Applications of Z-numbers and Neural Networks in Engineering

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Abstract. In the real world, much of the information on which decisions are based is vague, imprecise and incomplete. Artificial intelligence techniques can deal with extensive uncertainties. Currently, various types of artificial intelligence technologies, like fuzzy logic and artificial neural network are broadly utilized in the engineering field. In this paper, the combined Z-number and neural network techniques are studied. Furthermore, the applications of Z-numbers and neural networks in engineering are introduced.

Keywords: artificial intelligence, fuzzy logic, Z-number, neural network

1 Introduction

Intelligent systems are composed of fuzzy systems and neural networks. They have particular properties such as the capability of learning, modeling and resolving optimizing problems, suitable for specific kind of applications. The intelligent system can be named hybrid system in case that it combines a minimum of two intelligent systems. For example, the mixture of the fuzzy system and neural network causes the hybrid system to be called a neuron-fuzzy system.

Neural networks are made of interrelated groups of artificial neurons that have information which is obtainable by computations linked to them. Mostly, neural networks can adapt themselves to structural alterations while the training phase. Neural networks have been utilized in modeling complicated connections among inputs and outputs or acquiring patterns for the data [1–12].

Fuzzy logic systems are broadly utilized to model the systems characterizing vague and unreliable information [13–29]. During the years, investigators have proposed extensions to the theory of fuzzy logic. Remarkable extension includes Z-numbers [30]. The Z-number is defined as an ordered pair of fuzzy numbers (C, D), such that C is a value of some variables and D is the reliability which is a value of probability rate of C. Z-numbers are widely applied in various implementations in different areas [31–36].

In this paper, the basic principles and explanations of Z-numbers and neural networks are given. The applications of Z-numbers and neural networks in engineering are introduced. Also, the combined Z-number and neural network techniques are studied. The rest of the paper is organized as follows. The theoretical background of Z-numbers and artificial neural networks are detailed in Section 2. Comparison analysis of neural networks and Z-number systems is presented in Section 3. The combined Z-number and neural network techniques are given in Section 4. The conclusion of this work is summarized in Section 5.

2 Theoretical background

In this section, we provide a brief theoretical insight of Z-numbers and artificial neural networks.

2.1 Z-numbers

Mathematical preliminaries Here some necessary definitions of Z-number theory are given.

Definition 1. If q is: 1) normal, there exists $\omega_0 \in \Re$ where $q(\omega_0) = 1$, 2) convex, $q(\upsilon\omega + (1-\upsilon)\omega) \geq \min\{q(\omega), q(\tau)\}$, $\forall \omega, \tau \in \Re, \forall \upsilon \in [0,1]$, 3) upper semi-continuous on $\Re$, $q(\omega) \leq q(\omega_0) + \epsilon$, $\forall \omega \in N(\omega_0)$, $\forall \omega_0 \in \Re$, $\forall \epsilon > 0$, $N(\omega_0)$ is a neighborhood, 4) $q^+ = \{\omega \in \Re, q(\omega) > 0\}$ is compact, so q is a fuzzy variable, $q \in E : \Re \to [0,1]$.

The fuzzy variable q is defined as below

$$q = \left(\underline{q}, \overline{q}\right) \tag{1}$$

such that $\underline{q}$ is the lower-bound variable and $\overline{q}$ is the upper-bound variable.

Definition 2. The Z-number is composed of two elements $Z = [q(\omega), p]$. $q(\omega)$ is considered as the restriction on the real-valued uncertain variable ω and p is considered as a measure of the reliability of q. The Z-number is defined as Z^+-number, when $q(\omega)$ is a fuzzy number and p is the probability distribution of ω. If $q(\omega)$, and p, are fuzzy numbers, then the Z-number is defined as Z^--number.

The Z^+-number has more information in comparison with the Z^--number. In this work, we use the definition of Z^+-number, i.e., $Z = [q, p]$, q is a fuzzy number and p is a probability distribution.

