision rules representing a generalized description of the preference information of the decision makers.

The paper goes as follows. Section 2 presents the background. Section 3 introduces the approach. Section 4 presents the aggregation procedure. Section 5 deals with collective decision rules generation. Section 6 illustrates the approach through an application. Section 7 discusses some related work. Section 8 concludes the paper.

2 BACKGROUND

DRSA [3][4] is a rough sets-based multicriteria classification method. This method has been developed to overcome the shortcomings of rough set [5] in multicriteria classification problems. The idea of DRSA is to replace *indiscernibility* relation in rough approximations by *dominance* relation.

2.1 Basic notations and assumptions

Information about decision objects are often represented in terms of an *information table* where rows correspond to *objects* and columns correspond to *attributes*. The information table S is a 4-tuple $< U, Q, V, f >$ where: U is a finite set of objects, Q is a finite set of attributes, $V = \bigcup_{q \in Q} V_q$, V_q is a domain of attribute q, and $f : U \times Q \rightarrow V$ is an *information function* defined such that $f(x,q) \in V_q, \forall q \in Q, \forall x \in U$. The set of attributes Q is often divided into a sub-set C of *condition attributes* and a sub-set D of *decision attributes*. In this case, S is called *decision table*.

A series of assumptions are established first. The domain of condition attributes are supposed to be ordered to decreasing or increasing preference. Such attributes are called *criteria*. We assume that the preference is increasing with the value of $f(\cdot, q)$ for every $q \in C$. We also assume that the set of decision attributes D is a singleton $\{d\}$. Decision attribute d makes a partition of U into a finite number of decision classes **Cl** $= \{Cl_t, t \in T\}$, $T = \{0, \cdots, n\}$, such that each $x \in U$ belongs to one and only one class in **Cl**. Further, we suppose that the classes are preference-ordered, i.e. for all $r, s \in T$, such that $r > s$, the objects from Cl_r are preferred to the objects from Cl_s.

The idea of rough set approach is the approximation of knowledge generated by the decision attributes by "*granules of knowledge*" generated by condition attributes. The sets to be approximated are:

$$Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s, Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s, t = 0, \cdots, n.$$

Set $Cl_t^{\geq}$ is called the *upward union*. The assertion $x \in Cl_t^{\geq}$ means that "x belongs to at least class Cl_t". Set $Cl_t^{\leq}$ is called the *downward union*. The assertion $x \in Cl_t^{\leq}$ means that "x belongs to at most Cl_t".

2.2 Approximation of unions of classes

In DRSA the represented knowledge is a collection of upward and downward unions of classes and the "granules of knowledge" are sets of objects defined using a (weak) dominance relation. The dominance relation Δ_P, where $P \subseteq C$, is defined for each pair of objects x and y as follows:

$$x \Delta_P y \Leftrightarrow f(x,q) \geq f(y,q), \forall q \in P.$$

The "granules of knowledge" used for approximation in DRSA with respect to a set of criteria $P \subseteq C$ and object $x \in U$ are:

- $\Delta_P^+(x) = \{y \in U : y \Delta_P x\}$: the set of objects that dominate x,
- $\Delta_P^-(x) = \{y \in U : x \Delta_P y\}$: the set of objects dominated by x.

Δ_P^+ and Δ_P^- are respectively called P-*dominating* set and P-*dominated* set. For each set of criteria $P \subseteq C$, the P-lower and P-upper approximations of $Cl_t^{\geq}$ are defined as follows:

- $\underline{P}(Cl_t^{\geq}) = \{x \in U : \Delta_P^+(x) \subseteq Cl_t^{\geq}\}$,
- $\bar{P}(Cl_t^{\geq}) = \bigcup_{x \in Cl_t^{\geq}} \Delta_P^+(x) = \{x \in U : \Delta_P^-(x) \bigcap Cl_t^{\geq} \neq \emptyset\}$.

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P-lower approximation of $Cl_t^{\geq}$ contains all the objects with P-dominating set are assigned with certitude to classes at most as good as Cl_t. P-upper approximation of $Cl_t^{\geq}$ contains all the objects with P-dominating set is assigned to a class at least as good as Cl_t.

Similarly, the P-lower and P-upper approximations of $Cl_t^{\leq}$ are defined as follows:

- $\underline{P}(Cl_t^{\leq}) = \{x \in U : \Delta_P^-(x) \subseteq Cl_t^{\leq}\}$,
- $\bar{P}(Cl_t^{\leq}) = \bigcup_{x \in Cl_t^{\leq}} \Delta_P^-(x) = \{x \in U : \Delta_P^+(x) \bigcap Cl_t^{\leq} \neq \emptyset\}$.

P-lower approximation of $Cl_t^{\leq}$ contains all the objects with P-dominated set are assigned with certitude to a class at most as good as Cl_t. P-upper approximation of $Cl_t^{\leq}$ contains all the objects with P-dominated set is assigned to a class at least as good as Cl_t.

