On the similarity relation within fuzzy ontologies components

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Abstract— Ontology reuse is an important research issue. Ontology merging, integration, mapping, alignment and versioning are some of its subprocesses. A considerable research work has been conducted on them. One common issue to these subprocesses is the problem of defining similarity relations among ontologies components. Crisp ontologies become less suitable in all domains in which the concepts to be represented have vague, uncertain and imprecise definitions. Fuzzy ontologies are developed to cope with these aspects. They are equally concerned with the problem of ontology reuse. Defining similarity relations within fuzzy context may be realized basing on the linguistic similarity among ontologies components or may be deduced from their intentional definitions. The latter approach needs to be dealt with differently in crisp and fuzzy ontologies. This is the scope of this paper.

Index Terms— Ontology reuse, fuzzy set, fuzzy ontology, fuzzy description logic, similarity relations.

I. INTRODUCTION

In recent years, the number of online ontologies is on the increase. The need for sharing and reusing independently developed ontologies has become even more important and attractive. Ontology reuse is now one of the important research issues in the ontology field. In the following, we give a definition of some of its subprocesses:

- *Ontology merging*. The process of ontology merging creates a unique ontology that is a merged version of the original ontologies. The obtained ontology contains all the information from merged original ontologies, without indication of their former origin [7]. This process is usually performed when the original ontologies cover similar or overlapping domains.
- *Ontology integration*. The process of ontology integration creates one ontology by aggregation, assemblage, extension, combination, specialization or adaptation of ontologies on different subjects [7], [15].
- *Ontology mapping*. The process of ontology mapping determines the correspondence among ontologies entities. It has as output a set of mapping assertions denoting relations between these entities [21]. It allows ontologies to share, exchange and reuse information from one another.
- *Ontology alignment*. The process of ontology alignment between two ontologies consists on modifying one of them to make it more consistent and coherent with the other one [10].

- *Ontology versioning*. Ontology versioning consists on handling changes in different versions of an ontology, which implies versions recognition, ontologies update and versions relationships traceability [11].

In all these processes, we may have to deal with an important problem, which consists on the determination of similarity relations among ontologies components. In ontology merging or integration, concepts in different ontologies are combined when their corresponding similarity relation corresponds to equality. A mapping assertion returned by a process of ontology mapping may correspond to LessGeneral(C,D) when a concept C in the first ontology is subsumed by a concept D in the second one. In ontology alignment, the modification of one ontology depends on the relations existing between concepts on the modified ontology and the other. Allowing traceability in ontology versioning consists in establishing the relationships between versions of the same concept.

In this paper, four levels of similarity relations are used: subsumption, equivalence, overlapping and disjointness (see Section IV for the definition of these levels). These similarity relations are equally used in [8] to resolve semantic heterogeneity in databases and in [6] for semantic coordination between different models on the Semantic Web.

In all domains in which the concepts to be represented have imprecise definitions, crisp ontology becomes less suitable. Fuzzy ontology which is based on fuzzy description logics has been proposed to overcome this problem [20]. The main particularity of fuzzy ontologies is that a concept is considered as a fuzzy set and an instance does not fully belong or not to a given concept but possesses a membership degree being an instance of that concept. The same is for roles.

On the other hand, fuzzy ontologies are also concerned by the problem of ontology reuse: ontology mapping, merging, integration, alignment and versioning. To our knowledge, there is no research paper treating one of these reuse subprocesses for fuzzy ontologies. Some solutions proposed for merging, mapping, integration and alignment for crisp ontologies are easily applicable for fuzzy ones as they deal generally with linguistic similarities [12], [13]. Nevertheless, realizing these subprocesses by deducing similarity relations among ontologies components based on their intentional definitions has to be treated differently in crisp and fuzzy cases. This is the scope of this paper.

The paper goes as follows. Section II summarizes some relevant topics, namely, ontology, description logic and fuzzy set. Section III deals with fuzzy ontology and discusses some properties on it. Section IV shows how similarity relations among crisp ontologies components may be deduced. Then it explains the extension to fuzzy ontologies. Section V concludes the paper.

II. BACKGROUND

A. Ontology and description logic

An ontology is a shared model of some domain which is often conceived as a typically hierarchical data structure containing all relevant concepts, their relations, and instances within that domain. The commonly defined components of an ontology are *concepts*, *relations*, *roles*, *axioms* and *instances*. In the following we define each of these components.

- *Concepts* are descriptions of a group of individuals in the ontology's domain that share common properties (e.g. *Human* and *Animal* are two concepts). They are typically arranged in a taxonomy and each concept may have super- and sub-concepts (e.g. *Female* is a sub-concept of *Human*).
- *Relations* denote a type of interaction between concepts of the ontology's domain. Super- and sub-concepts are particular type of relations that may exist between concepts.
- *Roles* are associated with concepts to describe their features and attributes. They may have various restrictions defined on them.
- *Axioms* are model sentences that are always true. They are used to describe more precisely the semantics of the concepts and to constrain how the instances of the concepts could be created.
- *Instances* are used to represent specific elements in the ontology's domain. For example, "Penguin" is an instance of *Animal* concept.

