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Metaheuristics for rich portfolio optimisation and risk management: Current state and future trends

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ABSTRACT

Computational finance is an emerging application field of metaheuristic algorithms. In particular, these optimisation methods are becoming the solving approach alternative when dealing with realistic versions of several decision-making problems in finance, such as *rich* portfolio optimisation and risk management. This paper reviews the scientific literature on the use of metaheuristics for solving NP-hard versions of these optimisation problems and illustrates their capacity to provide high-quality solutions under scenarios considering realistic constraints. The paper contributes to the existing literature in three ways. Firstly, it reviews the literature on metaheuristic optimisation applications for portfolio and risk management in a systematic way. Secondly, it identifies the linkages between portfolio optimisation and risk management and presents a unified view and classification of both problems. Finally, it outlines the trends that have gradually become apparent in the literature and will dominate future research in order to further improve the state-of-the-art in this knowledge area.

1. Introduction

Since the last century, the direct relationship between financial decisions and wealth creation through capital accumulation and economic development has been widely accepted [134]. Thus, investments play an essential role in improvements of welfare standards. This quest for improvement is represented through the formulation of optimisation problems for most of the questions in financial economics. Traditionally, exact methods have been employed in determining optimal solutions to these problems. Considering multi-criteria problems, such as an unconstrained Markowitz model that maximises returns and minimises risk without further limitations, quadratic programming yields optimal solutions on an efficient frontier. Such exact methods, however, present certain limitations when solving realistic and large-scale combinatorial optimisation problems (COPs) of NP-hard nature. COPs are characterised by further constraints, such as market frictions, investor preferences, or investment bank policies. Under these circumstances, traditional analytical methods require either the use of simplifying (non-realistic) assumptions or extraordinarily long computing times. Because this approach neglects depicting the complex intricacies of the real-life problems that decision-makers face in their everyday actions, the results are predominantly not transferable to real-life operations without reservations, which highlights the need for alternative approaches. Furthermore, the ongoing internationalisation and integration of financial markets and institutions has caused financial decision-making processes to become even more complex, both in terms of associated constraints as well as in terms of the instances to solve. Advances in Operations Research and Computer Science have brought forward new solution approaches in optimisation theory, such as heuristics and metaheuristics. While the former are experience-based procedures, which usually provide 'good' solutions in short computing times, metaheuristics are general templates that can easily be tailored to address a wide range of problems. They have shown to provide near-optimal solutions in reasonable computing times to problems for which traditional methods are not applicable [118]. Since they usually require relatively little computational time, metaheuristics constitute an attractive alternative for problem-solving in several knowledge areas in which 'real-time' decisions are required. Besides the above, the pitfalls inherent to traditional analytical methods have also motivated the need to adopt metaheuristics for solving the complex constrained portfolio optimisation problems. A discussion on this can be found in Deb [39] and Coello et al. [32]. Among others, Talbi [161] provide an excellent overview of metaheuristic methodologies and their applications. In particular, applications of metaheuristics in the financial sector are

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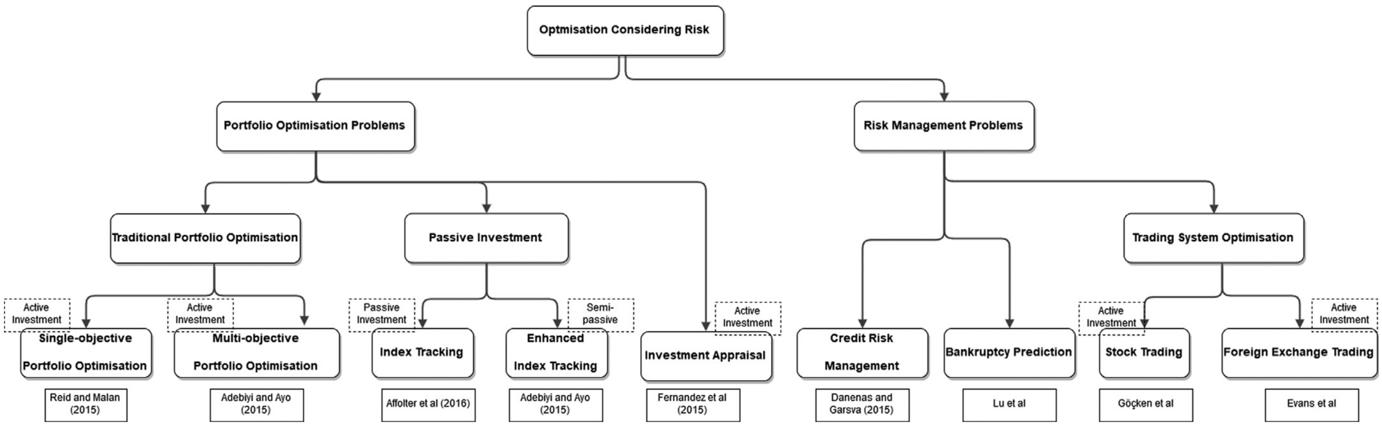

