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Spinflation

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Abstract: We study the cosmological implications of including angular motion in the DBI brane inflation scenario. The non-canonical kinetic terms of the Dirac-Born-Infeld action give an interesting alternative to slow roll inflation, and cycling branes can drive periods of accelerated expansion in the Universe. We present explicit numerical solutions demonstrating brane inflation in the Klebanov-Strassler throat. We find that demanding sufficient inflation takes place in the throat is in conflict with keeping the brane's total energy low enough so that local gravitational backreaction on the Calabi-Yau manifold can be safely ignored. We deduce that *spinflation* (brane inflation with angular momentum) can ease this tension by providing extra e-foldings at the start of inflation. Cosmological expansion rapidly damps the angular momentum causing an exit to a more conventional brane inflation scenario. Finally, we set up a general framework for cosmological perturbation theory in this scenario, where we have multi-field non-standard kinetic term inflation.

Keywords: Brane worlds, string cosmology.

Contents

1. Introduction

String theory, our best candidate for a theory of everything, has recently suggested a possible fundamental origin for the inflaton field in the context of brane inflation [1–3]. In a typical brane inflation scenario our three-dimensional Universe consists of a stack of D-branes embedded in an extra dimensional bulk spacetime. The identification of the inflaton field with the position of a probe brane moving in the extra dimensions is a considerable step towards addressing the problem of connecting the Inflationary Universe paradigm with fundamental particle physics.

The early constructions of brane inflation were somewhat *ad hoc*, fixing the compactification radius by hand, and using the inter-brane Coulomb attraction to provide an inflaton potential. However, using the recent improved understanding of flux compactifications and moduli stabilization in string theory, a more consistent and compelling picture has emerged in which brane inflation is realized by a mobile D3-(or anti-D3)-brane in a known stabilized warped compactified background.

In a previous paper, we examined the motion of probe D3-branes and D3-anti-branes in a typical warped geometry, allowing the branes to move freely in the angular directions [4]. These branes exhibited a rich variety of angular momentum dependent bouncing and cyclic trajectories which could be interpreted as cyclic cosmologies from the *mirage* perspective. In mirage cosmology, [7], the cosmological expansion of the brane arises as a result of the brane's motion through a curved bulk spacetime. The brane itself does not warp the background directly, but is treated as a 'probe', moving according to an effective action appropriate to the nature of the brane and background. The mirage approach has the distinct advantage of being exact in the case of codimension one cosmological branes, which do indeed move through warped backgrounds. However, the picture has shortcomings precisely because the brane does not back-react on the geometry, nor can it in higher codimension be associated with global effects like the domain wall. This lack of back-reaction makes it rather difficult to see how to localise generic matter on the brane and recover standard Einstein gravity at late times.

The mirage prescription has had a great impact on the development of brane cosmology, and can be viewed as an essential first step towards the understanding of the full brane universe. In the case of a brane moving on a background with angular isometries, the addition of angular motion, with its conserved momentum to the probe brane has a critical effect on the brane cosmological models thus derived. For example, in an adS background, a previously monotonic brane behaviour becomes dramatically altered into a 'slingshot' scenario, [5], and of course as already noted there is now the possibility of general bouncing and cyclic cosmologies [6]. Since these possibilities are so radically different to the simple radial motion brane inflation, it is vital to understand which features are solely a consequence of the mirage point of view, and which are preserved once one takes the first steps towards a more fully self consistent gravitational picture.

Ideally, one would like to go beyond the mirage approximation by finding a fully localised solution to the supergravity equations for the cosmological brane moving in an (appropriately deformed) warped throat. However, such a solution would have to reflect only the symmetries allowed, in this case, the three-dimensional non-compact spatial symmetry of the Universe. The internal six dimensions strictly speaking need not have any fixed symmetries, since placing a localised brane at an arbitrary point on the internal manifold will break the background isometries. In the case of codimension one, this is not an issue, as the only real degree of freedom is the scale factor of the Universe, which (when extrapolated to the bulk) depends on only two variables, and the system is integrable [8]. However, the additional codimensions, even in the absence of fluxes and warping, introduce new physical degrees of freedom, which in principle could be excited in the non-supersymmetric cosmological background. Even in the simple case of the codimension one Randall-Sundrum [9] scenario, cosmological solutions have the additional degree of freedom of a bulk black hole.

It is worth emphasizing that all known non-supersymmetric solutions in higher codimension are obtained by imposing an ansatz for the metric and/or bulk fields.

Such ansätze have the effect of restricting the degrees of freedom of the metric and bulk fields, rendering the problem tractable. For example, an analysis of cosmological solutions in higher codimension results in a full classification of solutions under the assumption of separability of metric functions [10], however, these most certainly are not the full set of solutions. In some cases, the imposition of even the mildest assumption catastrophically restricts the solution space, as in codimension two cosmology, where there is known to be no general FRW family [11] (although with Gauss-Bonnet terms the story is different [12]). On the other hand, when dealing with an isolated brane it is not clear one should be considering a local supergravity solution at high curvatures. The usual argument used in brane inflation is that one can keep the internal metric, excise a ball around the brane and simply replace with the metric appropriate to an isolated brane in asymptotically flat spacetime. While we note that this could be problematic (for example, a brane with a de Sitter worldvolume is *not* asymptotically flat), we suspect that a full computation would support the overall picture of brane inflation.

Therefore, in this paper, we adopt the usual approach in which our Universe resides on a stack of D3-branes embedded somewhere in the background spacetime and the motion of an additional three-brane probe acts as the inflaton field. We assume that the internal manifold metric is unchanged, except possibly very close to the brane, and that the only gravitational effect of the mobile brane is in the noncompact dimensions, where it sources cosmological expansion. We discuss the consistency of this hypothesis, finding for which regimes of the brane speed, the brane backreaction is reduced to a level compatible with our approximations. We allow the brane to have spiral motion (i.e. movement in the internal angular directions of the compactification [13]) in the throat of the warped space. In this framework, we find that the resulting cosmology corresponds to an expanding universe (that is, we do not find any more bounces in the scale factor as in the mirage approach). We show that large angular momentum may have a measurable effect on the brane inflation scenario, introducing the notion of *spinflation*. In the four-dimensional effective theory, the conserved angular momentum may appear as a field (or several fields). In particular, the presence of such *spinflatons* can augment the inflaton field producing a small number of additional e-foldings during the early stages of inflation. The angular momentum is rapidly inflated away. The presence of these fields can have important implications for cosmological perturbations since, non-adiabatic modes can play a non-trivial role. Precise observable predictions for brane inflation are only possible in very specific models [14], therefore, we concentrate on new general features induced by the addition of angular momentum into brane inflation scenarios. Finally, we develop the generalised theory of cosmological perturbations for scenarios involving multi-fields with non-standard kinetic terms and apply the results to the present case.

The layout of the paper is as follows: In section 2 we introduce the set-up, establish notation and conventions, and discuss the effect of angular momentum. In section 3, we discuss the gravitational consistency of the pseudo probe brane approach, and present an exact solution of a brane on a compactified spacetime at arbitrary velocity. We use this to derive a rigorous bound on the brane speed which can be applied to more complicated scenarios such as warped fluxed backgrounds. In section 4 we present numerical solutions for branes moving on the Klebanov-Strassler throat (an illustrative warped background for the deep IR), demonstrating how angular momentum or spinning can give extra e-foldings. In section 5 we present a formal calculation of cosmological perturbations in this DBI multi-field inflationary scenario. Finally, in a complete model of inflation a mechanism to excite standard model degrees of freedom is required at the end of the inflationary epoch (a so called *reheating* period). A rigorous discussion of reheating in the context of brane inflation remains a challenge. Some initial progress has been made in [15–17]. Here, we provide a short discussion of yet another mechanism to reheat the Universe in the context of our model.

2. DBI Brane Cosmology

The basic set-up is as follows: we consider a flux compactification of type IIB string theory on an orientifold of a Calabi-Yau three-fold (or an F-theory compactification of a Calabi-Yau four-fold), as in [18]. We further assume that the fluxes generate a warped throat region in the internal space, hence we adopt a metric of the form:

$$ds_{10}^2 \,=\, h^{-1/2}(r)\, g_{\mu\nu} dx^\mu dx^\nu \,+\, h^{1/2}(r)\, \mathrm{g}_{mn} dy^m dy^n\,, \qquad (2.1)$$

where $h(r)$ is the warp factor, $g_{\mu\nu}$ is the metric in four dimensions, g_{mn} is the metric of the internal space, and r is the proper (radial) length with respect to this internal metric (i.e. $\mathrm{g}_{rr} = 1$). We embed a mobile D3-brane in this ten-dimensional space whose world-volume dynamics are described by a Dirac-Born-Infeld (DBI) action. The D3-brane is free to move on the internal compact Calabi-Yau manifold, although we take the brane to be confined to the warped throat region. The basic setup is depicted in Fig. 1.