We also define the P-boundary sets of $Cl_t^{\geq}$ and $Cl_t^{\leq}$ as follows:

- $Bn_P(Cl_t^{\geq}) = \bar{P}(Cl_t^{\geq}) - \underline{P}(Cl_t^{\geq})$,
- $Bn_P(Cl_t^{\leq}) = \bar{P}(Cl_t^{\leq}) - \underline{P}(Cl_t^{\leq})$.

$Bn_P(Cl_t^{\geq})$ contains all the objects which are assigned both to a class better than Cl_t and to one or several classes worse than Cl_t. In other words, it contains objects with P-dominating set cannot be assigned with certitude to classes at least as good as Cl_t. Similarly, $Bn_P(Cl_t^{\leq})$ is the set of objects with P-dominated set cannot be assigned with certitude to classes at most as good as Cl_t.

2.3 Decision rules

The approximations of upward and downward unions of classes can serve to induce a set of "$if \cdots, then \cdots$" decision rules relating condition and decision attributes. An object $x \in U$ *supports* a decision rule if its description matchs both the condition part and the decision part of this rule. A decision rule *covers* object x if the description of x matches at least the condition part of the rule. The *strength* of a decision rule is the number of objects supporting this rule.

2.4 Quality of classification

The *quality of classification* is defined by the following ratio:

$$\begin{aligned} \gamma_P &= \frac{card(U - (\bigcup_{t=1,\cdots,n} Bn_P(Cl_t^{\geq})))}{card(U)} \\ &= \frac{card(U - (\bigcup_{t=0,\cdots,n-1} Bn_P(Cl_t^{\leq})))}{card(U)}. \end{aligned} \tag{1}$$

It expresses the pourcentage of objects that are assigned with certitude to a given class.

3 COLLECTIVE DECISION RULES CONSTRUCTION APPROACH

The proposed approach is composed of three phases: individual classification, aggregation, and generation of collective decision rules. The main input of the approach is a common information table I defined as $< U, C, V, f >$ with a finite set U of objects and a finite set C of criteria. The output is a collection of collective decision rules representing a generalized description of the preference information provided by the decision makers. Let $H = \{1, \cdots, i, \cdots, h\}$ be a finite set of decision makers corresponding to h decision attributes $D_1, \cdots, D_i, \cdots, D_h$. Further, we suppose that decision attributes are defined on the same domain. We also assume that each decision maker $i \in H$ has a preference order on the universe U and that this preference order is represented by a finite set of preference ordered classes:

$$\mathbf{Cl}_i = \{Cl_{t,i}, t \in T_i\}, T_i = \{0, \cdots, n_i\},$$

such that $\bigcup_{t=1}^{n_i} Cl_{t,i} = U, Cl_{t,i} \cap Cl_{r,i} = \emptyset, \forall r, t \in T_i, r \neq t$, and if $x \in Cl_{r,i}, y \in Cl_{s,i}$ and $r > s$, then x is better than y for decision maker i.

3.1 Phase 1: Individual classification

In this first phase, each decision maker uses the common information table I to construct its own decision table S_i defined as $< U, C \cup D_i, V, f_i >$ where D_i is a new decision attribute and f_i is an information function, both associated with decision maker i. Then, each decision maker runs the DRSA method using its decision table S_i as input. In terms of this phase, the classification conducted by each decision maker is characterized, among others, by:

- the P-lower approximation and and P-boundary of $Cl_{t,i}^{\leq}$ and $Cl_{t,i}^{\geq}$, for each $t \in T_i$, and
- the quality of classification γ_P^i defined in similar way to Eq. (1).

3.2 Phase 2: Aggregation

The objective of this phase is to combine the outputs of the first phase in order to assign to each object $x \in U$ a collective assignment interval by using an aggregation procedure detailed in Section 4. First, we design by **Cl** the collective preference order obtained by the union of individual preference orders:

$$\mathbf{Cl} = \{Cl_t, t \in T\}, T = \{0, \cdots, n\},$$

such that each $x \in U$ belongs to one and only one class $Cl_t \in$ **Cl**. This operation is correct since, as stated before, decision attributes are defined on the same domain. According to this definition, we have: $x \in Cl_{t,i} \Leftrightarrow x \in Cl_t, \forall x \in U, \forall t \in T$, and $\forall i \in H$.

The aggregation procedure can be represented as follows:

$$\begin{array}{lcl} U & \rightarrow & \mathbf{Cl} \times \mathbf{Cl} \\ x & \rightarrow & I(x) = [l(x), u(x)] \end{array}$$

It is a mapping from U to **Cl** $\times$ **Cl** that associates to each $x \in U$ a *collective assignment interval* $I(x) = [l(x), u(x)]$, where $l(x)$ and $u(x)$ are respectively the lower and upper classes to which object x can be assigned. Details are given in Section 4.3.