The increasing importance of ontologies in many applications, especially the Semantic Web, make their construction, maintenance, integration and evolution very crucial tasks. Efficient realization of these tasks greatly depends on the availability of an unambiguous language such as Description logics (DLs) [1]. One of the key advantages of using DLs is that it is possible to support correct ontology design, to decrease the risk of confusion among the domain experts and to make it more accessible to automated processes. This combination of features make DLs well-suited to the representation and reasoning about ontologies and the base of many ontology modelling languages [9].

Description logics are a logic-based knowledge representation formalisms designed to represent the knowledge of an application domain in a structured way [17]. They are characterized by a set of constructors provided for building complex concepts and roles from atomic ones. Atomic concepts (unary predicates), atomic roles (binary predicates) and individuals (constants or instances) are the basic syntax building blocks. To indicate that an individual or a couple of individuals is an instance of a given concept or role we use assertions. Let $\{a, b, c...\}$ be a set of individual names and C and R are a concept and a role, respectively. Then, an assertion is of the form $\langle a:C \rangle$ or $\langle (b,c):R \rangle$.

The semantic for description logic is based on interpretations. An interpretation $I = (\Delta^I, \bullet^I)$ consists of a nonempty set Δ^I called the *domain* and an interpretation function $\bullet^I$ over such domain. $\bullet^I$ maps every concept to a subset of Δ^I, every role to a subset of $\Delta^I \times \Delta^I$ and every individual to an element of Δ^I.

We provide in Table I some examples of constructors, their corresponding DL syntax and the corresponding semantic given by an interpretation $I = (\Delta^I, \bullet^I)$.

TABLE I
EXAMPLES OF DL CONSTRUCTORS

Constructor	DL Syntax	Example	Semantics
Atomic concept	A	Scientific	$A^I \subseteq \Delta^I$
Atomic role	R	DISCOVERS	$R^I \subseteq \Delta^I \times \Delta^I$
Conjunction	C⊓D	happyman⊓famous	$C^I \sqcap D^I$
Disjunction	C⊔D	happyman⊔famous	$C^I \sqcup D^I$
Negation	¬C	¬famous	$D^I \setminus C^I$
Exists restriction	∃R.C	∃WRITE.article	$\{x \mid \exists y. \langle x,y \rangle \in R^I \wedge y \in C^I\}$
Value restriction	∀R.C	∀WRITE.article	$\{x \mid \forall y. \langle x,y \rangle \in R^I \Rightarrow y \in C^I\}$

The architecture of a knowledge representation system based on a description logic comprises two components: the TBox (Terminological box) and the ABox (Assertional Box). The TBox contains concept definitions and axioms establishing equivalence and subsumption relationships between concepts and roles. It is used to introduce the terminology (i.e vocabulary of an application domain). On the other hand, the ABox contains concept/role assertions and it is used to describe the state of an application domain.

An ontology can be formalized in a TBox and an ABox. Thus, we can consider that a TBox corresponds to a specification of the intensional level of the ontology and an ABox corresponds to a specification of the extensional one.

B. Fuzzy set

Fuzzy set concept introduced by Zadeh [23] is a natural extension of the classical crisp set where either an object is a member of a set or it is not a member of a set. Classical two-valued logic apply when the set has crisp boundaries but in real-world this is rarely the case.

Each fuzzy set is fully defined through its *membership function* that maps the elements of the interest domain—often called *universe of discourse*—to [0,1]. Mathematically, let U be the universe of discourse and F a fuzzy set defined on U. Then, the membership function associated with fuzzy set F is defined as follows:

$$\mu_F: \quad U \quad \longrightarrow \quad [0,1]$$
$$u \quad \longrightarrow \quad \mu_F(u)$$

The function μ_F associate to each element u of U a degree of membership (d.o.m) $\mu_F(u)$ in the range [0,1]; where 0 implies no-membership and 1 implies full membership. A value between 0 and 1 indicates the extent to which u can be considered as an element of fuzzy set F.

Alternatively, a fuzzy set F may be represented by its possibility distribution:

$$\pi_X = \{\mu(u_1)/u_1, \cdots, \mu(u_i)/u_i, \cdots, \mu(u_n)/u_n\};$$

where $\mu(u_i) \in [0, 1]$ represents a measure of the possibility that a variable X that takes its values in U has the value u_i. It is easy to see that $\mu_F = \pi_X$.

All crisp set operations can be easily extended to deal with fuzzy sets. Let A and B two fuzzy sets defined on U. The union, intersection and complement set operations are extended to fuzzy sets as follows:

$$\mu_{A \cup B} \quad = \quad \max_u(\mu_A(u), \mu_B(u)) \tag{1}$$

$$\mu_{A \cap B} \quad = \quad \min_u(\mu_A(u), \mu_B(u)) \tag{2}$$

$$\mu_{\overline{A}} \quad = \quad 1 - \mu_A(u) \tag{3}$$

Equation (1) and (2) can be easily extended to more than two fuzzy sets.