In our explicit computations, we take the geometry of the throat to be the non-singular warped deformed conifold (or Klebanov-Strassler) geometry [19], the details of which we discuss in section 4. Mid-throat, the geometry is well approximated by the singular warped conifold (or Klebanov-Tseytlin) geometry [20]; however, we shall not need to make use of this simplification in our analysis. The brane's energy-momentum produces a gravitational effect on the non-compact spacetime dimensions in which it is extended, as well as a possible local distortion of spacetime. We adopt the standard approximation that this local distortion is negligable, and the brane moves effectively as a test particle on the compact manifold. It is natural

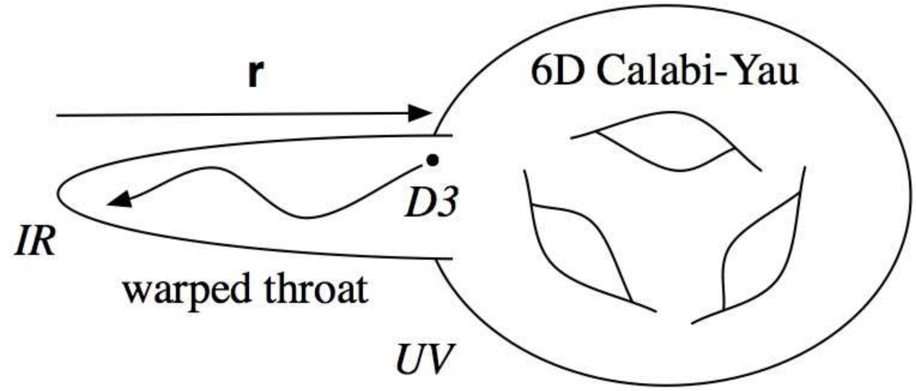

Figure 1: A mobile D3-brane spiraling down the warped throat region of a flux compactification. The throat is smoothly glued to a Calabi-Yau threefold in the UV region. In principle, our Universe may exist in various parts of compactification, including other warped throats not shown in the diagram. The construction involves wrapped D-branes and orientifold planes not shown here.

to transform to field theory variables for an effective four-dimensional description obtained by integrating out the internal coordinates:

$$\begin{aligned} S = & \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\,R \\ & - g_s^{-1}\int d^4x\sqrt{-g}\left[f^{-1}\sqrt{1+f\,\mathrm{g}_{mn}g^{\mu\nu}\partial_\mu\phi^m\partial_\nu\phi^n} - q\,f^{-1} + V(\phi^m)\right]. \end{aligned} \tag{2.2}$$

The first term is the ordinary four-dimensional Einstein-Hilbert action, which arises from dimensional reduction of the closed string sector of the ten dimensional action, (see discussion in section 4). The second line contains the action that controls the dynamics of the fields, parameterizing the position of the brane along the internal coordinates, $\phi^m = (\phi, \phi^\alpha)$ where ϕ^α are angular variables, and the relation between the radial geometrical and field theory variable is:

$$\phi = \sqrt{T_3}\,r\,, \qquad f(\phi) = T_3^{-1}h\left(\frac{\phi}{\sqrt{T_3}}\right) \tag{2.3}$$

where $T_3 = ((2\pi)^3\alpha'^2)^{-1}$ is the D-brane tension. The parameter $q = 1$ ($q = -1$) indicates that we are considering a brane (anti-brane). Although we include q to maintain generality of the formulae, in this paper we will only consider brane motion, i.e. $q = 1$. g_{mn} corresponds to the moduli space metric, while $g_{\mu\nu}$ is the 4D-effective spacetime metric, that we take to be flat Friedmann-Robertson-Walker (FRW):

$$ds^2 = -dt^2 + a^2(t)\,dx^i dx_i\,, \tag{2.4}$$

where $a(t)$ is the scale factor. Finally, we add a potential for the scalar fields $V(\phi)$ which can arise once the system is coupled to other sectors of the theory. We are interested in the case where the potential corresponds to a mass term for the radially associated scalar field. In general, the computation of the potential $V(\phi)$ is rather involved and has only been done explicitly in a few specific models. The potential receives contributions from gravitational and Coulomb interactions, in addition to (non-negligible) correction terms from various compactification effects, for example, non-perturbative corrections to the superpotential from moduli stabilizing wrapped D7 (or Euclidean D3 brane instantons) [1, 14]. See also [21] for papers in a similar framework, [22] for related ideas with wrapped branes, and [23] for multiple brane scenarios.

We assume the scalar fields ϕ^n are homogeneous, that is $\phi^n = \phi^n(t)$. The variation of the DBI part of (2.2) with respect to the metric gives the energy-momentum tensor. This has the form of a perfect fluid, with energy density and pressure:

$$E = \frac{1}{f}\,[\gamma - q] + V \tag{2.5}$$

$$P = \frac{1}{f}\,\left[q - \gamma^{-1}\right] - V\,. \tag{2.6}$$

Here we have defined a quantity that plays a key role in the following discussion:

$$\gamma \equiv \frac{1}{\sqrt{1 - f v^2}} \tag{2.7}$$

which lies in the range $\gamma \in [1, \infty)$ and $v^2 = \mathrm{g}_{mn}\dot\phi^m\dot\phi^n$. Clearly γ is a generalization of the usual relativistic Lorentz factor to the warped background. The $(n+1)$-equations of motion for the scale factor $a(t)$, and the scalar fields ϕ^n, become

$$\frac{\ddot a}{a} = -\frac{1}{6\,M_{Pl}^2 g_s}\,(E + 3\,P) \tag{2.8}$$

$$\frac{1}{a^3}\frac{d}{dt}\left[a^3\,\mathrm{g}_{mn}\,\dot\phi^n\gamma\right] = \gamma\,\left(\gamma^{-1} - q\right)^2\,\frac{\partial_m f}{2\,f^2} + \frac{\gamma}{2}\left(\frac{\partial\,\mathrm{g}_{ln}}{\partial\phi^m}\right)\dot\phi^l\dot\phi^n - \partial_m V\,. \tag{2.9}$$

These equations are accompanied by the Friedmann constraint

$$H^2 = \frac{1}{3\,g_s\,M_{Pl}^2}\,E = \beta E\,, \tag{2.10}$$

and the equation of conservation of energy:

$$\dot E + 3H\,(E + P) = 0\,. \tag{2.11}$$

We are interested in the case where the potential is independent of the scalar fields associated with the angular variables, so that there are conserved angular momenta. For simplicity, we restrict to a $U(1)$ subgroup, choosing an angle which corresponds to an isometry of the metric.[1] The corresponding angular momenta, l_α, is defined as:

$$\frac{1}{a^3}\frac{d}{dt}\left[a^3\, \mathrm{g}_{\alpha\beta}\,\dot{\theta}^\beta \gamma\right] = 0 \qquad \Rightarrow \qquad l_\alpha \equiv a^3\, \mathrm{g}_{\alpha\beta}\,\dot{\theta}^\beta \gamma\,, \tag{2.12}$$

The only dynamical variables that remain (to be determined from the equations of motion) are the four-dimensional scale factor $a(t)$, and the position of the brane along the radial coordinate, $\phi(t)$. In terms of the above, the velocity v^2 may be written as

$$v^2 \; = \; \frac{\left[\dot{\phi}^2 + \ell^2(\phi)/a^6\right]}{\left[1 + f\,\ell^2(\phi)/a^6\right]}\,, \tag{2.13}$$

where, $\ell^2(\phi) = T_3\, \mathrm{g}^{\alpha\beta}\, l_\alpha l_\beta$. Therefore,

$$\gamma \; = \; \sqrt{\frac{1 + f\,\ell^2(\phi)/a^6}{1 - f\,\dot{\phi}^2}}\,. \tag{2.14}$$

The DBI action places a velocity bound on the radial field ϕ, forcing $1 - f\,v^2 > 0$. This translates into the requirement $f\,\dot{\phi}^2 < 1$, *independent* of the value of the angular momentum ℓ [4].