3.3 Phase 3: Generation of collective decision rules

The objective of this phase is to use the DRSA method to infer a set of collective decision rules representing a generalized description of the preference information provided by the different decision makers. The application of DRSA method requires that the decision attribute be mono-valued. Thus, some simple rules are first used to construct a collective decision table with a mono-valued decision attribute (Section 5.1). Then, the DRSA method may be applied using the obtained collective decision table as input (Section 5.2).

4 AGGREGATION PROCEDURE

The aggregation procedure is composed of three steps.

4.1 Step 2.1: Normalization

The objective of this first step is to standardize the quality of classifications γ_P^i ($\forall i \in H$) using the following formula:

$$ {}^i\gamma'_P = \frac{1}{h} \cdot \sum_{i=1}^{h} \gamma_P^i, \quad (i = 1, \cdots, h). \tag{2} $$

4.2 Step 2.2: Computing the concordance and discordance powers

The aggregation procedure is based on the majority principle which is defined through the concordance and discordance powers. The semantic interpretation of these powers is similar to the same concepts employed in ELECTRE family of multicriteria methods; see[2]. However, they are defined, computed and used differently in the present paper.

4.2.1 Concordance power

First, we define the sets $L(x, Cl_t^{\leq})$ and $L(x, Cl_t^{\geq})$ as follows:

- $L(x, Cl_t^{\leq}) = \{i : i \in H \wedge x \in \underline{P}(Cl_{t,i}^{\leq})\}$,
- $L(x, Cl_t^{\geq}) = \{i : i \in H \wedge x \in \underline{P}(Cl_{t,i}^{\geq})\}$.

The first set represents the decision makers for which object x belongs to the lower approximation of $Cl_t^{\leq}$. The second one represents the decision makers for which object x belongs to the lower approximation of $Cl_t^{\geq}$. Next, the *concordance powers* for the assignment of x to $Cl_t^{\leq}$ and to $Cl_t^{\geq}$ are computed as follows:

$$ L^+(x, Cl_t^{\leq}) = \sum_{i \in L(x, Cl_t^{\leq})} {}^i\gamma'_P. \tag{3} $$

$$ L^+(x, Cl_t^{\geq}) = \sum_{i \in L(x, Cl_t^{\geq})} {}^i\gamma'_P. \tag{4} $$

$L^+(x, Cl_t^{\leq}) \in [0, 1]$ measures the power of coalition of decision makers that assign x to the lower approximation of $Cl_t^{\leq}$. $L^+(x, Cl_t^{\geq}) \in [0, 1]$ measures the power of coalition of decision makers that assign x to the lower approximation of $Cl_t^{\geq}$.

4.2.2 Discordance power

First, we define the sets $B(x, Cl_t^{\leq})$ and $B(x, Cl_t^{\geq})$ as follows:

- $B(x, Cl_t^{\leq}) = \{i : i \in H \wedge x \in Bn_P(Cl_{t,i}^{\leq})\}$,
- $B(x, Cl_t^{\geq}) = \{i : i \in H \wedge x \in Bn_P(Cl_{t,i}^{\geq})\}$.

The first set represents the decision makers for which object x belongs to the boundary of $Cl_t^{\leq}$. The second one represents the decision makers for which object x belongs to the boundary of $Cl_t^{\geq}$. Then, the *discordance powers* for the assignment of x to the boundary of $Cl_t^{\leq}$ and $Cl_t^{\geq}$ are computed as follows:

$$ B^+(x, Cl_t^{\leq}) = \sum_{i \in B(x, Cl_t^{\leq})} {}^i\gamma'_P. \tag{5} $$

$$ B^+(x, Cl_t^{\geq}) = \sum_{i \in B(x, Cl_t^{\geq})} {}^i\gamma'_P. \tag{6} $$

$B^+(x, Cl_t^{\leq}) \in [0, 1]$ measures the power of coalition of decision makers that assign x to the boundary of $Cl_t^{\leq}$. $B^+(x, Cl_t^{\geq}) \in [0, 1]$ measures the power of coalition of decision makers that assign x to the boundary of $Cl_t^{\geq}$.

4.3 Step 2.3: Definition of assignment intervals

Let $\theta \in [.5, 1.0]$ be a majority threshold and $\theta' \in [0, .5]$ be a veto threshold. Based on the concordance and discordance powers, we may distinguish four situations for the assignment of x to $Cl_t^{\leq}$:

	$B^+(x, Cl_t^{\leq}) < \theta'$	$B^+(x, Cl_t^{\leq}) \geq \theta'$
$L^+(x, Cl_t^{\leq}) \geq \theta$	$x \in Cl_t^{\leq}$	$x \notin Cl_t^{\leq}$
$L^+(x, Cl_t^{\leq}) < \theta$	$x \notin Cl_t^{\leq}$	$x \notin Cl_t^{\leq}$

These situations are summarized by the following assignment rule:

`if` $L^+(x, Cl_t^{\leq}) \geq \theta \wedge B^+(x, Cl_t^{\leq}) < \theta'$, `then` $x \in Cl_t^{\leq}$
`else` $x \notin Cl_t^{\leq}$ *(rule 1)*

This assignment rule can be explained as follows. An object x is assigned to $Cl_t^{\leq}$ if and only if:

- there is a "sufficient" majority of decision makers (in terms of their quality of classification) that assign x to $Cl_t^{\leq}$, and
- when the first condition holds, none of the minority of decision makers shows an "important" opposition to the assignment of x to $Cl_t^{\leq}$.