The definition of the membership function is a crucial step in all applications of fuzzy logic. Basically, we may distinguish three types of membership functions: trapezoidal, triangular or sinusoidal. Several other special membership function my be derived from the basic ones. In Table IV we provide a rich set of membership functions along with some examples and their graphical representations. The fuzzy set on which a fuzzy data is based is denoted A in Table IV. As it is shown in Table IV, each of these functions is fully defined in terms of a series of parameters.

In the rest of the paper, ontology components within fuzzy context may be defined basing on linguistic attributes and/or fuzzy mumblers and may use any model from Table IV. Note that models of Table IV are a subset of several data types proposed in [3]–[5] and devoted to model imperfect information (i.e. vague, imprecise and/or uncertain) within fuzzy databases.

III. FUZZY ONTOLOGY

Fuzzy ontology has been introduced by [20] to represent knowledge in all domains in which the concepts to be represented have an imprecise definition. It is based on fuzzy description logic introduced in [18], [19]. In fuzzy description logic, concepts and roles are considered as fuzzy sets. Thus an instance does not fully belong or not to a given fuzzy concept (resp. fuzzy role) but possesses a membership degree being an instance of that concept (resp. role). For instance, $GreatCountry$ is a fuzzy concept and is defined as follows:

$$GreatCountry = Country \sqcap \exists width.Large$$

The term "*width*" is a fuzzy attribute and has as value the fuzzy linguistic term *Large*. The linguistic term *Large* may be defined by a trapezoidal function as shown graphically in Figure 1. Mathematically, it is defined as follows:

$$\mu_{Large}(c.width) = \begin{cases} 1, & \text{if } \beta \le c.width \le \gamma; \\ \frac{\lambda - c.width}{\lambda - \gamma}, & \text{if } \gamma < c.width < \lambda; \\ \frac{c.width - \alpha}{\beta - \alpha}, & \text{if } \alpha < c.width < \beta; \\ 0, & \text{otherwise.} \end{cases}$$

This function is used to compute, for a given country c, the degree of satisfaction of $\exists width.Large$ associated with fuzzy concept $GreatCountry$. In other words, it measures the degree of largeness of the width of the country c.

The semantic of fuzzy description logic is based on an interpretation $I = (\Delta^I, \bullet^I)$. As for crisp description logic interpretation, Δ^I is a nonempty set called the domain whereas $\bullet^I$ is a function that associates to every concept C a membership function $C^I : \Delta^I \to [0, 1]$; and to every role R a membership function $R^I : \Delta^I \times \Delta^I \to [0, 1]$; and as for the crisp case, to every individual an element of Δ^I [18]. C^I (resp. R^I) is thus interpreted as the membership degree function of fuzzy concept C (resp. role R). $C^I(d)$ gives the degree of d ($d \in \Delta^I$) being an element of the fuzzy concept C under interpretation I.
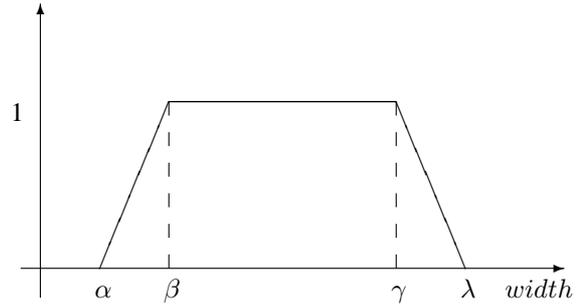
$\mu_{Large}(width)$

Fig. 1.   Trapezoidal representation of the linguistic term *Large*

The conjunction, disjunction and negation of fuzzy sets can be easily applied to the interpretation of fuzzy concepts. Thus, for all $d \in \Delta^I$, $\bullet^I$ has to satisfy the following properties [18], [22]:

$$
\begin{aligned}
\top^I(d) &= 1 \\
\bot^I(d) &= 0 \\
(C \sqcap D)^I(d) &= min\{C^I(d), D^I(d)\} \\
(C \sqcup D)^I(d) &= max\{C^I(d), D^I(d)\} \\
(\neg C)^I(d) &= 1 - C^I(d) \\
(\forall R.C)^I(d) &= min_{d' \in \Delta^I}\{max\{1 - R^I(d, d'), C^I(d')\}\} \\
(\exists R.C)^I(d) &= max_{d' \in \Delta^I}\{min\{R^I(d, d'), C^I(d')\}\}
\end{aligned}
$$

Fuzzy ontology can be formalized in a fuzzy TBox and a fuzzy ABox. As for crisp TBox, a fuzzy TBox contains concept definitions and axioms establishing equivalence and subsumption relationships between concepts and roles. In a fuzzy TBox we may have fuzzy or crisp concepts. Note that a crisp concept is a particular case of a fuzzy concept and a primitive concept is a crisp one. For more flexibility, we consider that definition of fuzzy concepts may be based on fuzzy attribute(s) or number restriction(s). For instance,

GreatCountry defined earlier is a fuzzy concept based on the fuzzy attribute *width*. Models I.1, I.4, I.5, II.1, II.2, II.3 and II.4 in Table IV are different forms of fuzzy attributes that may be associated with fuzzy concepts or roles.