An alternate way of writing equations (2.8 - 2.10) is in the first order formalism:

$$\dot{a} = Ha \tag{2.15}$$

$$f\,\dot{\phi}^2 = 1 - \left(1 + \frac{f\ell^2(\phi)}{a^6}\right)\cdot\left(q + f\left(\frac{H^2}{\beta} - V\right)\right)^{-2} \tag{2.16}$$

$$\dot{H} = -\frac{3\beta}{2}\left[2q\left(\frac{H^2}{\beta} - V\right) + f\left(\frac{H^2}{\beta} - V\right)^2\right]\cdot\left(q + f\left(\frac{H^2}{\beta} - V\right)\right)^{-1} \tag{2.17}$$

It is often useful in inflationary theory to use the Hamilton-Jacobi formulation where the dynamical variable, H, is written as a function of ϕ [24]. Provided the scale factor does not explicitly appear in the main equations of motion, the Friedman and Raychaudhuri equations, (2.16) and (2.17), can be rewritten as a single nonlinear first order equation for $H(\phi)$. Often, given a potential $V(\phi)$, this leads to a direct solution for H. Unfortunately, in the present case, the scale factor does appear in (2.16), and therefore we cannot perform a Hamilton-Jacobi reduction, this is because the system is now dependent not only on the field ϕ, but also on additional fields θ^α. In addition, there is the added problem that the radial field ϕ is now no longer necessarily a monotonic function of time, the brane can, and in general does, have several oscillations up and down the throat, thus leading to a multivalued $H(\phi)$. There is however a way of rewriting the equations in a pseudo Hamilton-Jacobi form

[1] Note, we will use $m, n, \ldots$ to denote all the internal indices, and $\alpha, \beta, \ldots$ to denote the angular internal directions.

by altering our field-space coordinates (see section 5 for a full description of this). Since the inflationary trajectory is simply a curve in the internal Calabi-Yau manifold we can define a new coordinate, σ, which is the proper distance along this trajectory:

$$\sigma = \int \sqrt{\dot{\phi}^2 + T_3\, g_{\theta\theta}\, \dot{\theta}^2} dt \tag{2.18}$$

thus σ is a monotonic function of time, and we can now proceed with rewriting the equations of motion along the inflationary trajectory. In terms of this coordinate, one can readily derive $\dot{\sigma}$ in terms of $H'(\sigma)$ and f, leading to

$$\frac{4H'(\sigma)^2}{9\beta^2} = f\left(\frac{H^2}{\beta} - V\right)^2 + 2\left(\frac{H^2}{\beta} - V\right) \tag{2.19}$$

Because f and V are both functions of ϕ, this is not a true differential equation for H, nonetheless, some useful general qualitative information about the inflationary trajectory can be extracted. For example, since $|\dot{\phi}| \leq \dot{\sigma}$, f increases less rapidly, and V decreases less rapidly along an inflationary trajectory with angular momentum, which in turn means that H', and hence $\dot{\sigma}$ is lowered. This feeds into the integral for the number of e-foldings, as we show in the next section.

3. Cosmology and Gravity of the brane

The picture outlined in the previous section is that of a D3 brane moving on a warped background, and driving cosmological expansion in the noncompact directions via the energy-momentum of its motion. As yet, we have not specified whether the brane is slowly or relativistically rolling, however, one simple fact is clear from (2.10): the four-dimensional scale factor is always expanding. This is because the scale factor refers to the metric on the noncompact spacetime, and not the induced metric on the brane. The expansion is driven by the brane moving up *and* down along (damped) cyclic trajectories through the throat.

In the following section, we explore some general gravitational features of this set-up. Specifically, we are interested in when the brane sources accelerated expansion, and when the gravitational back reaction of the brane becomes significant.

3.1 Brane (Sp)Inflation

The acceleration is quantified by the "acceleration" parameter:

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}$$

and the acceleration equation may be written as:

$$\frac{\ddot{a}}{a} = H^2(1-\varepsilon)\,. \tag{3.1}$$

To achieve accelerated expansion of the scale factor a, ε must be smaller than unity.

Using equations (2.16) and (2.17), the acceleration parameter may be expressed as

$$\varepsilon = \frac{3\beta}{2H^2}\left\{\left[q + f\left(\frac{3H^2}{\beta} - V\right)\right]\dot{\phi}^2 + \frac{l^2(\phi)}{a^6}\left[q + f\left(\frac{3H^2}{\beta} - V\right)\right]^{-1}\right\}. \qquad (3.2)$$

Thus, the angular momentum plays a significant role in determining the degree of acceleration of the cosmological expansion. Naively, the angular momentum appears to hinder acceleration (because it provides a positive contribution to ε (3.2)); however, the angular momentum *reduces* the brane speed $\dot{\phi}$ (see eq. 2.16) acting to decrease the value of the first term in (3.2). Therefore, there is a nontrivial interplay between the $\dot{\phi}$ and ℓ terms. We shall demonstrate that the suppression of $\dot{\phi}$ will typically win out, and the acceleration parameter ε is effectively reduced by the angular momentum, yielding *spinflation.* The total number of e-foldings, N_e is given by:

$$N_e(\phi) = \int H dt = \int_{\phi_i}^{\phi} \frac{H(\phi)}{\dot{\phi}} d\phi = \int_{\sigma_i}^{\sigma} \frac{H(\sigma)}{\dot{\sigma}} d\sigma\,, \qquad (3.3)$$

where the intermediate ϕ integral is understood as a sum over intervals where $H(\phi)$ is monotonic. Although the actual behaviour of $H(\sigma)$ is rather involved, one can see immediately that the overall interval for the integration in the presence of angular momentum is increased over its absence. Since the value of $\dot{\sigma}$ is not increased by angular momentum without a corresponding increase in H, we see that this increase in integration interval will translate to an increase in N_e.

Here we briefly summarize the different ways cosmological acceleration can arise, before presenting explicit numerical examples of these possibilities. Note that ε can be written as

$$\varepsilon = \frac{3}{2}\frac{E+P}{E} = \frac{3}{2}\frac{\gamma f v^2}{\gamma - 1 + fV}\,. \qquad (3.4)$$

In ordinary slow-roll acceleration ε is small because $fv^2 \ll 1$, i.e. the field is moving slowly. This is the most well studied model of stringy brane inflation, and generally requires a flat, dominant, potential. See [14] for interesting recent work exploring the validity of this set-up.

In D-acceleration it is assumed that the field is 'rapidly' moving, $fv^2 \simeq 1$, and $\gamma \gg 1$, but that $fV \gg \gamma$, as in the set-up of [25]. This effect is caused by the large warp factor, rather than a large bare velocity, and again, in past investigations there is no angular momentum $\ell = 0$. As we have shown in our discussion of Eq. (3.2), the angular momentum plays an active role in determining the degree of acceleration of the system. Aside from decreasing the value of the slow-roll paramter ε, angular momentum contributes to the bouncing and cyclic features of the brane trajectories [4].

In the proximity of a turning point in the brane trajectory, when $\dot{\phi} \to 0$, the size of ε is reduced because the $\dot{\phi}^2$ contribution vanishes (3.2). Only the ℓ contribution remains:

$$\varepsilon = \frac{3\beta}{2H^2}\frac{\ell^2(\phi)}{a^6}\left[1 + h\left(\frac{3H^2}{\beta} - V\right)\right]^{-1} . \tag{3.5}$$

The obvious bouncing points occur whenever the brane passes through the tip of the KS geometry and then is pulled back down by the force generated from the effective potential V. During each pass up and down the throat an accelerating phase is induced (whenever $\dot{\phi} \to 0$) leading to a multi-inflationary type of scenario [26]. This occurs independently from motion in the angular directions. Such oscillations, however, are heavily damped and it is not possible to gain significant additional e-foldings from these short bursts of acceleration.

In principle, angular momentum can induce new bouncing points *before* the brane reaches the tip of the conifold, resulting in cyclic trajectories [4]. One may therefore hope that such an angular momentum induced bounce could give rise to a prolonged accelerating period. Unfortunately, it is not possible to have sufficient angular momentum to create such a bounce while simultaneously generating enough e-foldings to solve the problems of the Standard Big-Bang model. In a successful inflationary model, the angular momentum is inflated away too quickly[2] to induce the desired bounce before the brane reaches the tip of the KS geometry. Recall, from Eq. (3.2) that the angular momentum contribution dilutes with the scale factor as a^{-6}. During prolonged accelerated expansion, the angular momentum is inflated away faster than most other matter which dilute as

$$\rho_i \sim a^{-3(1+w_i)} , \tag{3.6}$$

where the w_i are the effective equation of state parameters for the various matter sources designated by the index i.

3.2 Gravitational back reaction

A concern with the pseudo probe brane approximation for DBI inflation is that the local gravitational effect of the brane on the throat is not generally considered. One point of view is that this effect is extremely localized, and may be approximated by gluing the asymptotically flat solution in along a tube surrounding the brane world-line. For small velocities, this should be a good approximation, but in DBI-inflation, the brane is typically moving relativistically, and one must check the corresponding increase in proper energy does not ruin this picture. Silverstein and Tong [25] gave two separate estimates of this effect, with two different bounds. Here we show how to compute this bound precisely.

[2]See the online talk *http://online.itp.ucsb.edu/online/strings03/quevedo/* for a discussion of this effect on a similar configuration.

In order to investigate the back reaction we consider the exact solution of the D3-brane on the compact manifold S^1:

$$ds^2 = h(r,x)^{-1/2}\left[-dt^2 + d\mathbf{X}^2\right] + h(r,x)^{1/2}\left[dr^2 + r^2 d\Omega_{\rm IV}^2 + dx^2\right] \tag{3.7}$$

where x has periodicity $2\pi R$, and $h = 1 + \mathcal{H}(r,x)$, with

$$\mathcal{H}(r,x) = \frac{\pi g_s \alpha'^2}{Rr^3}\left\{\frac{r}{R}\frac{\cosh(r/R)\cos(x/R) - 1}{(\cosh(r/R) - \cos(x/R))^2} + \frac{\sinh(r/R)}{\cosh(r/R) - \cos(x/R)}\right\}. \tag{3.8}$$

This is a different physical set-up to the CY internal manifold, however, it is similar in the sense that the brane is moving in a compact direction, and crucially, allows for an exact computation of the gravitational field.