The triangular membership function is defined as

$$\mu_q = G\left(a, b, c\right) = \begin{cases} \frac{\omega - a}{b - a} \; a \leq \omega \leq b \\ \frac{c - \omega}{c - b} \; b \leq \omega \leq c \end{cases} \; otherwise \; \mu_q = 0 \tag{2}$$

and the trapezoidal membership function is defined as

$$\mu_q = G\left(a, b, c, d\right) = \begin{cases} \frac{\omega - a}{b - a} \; a \leq \omega \leq b \\ \frac{d - \omega}{d - c} \; c \leq \omega \leq d \\ \;\; 1 \;\;\; b \leq \omega \leq c \end{cases} \; otherwise \; \mu_q = 0 \tag{3}$$

The probability measure of q is defined as

$$P(q) = \int_{\Re} \mu_q(\omega)p(\omega)d\omega \tag{4}$$

such that p is the probability density of ω. For discrete Z-numbers we have

$$P(q) = \sum_{j=1}^{n} \mu_q(\omega_j)p(\omega_j) \tag{5}$$

Definition 3. The α-level of the Z-number $Z = (q, p)$ is stated as below

$$[Z]^\alpha = ([q]^\alpha, [p]^\alpha) \tag{6}$$

such that $0 < \alpha \leq 1$. $[p]^\alpha$ is calculated by the Nguyen's theorem

$$[p]^\alpha = p([q]^\alpha) = p([\underline{q}^\alpha, \overline{q}^\alpha]) = \left[\underline{P}^\alpha, \overline{P}^\alpha\right] \tag{7}$$

such that $p([q]^\alpha) = \{p(\omega)|\omega \in [q]^\alpha\}$. Hence, $[Z]^\alpha$ is defined as

$$[Z]^\alpha = \left(\underline{Z}^\alpha, \overline{Z}^\alpha\right) = \left(\left(\underline{q}^\alpha, \underline{P}^\alpha\right), \left(\overline{q}^\alpha, \overline{P}^\alpha\right)\right) \tag{8}$$

such that $\underline{P}^\alpha = \underline{q}^\alpha p(\underline{\omega}_j^\alpha)$, $\overline{P}^\alpha = \overline{q}^\alpha p(\overline{\omega}_j^\alpha)$, $[\omega_j]^\alpha = (\underline{\omega}_j^\alpha, \overline{\omega}_j^\alpha)$.

Let $Z_1 = (q_1, p_1)$ and $Z_2 = (q_2, p_2)$, we have

$$Z_{12} = Z_1 * Z_2 = (q_1 * q_2, p_1 * p_2) \tag{9}$$

where $* \in \{\oplus, \ominus, \odot\}$. $\oplus$, $\ominus$ and $\odot$, indicate sum, subtract and multiply respectively.

The operations utilized for the fuzzy numbers $[q_1]^\alpha = [q_{11}^\alpha, q_{12}^\alpha]$ and $[q_2]^\alpha = [q_{21}^\alpha, q_{22}^\alpha]$ are defined as [37],

$$\begin{array}{l} [q_1 \oplus q_2]^\alpha = [q_1]^\alpha + [q_2]^\alpha = [q_{11}^\alpha + q_{21}^\alpha, q_{12}^\alpha + q_{22}^\alpha] \\ [q_1 \ominus q_2]^\alpha = [q_1]^\alpha - [q_2]^\alpha = [q_{11}^\alpha - q_{22}^\alpha, q_{12}^\alpha - q_{21}^\alpha] \\ [q_1 \odot q_2]^\alpha = \begin{pmatrix} \min\{q_{11}^\alpha q_{21}^\alpha, q_{11}^\alpha q_{22}^\alpha, q_{12}^\alpha q_{21}^\alpha, q_{12}^\alpha q_{22}^\alpha\} \\ \max\{q_{11}^\alpha q_{21}^\alpha, q_{11}^\alpha q_{22}^\alpha, q_{12}^\alpha q_{21}^\alpha, q_{12}^\alpha q_{22}^\alpha\} \end{pmatrix} \end{array} \tag{10}$$

For the discrete probability distributions, the following relation is defined for all $p_1 * p_2$ operations

$$p_1 * p_2 = \sum_{\iota} p_1(\omega_{1,j})p_2(\omega_{2,(n-j)}) = p_{12}(\omega) \tag{11}$$