Concerning number restrictions, the membership degree of an individual being an instance of a concept defined based on a fuzzy number restriction. For instance $(\geq n\ R)$, depends partially on the "satisfaction degree" of $(\geq n\ R)$ for this instance. The parameter n used on a number restriction $(\geq n\ R)$ may be a fuzzy number (Model I.2 in Table IV) and the corresponding number restriction is a fuzzy one. Fuzzy number restriction have the same definitions as crisp ones:

$$\geq n\ R \text{ for fuzzy at-least restriction}$$
$$\leq n\ R \text{ for fuzzy at-most restriction}$$

where n may be a nonnegative integer as in a crisp TBox or to a fuzzy number (e.g. about 20). Basing on the interpretations of crisp number restrictions of fuzzy description logics defined in [18], [19], we define the interpretations for these two fuzzy number restrictions as follows:

$$(\geq n\ R)^I(d) = sup_{d'_1, d'_2, \ldots, d'_m \in \Delta^I} \bigwedge_{i=1}^{m} R^I(d, d'_i) \qquad (4)$$

where $m = |\{d' \in \Delta^I : R^I(d, d') > 0\}|;\ m \geq n$, and

$$(\leq n\ R)^I(d) = inf_{d'_1, d'_2, \ldots, d'_m \in \Delta^I} \bigvee_{i=1}^{m} \neg R^I(d, d'_i) \qquad (5)$$

where $m = |\{d' \in \Delta^I : R^I(d, d') > 0\}|;\ m \leq n$.

We can define these interpretations basing on the extension of the binary operators $\geq$ and $\leq$ to fuzzy context initially proposed in [14] and further extended in [5]:

$$(\geq n\ R)^I(d) = \mu_\geq(n, m) \qquad (6)$$

and

$$(\geq n\ R)^I(d) = \mu_\leq(n, m) \qquad (7)$$

Function $\mu_\geq(n, m)$ (resp. $\mu_\leq(n, m)$) returns a value in the range [0,1] that measures the degree to which fuzzy data n is greater or equal (resp. less or equal) to fuzzy data m (see [14] and [5] for a complete definition of these extended operators). Let C and D be two fuzzy concepts and R and S be two fuzzy roles. Basing on the Zadeh's definition of fuzzy subset, a fuzzy interpretation I satisfies $C \sqsubseteq D$ (and $R \sqsubseteq S$) iff:

$$\forall d \in \Delta^I, C^I(d) \leq D^I(d) \qquad (8)$$

and

$$\forall (d, d') \in \Delta^I \times \Delta^I, R^I(d, d') \leq S^I(d, d') \qquad (9)$$

In the other hand, I satisfies $C \equiv D$ (and $R \equiv S$) iff:

$$\forall d \in \Delta^I, C^I(d) = D^I(d) \qquad (10)$$

and

$$\forall (d, d') \in \Delta^I \times \Delta^I, R^I(d, d') = S^I(d, d') \qquad (11)$$

Terminological axioms in a crisp TBox are of the forms $C \sqsubseteq D$ ($R \sqsubseteq S$) or $C \equiv D$ ($R \equiv S$) where C and D are concepts (and R and S are roles). In a fuzzy TBox, concepts and roles are considered as fuzzy sets and C (resp. R) may have a degree $n \in [0, 1]$ to subsume or to be equivalent to D (resp. S). Fuzzy axioms are then of the form $\langle C \sqsubseteq D\ n \rangle$ ($\langle R \sqsubseteq S\ n \rangle$) or $\langle C \equiv D\ n \rangle$ ($\langle R \equiv S\ n \rangle$). An interpretation I satisfies a fuzzy subsumption $\langle C \sqsubseteq D\ n \rangle$ (resp. $\langle R \sqsubseteq S\ n \rangle$) if and only if $(C \sqsubseteq D)^I \geq n$ (resp. $(R \sqsubseteq S)^I \geq n$). The same is for equivalence. The computing details of the degree of subsumption or equivalence between two fuzzy concepts or roles is given in the next section.