It is now simple to explore the effect of motion on the internal manifold: one performs a lorentz boost before making the identification. This has the effect of replacing the time and internal coordinate with the new coordinates

$$t(\tau,\xi) = \gamma(\tau + v\xi)\ , \qquad x(\tau,\xi) = \gamma(\xi + v\tau) \tag{3.9}$$

thus rendering the metric explicitly time dependent. In the new coordinates:

$$ds_{\tau,\xi} = -h^{-1/2}(1 - \gamma^2\mathcal{H}v^2)d\tau^2 + 2v\mathcal{H}h^{-1/2}\gamma^2 d\tau d\xi + h^{-1/2}(1 + \gamma^2\mathcal{H})d\xi^2 \tag{3.10}$$

Demanding, that at $r = 0$, the effect of the brane is negligable at the scale of the compactification implies

$$\frac{4\pi g_s \alpha'^2}{R^4}\gamma^2 \ll 1 \tag{3.11}$$

On the other hand, asymptotically, the Newtonian potential can be read off as

$$g_{\tau\tau} \sim 1 - \frac{\pi g_s \alpha'^2 \gamma^2}{Rr^3} = 1 - \frac{3\pi G_6 M_{ADM}}{2\pi^2 r^3} \tag{3.12}$$

where M_{ADM} is the ADM mass of the brane. Now we use $G_7 = 2\pi R G_6$, and $G_7 M_{ADM} = G_{10}\mu/g_s$, where μ is the proper energy per unit worldvolume of the D3 to obtain:

$$\frac{3\kappa_{10}^2 \mu}{16\pi^3 g_s R^4} \ll 1 \tag{3.13}$$

This is an exact relation for the single D3 moving and localized on an S^1. Now we apply this to the D3 in the warped throat, where for the proper density of the probe we have $\mu = T_3 f E$ in field theory variables, and R now refers to the local ambient curvature of the throat. This gives a bound

$$\gamma(\phi) - 1 \ll \frac{R^4}{l_s^4 g_s}\,, \tag{3.14}$$

in agreement with the second of Silverstein and Tong's bounds [25]. In other words, since $g_s \ll 1$, and $R \gg 1$ for the supergravity description to be valid, the probe brane can move hyperrelativistically before local gravitational backreaction would seem to be an issue. Nevertheless, we typically find that γ grows to large values extremely quickly, making this bound difficult to satisfy in a successful inflationary scenario.

4. The Klebanov-Strassler Throat

The Klebanov-Strassler (KS) throat is an exact nonsingular supergravity solution with fluxes sourced by D3 and wrapped D5 branes. We use this explicit warped geometry in order to explore the behaviour of DBI-inflation in the deep infrared region. In fact, a great deal of interesting cosmological behaviour occurs in regions where we cannot use a pure AdS or Klebanov-Tseytlin approximation to the geometry.

The background geometry has the form [19]:

$$ds^2 = h^{-1/2} g_{\mu\nu}\, dx^\mu dx^\nu + h^{1/2}\, ds_6^2 \tag{4.1}$$

where

$$ds_6^2 = \frac{\epsilon^{4/3}}{2} K(\eta) \Bigg[\frac{1}{3\, K(\eta)^3} \left\{ d\eta^2 + (g^5)^2 \right\} + \cosh^2(\eta/2) \left\{ (g^3)^2 + (g^4)^2 \right\} + \sinh^2(\eta/2) \left\{ (g^1)^2 + (g^2)^2 \right\} \Bigg], \tag{4.2}$$

and

$$K(\eta) = \frac{(\sinh(2\eta) - 2\,\eta)^{1/3}}{2^{1/3} \sinh \eta} \tag{4.3}$$

$$h(\eta) = 2^{2/3} (g_s M \alpha')^2 \epsilon^{-8/3} \int_\eta^\infty dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh x - 2x)^{1/3} . \tag{4.4}$$

Here, M quantifies the amount of three-form flux present in the background (for details see [19]), the parameter $\epsilon^{2/3}$ has dimensions of length and defines the UV scale of the throat [27], and the explicit form of the one-form basis $\{g^i\}$ (given in [4,19]) is not important for our present investigation. The metric (4.2) has angular isometries, and therefore conserved charges are present. For concreteness, we take the motion to be in the angular variable which contributes identically to g^2 and g^4. The warp factor for KS (4.4) is shown in Fig. 2.

Before discussing the brane motion, we estimate the ranges for the parameters appearing in the geometry and equations of motion. The effective four-dimensional Planck mass is obtained from a reduction of the 10D supergravity action:

$$\frac{1}{\kappa_{10}^2} \int d^{10}X \sqrt{G_{10}} R_{10} = \frac{1}{\kappa_{10}^2} \int d^4x d^6y \sqrt{\det g_{\mu\nu}} \sqrt{\det g_{mn}} h^{1/2} \{ h^{1/2} R(g_{\mu\nu}) + \dots \} \tag{4.5}$$

Thus the Planck scale is given by

$$M_{Pl}^2 = \frac{V_6^{(w)}}{\kappa_{10}^2} = \frac{\mathcal{V}_6}{(2\pi)^6 \pi g_s^2} M_s^2 \tag{4.6}$$

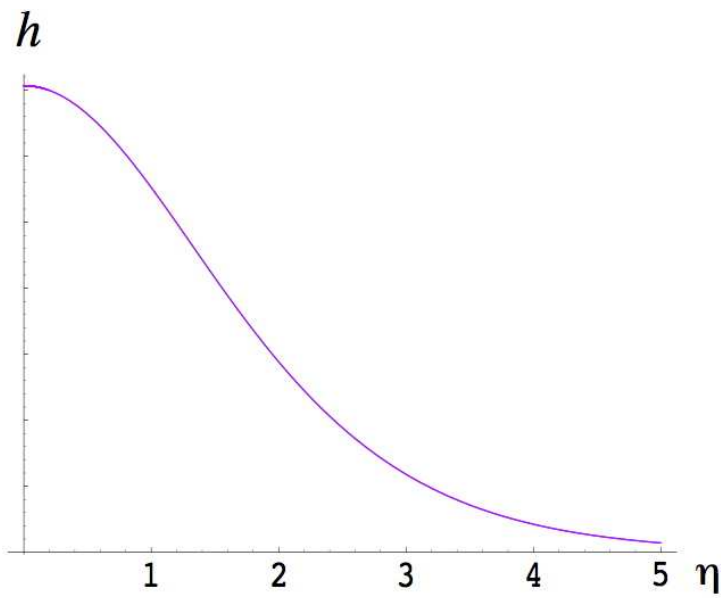

Figure 2: Warp factor for the KS geometry.

where

$$V_6^{(w)} = \int d^6y\, \sqrt{g_6}\, h > \frac{\sqrt{2}\epsilon^4\pi^3}{3} \int_0^{\eta_{UV}} h(\eta) \sinh^2\eta\; d\eta \tag{4.7}$$

is the weighted volume of the Calabi-Yau. We have also used $\kappa_{10}^2 = (2\pi)^7\, \alpha'^4\, g_s^2/2$, and expressed the Planck mass in terms of the string scale $M_s^{-2} = \ell_s^2 = \alpha'$, and the volume in string units $\mathcal{V}_6 \equiv V_6/\ell_s^6$.

Performing this integral for the KS throat shows that for $\eta_{UV} \sim \mathcal{O}(1-10)$, the volume is well approximated by

$$V_6^{(w)} \simeq \frac{\epsilon^{4/3}\pi^3}{3}(g_s M\alpha')^2\eta_{UV}^3\,. \tag{4.8}$$

We can understand this intuitively since the metric near the tip of the throat is approximately that of a three-sphere times the neighborhood of the origin of $\mathbf{R}^3$. The flux parameter, g_sM, is related to M_{Pl}, by

$$g_sM < \left(\frac{4}{\eta_{UV}}\right)^{3/2} \frac{\ell_s}{\epsilon^{2/3}}\sqrt{3}g_s\pi^2\frac{M_{pl}}{M_s}\,. \tag{4.9}$$

Thus in order to have large g_sM, which we require for the supergravity approximation to be valid, we must take either M_{Pl} large or ϵ small. On the other hand, the parameter that measures the coupling to gravity is

$$\beta = \frac{1}{3\, g_s\, M_{Pl}^2}\,, \tag{4.10}$$

and the bigger β, the stronger the backreaction caused by the moving brane, and hence the greater the inflationary effect. However, β cannot be too large, otherwise we are outside the regime of perturbative string gravity. A reasonable choice of parameters is $g_s \sim 0.1$ and $M_{Pl} \sim 10^2$. With this choice of parameters and an $\eta_{UV} \sim 10$, we see that $g_s M \simeq 40 - 50\epsilon^{-2/3}$.