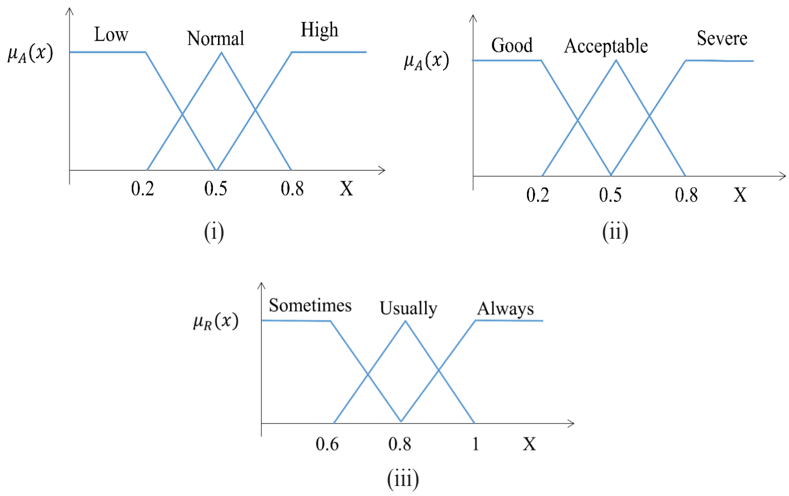

Fig. 1. Membership functions applied for (a) cereal yield, cereal production, economic growth, (b) threat rate, and (c) reliability

Background and related work The implementations of Z-numbers based techniques are bounded because of the shortage of effective approaches for calculation with Z-numbers.

In [38], the capabilities of the Z-numbers in the improvement of the quality of risk assessment are studied. Prediction equal to (High, Very Sure) is institutionalized in the form of Z-evaluation "y is $Z(c, p)$", such that y is considered as a random variable of threat probability, c and p are taken to be fuzzy sets, demonstrating soft constraints on a threat probability and a partial reliability, respectively. The likelihood of risk is illustrated by Z-number as: Probability= Z_1(High, Very Sure), such that c is indicated through linguistic terms High, Medium, Low, also, p is indicated through terms Very Sure, Sure, etc. Likewise, consequence rate is explained as: Consequence measure= Z_2(Low, Sure). Threat rates (Z_{12}) is computed as the product of the probability (Z_1) and consequence measure (Z_2).

In [39], Z-number-based fuzzy system is suggested to determine the food security risk level. The proposed system is relying on fuzzy If-Then rules, which applies the basic parameters such as cereal production, cereal yield, and economic growth to specify the threat rate of food security. The membership functions applied to explain input, as well as output variables, are demonstrated in Figure 1.

In [40], the application of the Z-number theory to selection of optimal alloy is illustrated. Three alloys named Ti12Mo2Sn alloy, Ti12Mo4Sn alloy, and Ti12Mo6Sn alloy are examined and an optimal titanium alloy is selected using the proposed approach. The optimality of the alloys is studied based on three criteria: strength level, plastic deformation degree, and tensile strength.

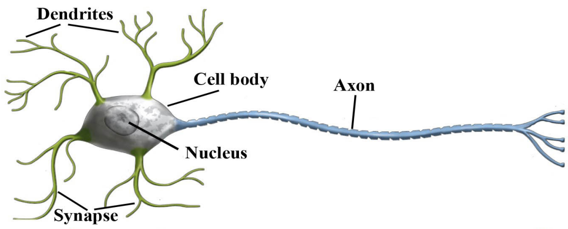

Fig. 2. The structure of a biological neuron

2.2 Neural networks

Neural networks are constructed from neurons and synapses. They alter their rates in reply from nearby neurons as well as synapses. Neural networks operate similar to computer as they map inputs to outputs. Neurons, as well as synapses, are silicon members, which mimic their treatment. A neuron gathers the total incoming signals from other neurons, afterward simulate its reply represented by a number. Signals move among the synapses, which contain numerical rates. Neural networks learn once they vary the value of their synapsis. The structure of a biological neuron or nerve cell is shown in Figure 2. The processing steps inside each neuron is demonstrated in Figure 3.