In a crisp ABox, assertions are of the form *a:A* and *(a,b):R* where *a* and *b* are individuals and *A* and *R* are a concept and a role, respectively. An assertion is added to an ABox if it is proven that it is satisfiable with respect to the corresponding TBox. In the case of fuzzy ABox, because *A* and *R* are considered as fuzzy sets, then we consider that *a:A* (resp. *(a,b):R*) is satisfiable with respect to the corresponding fuzzy TBox if *a* (resp. *(a,b)*) has a membership degree to be an instance of the fuzzy concept *A* (resp. fuzzy role *R*) strictly greater than zero; otherwise, it is not satisfiable. A fuzzy ABox contains then assertions of the form $\langle a{:}A\ n \rangle$ and $\langle (a,b){:}R\ n \rangle$. These assertions denote that *a* (resp. *(a,b)*) has a membership degree of being an instance of the fuzzy concept *A* (resp. fuzzy role *R*) equal to n. It corresponds to which we call the *degree of satisfiability* of the assertion *a:A* (resp. *(a,b):R*) with respect to the corresponding fuzzy TBox. An interpretation I satisfies a fuzzy assertion $\langle a{:}A\ n \rangle$ (resp. $\langle (a,b){:}R\ n \rangle$) if and only if $A^I(a) \geq n$ (resp. $R^I(a,b) \geq n$) [18].

IV. SIMILARITY RELATIONS

Determining the similarity relation among ontology components is crucial for many ontology reuse subprocesses. In this section we propose an approach to determine similarity relation among fuzzy ontologies components basing on their intentional definitions. An intentional definition of an ontology component is a set of description logic formulae that represent the meaning of that component. As we work with fuzzy ontologies, our approach measures the similarity relations among ontologies components in terms of a degree of satisfaction in the range [0,1]. For more clarity, we illustrate our approach only on concept component. The same idea applies for the other fuzzy ontology components.

As in [6], [8], four levels of similarity relations are adopted in this paper:

- *Subsumption*. A concept C is subsumed by a concept D ($C \sqsubseteq D$) if the intentional definition of C implies the intentional definition of D. It also means that the concept C (resp. D) is more general (resp. less general) than the concept D (resp. C). A fuzzy concept C has a degree $n \in [0,1]$ to be subsumed by a fuzzy concept D.
- *Equivalence*. Two concepts C and D are equivalent ($C \equiv D$) if each of them is subsumed by the other one. A fuzzy concept D has a degree $n \in [0,1]$ to be equivalent to a fuzzy concept C.

- *Disjointness.* Two concepts C and D are disjoint if the conjunction of their intentional definitions implies false. We do not give a degree of disjointness between two fuzzy concepts.
- *Overlapping.* Two concepts C and D overlap if the conjunction of their intentional definitions can not be proven to be false and if $C \sqcap D$ is satisfiable with respect to the corresponding terminological model. Two fuzzy concepts C and D have a degree $n \in [0,1]$ to overlap each other.

In the rest of this section, we first illustrate how similarity relations are deduced in crisp ontologies. We then propose our extension to fuzzy ontologies.

A. Deducing similarity relations in crisp ontologies

Because ontologies are based on description logics, it is possible to perform specific kinds of reasoning on them. We can for instance deduce by inference some implicit knowledge that are not explicitly defined on the corresponding TBox or Abox. Basing on this fact, it is possible to use an appropriate inference mechanism to deduce similarity relation among two concepts belonging to the same ontology. As it is mentioned in [2], checking satisfiability of concepts is a key aspect of inference as many important inference mechanisms can be reduced to the (un)satisfiability. We can then reduce the problem of determining similarity relations among concepts belonging to the same ontology to an (un)satisfiability problem. We illustrate in Table II how subsumption (less general, more general), equivalence and disjointness of concepts can be reduced to the unsatisfiability checking problem.

TABLE II
REDUCTION TO UNSATISFIABILITY

Similarity relation	Unsatisfiability checking
MoreGeneral(D,C)	$\neg D \sqcap C$
LessGeneral(D,C)	$D \sqcap \neg C$
Equivalent(D,C)	($\neg D \sqcap C$) and ($D \sqcap \neg C$)
Disjoint(D,C)	$D \sqcap C$

If we want to adopt the same idea to determine similarity relations among concepts belonging to different ontologies we have to construct a common terminology to perform needed inferences (inferences for deducing subsumption, equivalence, disjointness and overlapping). This terminology may correspond to the union of different ontologies TBoxes. Nevertheless, we may have an important terminology that we may not used on its totality. Alternatively, this terminology may correspond to the intersection of different ontologies TBoxes. Some axioms contained on this intersection may not be useful if they are not related to the concepts for which we want deduce the similarity relation. The solution consists on using the smallest terminology that contains only these axioms that are directly or indirectly related to the concepts for which we want to determine similarity relations.

A similar approach is adopted in [6] where the authors reduce the problem of determining similarity relation among concepts to subsumption. We have chosen to reduce this problem to unsatisfiability checking because we think that it can be easily extended to the fuzzy case as we will see in the next subsection.

In order to perform the needed inferences shown in Table II, we have to use a DL system that supports the conjunction operator "$\sqcap$" and allows to form the negation "$\neg$" of a description. Even so, the systems having these properties are based on crisp DL and are not suitable for deducing similarity relations on fuzzy ontologies. Then, in order to adopt the same approach to deduce similarity relations in fuzzy ontologies, we have to use/define fuzzy conjunction and negation operators. We show in the next section how the approach is extended to deduce similarity relations in fuzzy ontologies. We show equally how the degree of satisfiability of a given similarity relation is computed.