4.1 Numerical solutions

We consider the motion of a D3-brane, allowing it to spiral along an angular coordinate. Transforming to field theory variables, the radial coordinate is defined as:

$$r_{KS} = \frac{\epsilon^{2/3}}{\sqrt{6}} \int_0^{\eta} \frac{\sinh x \, dx}{(\sinh x \cosh x - x)^{1/3}} \tag{4.11}$$

with $\phi = \sqrt{T_3}\, r$, and $f = h/T_3$ as before. Note that we now have a rather convoluted relation between the "natural" geometric coordinate η, in which the metric is expressible in terms of analytic functions (or integrals), and the field theory coordinate ϕ, which is the canonically normalized inflation field, and a natural coordinate to use for cosmological inflation. Most work on brane inflation uses an internal metric in which the natural radial coordinate coincides with the proper distance along the throat, however, because we are neither slow-rolling, nor restricting ourselves to a small region of the throat, we must necessarily consider the proper normalization and definition of the inflaton field, which leads to the use of (4.11). For the angular field, we use the coordinate θ, which remains dimensionless.

In principle, the potential $V(\phi)$ could have contributions from a variety of sources, so in order to extract generic features of introducing angular momentum into brane inflation scenarios we consider the simple quadratic potential, $V = m^2 \phi^2$. We integrated the brane equations for a variety of parameter values and initial conditions. The number of e-foldings is mainly determined by $g_s M$ and m, and is largely insensitive to the value of ϵ. We find that the larger the inflaton mass, the greater the amount of inflation, as in the original DBI inflation model [25], with (roughly) $N_e \propto m g_s M$. We also find that the number of e-foldings increases the further up the throat we start the brane rolling, as expected. When the angular momentum is turned on, the acceleration is increased. The amount of acceleration provided by the angular motion (although not enough to account for the entire number of e-folds necessary to solve the horizon problem, etc.) can contribute with a handful of e-foldings. Figures 3 and 4 show a sample inflating solution with $g_s M = 300$, $\epsilon = 0.05$, $l_\theta \sim 5.8 \times 10^6$.

The solution describes a brane moving down the throat from the CY bulk. The brane accelerates as it falls, and arrives at the tip of the cone with non-zero velocity and continues its motion smoothly back up the throat until it reaches a turning point, where it is pulled down again due to the attractive potential. The scale factor stops accelerating *before* the brane reaches the tip of the throat without requiring the presence of anti-$\overline{D}3$-branes. Therefore, inflation ends in a natural way, simply because of a geometric constraint. The KS warp factor approaches a constant value

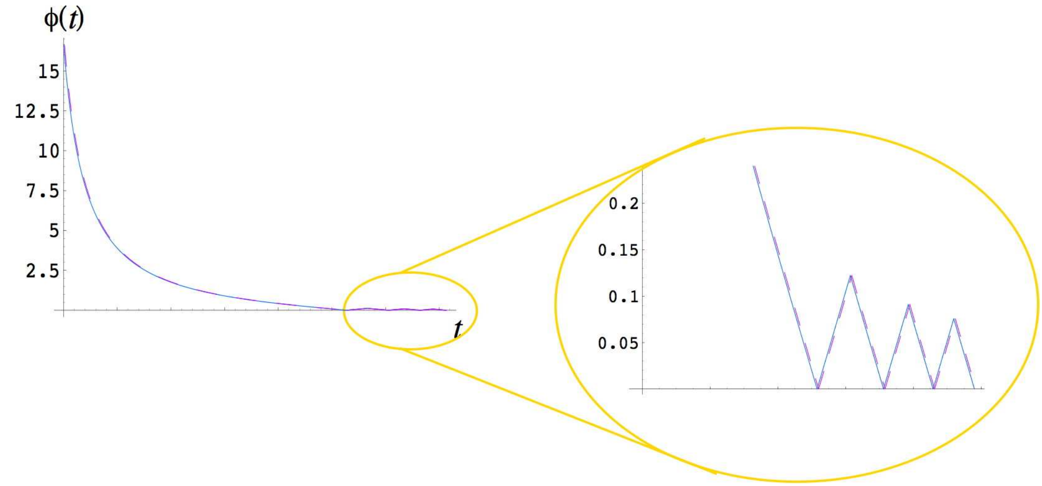

Figure 3: A sample plot of $\phi(t)$ for an inflating solution with (light purple) and without (blue) angular momentum l_θ.

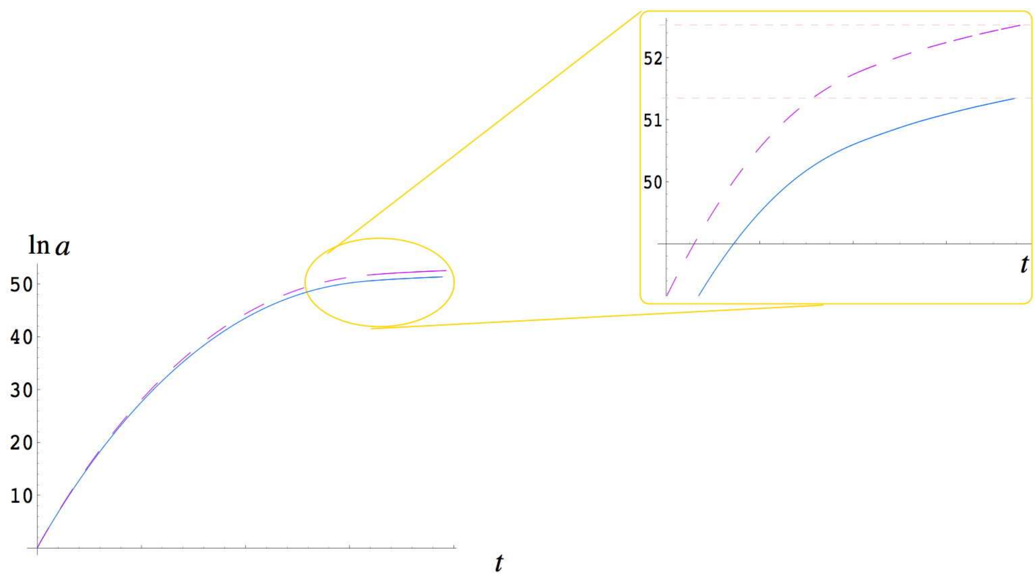

Figure 4: Behaviour of the scale factor for the solution in figure 3.

at the tip of the conifold. The brane bouncing motion continues, with the brane moving up and down for a few more cycles, causing the Universe to expand and accelerate briefly near the turning points. As the brane loses energy and reaches the minimum of its potential, the Universe expands as a stiff fluid-dominated Universe. Equation (2.16) implies that at the tip,

$$H^2 \to \frac{\beta \ell_0^2}{2a^6}\,, \tag{4.12}$$

where $\ell_0 = \ell(\phi = 0)$ is the value of the angular momentum at the tip. The remnant energy density in the angular momentum acts as a source that redshifts as $\rho_\ell \sim a^{-6}$,

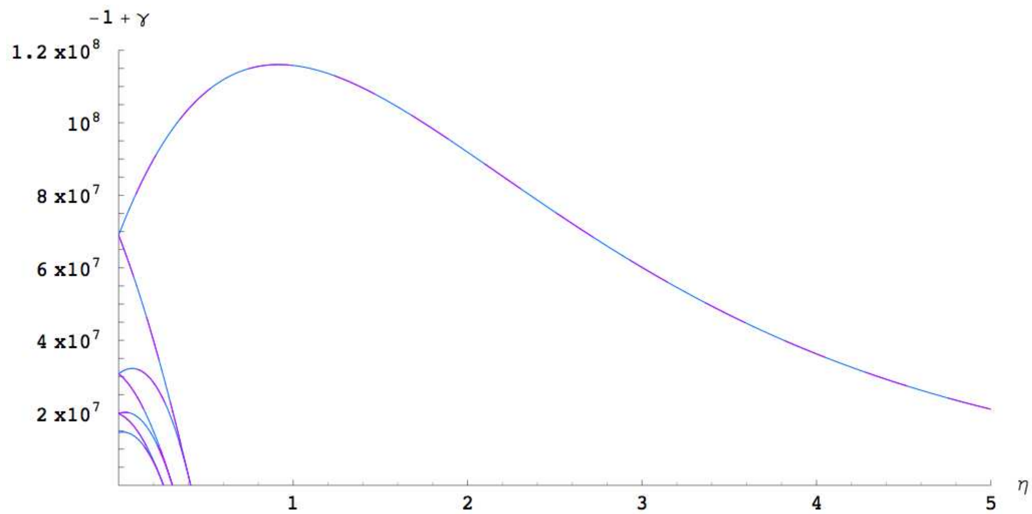

Figure 5: Behaviour of the gamma factor for the solution in figure 3.

and therefore, has a stiff equation of state $w_{eff} = 1$ (see Eq. (3.6)).

When the angular momentum is turned on, it provides the brane with an extra kick of acceleration (*spinflation*). This gives a couple of additional e-foldings, although the dominant number of e-foldings is provided by radial motion.