Fig. 1. Classification of two core areas regarding the use of metaheuristics in finance

presented in [68,111]. While metaheuristics do not guarantee finding a globally optimal solution, Gilli and Schumann [67] point out that the goal of optimisation in most real-life instances is not to provide an optimal solution, but one that fulfils the decision-maker's objectives to a highly satisfactory extent. Hence, these authors promote the use of metaheuristic approaches in practical applications. In effect, with respect to exact methods that provide an optimal solution to a simplified model of a real-life problem, metaheuristics can provide a near-optimal solution to a realistic model of the same problem, which might be preferable for most decision-makers. The contribution of this work to the existing literature is threefold. Firstly, it reviews the literature on metaheuristic optimisation applications for portfolio and risk management in a systematic way. This classification, in conjunction with the corresponding sub-problems, is depicted in Fig. 1. For investment decisions it is indicated whether the corresponding problem refers to an active or passive strategy. Further, an exemplary recent paper is provided for each sub-problem.

Secondly, the work identifies the linkages between portfolio optimisation and risk management and presents a unified view and classification of these problems. It is expected that the revocation of the strict classification of financial COPs can lead to a methodological transfer of knowledge in between different applications that enable more effective and efficient selections of decision-makers. Thirdly, the work also outlines the trends that have gradually become apparent in the literature and are expected to dominate future research in this knowledge area. Table 1 presents an overview of these trends and challenges classified into problem-specific and methodology-specific dimensions.

The remainder of the paper is structured as follows: Section 2 presents the research methodology and an overview of recent publications. Section 3 consists of a short overview of metaheuristics for those readers who are less familiar with these methods. Following this, a review of the recent literature on portfolio optimisation and the corresponding sub-problems is presented in Section 4, while Section 5 reviews the research on risk management problems. Further, the linkage between the two is discussed in Section 6. Future trends in the application of metaheuristics in the areas of portfolio optimisation and risk management are analysed in Section 7. Finally, Section 8 highlights the main findings and contributions of this work and concludes it.

2. Review strategy

The increasing popularity of the application of metaheuristics to portfolio optimisation problems (POPs) and risk management problems (RMPs) is depicted below in Fig. 2 based on Scopus-indexed publications that explicitly consider metaheuristics as an approach in solving different financial COPs.

The search for POPs was conducted by examining the articles that explicitly consider portfolio optimisation (or the American English equivalent), index tracking or project selection in the abstract, title or keywords and make use of metaheuristics. For risk management problems, the search terms were bankruptcy, credit risk or stock or foreign exchange trading. In the case of portfolio optimisation, it becomes obvious that the trend in publications is increasing, i.e., metaheuristics have received increased attention as solving approaches. This has previously been predicted by researchers due to their power in obtaining high quality solutions to many real world complex problems [130]. More specifically, continuing increases in computing power, the advancement of metaheuristic frameworks and parallelisation strategies favour these methodologies when dealing with NP-hard financial COPs. On the contrary, while risk management problems seem to have

Table 1
Open research challenges associated with portfolio optimisation and risk management problems

Dimension		Research trends and challenges
Problem-specific	Realistic problem modelling	(1) Deviating from the traditional formulation, more accurate risk measures –such as value-at-risk variations– are to be evaluated with regard to their ability in improving the depiction of portfolio risk. (2) Hybridisation of simulation, machine learning, and optimisation should be employed to include in the optimisation model the macro- and micro-level uncertainty of financial markets.
	Problem complexity	(1) The introduction of additional required constraints and a more narrow execution of traditional constraints in a uniform way are yet to be presented. (2) The internationalisation and integration of financial markets call for the inclusion of an extended asset pool in portfolio optimisation problems.
Methodology-specific	Computational times	(1) The increasing complexity of the problem modelling calls for faster metaheuristic approaches. (2) Especially for large-scale problems, distributed and parallel computing techniques could be explored for real-time problem-solving.
	Methodological complexity	(1) The predominance of population-based metaheuristics is not uniformly justified by the quality of the results; thus single-point metaheuristic approaches can be further explored. (2) The hybridisation of methodologies is a clear trend; however, this hybridisation should be done with care to avoid developing methods of increasing complexity that are difficult to reproduce in practice.