B. Extension to fuzzy ontologies

Two fuzzy concepts have a degree $n \in [0, 1]$ to be related by a given similarity relation. When n is equal to 0 the two concepts have no similarity at all and when n is equal to 1 the two concepts are fully similar. A value between 0 and 1 measures the level to which the two concepts are similar. In the previous section we have reduced the problem of determining similarity relation within two crisp concepts to unsatisfiability checking. Then determining if a concept C is subsumed by a concept D is reduced to unsatisfiability checking of $C \sqcap \neg D$. This means that a concept C is subsumed by a concept D if and only if $C \sqcap \neg D$ is *false*. When $C \sqcap \neg D$ is *true*, there is no subsumption between concepts C and D.

Within fuzzy context, the degree of subsumption of a fuzzy concept C by a fuzzy concept D, denoted $\mu_\sqsubseteq(C,D)$, is computed as follows:

$$\mu_\sqsubseteq(C, D) = 1 - \mu_\sqcap(C, \neg D) \qquad (12)$$

This equation apply also for crisp cases. In fact, when $\mu_\sqcap(C, \neg D)$ is equal to 0 (equivalent to $C \sqcap \neg D$ is *false* in the crisp case), then $\mu_\sqsubseteq(C,D)$ is equal to 1 (equivalent to C is subsumed by D in the crisp case); and when $\mu_\sqcap(C, \neg D)$ is equal to 1 (equivalent to $C \sqcap \neg D$ is *true* in the crisp case), then $\mu_\sqsubseteq(C,D)$ is equal to 0 (equivalent to C is not subsumed by D in the crisp case). The crisp case is simply a special case of fuzzy reasoning which is reduced to the extreme values of the range [0,1].

The same idea is used for the other types of relationships. In Table III we provide the different formulae for computing the satisfiability degree of these relationships. As explained earlier, these formulae apply for both crisp and fuzzy cases. We call satisfiability degree of MoreGeneral(D,C) the degree of subsumption of C by D. The same is for the other similarity relations.

TABLE III
SATISFIABILITY DEGREE OF SIMILARITY RELATIONS

Similarity relation	Satisfiability degree
MoreGeneral(D,C)	$\mu_\sqsubseteq(D, C) = 1 - \mu_\sqcap(\neg D, C)$
LessGeneral(D,C)	$\mu_\sqsupseteq(D, C) = 1 - \mu_\sqcap(D, \neg C)$
Equivalent(D,C)	$\mu_\equiv(D, C) = 1 - \min(\mu_\sqcap(\neg D, C), \mu_\sqcap(D, \neg C))$
Disjoint(D,C)	$\mu_\sqcup(D, C) = 1 - \mu_\sqcap(D, C)$
Overlap(D,C)	$\mu_\sqcap(D, C) = \min(\mu_\sqcap(D, C), \mu_\sqcap(C, D))$

To compute $\mu_\sqcap(X,Y)$—where $(X,Y) \in \{(D,C),(D,\neg C),(\neg D,C)\}$—in Table III, we use the following formula [5]:

$$\mu_\sqcap(\tilde{x},\tilde{y}) = \sup_{z \in X \bigcap Y} \min(p(\tilde{x},\tilde{y}), \pi_{\tilde{x}}(z), \pi_{\tilde{y}}(z)) \qquad (13)$$

where $\tilde{x}$ and $\tilde{y}$ are two fuzzy data; and $\pi_{\tilde{x}}(z)$ and $\pi_{\tilde{y}}(z)$ are the possibility distributions defined on U; and $p(\tilde{x},\tilde{y})$ is the "proximity relation" defined on the domains X and Y. The proximity relation measures the level to which two fuzzy data are semantically equivalent (see e.g. [16]). For fuzzy numbers, $p(\tilde{x},\tilde{y})$ is equal to $\delta(\tilde{x},\tilde{y})$ where δ is a Diracs delta [14].

As mentioned earlier, a fuzzy concept is defined basing on fuzzy attribute(s) or number restriction(s). Thus, determining the degree of satisfiability of a similarity relation among two fuzzy concepts is function of the degree of conjunction, disjunction and/or negation of their corresponding fuzzy attribute(s) values and/or their number restriction(s). In the following, we illustrate through two examples how we determine the degree of subsumption among two concepts defined basing on fuzzy attributes and number restrictions, respectively.
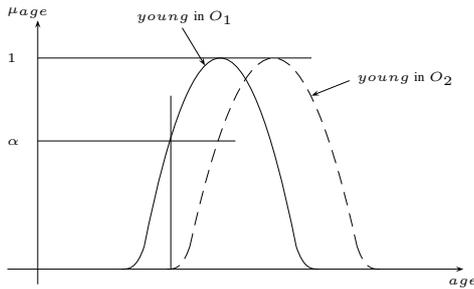