Background and related work In [41], artificial neural network technique is utilized for modeling the void fraction in two-phase flow inside helical vertical coils with water as work fluid. In [42] artificial neural network and multi-objective genetic algorithm are applied for optimizing the subcooled flow boiling in a vertical pipe. Pressure, the mass flux of the water, inlet subcooled temperature, as well as heat flux are considered as inlet parameters. The artificial neural network utilizes inlet parameters for predicting the objective functions, which are the maximum wall surface temperature as well as averaged vapor volume fraction at the outlet. The optimization procedure of design parameters is shown in Figure 4.

In [43], artificial neural network technique is applied for predicting heat transfer in supercritical water. The artificial neural network is trained on the basis of 5280 data points gathered from experimental results. Mass flux, heat flux, pressure, tube diameter, as well as bulk specific enthalpy are taken to be the inputs of the proposed artificial neural network. The tube wall temperature is taken to be the output, see Figure 5.

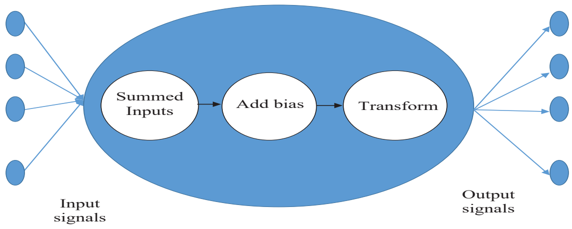

Fig. 3. Processing steps inside each neuron

3 Comparison analysis of neural networks and Z-number systems

Neural networks and Z-number systems can be considered as a part of the soft computing field. The comparison of Neural networks and Z-number systems is represented in Table 1. Neural networks have the following advantageous:

Table 1. The comparison of Neural networks and Z-number systems.

	Z-number systems	Neural networks
Knowledge presentation	Very good	Very bad
Uncertainty tolerance	Very good	Very good
Inaccuracy tolerance	Very good	Very good
Compatibility	Bad	Very good
Learning capability	Very bad	Very good
Interpretation capability	Very good	Very bad
Knowledge detection and data mining	Bad	Very good
Maintainability	Good	Very good

i Adaptive Learning: capability in learning tasks on the basis of the data supplied to train or initial experience.
ii Self-organization: neural networks are able to create their organization while time learning.
iii Real-time execution: the calculations of neural networks may be executed in parallel, also specific hardware devices are constructed, which can capture the benefit of this feature.

Neural networks have the following drawbacks:

i The utilization of neural networks is in direct connection with the availability of the training data.
ii The acquired solution from the learning procedure may not be often explained.

iii Almost all the neural network systems contain black boxes such that the ultimate state may not be explained.

Fuzzy logic has the following advantageous:

i Simple to learn and apply.
ii A user-friendly procedure to produce.
iii Generation of more effective performance.

Fuzzy logic has the following drawbacks:

i Constructing an uncertain system is complex.
ii It is not easy to define proper membership values for uncertain systems.

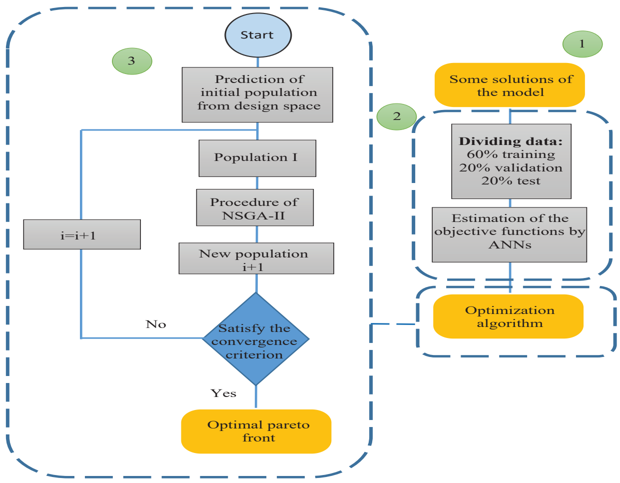

Fig. 4. The optimization procedure of input parameters

4 Combined Z-number and neural network techniques

4.1 Why apply Z-numbers in neural networks

Each neuron in the artificial neural network is linked with another neuron via a connection link in such a manner that the connecting link is related to a weight with the information regarding the input signal. Therefore, the weights contain beneficial information regarding input to resolve the problems. Some reasons for applying Z-numbers in neural networks are as follows:

i In a case that crisp values cannot be implemented, uncertain values such as Z-numbers are utilized.