Insisting that the full e-foldings of inflation take place in the KS throat leads to a rather artificial set of parameter values, which induce extremely strong relativistic motion (see figure 5). To obtain sufficient e-foldings large $g_s M$ is required, which is consistent with the supergravity approximation. On the other hand, to keep the volume of the throat under control and consistent with the value of M_{Pl}, we require that ϵ be small, which means that the throat is very strongly warped. This is problematic because it leads to an unacceptably large relativistic γ-factor, violating the back-reaction bound (3.14). We were unable to find a set of parameter values and initial conditions which could both satisfy this bound *and* supply the full e-foldings required by inflation in the KS throat. We suspect therefore that DBI inflation in the deep IR is not a viable scenario, although we have not been able to prove this result. It is possible that a different potential or background may evade these difficulties.

5. Cosmological perturbations and angular motion

In the previous section, we found that angular momentum acts to prolong inflation. We also noted that the main contribution from spinning occurs at the start of inflation, before the spinflaton is damped away. However, even though the coherent effect of spinning may damp away, it is possible that perturbations in the spinflaton may have important cosmological consequences. We therefore explore the perturbation theory of DBI-spinflation in this section.

Garriga and Mukhanov, [28], developed a treatment of cosmological perturbations in single field inflationary models with non-standard kinetic terms, showing that the dynamics of linearized perturbations provides a spectral index for curvature perturbations very near to one. Building on these results, Alishahiha, Silverstein and Tong [29] applied their analysis to the the D-acceleration framework, also in the single field case. Typically, although Gaussian perturbations provide a spectral index compatible with observations, non-Gaussianities are generally non-negligible, and provide an observational signature for these models. Interestingly, bounds from non-Gaussianities considerably limit the range of the field ϕ [29, 30].

If angular momentum is turned on, we enter into a multi-field inflationary scenario, and non-adiabatic fluctuations can be generated. The issue of how to treat perturbations associated to non-adiabatic modes, in the context of DBI cosmology, has not been studied in full detail up to now. There are two key differences between this set-up and standard slow-roll perturbation analysis. One is the non-standard DBI kinetic term, which alters the analysis and typically gives rise to large non-gaussianities. Another is that the geometry of the spinflaton target space (the KS or other warped throat) is strongly curved; even aside from the warp factor, $h(\eta)$, the intrinsic metric g_{mn} is not flat. Clearly, in slow-roll brane inflationary models this curvature is not important, as all manifolds are locally flat by definition. However, in "fast-roll" inflation (i.e. $fv^2 \gg 1$) the curvature of the target space becomes important. We emphasize that this is the first model in which the spinflaton manifold is not flat, or conformally flat. This complicates the perturbation analysis considerably. Our focus is to set up the formalism, and extract general properties for multi-field inflationary models with non-standard kinetic terms.

In standard, slow-roll inflation the presence of entropy modes normally seeds curvature perturbations, and, due to this fact the latter are not constant, even in the long wavelength limit. We examine the contributions of entropy modes to curvature perturbations in our multi-field DBI framework, focusing on the long wavelength limit in order to understand whether similar features occur here, and to explore the general characteristics of curvature perturbations. We show that adiabatic and non-adiabatic perturbations move with different speeds, commenting on possible consequences of this fact. We also explicitly prove that all perturbations remain well behaved at the brane motion turning points, suggesting that our time dependent trajectories are stable under perturbations at such points. The resulting formalism has a natural application in cosmological models such as the ones we are considering here. The analysis is completely general and has broad applications. Our formalism extends the seminal work of [31] for the canonical kinetic term case (see [32] for nice reviews, and [33] for recent applications to string and brane cosmology).

As in standard multifield inflation, it is useful to perform a rotation in field space in order to rewrite the scalar equations in a convenient way. We re-define coordinates based on inflationary trajectories and their normals. For simplicity, we

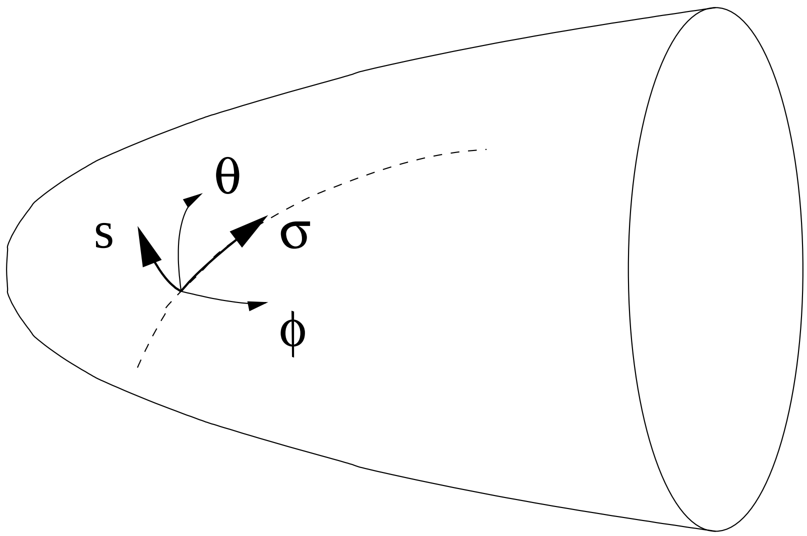

Figure 6: Illustration of the multifield space.

consider the effect of only one angular degree of freedom, although this formalism is easily generalized.

We define the angle α as:

$$\cos\alpha = \frac{\dot{\phi}}{\sqrt{2X}} \qquad \sin\alpha = \frac{\sqrt{B}\dot{\theta}}{\sqrt{2X}} \tag{5.1}$$

where

$$X \equiv \frac{1}{2}\left(\dot{\phi}^2 + T_3\, g_{\theta\theta}\,\dot{\theta}^2\right) \equiv \frac{1}{2}\left(\dot{\phi}^2 + B\dot{\theta}^2\right) . \tag{5.2}$$

When $\dot{\theta} = 0$, the angle α vanishes. We define the averaged trajectory field σ (see 2.18) and cf. [31]):

$$d\,\sigma \;=\; \cos\alpha\, d\phi + \sqrt{B}\sin\alpha\, d\theta\,, \tag{5.3}$$

which is the geodesic length introduced in (2.18), and an orthogonal, entropy field s:

$$d\,s \;=\; \sqrt{B}\cos\alpha\, d\theta - \sin\alpha\, d\phi\,. \tag{5.4}$$

This leads to the equalities

$$\begin{aligned}\dot{\sigma}^2 &= 2X \\ \dot{s} &= 0\end{aligned} \tag{5.5}$$

in exact analogy to the flat multifield case.

The equations for ϕ and θ can be rewritten in these new variables

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3\,\gamma\,\dot{\sigma}\right) \;=\; -\frac{\partial V}{\partial\sigma} + \frac{\gamma(\gamma^{-1}-1)^2}{2f^2}\frac{\partial f}{\partial\sigma} \tag{5.6}$$

$$\gamma\,\dot{\sigma}\,\dot{\alpha} \;=\; -\frac{\partial V}{\partial s} + \frac{\gamma(\gamma^{-1}-1)^2}{2f^2}\frac{\partial f}{\partial s} + \frac{\gamma\dot{\sigma}^2}{2B}\frac{\partial B}{\partial s} \tag{5.7}$$

These expressions show that σ can be regarded as an averaged brane trajectory: it satisfies a dynamical equation, (5.6), typical of dynamical evolution under a potential, with modifications from the warped geometry (see Figure 6). Equation (5.7) is first order, and describes the dynamics of the angular field α, that is responsible for the departure of the brane trajectory from the radial motion. Note that conservation of the angular momentum gives the relation

$$\sin\alpha = \frac{l}{a^3\gamma\dot{\sigma}\sqrt{B}} \tag{5.8}$$

Now consider linear perturbation theory. We express the perturbations for the scalar fields in terms of perturbations in these new variables:

$$\frac{\delta\sigma}{\dot{\sigma}} \equiv \cos^2\alpha\left(\frac{\delta\phi}{\dot{\phi}}\right) + \sin^2\alpha\left(\frac{\delta\theta}{\dot{\theta}}\right) \tag{5.9}$$

$$\frac{\delta s}{\dot{\sigma}\,\sin\alpha\cos\alpha} \equiv \frac{\delta\theta}{\dot{\theta}} - \frac{\delta\phi}{\dot{\phi}}\,. \tag{5.10}$$

Since the spatial part of the energy-momentum perturbation is proportional to the metric, the scalar metric perturbations can be written in longitudinal gauge as [36]:

$$ds^2 \;=\; -\left(1+2\Phi\right)\,dt^2 + \left(1-2\Phi\right)a^2(t)\gamma_{ij}dx^i dx^j\,. \tag{5.11}$$

Following [28], we define gauge invariant combinations of the perturbations that simplify the equations. Let ξ and ζ be:

$$\Phi a \equiv \frac{3\beta}{2}H\xi, \tag{5.12}$$

$$\cos^2\alpha\left(\frac{\delta\phi}{\dot{\phi}}\right) + \sin^2\alpha\left(\frac{\delta\theta}{\dot{\theta}}\right) = \frac{\delta\sigma}{\dot{\sigma}} \equiv \frac{\zeta}{H} - \frac{3\beta\xi}{2a}. \tag{5.13}$$

The perturbation ζ is usually called the curvature perturbation (in uniform-density hypersurfaces). The perturbation ξ corresponds to the Newtonian potential in longitudinal gauge, while δs is the entropy perturbation (notice that it is gauge invariant). It is common to work with the canonical Sasaki-Mukhanov variable used to quantise the field fluctuations coupled to first order metric perturbations during inflation

$$\delta\sigma_\Phi \;\equiv\; \frac{\dot{\sigma}}{H}\,\zeta \tag{5.14}$$

which corresponds to a gauge invariant perturbation for a spatially flat gauge. In the long wavelength limit, it is equivalent to the curvature perturbation.