Fig. 2. Graphical representation of the linguistic term "*young*" in O_1 and O_2

1) Degree of subsumption of fuzzy attributes-based concepts: Given two fuzzy concepts *YoungPerson* and *Teenager* defined in two fuzzy ontologies O_1 and O_2 as follows:

$$\begin{aligned} Younger &= Person \sqcap \exists\ age.young \\ Teenager &= Person \sqcap \exists\ age.young \end{aligned}$$

Figure 2 shows the definition of fuzzy linguistic term *young* in ontology O_1 and O_2. We use y_1 and y_2 to denote the linguistic term *young* in ontology O_1 and O_2, respectively. We want to determine the degree of subsumption of the fuzzy concept *Younger* by the fuzzy concept *Teenager*. Basing on Table III, the satisfiability degree of MoreGeneral(*Teenager,Younger*) is equal to 1-$\mu_\sqcap$(¬*Teenager,Younger*). We suppose that the concept *Person* defined both in O_1 and O_2 is primitive one (i.e. crisp) and that it models the same thing. Thus, $\mu_\sqcap$(¬*Teenager,Younger*) may be computed as follow:

$$\mu_\sqcap(\neg Teenager, Younger) = \mu_\sqcap(\neg y_2, y_1) \qquad (14)$$

and $\mu_\sqcap(\neg y_2, y_1)$ is computed through Equation (13), i.e.

$$\mu_\sqcap(\neg\tilde{y_2}, \tilde{y_1}) = \sup_{z \in Y_2 \bigcap Y_1} \min(p(\neg\tilde{y_2}, \tilde{y_1}), \pi_{\neg\tilde{y_2}}(z), \pi_{\tilde{y_1}}(z)) \qquad (15)$$

and where

$$\mu_{\neg y_2}(d) = 1 - \mu_{y_2}(d). \qquad (16)$$

Then, we obtain

$$\mu_\sqcap(\neg\tilde{y_2}, \tilde{y_1}) = \min(p(\neg\tilde{y_2}, \tilde{y_1}), \alpha); \qquad (17)$$

and then

MoreGeneral(*Teenager,Younger*)=$1 - min(p(\neg\tilde{y_2}, \tilde{y_1}), \alpha)$. $\qquad (18)$
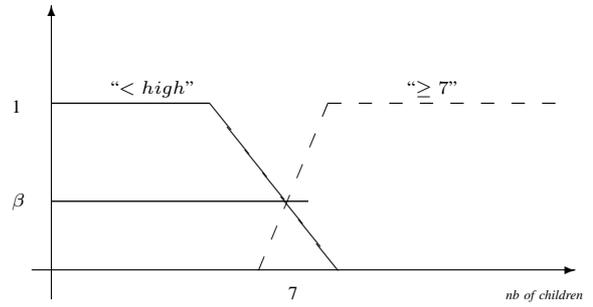

Fig. 3. Graphical representation of "$< high$" and "≥ 7" number restrictions

2) Degree of subsumption of fuzzy numbers restriction-based concepts: Given two fuzzy concepts *GreatFamily* and *NumFamily* defined in two ontologies O_1 and O_2 as follows:

$$\begin{aligned} GreatFamily &= Family \sqcap (\geq 7\ hasChild) \\ NumFamily &= Family \sqcap (\geq high\ hasChild) \end{aligned}$$

We want to determine the degree of subsumption of the fuzzy concept *GreatFamily* by the fuzzy concept *NumFamily*. Basing on Table III, the satisfiability degree of MoreGeneral(*NumFamily,GreatFamily*) is equal to 1-$\mu_\sqcap$(¬*NumFamily,GreatFamily*). We suppose that the concept *Family* defined in the two ontologies is primitive one and that it has the same signification. Then, $\mu_\sqcap$(¬*NumFamily,GreatFamily*) is computed as follow:

$$\begin{aligned} \mu_\sqcap(\neg NumFamily, GreatFamily) &= \mu_\sqcap(\neg(\geq high), \geq 7) \\ &= \mu_\sqcap(< high, \geq 7). \end{aligned} \qquad (19)$$

The right side of Equation (19) may be computed as follows:

$$\mu_\sqcap(< high, \geq 7) =$$
$$\sup_{z \in X \bigcap Y} \min(p(\tilde{x},\tilde{y}), \pi_{\tilde{x};z<high}(z), \pi_{\tilde{x};z\geq 7}(z)). \qquad (20)$$

The graphical representations of "$< high$" and "≥ 7" number restrictions are shown in Figure 3. Then, we obtain

$$\mu_\sqcap(< high, \geq 7) = \min(p(\tilde{x},\tilde{y}), \beta). \qquad (21)$$