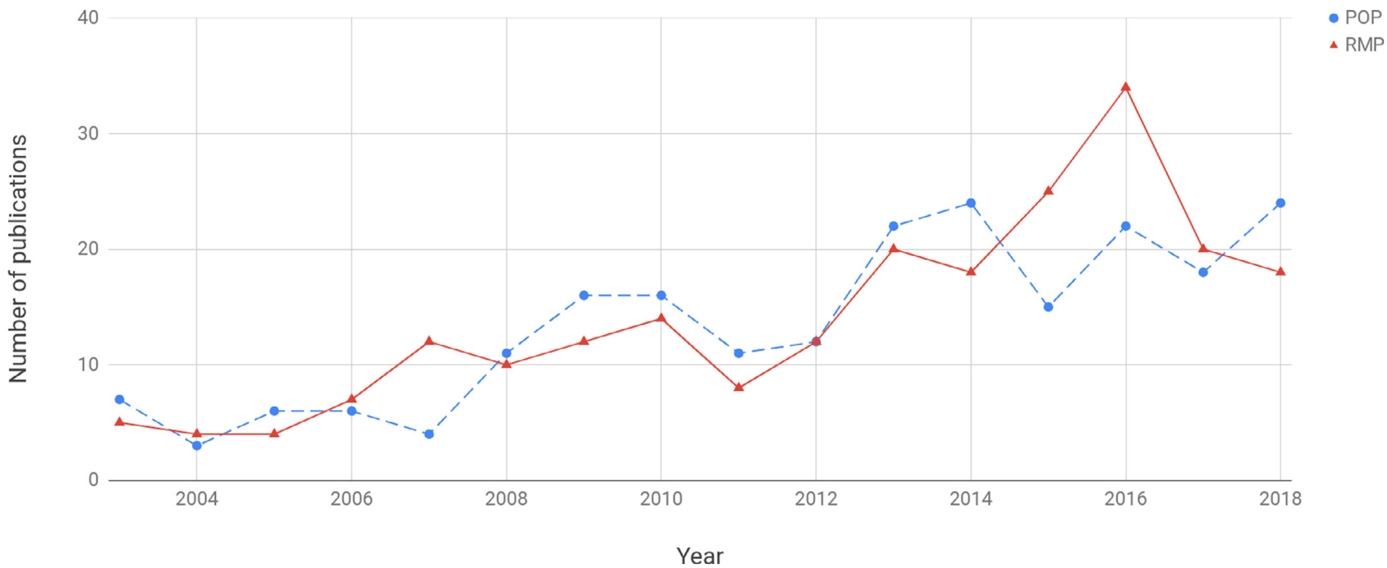

Fig. 2. Scopus-indexed publications applying metaheuristics to POPs and RMPs for the period 2003–18

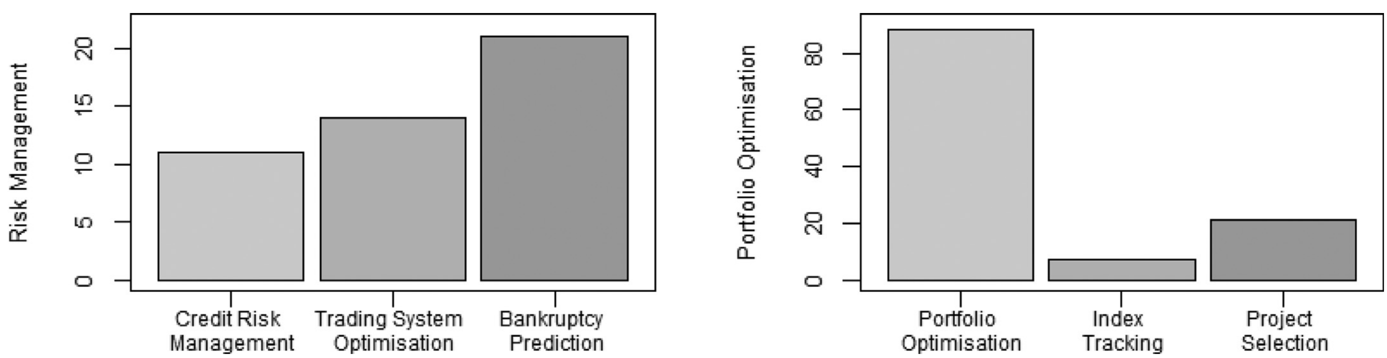

Fig. 3. Number of publications on the respective sub-problems

received similar attention, the individual sub-problems of risk management received much less attention than traditional portfolio optimisation. These proportions are broken down in Fig. 3, which shows that traditional portfolio optimisation represents the majority of metaheuristic applications.