In similar way, four situations can be distinguished for the assignment of x to $Cl_t^{\geq}$:

	$B^+(x, Cl_t^{\geq}) < \theta'$	$B^+(x, Cl_t^{\geq}) \geq \theta'$
$L^+(x, Cl_t^{\geq}) \geq \theta$	$x \in Cl_t^{\geq}$	$x \notin Cl_t^{\geq}$
$L^+(x, Cl_t^{\geq}) < \theta$	$x \notin Cl_t^{\geq}$	$x \notin Cl_t^{\geq}$

These situations are summarized by the following assignment rule:

`if` $L^+(x, Cl_t^{\geq}) \geq \theta \wedge B^+(x, Cl_t^{\geq}) < \theta'$, `then` $x \in Cl_t^{\geq}$
`else` $x \notin Cl_t^{\geq}$ *(rule 2)*

This assignment rule can be explained as follows. An object x is assigned to $Cl_t^{\geq}$ if and only if:

- there is a "sufficient" majority of decision makers (in terms of their quality of classification) that assign x to $Cl_t^{\geq}$, and
- when the first condition holds, none of the minority of decision makers shows an "important" opposition to the assignment of x to $Cl_t^{\geq}$.

The application of these assignment rules on the set of objects U permits to associate to each object x a collective assignment interval $I(x) = [l(x), u(x)]$ where:

$$ l(x) = \begin{cases} \text{argmax}_{Cl_t} N_1(x), & \text{if } N_1(x) \neq \emptyset, \\ Cl_0, & \text{otherwise.} \end{cases} \tag{7} $$

$$ u(x) = \begin{cases} \text{argmin}_{Cl_t} N_2(x), & \text{if } N_2(x) \neq \emptyset, \\ Cl_n, & \text{otherwise.} \end{cases} \tag{8} $$

where $N_1(x) = \{Cl_t : x \in Cl_t^{\geq}\}$ and $N_2(x) = \{Cl_t : x \in Cl_t^{\leq}\}$. Set $N_1(x)$ contains the set of classes to which x is assigned by applying *rule 2*, while set $N_2(x)$ contains the set of classes to which x is assigned by applying *rule 1*.

The aggregation procedure is summed up in Algorithm 1. This algorithm runs in $O(|U| \cdot n \cdot h)$ where $|U|$ is the cardinality of U, n is the number of classes and h is the number of decision makers.

Algorithm 1 **AggregationProcedure**

Input: $\underline{P}(Cl^{\leq}_{t,i})$, $\underline{P}(Cl^{\geq}_{t,i})$: P-lower approx. ($i \in H; t \in T_i$)
$Bn_P(Cl^{\leq}_{t,i})$, $Bn_P(Cl^{\geq}_{t,i})$: P-boundary ($i \in H; t \in T_i$)
γ^i_P: quality of classification ($i \in H$)
Output: $I(x)$: Collective assignment interval ($\forall x \in U$)
1. Normalize γ^i_P ($i \in H$)
2. **for** each $x \in U$
3. **for** each $t \in T$
4. compte $L(x, Cl^{\leq}_t)$, $B(x, Cl^{\leq}_t)$, $L(x, Cl^{\geq}_t)$, $B(x, Cl^{\geq}_t)$
5. compte $L^+(x, Cl^{\leq}_t)$, $B^+(x, Cl^{\leq}_t)$, $L^+(x, Cl^{\geq}_t)$, $B^+(x, Cl^{\geq}_t)$
6. **if** $L^+(x, Cl^{\leq}_t) \geq \theta$ and $B^+(x, Cl^{\leq}_t) < \theta'$, **then** $x \in Cl^{\leq}$
7. **else** $x \notin Cl^{\leq}$ **end if**
8. **if** $L^+(x, Cl^{\geq}_t) \geq \theta$ and $B^+(x, Cl^{\geq}_t) < \theta'$, **then** $x \in Cl^{\geq}$
9. **else** $x \notin Cl^{\geq}$ **end if**
10. **end for**
11. $N_1(x) \leftarrow \{Cl_t : x \in Cl^{\geq}_t\}$
12. $N_2(x) \leftarrow \{Cl_t : x \in Cl^{\leq}_t\}$
13. **if** $N_1(x) \neq \emptyset$, **then** $l \leftarrow \text{argmax}_{Cl_t} N_1(x)$
14. **else** $l \leftarrow Cl_0$ **end if**
15. **if** $N_2(x) \neq \emptyset$ **then** $u \leftarrow \text{argmin}_{Cl_t} N_2(x)$
16. **else** $u \leftarrow Cl_n$ **end if**
17. $I(x) \leftarrow [l, u]$
18. **end for**