TABLE IV

DIFFERENT MEMBERSHIP FUNCTIONS

Linguistic label	Model	Representation	Parameters	$\mu_A(z)$
Fuzzy range label e.g. *age*= more or less between 20 and 30	I.1		$\alpha, \beta, \gamma, \lambda$	$\mu_A(z) = \begin{cases} 1, & \text{if } \beta \leq z \leq \gamma; \\ \frac{\lambda-z}{\lambda-\gamma}, & \text{if } \gamma < z < \lambda; \\ \frac{z-\alpha}{\beta-\alpha}, & \text{if } \alpha < z < \beta; \\ 0, & \text{Otherwise.} \end{cases}$
Approximate value e.g. *age*=about 35	I.2		c, c^-, c^+	$\mu_A(z) = \begin{cases} 1, & \text{if } z = c; \\ \frac{c^+-z}{c^+-c}, & c < z < c^+; \\ \frac{z-c^-}{c-c^-}, & c^- < z < c; \\ 0, & \text{Otherwise.} \end{cases}$
Interval e.g. *age* $\in [25, 35]$	I.3		α, β	$\mu_A(z) = \begin{cases} 1, & \text{if } \alpha \leq z \leq \beta; \\ 0, & \text{Otherwise.} \end{cases}$
Less than value e.g. age= less than 35	I.4		γ, λ	$\mu_A(z) = \begin{cases} 1, & \text{if } z \leq \gamma; \\ 0, & \text{if } z < \lambda; \\ \frac{\lambda-z}{\lambda-\gamma}, & \text{if } \gamma \leq z \leq \lambda. \end{cases}$
More than value e.g. age= more than 35	I.5		α, β	$\mu_A(z) = \begin{cases} 1, & \text{if } z \geq \beta; \\ 0, & \text{if } z \leq \alpha; \\ \frac{z-\alpha}{\beta-\alpha}, & \text{if } \alpha < z < \beta. \end{cases}$
Real number e.g. *age*=30	I.6		c	$\mu_A(z) = \begin{cases} 1, & \text{if } z = c; \\ 0, & \text{Otherwise.} \end{cases}$
Linguistic label e.g. *age*=young	II.1		a, c	$\mu_A(z) = \frac{1}{(1+(a(z-c)^2)}; \ z \geq 0$
Linguistic label e.g. *age*=young	II.2		a_1, a_2, b_1, b_2	$\mu_A(z) = \begin{cases} \frac{1}{1+\frac{z-a_1-b_1}{b_1}}, & \text{if } z < a_1 + b_1; \\ 1, & \text{if } a_1 + b_1 \leq z \leq a_2 - b_2; \\ \frac{1}{1+\frac{z-a_2+b_2}{b_2}}, & \text{if } z > a_2 - b_2. \end{cases}$
Linguistic label *age*=very old	II.3		a_1, b_1	$\mu_A(z) = \begin{cases} \frac{1}{1+\frac{z-a_1-b_1}{b_1}}, & \text{if } z < a_1 + b_1; \\ 1, & \text{if } a_1 + b_1 \leq z; \end{cases}$
Linguistic label e.g. *age*=very young	II.4		a_2, b_2	$\mu_A(z) = \begin{cases} 1, & \text{if } z \leq a_2 - b_2; \\ \frac{1}{1+\frac{z-a_2+b_2}{b_2}}, & \text{if } z > a_2 - b_2. \end{cases}$

Finally we obtain

$$\text{MoreGeneral}(NumFamily, GreatFamily) = 1 - \min(p(\tilde{x}, \tilde{y}), \beta). \tag{22}$$

These two examples show how the degree of satisfiability of subsumption similarity relation is computed. The same reasoning applies for the other similarity relations since all of them are expressed (as shown in Table III) in terms of fuzzy conjunction and negation operators which their definition and use are illustrated in the two previous examples.

Finally, we note that when two concepts are related by more than one similarity relation, one intuitive and simple solution consists in returning the one that has the highest degree of satisfiability.

V. CONCLUSION AND FUTURE WORKS

Ontology reuse is an important issue in the ontology field. Determining similarity relation among ontology components is essential for many reuse subprocesses. In all domains in which the concepts to be represented have an imprecise definition, crisp ontology becomes less suitable. Fuzzy ontology which is based on fuzzy description logics has been proposed to overcome this problem. Both crisp and fuzzy ontologies are concerned with the problem of ontology ruse. Deducing sim-

ilarity relations among ontology components basing on their intentional definitions need to be treated differently in crisp and fuzzy cases.

In this paper our description of fuzzy ontologies is based on fuzzy logic descriptions proposed in [18], [19]. We have added which we have called fuzzy number restrictions but omitted fuzzy constraints. We have also proposed a solution to deduce similarity relation among crisp ontology components. A similar approach is adopted in [6] where the authors reduce the problem of determining similarity relation among concepts to subsumption. We have chosen to reduce this problem to unsatisfiability checking since it can be easily extended to the fuzzy case as it is shown in paragraph IV-B. In this paragraph, we have shown the way the degree of satisfiability of a given similarity relation among fuzzy concepts is computed.

Actually we work on the implementation of the proposed solution on Protege. We tend also to apply this solution for fuzzy spatial ontologies merging.

Similarity relations among ontology components may be used as in [6] to resolve semantic heterogeneity in conventional databases. In the same way, our approach may be used to resolve semantic heterogeneity in fuzzy databases.

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