ii Since the training, as well as learning, assist neural network to have a high performance in unanticipated status, therefore in such status, uncertain values like Z-numbers are more suitable than crisp values.

iii In neural networks, Z-numbers are more applicable than fuzzy numbers. Z-numbers are more precise when compared with fuzzy numbers. Also, Z-numbers have less difficulty in computation in comparison with nonlinear system modeling approaches.

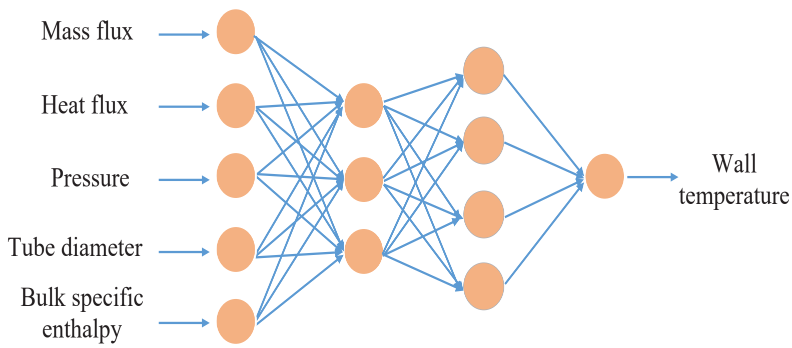

Fig. 5. Proposed artificial neural network for predicting heat transfer in supercritical water

4.2 Complexity in applying Z-numbers in neural networks

There exist some troubles when utilizing Z-numbers in neural networks. The complexity is associated with membership rules, the requirement to construct an uncertain system since it is often difficult to derive it by supplied set of complicated data.

Neural networks can be used to train Z-numbers. The advantageous of using neural networks for training Z-numbers are as follows:

i Novel patterns of data may be learned simply using neural networks therefore, it may be utilized for preprocessing data in uncertain systems.

ii Neural networks due to their abilities in learning new relation with new input data may be utilized for refining fuzzy rules to generate the fuzzy adaptive system.

4.3 Examples of combined Z-number and neural network techniques

Example 1. The following system is designed such that inputs and outputs are in the form of Z-numbers [44],

$$\begin{aligned} \zeta(t) &= \vartheta cos(\varphi \Delta k t) \\ v(t+1) &= \tfrac{\Delta k^2[\zeta(t)-\psi v^3(t)]-v(t-1)+\rho v(t)}{(1+\omega\Delta k)} \end{aligned} \quad (12)$$

such that $\rho = \omega\Delta k - \theta\Delta k^2 + 2$. $\Delta k, \omega, \theta, \psi, \vartheta$ are Z-number parameters. φ is

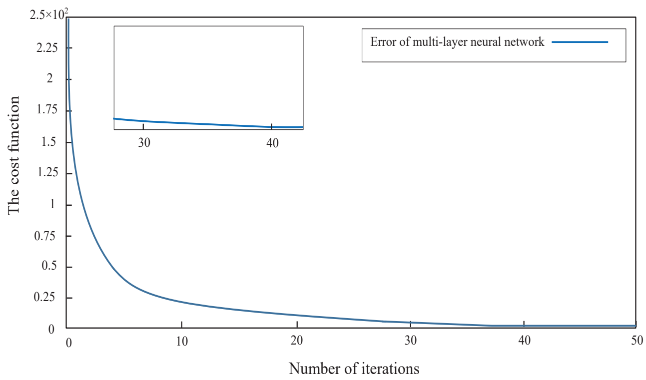

Fig. 6. Approximated error of multi-layer neural network

taken to be a random variable uniformly distributed in the interval $[0.1, 2.9]$ with mean $E\{\varphi\} = 1.5$, as well as the initial conditions being $v(0) = v(1) = 1$. The following are assumed,

$$\begin{aligned} \Delta k &= [(0.03, 0.05, 0.06), p(0.6, 0.8, 0.86)] \\ \omega &= [(0.1, 0.3, 0.5), p(0.6, 0.7, 0.87)] \\ \theta &= [(-4.2, -4, -3.8), p(0.6, 0.8, 86)] \\ \psi &= [(0.8, 1, 1.2), p(0.7, 0.8, 0.85)] \\ \vartheta &= [(0.2, 0.5, 0.7), p(0.7, 0.8, 0.85)] \end{aligned} \quad (13)$$