5 INFERENCE OF DECISION RULES

5.1 Construction of a collective decision table

The objective of this step is to construct a collective decision table S defined as $< U, C \cup D, V, g >$ where D is a collective decision attribute and g is a collective information function defined as follows:

$$g(x, q) = \begin{cases} f(x, q), & \text{if } q \in C, \\ g(x, D), & \text{if } q = D. \end{cases} \quad (9)$$

Two cases may be distinguished for the definition of $g(x, D)$. The first holds when $l(x) = u(x)$. Here, object x is assigned to a single class and consequently we can set $g(x, D) = l(x)$ (or similarly $g(x, D) = u(x)$). The second case holds when $l(x) < u(x)$. This corresponds to the situation where object x is assigned to more than one class. To define $g(x, D)$ we may apply one of the following rules to reduce the collective assignment interval $I(x)$ to a single class:

- use the "min" operator on the collective assignment interval $I(x)$. This leads to $g(x, D) = l(x)$. (*rule 3*)
- use the "max" operator on the collective assignment interval $I(x)$. This leads to $g(x, D) = u(x)$. (*rule 4*)
- use the "median" operator on $l', \cdots, u'$, where $l', \cdots, u'$ is an ordered list issued from $l(x), \cdots, u(x)$. (*rule 5*)

The proposed approach assumes an ordinal measurement scale. Hence, the median value may correspond to no decision class (when there is an even number of values). To avoid this problem, *rule 5* can be subdivided into two rules:

- use the "floor" of the median value: $g(x, D) = \lfloor \mu(l', \cdots, u') \rfloor$. (*rule 5.1*)
- use the "ceil" of the median value: $g(x, D) = \lceil \mu(l', \cdots, u') \rceil$. (*rule 5.2*)

Function $\mu(\cdot)$ returns the median value. The collective assignment interval reduction step is formalized in Algorithm 2. `OrderedList` in Algorithm 2 returns an ordered list from $(l(x), \cdots, u(x))$. Algorithm 2 runs in $O(|U| \cdot k \log k)$ where $|U|$ is the cardinality of U and k is the number of values in $(l(x), \cdots, u(x))$.

Algorithm 2 **AssignmentIntervalReduction**

Input: $I(x)$: Collective assignment interval ($\forall x \in U$)
$rule$: Interval reduction rule
Output: $g(x, D), \forall x \in U$
1. **for each** $x \in U$
2. $l \leftarrow l(x)$
3. $u \leftarrow u(x)$
4. **if** $l = u$**, then** $g(x, D) \leftarrow l$
5. **else if** $rule$ **is 'min', then** $g(x, D) \leftarrow l$
6. **else if** $rule$ **is 'max', then** $g(x, D) \leftarrow u$
7. **else** $(l', \cdots, u') \leftarrow$ **OrderedList**$(l(x), \cdots, u(x))$
8. $m \leftarrow \text{median}(l', \cdots, u')$
9. **if** $rule$ **is 'floor', then** $g(x, D) \leftarrow \lfloor m \rfloor$ **end if**
10. **if** $rule$ **is 'ceil', then** $g(x, D) \leftarrow \lceil m \rceil$ **end if**
11. **end if**
12. **end if**
13. **end if**
14. **end for**

5.2 Inference of collective decision rules

The objective here is to apply DRSA using the collective decision table S as input. The application of DRSA at this level is the same as for a single decision maker. The output is a collection of decision rules synthesizing the preference information of the different decision makers. These rules can then be included in a knowledge-based decision support system [6] and used as basis for decision making.

6 APPLICATION

The problem considered concerns post-accident nuclear risk management in the southern France in which one of the authors was implied. For the purpose of this paper, only a subset of data is used. Further decision objects and names of decision makers are codified (confidentiality reasons). The problem considered involves 10 decision objects, 7 evaluation criteria, 3 decision makers (CM, PP and CAL), and six decision classes (Cl_0 to Cl_5). Decision objects correspond to a subset of the districts of the study area. The list of evaluation criteria is given in Table 1 and decision classes are given in Table 2.

Table 1. List of evaluation criteria

Code	Description
C_1	Radioecological vulnerability of agricultural area
C_2	Radioecological vulnerability of forest area
C_3	Radioecological vulnerability of urban area
C_4	Real estate vulnerability
C_5	Tourism vulnerability
C_6	Economic vulnerability of companies
C_7	Employment vulnerability

Table 2. Decision classes

Level	Class	Name
0	Cl_0	Normal situation
1	Cl_1	Very minor
2	Cl_2	Minor
3	Cl_3	Moderate
4	Cl_4	Major
5	Cl_5	Major and long-lasting

6.1 Phase 1: Individual classification

First, each decision maker runs the DRSA[3] method using its own decision table obtained by adding a new decision attribute to the common information table. Decision tables used here are given in Table 3 where decision attributes D_1, D_2 and D_3 correspond to decision makers CM, PP and CAL. The obtained quality of classifications are $\gamma_P^1 = 0.61$ (CM), $\gamma_P^2 = 0.33$ (PP), and $\gamma_P^3 = 0.33$ (CAL).