One of the major contributions of this work is to discuss the idea that most risk management variants are strongly correlated with portfolio optimisation, i.e., that risk management problems can oftentimes be partially expressed as portfolio optimisation problems. To exemplify this assumption, imagine a decision-maker in a loan decision process who is choosing a portfolio of successful applicants from a pool of potential loan receivers based on his acceptance criteria and budget. Accordingly, it is possible to transfer methodological knowledge from the well-studied portfolio optimisation problem to the less explored area of risk management problems.

3. An overview of metaheuristics

Heuristics may be described as intelligent search strategies for solving problems [135]. They tend to be used in applications where exact methods fail to find a solution to a computationally hard problem and to speed up the search for high-quality solutions. Despite not guaranteeing optimal solutions, heuristics have been extensively employed due to the high number of successful applications. Their main disadvantage is that most are problem- or even instance-dependent [6]. As a consequence, considerable efforts to adapt them for addressing different problems or instances are needed. Metaheuristics are intended to overcome this drawback. The term, first introduced by Glover [70], can be described as a set of guidelines or strategies to develop heuristic optimisation algorithms. It is important to note that while metaheuristics (as frameworks) are domain-independent, their implementation is domain-specific. According to Feo and Resende [56], the effectiveness of these methods greatly depends on their ability to adapt to a specific instance to solve, to avoid getting stuck in local optima, and to exploit the structure of a problem. The authors discuss the relevant role of restart procedures, controlled randomisation, efficient data structures, and pre-processing. The popularity of metaheuristics has grown rapidly among both the scientific community and practitioners. Research fields in which they are commonly and highly successfully employed include logistics and transportation, finance, machine learning, computer vision, cryptology, and healthcare sciences. Metaheuristics can be classified into population-based metaheuristics, which work with a set of individual solutions that form a population, and single-solution metaheuristics, which maintain a single solution. For a description of the most widely used metaheuristics, the interested reader is referred to the works of Holland [82], Rechenberg et al. [141], Glover [69], Farmer et al. [54], Dorigo [49], Eberhart and Kennedy [50] and Seeley [150] for population-based metaheuristics and to the works of Kirkpatrick et al. [89], Glover [70], Feo and Resende [55], Martin et al. [113] and Mladenovic [121] for single-solution metaheuristics. While the former focus on exploration (diversification), searching a relatively large area of the search space, the latter centre on exploitation (intensification), applying local search within a limited region. Fig. 4 lists the most employed metaheuristics with regards to this classification and includes the references of the first applications.

Despite this list being relatively short, a vast variety of

Single-solution	Iterated local search (ILS) (Martin et al, 1991) Greedy randomized adaptive search procedure (GRASP) (Feo and Resende, 1989) Simulated annealing (SA) (Kirkpatrick et al, 1983) Tabu search (TS) (Glover, 1986)
Population-based	Ant colony optimisation (ACO) (Dorigo, 1992) Artificial immune systems (AIS) (Farmer et al, 1986) Bee colony (BC) (Seeley, 1995) Evolutionary strategies (ES) (Rechenberg, 1965) Genetic algorithms (GA) (Holland, 1962) Particle swarm optimisation (PSO) (Eberhart and Kennedy, 1995)

Fig. 4. Scheme of the most popular traditional metaheuristics

Table 2
Nature-inspired vs. non-nature metaheuristics

Nature-inspired metaheuristics		Non-nature metaheuristics
SI Based Metaheuristics	**Evolutionary Algorithms**	
Particle Swarm Optimisation (PSO)	Genetic Algorithms (GA)	Variable Neighbourhood Search (VNS)
Ant Colony Optimisation (ACO)	Genetic Programming (GP)	Iterated Local Search (ILS)
Cuckoo Search (CS)	Differential Evolution (DE)	Greedy Adaptive Search Procedure (GRASP)
Harmony Search (HS)		Simulated Annealing (SA)
Bat-Inspired Algorithm (BA)		Iterated Greedy (IG)
Firefly Algorithm (FA)		Tabu Search (TS)

metaheuristic methodologies exist, especially through hybridisation, and frequently, research communities focus only on a subset. In a similar vein, metaheuristics can be classified into two groups: *(i) nature-inspired metaheuristics*, such as genetic algorithms, particle swarm optimisation, colony optimisation, cuckoo search, harmony search, bat-inspired algorithms, firefly algorithms, etc.; and *(ii) non-nature metaheuristics*, such as tabu search, variable neighbourhood search, iterated local search, iterated greedy, simulated annealing, greedy adaptive search procedure, etc. The first group further bifurcates into evolutionary algorithms (EA) and swarm intelligence (SI) based metaheuristics. Table 2 presents this classification of well-known metaheuristics.