In order to model the uncertain nonlinear system (12), a multi-layer neural network is used such that obtains the Z-number coefficients of (12). The error plot is demonstrated in Figure 6.

Example 2. A liquid tank system is demonstrated in Figure 7, which is modeled as below

$$\frac{d}{dt}v(t) = -\frac{1}{SO}v(t) + \frac{d}{S} \quad (14)$$

where $d = t + 1$ is inflow disturbances of the tank that generates vibration in liquid level v, $O = 1$ is the flow obstruction which can be curbed utilizing the

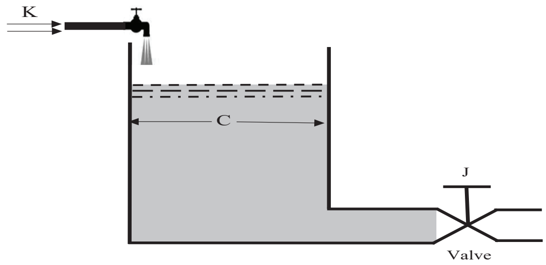

Fig. 7. Liquid tank system

valve, also $S = 1$ is the cross-section of the tank. Two types of neural networks, static Bernstein neural network (SBNN) and dynamic Bernstein neural network (DBNN) [45], are used to estimate the Z-number solutions of (14). The error plots of SBNN and DBNN are demonstrated in Figure 8.

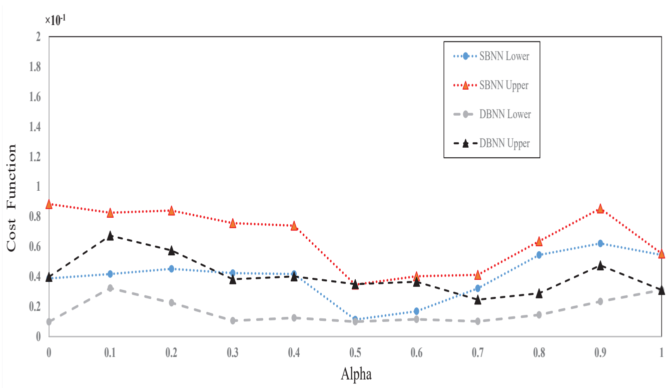

Fig. 8. Approximated errors of SBNN and DBNN

Example 3. The heat source by insulating materials is demonstrated in Figure 9, which is modeled as below

$$\frac{M_1}{N_1} \oplus \frac{M_2}{N_2} = \frac{M_3}{N_3} \oplus \frac{M_4}{N_4} \oplus J \tag{15}$$

A heat source is placed in the center of insulating materials. The widths of the insulating materials are in the form of Z-numbers. The coefficients of conductivity materials are $N_1 = h, N_2 = h\sqrt{h}, N_3 = h^2, N_4 = \sqrt{h}$, such that h is elapsed time. J is thermal resistance. Neural network technique is used to approximate Z-number solutions of (15)[46].

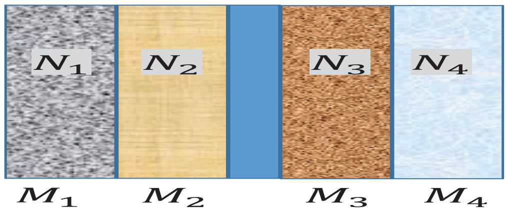

Fig. 9. The heat source

5 Conclusion

The notion of Z-numbers is rather naturally obtained while gathering vague information in a linguistic appearance. In this paper, the combined Z-number and neural network techniques are studied. Furthermore, the applications of Z-numbers and neural networks in engineering are introduced. As some researchers have effectively used Z-numbers, in-depth discussions are given for stimulating future studies.

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