We now study the three dynamical perturbations ζ, ξ, and δs. While the perturbations ζ and ξ are familiar from the analysis of [28], here we must consider entropy perturbations associated with the generation of non-adiabatic fluctuations for the curvature perturbations. The perturbed Einstein equations for energy and momentum readily give

$$\dot{\xi} = \frac{a\,(E+P)}{H^2}\zeta, \tag{5.15}$$

$$\begin{aligned}(E+P)\,\frac{\dot{\zeta}}{H} &= \frac{H\,c_S^2}{a^3}\,(\Delta\xi)\ -\ \tan\alpha\,\left(\dot{P}-c_S^2\dot{E}\right)\,\frac{\delta s}{\dot{\sigma}}\\ &= \frac{H\,c_S^2}{a^3}\,(\Delta\xi)\ -\ \sin\alpha\,\left(-V_{,\phi}(1+c_S^2)+\frac{f_{,\phi}}{f^2}(1-\gamma^{-1})^2\right)\,\delta s\end{aligned} \tag{5.16}$$

Equation (5.16) shows how the entropy perturbation couples to the curvature perturbation ζ, when $\alpha \neq 0$. This induces non-adiabatic entropy modes that do not vanish even in the long wavelength limit, $k^2 \ll H^2a^2$, or when the potential V vanishes (this is different to the case discussed in [35]). Clearly, (5.15) and (5.16) can be combined to give a second order equation, however, this is more conveniently expressed in terms of the Sasaki-Mukhanov variable, (5.14), as:

$$\begin{aligned}\delta\ddot{\sigma}_\Phi + \left(3H+3\frac{\dot{\gamma}}{\gamma}\right)\delta\dot{\sigma}_\Phi\ +\ \left(U_{\sigma_\Phi}+c_s^2\,\frac{k^2}{a^2}\right)\delta\sigma_\Phi&\\ = -\left(\frac{H\,\dot{\sigma}c_S^2}{a^3\,(E+P)}\right)&\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P}-c_S^2\dot{E}\right)\,\frac{\delta s}{\dot{\sigma}}\right]^{\cdot}\end{aligned} \tag{5.17}$$

with

$$\begin{aligned}U_{\sigma_\Phi} &\equiv \frac{\dot{\sigma}Hc_S^2}{a^3\,(E+P)}\,\left[\left(\frac{\dot{H}}{H}-\frac{\ddot{\sigma}}{\dot{\sigma}}\right)\,\frac{a^3(E+P)}{\dot{\sigma}Hc_S^2}\right]^{\cdot}\\ &= \left(\frac{\dot{H}}{H}-\frac{\ddot{\sigma}}{\dot{\sigma}}\right)\left(3H+\frac{\ddot{\sigma}}{\dot{\sigma}}+3\frac{\dot{\gamma}}{\gamma}-\frac{\dot{H}}{H}\right)+\left(\frac{\dot{H}}{H}-\frac{\ddot{\sigma}}{\dot{\sigma}}\right)^{\cdot}.\end{aligned} \tag{5.18}$$

A third, independent equation of motion that controls the evolution of δs is found from the perturbed $\theta-$equation

$$\delta\ddot{s}\ +\ \left(3H+\frac{\dot{\gamma}}{\gamma}\right)\delta\dot{s}\ +\ \left(U_s+\frac{k^2}{a^2}\right)\delta s\ =\ -\frac{k^2}{a^2}\,\frac{\dot{\sigma}\,\tan\alpha\,H}{a\,(E+P)^2}\,\left(\dot{P}-c_S^2\dot{E}\right)\,\xi \tag{5.19}$$

where

$$\begin{aligned}U_s\ =\ \tan\alpha&\left[3H\tan\alpha\left(\frac{\ddot{\sigma}}{\dot{\sigma}}-\frac{\dot{\alpha}}{\tan\alpha}+3Hc_S^2-\cos\alpha\frac{B'\,\dot{\sigma}}{2B}\right)\right.\\ &\left.+\ \dot{\sigma}\left(\frac{\dot{\alpha}}{\dot{\sigma}}-\frac{B'\,\cos\alpha}{2B\,\tan\alpha}\right)^{\cdot}\right].\end{aligned} \tag{5.20}$$

Even without solving these equations, we can extract a great deal of useful qualitative information about the behaviour of perturbations in multifield DBI inflation.

First, the entropy perturbation evolves independently of the curvature perturbation at large scales ($k^2/a^2 \ll 1$): it couples to the curvature perturbation only through the Newtonian potential ξ. (This remains true even if the potential is not angularly invariant, see (A.4).) Interestingly, the converse is not true: at large scales the entropy perturbation actually seeds curvature perturbations. Equation (5.17) (or (A.4) for the case of angular V dependence) contains contributions from δs that do not vanish even in the long wave-length limit.

Another important observation is that curvature and entropy perturbations move with *different speeds*. While curvature perturbations move with a speed of sound much smaller than the speed of light (indeed $c_S^2 \ll 1$), the entropy perturbations move at light speed. An interpretation of this fact relies on the observation that the speed of the perturbations is related with the speed of the background quantity they are associated with. Curvature perturbations are associated with the speed of the average trajectory $\dot{\sigma}$, that enters the speed of sound as $c_S^{-2} = 1 - h\dot{\sigma}^2$. Entropy perturbations refer to $\dot{s} = 0$, and the corresponding sound speed is $c_S^{-2} = 1 - h\dot{s}^2 = 1$. [3] A consequence of this fact is that entropy perturbations can re-enter the horizon much earlier than the curvature perturbations in DBI models. It would be interesting to study consequences and applications of this fact.

Finally, coupling between adiabatic and isocurvature modes generally becomes stronger at the turning points, if the angular momentum does not vanish. Since $\sin\alpha = 1$ at these points, the coefficient of the term proportional to δs in (5.17) acquires its maximal size. This raises an important physical and technical issue: are the perturbations finite at turning points? We have learned that the brane trajectory will encounter bounces during its evolution, and this can have interesting applications for cosmology. In the absence of angular momentum, the entropy perturbations decouple from curvature perturbations, however, in the presence of angular momentum this is no longer the case, and at the bounce when $\cos\alpha = 0$, there is a potential divergence in this coupling from the $\tan\alpha$ term on the RHS of (5.19). In addition, the expression for U_s, (5.20), is potentially singular for the same reason. Clearly, such a singularity would be catastrophic for cosmological perturbation theory, and of course does not arise in conventional inflationary models. We must therefore check that this is not an issue for spinflation.

Physically, we do not expect the turning point to exhibit any particular pathology. Inflation is produced by the motion of a D3-brane on an internal manifold, and cosmological perturbation theory is a combination of gravitational perturbations in the noncompact manifold and internal fluctuations in the trajectory of the brane, in other words the brane "flutters" on the internal manifold as a function of the spacetime coordinates. Picturing the moving D3-brane as a fluttering sheet shows that although we might not expect every point on the brane to turn at the same time,

[3] We thank Kazuya Koyama for discussions on this point.

overall the brane will turn and there should be no associated singularity. Similarly, from the field theory point of view, a field oscillating in a potential well experiences several turning points, and in no way are these associated with singularities. Therefore, although the formulae appear ill-behaved at $\cos\alpha = 0$, we do not expect that there is a technical problem.

The equation for curvature perturbations, (5.16), is regular at any bounce, however, the entropy perturbation equation is somewhat more subtle. The RHS of (5.19) is regular, for the same reason as (5.16). At first sight (5.20) seems problematic, containing terms proportional to $\tan^2\alpha$. Using (5.7), the derivative in the second term of (5.20) is:

$$\dot{\sigma}\frac{d}{dt}\left(\frac{\dot{\alpha}}{\dot{\sigma}} - \frac{B'\ \cos\alpha}{2B\ \tan\alpha}\right) = -\ddot{\sigma}\sin\alpha\left(\gamma V_{,\phi}\frac{(1+c_S^2)}{\dot{\sigma}^2} + \frac{f_{,\phi}}{f}\frac{(\gamma-1)}{(\gamma+1)}\right). \tag{5.21}$$

Using conservation of angular momentum:

$$\frac{\dot{\alpha}}{\tan\alpha} + \frac{\ddot{\sigma}}{\dot{\sigma}} + 3H + \frac{B'\,\dot{\phi}}{2B} + \frac{\dot{\gamma}}{\gamma} = 0 \tag{5.22}$$

to substitute for H in the first term of (5.20), and tracking the V and f derivatives carefully, we find

$$U_s = -\tan\alpha\sin\alpha\dot{\sigma}\left(\gamma V_{,\phi}\frac{(1+c_S^2)}{\dot{\sigma}^2} + \frac{f_{,\phi}}{f}\frac{(\gamma-1)}{(\gamma+1)}\right)\left(\frac{\ddot{\sigma}}{\dot{\sigma}} + 3Hc_S^2\right) = \mathcal{O}(1)\,. \tag{5.23}$$

Thus the perturbations are manifestly well-behaved at any bounces, in agreement with our expectations.