Table 3. Decision tables

Object	C_1	C_2	C_3	C_4	C_5	C_6	C_7	D_1	D_2	D_3
x_1	4	5	5	5	4	1	1	4	4	5
x_2	4	5	5	5	4	2	2	4	4	5
x_3	4	5	5	5	4	2	1	4	4	5
x_4	4	5	5	5	4	3	1	5	4	5
x_5	3	2	2	4	4	2	0	3	2	3
x_6	1	1	1	2	4	1	0	0	0	1
x_7	2	2	1	2	4	1	0	3	2	2
x_8	1	2	1	2	2	1	0	0	0	1
x_9	3	2	2	4	4	2	0	3	2	2
x_{10}	3	3	3	4	4	1	0	3	2	3

6.2 Phase 2: Aggregation

6.2.1 *Step 2.1: Normalization*

First, Eq. (2) is used to normalize the quality of classifications γ_P^1, γ_P^2, and γ_P^3, which leads to: ${}^1\gamma'_P = .48.$, ${}^2\gamma'_P = .26.$ and ${}^3\gamma'_P = .26.$

6.2.2 *Step 2.2: Computing the concordance/discordance powers*

Concordance power For illustration, we only show the computing of $L^+(x_5, Cl_3^{\leq})$. The lower approximations for $Cl_3^{\leq}$ according to decision makers CM, PP and CAL are as follows:

- $\underline{P}(Cl_3^{\leq}) = \{x_5, x_6, x_7, x_8, x_9, x_{10}\}$. (CM)
- $\underline{P}(Cl_3^{\leq}) = \{x_5, x_6, x_7, x_8, x_9, x_{10}\}$. (PP)
- $\underline{P}(Cl_3^{\leq}) = \{x_8\}$. (CAL)

Hence, we have $L(x_5, Cl_3^{\leq}) = \{1, 2\}$. This means that only decision makers CM and PP assign x_5 to the lower approximation of $Cl_3^{\leq}$. Now, Eq. (3) can be used to compute the concordance power for object x_5 with respect to $Cl_3^{\leq}$:

$$L^+(x_5, Cl_3^{\leq}) \quad = \quad \sum_{i \in L(x_5, Cl_3^{\leq})} {}^i\gamma'_P = {}^1\gamma'_P + {}^2\gamma'_P = .48 + .26 = .74.$$

The concordance powers of decision object x_5 with respect to $Cl_t^{\leq}$ $(t = 0, \cdots, 4)$ and $Cl_t^{\geq}$ $(t = 1, \cdots, 5)$ are given in Table 4.

Discordance power For illustration, we only show the computing of $B^+(x_5, Cl_4^{\geq})$. The boundaries for $Cl_4^{\geq}$ according to decision makers CM, PP and CAL are as follows:

- $Bn_P(Cl_4^{\geq}) = \emptyset$. (CM)
- $Bn_P(Cl_4^{\geq}) = \emptyset$. (PP)
- $Bn_P(Cl_4^{\geq}) = \{x_5, x_6, x_7, x_9, x_{10}\}$. (CAL)

Then, we get $B(x_5, Cl_4^{\geq}) = \{3\}$. This means that only decision maker CAL assigns x_5 to the boundary of $Cl_4^{\geq}$. By Eq. (6), the discordance power for the assignment of object x_5 to $Cl_4^{\geq}$ is:

$$B^+(x_5, Cl_4^{\geq}) \quad = \quad \sum_{i \in B(x_5, Cl_4^{\geq})} {}^i\gamma'_P = {}^3\gamma'_P = .26$$

The boundary powers of decision object x_5 with respect to $Cl_t^{\leq}$ $(t = 0, \cdots, 4)$ and $Cl_t^{\geq}$ $(t = 1, \cdots, 5)$ are summed up in Table 4.

[3] Using 4eMKa, which is a stand-alone and free software implementing the DRSA method. See: http://idss.cs.put.poznan.pl/site/4emka.html.

Step 2.3: Definition of assignment intervals Here, assignment rules *rule 1* and *rule 2* given in Section 4.3 are used to associate to each object $x \in U$ a collective assignment interval $I(x)$. The majority and veto thresholds used in this application are $\theta = .5$ and $\theta' = .25$, respectively. Then, assignment *rule 1* and *rule 2* become:

$$\texttt{if } L^+(x, Cl_t^{\leq}) \geq .5 \wedge B^+(x, Cl_t^{\leq}) < .25, \texttt{ then } x \in Cl_t^{\leq} \\ \texttt{else } x \notin Cl_t^{\leq}$$

$$\texttt{if } L^+(x, Cl_t^{\geq}) \geq .5 \wedge B^+(x, Cl_t^{\geq}) < .25, \texttt{ then } x \in Cl_t^{\geq} \\ \texttt{else } x \notin Cl_t^{\geq}$$

The application of these rules to x_5 is summarized in Table 4 (fourth row). According to this table, it is easy to see that the first assignment rule is verified only for $Cl_3^{\leq}$ and $Cl_4^{\leq}$ while the second assignment rule is verified only for $Cl_1^{\geq}$ and $Cl_2^{\geq}$. In conclusion, we obtain: $x_5 \in Cl_3^{\leq}$, $x_5 \in Cl_4^{\leq}$, $x_5 \in Cl_1^{\geq}$ and $x_5 \in Cl_2^{\geq}$.

Table 4. Application of assignment rules (*rule 1* and *rule 2*) to object x_5

$Cl_t^{\cdot}$	$Cl_0^{\leq}$	$Cl_1^{\leq}$	$Cl_2^{\leq}$	$Cl_3^{\leq}$	$Cl_4^{\leq}$	$Cl_1^{\geq}$	$Cl_2^{\geq}$	$Cl_3^{\geq}$	$Cl_4^{\geq}$	$Cl_5^{\geq}$
$L^+(x_5, Cl_t^{\cdot})$	0	0	0	.74	1	1	1	.48	0	0
$B^+(x_5, Cl_t^{\cdot})$	0	0	.52	.26	0	0	0	.52	0.26	0
Decision	No	No	No	Yes	Yes	Yes	Yes	No	No	No

Now, to define the assignment interval $I(x_5) = [l(x_5), u(x_5)]$, we use Eqs. (7) and (8) to define $l(x_5)$ and $u(x_5)$. Based on Table 4, we get: $N_1(x_5) = \{Cl_t : x_5 \in Cl_t^{\geq}\} = \{Cl_1, Cl_2\}$, and $N_2(x_5) = \{Cl_t : x_5 \in Cl_t^{\leq}\} = \{Cl_3, Cl_4\}$. Then, Eqs. (7) and (8) lead to:

- $l(x_5) = \text{argmax}_{Cl_t} N_1(x_5) = \text{argmax}_{Cl_t} \{Cl_1, Cl_2\} = Cl_2$.
- $u(x_5) = \text{argmin}_{Cl_t} N_2(x_5) = \text{argmin}_{Cl_t} \{Cl_3, Cl_4\} = Cl_3$.

Finally, the assignment interval for decision object x_5 is $I(x_5) = [Cl_2, Cl_3]$. For convenience, the assignment intervals for all decision objects are given in Table 5 (second column).

6.3 Phase 3: Generation of collective decision rules

Step 3.1: Construction of a collective decision table The objective here is to construct the collective decision table $< U, C \cup D, V, g >$. The definition of $g(x, D)$, $\forall x \in U$ is summarized in Table 5 where columns "min", "max", "floor" and "ceil" refer to interval reduction rules *rule 3*, *rule 4*, *rule 5.1* and *rule 5.2*.

Table 5. The definition of $g(x, D)$ for different interval reduction rules

x_i	$I(x_i)$	min	max	floor	ceil
x_1	$[Cl_4, Cl_4]$	Cl_4	Cl_4	Cl_4	Cl_4
x_2	$[Cl_4, Cl_4]$	Cl_4	Cl_4	Cl_4	Cl_4
x_3	$[Cl_4, Cl_4]$	Cl_4	Cl_4	Cl_4	Cl_4
x_4	$[Cl_5, Cl_5]$	Cl_5	Cl_5	Cl_5	Cl_5
x_5	$[Cl_2, Cl_3]$	Cl_2	Cl_3	Cl_2	Cl_3
x_6	$[Cl_0, Cl_0]$	Cl_0	Cl_0	Cl_0	Cl_0
x_7	$[Cl_3, Cl_3]$	Cl_3	Cl_3	Cl_3	Cl_3
x_8	$[Cl_0, Cl_0]$	Cl_0	Cl_0	Cl_0	Cl_0
x_9	$[Cl_3, Cl_3]$	Cl_3	Cl_3	Cl_3	Cl_3
x_{10}	$[Cl_2, Cl_3]$	Cl_2	Cl_3	Cl_2	Cl_3

Step 3.2: Inference of collective decision rules The quality of classifications according to different interval reduction rules are given in Table 6. As it is shown in this table, interval reduction using the "max criterion" (*rule 4*) leads to the highest quality of classification (.83). The quality of classifications obtained by *rule 5.1* (floor) and *rule 5.2* (ceil) are equal to .72. In the three cases, we can conclude that the number of objects assigned with certitude to a given

class is acceptable. In the contrary, the quality of classification obtained by the "min" criterion (*rule 3*) is relatively low. Hence, the use of *rule 3* is not recommended in this illustrative application.