Finally, in the case of conserved angular momentum, during inflationary expansion entropy perturbations become rapidly decoupled from the adiabatic ones. Consequently, they are not likely to drastically modify the evolution of curvature perturbations. This is particularly evident from equation (5.16): indeed, the angle α decreases as the cube of the scale factor (see eq. (5.8)). During inflation the scale factor increases rapidly and the angle α shrinks to negligible size within a handful of e-folds. A different situation may occur when the angular momentum is *not* conserved, and the potential V explicitly depends on the angular coordinates. In Appendix A we derive the general form of the perturbation equations. We leave the analysis of the imprints of entropy perturbations to inflationary parameters in this general case for future work.

6. Conclusions and Outlook

In this paper we studied various aspects of angular motion of branes travelling in a warped throat. Our main focus was on the impact of angular momentum in a DBI-inflationary scenario, where the brane is typically moving at relativistic proper velocities. We used the KS throat as a stereotypical example to study the cosmologies of such trajectories numerically, and developed an analytic perturbation formalism for general multi-field DBI-inflation. We find that the angular momentum enriches the qualitative features of the brane motion.

The four-dimensional low energy cosmology is controlled by a standard Einstein-Hilbert term for gravity, coupled to a DBI-like action for the scalar fields that parameterise the brane position in the higher dimensional background. The cosmological evolution is characterised by an always increasing scale factor. Periods of accelerated expansion can occur in various ways. In addition to standard slow-roll inflation, the non-canonical form of the kinetic terms impose a bound on the brane velocity. This leads to the D-acceleration scenario of [25]: in the regions where the brane speed is reduced, by means of this bound, the potential energy dominates and drives inflation. Short periods of acceleration can occur when the brane trajectories have turning points: since the brane speed in the radial direction vanishes, the total velocity is reduced and potential terms are more likely to dominate, providing acceleration.

We took the specific example of a warped KS throat for a numerical study of the cosmologies arising from a spinning brane. Our investigations indicated that building a successful inflationary model based on the DBI-action is in tension with the constraints coming from the validity of the supergravity approximation and fine tuning of parameters is required. We found that in order to have sufficient inflation, the relativistic γ-factor was driven to extremely large values, well in excess of any non-gaussianity or other bounds [29, 34, 37], and typically at least an order of magnitude in excess of the back-reaction bound. This would appear to be a major challenge for the DBI-inflation scenario within string theory. It would be interesting to investigate how generic this conclusion is. For example, making the effective potential $V(\phi)$ steeper by using a ϕ^4 term seems to ameliorate the problem. We could also include a conformal coupling term (see [38]) or alter the background throat geometry.

Non-standard kinetic terms reveal novel features when analysing Gaussian cosmological perturbations. Brane motion along angular trajectories, in addition to the radial one, implies that cosmological expansion is driven by more than one field. Moreover, the strong warping of the throat means that the curvature of the target space can also contribute to the perturbation theory. Non-adiabatic modes may play an important role on the evolution of curvature perturbations. We derive the equations for cosmological perturbations in this set-up, finding that non-adiabatic entropy modes move with different speeds with respect to adiabatic, curvature perturbations. This may have important consequences when studying inflationary models, for example since different perturbations enter the horizon at different times. Also, the features of the curved target space appear in the effective potentials that drive the perturbations, that are sensitive to the underlining geometry.

We focused mainly on potentials with no angular dependence, since that was the example used in the explicit numerical cosmologies. However, for completeness, we include in an appendix the calculations in the presence of angular dependence. In this situation, the coupling between entropy and curvature modes is likely to be suppressed during inflationary expansion. In general, one expects that wrapped branes used to stabilise the throat moduli induce angular dependence, and indeed this has been demonstrated explicitly for a particular embedding of D7-branes in the KT background [14], thus any concrete model of DBI-inflationary perturbations must include such an analysis. A complete analysis of angular dependence is clearly the next step in exploring general inflationary trajectories, however, given that the number of e-foldings seems to be related to the length of the inflationary trajectory it is quite likely that even in the presence of angular dependence, the number of e-foldings will be increased.

Finally, it is possible that angular momentum may have additional interesting consequences for reheating the Universe at the end of inflation. In conventional brane inflationary models an anti-$\overline{D}$-brane is placed at the tip of the throat. The reheating mechanism involves tachyon condensation, when the inflaton brane reaches the $\overline{D}$-brane causing an explosive reheating process. In our case, a $\overline{D}$-brane is not necessary for successful reheating.

We can envision a natural mechanism for an inflationary exit, and an alternative scenario for reheating. It is possible that damped brane oscillations through the tip of the KS throat cause the three non-compact dimensions to heat up (similar scenarios have been proposed in [15,17]). The brane inflaton fields ϕ may have couplings to a scalar ξ or spinor ψ through an interaction Lagrangian of the form

$$\mathcal{L} = -\frac{1}{2}g^2\,\phi^2\xi^2 - h\,\phi\bar{\psi}\psi\,. \tag{6.1}$$

Each time the brane bounces in the throat, the inflaton can decay *via* the above interactions into ξ and ψ particles. The total decay rate is consequently

$$\Gamma = \Gamma(\phi \to \xi\xi) + \Gamma(\phi \to \psi\psi)\,. \tag{6.2}$$

Details of this process depend on the model and on specific features of the compactification.

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A. Cosmological perturbations for an angularly dependent potential

We now generalize the equations that govern the cosmological perturbations to the case where the potential depends explicitly on the angular variable θ. We start by defining a combination Q that will frequently appear:

$$Q \equiv a^3 \gamma \sqrt{B}\, \dot{\sigma} \sin \alpha\,. \tag{A.1}$$

This quantity, at the background level, satisfies the relation

$$\partial_t Q = -a^3\, \partial_\theta V$$

so that if $\partial_\theta V = 0$, Q is constant and defines the conserved angular momentum. In the more general case in which $\partial_\theta V \neq 0$, it is straightforward to obtain the equations that determine the curvature perturbations. We find

$$\dot{\xi} = \frac{a\,(E+P)}{H^2} \zeta, \tag{A.2}$$

$$\begin{aligned} &(E+P)\frac{\dot{\zeta}}{H} = \\ &\qquad \frac{H c_S^2}{a^3}\,(\Delta\xi) - \frac{\tan\alpha}{\dot{\sigma}}\left[\left(\dot{P} - c_S^2 \dot{E}\right) - \left(1+c_S^2\right)(E+P)\,\frac{\partial_t Q}{Q}\right]\delta s, \end{aligned} \tag{A.3}$$

from which a few additional steps lead to a second order equation in terms of the Sasaki-Mukhanov variable.

The equation governing entropy perturbations is:

$$\begin{aligned} &\delta\ddot{s} + \left(3H + \frac{\dot{\gamma}}{\gamma}\right)\delta\dot{s} + \left(U_s + \frac{k^2}{a^2}\right)\delta s = \\ &\qquad -\frac{k^2}{a^2}\frac{\dot{\sigma}\tan\alpha H}{a\,(E+P)^2}\left[\dot{P} - c_S^2\dot{E} - \left(1+c_S^2\right)(E+P)\,\frac{\partial_t Q}{Q}\right]\xi \end{aligned} \tag{A.4}$$

where U_s is

$$\begin{aligned} U_s = {}& \tan\alpha\left[3H\tan\alpha\left(\frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan\alpha} + 3Hc_S^2 - \cos\alpha\frac{B'\,\dot{\sigma}}{2B}\right) + \dot{\sigma}\left(\frac{\dot{\alpha}}{\dot{\sigma}} - \frac{B'\,\cos\alpha}{2B\,\tan\alpha}\right)^{\boldsymbol{\cdot}}\right] + \\ &+ \tan\alpha\frac{\partial_t Q}{Q}\left\{\dot{\alpha} - \frac{B'\,\cos\alpha\,\dot{\sigma}}{2B\,\tan\alpha} - \tan\alpha\left[\dot{P} - c_S^2\dot{E} - \left(1+c_S^2\right)\,(E+P)\,\frac{\partial_t Q}{Q}\right]\right\} + \\ &+ \frac{a^3\dot{\sigma}\tan\alpha}{Q}\left[\frac{\partial^2 V}{\partial\theta^2}\frac{B'\,\cos\alpha}{2B} - \frac{\partial^2 V}{\partial\theta\partial\phi}\sin\alpha\right]. \end{aligned} \tag{A.5}$$

With the aid of these equations, it is possible to study the evolution of cosmological perturbations for a system of two fields with DBI action in full generality.

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