Table 6. The classification quality for different interval reduction rules

Rule	min	max	floor	ceil
γ_P	.28	0.83	.72	.72

A selection of collective decision rules generated using *rule 5.1* for interval reduction is given in Table 7. The first column in this table contains the decision rule. The second column contains objects supporting the rule. The last column indicates the strength of the rule. The description of these rules is straightforward. For illustration, we briefly comment two ones:

- **Rule 4**: if $f(x, q_5) \leq 3$, then $Cl_2^{\leq}$
- **Rule 20**: if $f(x, q_2) \leq 2 \wedge f(x, q_5) \leq 4$, then $Cl_2^{\geq}$

Rule 4 means that an object x is assigned to $Cl_2^{\leq}$ if its evaluation with respect to "Tourism vulnerability" criterion (q_5) is less or equal to 3. Rule 4 is supported only by decision objects x_8 and x_{15}. Its strength is equal to 40%.

Rule 20 says that object x is assigned to $Cl_2^{\geq}$ once (i) its evaluation with respect to "Radioecological vulnerability of forest area" criterion (q_2) is less or equal to 2, and (ii) its evaluation with respect to "Tourism vulnerability" criterion (q_5) is less or equal to 4. The decision objects supporting Rule 20 are: x_1, x_2, x_3, x_4, x_5, x_7, x_9, and x_{10}. The strength of Rule 20 is equal to 92.86%.

Table 7. A selection of collective decision rules

Rule	**Supporting objects**	**Strength**
Rule1: if $f(x, q_5) \leq 2$, then $Cl_0^{\leq}$	x_8	100%
Rule2: if $f(x, q_1) \leq 1$, then $Cl_2^{\leq}$	x_6,x_8	80%
Rule4: if $f(x, q_5) \leq 3$, then $Cl_2^{\leq}$	x_8,x_{15}	40%
Rule13: if $f(x, q_6) \leq 3$, then $Cl_5^{\geq}$	x_4	100%
Rule19: if $f(x, q_1) \leq 2$,then $Cl_2^{\geq}$	$x_1, x_2, x_3, x_4, x_5, x_7, x_9, x_{10}$	100%
Rule20:if $f(x, q_2) \leq 2 \wedge f(x, q_5) \leq 4$, then $Cl_2^{\geq}$	$x_1, x_2, x_3, x_4, x_5, x_7, x_9, x_{10}$	92.86%
Rule22: if $f(x, q_2) \leq 2 \wedge f(x, q_5) \leq 3$, then $Cl_2^{\geq}$	$x_1, x_2, x_3, x_4, x_5, x_7, x_9, x_{10}$	100%

7 RELATED WORK

In [7], the authors propose a DRSA-based methodology to group decision making with application to knowledge management. It contains four steps. First, a common decision table is constructed. Second, decision rules for each assignment example determined in the first step are inferred. The obtained results are checked for inconsistencies problems. Third, each decision maker solves the eventual inconsistence problems. Fourth, the analyst identifies collectively accepted decision rules. The main shortcoming of [7]'s methodology is its time consuming. In fact, the methodology requires, in conflicting situations, that the analyst conducts an in-depth discussion with the different decision makers in order to solve the conflicts. This is a time-consuming and difficult task.

The authors in [1] propose an argumentative multi-agent model based on a mediator agent in order to automate the resolution of conflicts between decision makers in [7]'s methodology. This approach allows the mediator agent to elicit preference of decision makers while exploiting and managing their points of view. Although this multi-agent system-based approach permits to automatize conflict resolution, it has one major shortcoming. In fact, the aggregation rule used in [1] is defined as a weighted-sum of four criteria: the number of agents, the quality of classification, the number of rules and the average strength of rules. However, we think that the second and fourth criteria are similar, which may lead to over-evaluation.

Another extension of DRSA to support multiple decision makers is reported in [4] where the authors extend the lower and upper approximations and boundary concepts. More specifically, they introduce the concepts of downward and upward multi-union and mega-union. These concepts are then used to define lower and upper approximation for unions of classes. We think that this extension has three main shortcomings. First, it is difficult for decision makers to understand the aggregation mechanism adopted in [4]. Second, [4]'s approach is expensive in computational time. Third, there is no dialogue between the different decision makers.

8 CONCLUSION

We proposed a three-phase DRSA-based approach for group multi-criteria classification problems. The proposed approach takes as input a common information table and generates a set of collective decision rules representing a generalized description of the preference information of the decision makers. The paper detailed the approach and illustrates it through a real-world application. The proposed approach has several merits. First, as it is based on DRSA, the approach: (i) does not require any preference parameter, (ii) is able to deal with lack of information, and (iii) is able to detect and handle inconsistency problems in the decision table. Second, the approach uses the majority rule which is characterized by (i) its simplicity, anonymity and neutrality, and (ii) its low-demanding in terms of computational time. Third and in contrary to [7][1] (which are very demanding in terms of dialogue) and [4] (which requires no dialogue), the proposed approach is not very demanding in terms of dialogue between the different decision makers.

Several topics need to be investigated in the future. The first one concerns the use of decision rules-related information to define the assignment rules. The second one is related to the use of other classification methods that accept interval-based assignment for decision objects. The third one concerns the use of input level aggregation-oriented schema.

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