The successful application of metaheuristics has led to an increased interest in improvements and new developments of these methodologies in the academic community. However, more recently, while recognising the high value of many modern contributions, researchers occasionally criticise the lack of a scientific base, which is replaced by the most diverse metaphors [154], and leads to irreproducibility of the results and thus lack of reliability of the computational experiments. This has also been the case for individual papers reviewed in this article. This is why the application of simplified, reproducible metaheuristics is a pressing open line of further research. Finally, we refer the reader interested in an extensive review of metaheuristics to Talbi [161], Gendreau et al. [64], and Siarry [152].

4. Portfolio optimisation

Since Markowitz [112] developed the portfolio optimisation theory centred around the mean-variance approach, the academic community has been highly engaged in advancing the tools for portfolio optimisation. Central to this theory is the assumption that financial investors prefer (dislike) assets with higher returns (lower risks), *ceteris paribus*. It is thus the goal to minimise the level of portfolio risk measured by means of the portfolio variance for a given expected return level, resulting in the so-called unconstrained efficient frontier, from which the portfolio choice is determined by the risk awareness of the investor. This established the portfolio optimisation problem, which is a strategy of: *(i)* selection of financial assets; and *(ii)* determination of the optimal weights allocated to those assets that results in a desired portfolio return and associated minimum level of risk. For an overview of traditional methods applied to portfolio optimisation and risk management problems, the interested reader is referred to Best [13] and Pfaff [136]. Based on the investor's involvement with the asset selection, two types of investment management strategies can be identified. On the one hand, active investment strategies aim at beating market returns. On the other hand, passive investment strategies aim at replicating a benchmark index. This strategy has become specifically popular with equity funds and although it is originally based on the efficient market hypothesis, passively indexed funds can still outperform active funds and have shown to do so on average due to the increased management costs of active funds in the presence of market failures [105]. According

Table 3
The application of traditional metaheuristics and hybridisation to sub-problems of portfolio optimisation

Optimisation problem	Single-solution search				Population-based search											Hybrid
	SA	TS	FD	SD	GA	FA	ACO	DE	EA	ABC	PSO	IWO	AIS	SS	HS	
Single-objective portfolio optimisation	4	3	1	1	6			2	1	2	6					10
Multi-objective portfolio optimisation					2	1		4	1	3	6	1				2
Index tracking	1				5			2				1				3
Enhanced index tracking	1	1	1	1	3			1	1		1		2			2
Project selection	1	2			4		6				1			2	1	5

to these conclusions, index replication is not solely a hedging strategy, but provides stable profitability. Table 3 presents a summary of the metaheuristics applied to each of the problems reviewed in this section: single-objective portfolio optimisation, multi-objective portfolio optimisation, index tracking, enhanced index tracking, and project portfolio selection. The number of articles found on each topic and metaheuristic is included inside each cell. The classical portfolio optimisation is an active investment strategy, particularly when active re-balancing of the portfolio takes place in multi-period observations and, by its nature, investment appraisal requires the active selection of project portfolios. Index tracking is traditionally a passive strategy, while enhanced index tracking involves active management to some extent. Different metaheuristics can be classified with respect to different characteristics. As previously pointed out, they are classified in the following depending on whether they conduct a population-based or a single-solution search. While the latter can be categorised as trajectory and perform a closed walk on the neighbourhood graph with the possibility of accepting a worse solution temporarily to escape local minima, the former are discontinuous and tend to jump through the search space [14].

From Table 3, the following conclusions can be drawn: PSO, GA followed by SA and TS are the favoured single-solution search metaheuristics to approach POPs, while, in general, population-based metaheuristics, especially GA, are the most employed methodology. It also becomes evident that the methodologies employed to approach index tracking have been applied to enhance index tracking, single-objective portfolio optimisation has received more attention than multi-objective portfolio optimisation with respect to population-based methodological coverage. Furthermore, no single-solution approach has been used to address the multi-objective POP. It is further striking that ACO is by most the favoured metaheuristic to address the selection of project portfolios. Lastly, it is noteworthy that hybridisation of different metaheuristics in order to improve the optimisation solutions has increased together with the complexity of the methodologies. In the following, the metaheuristic approaches to solving the individual subproblems are reviewed.

4.1. Traditional portfolio optimisation

While the classical POP can be solved by means of quadratic programming, this methodology may become invalid when constraints are introduced to account for a range of realistic scenarios. The ensuing POP with realistic constraints becomes NP-hard, which has spawned the growing use of metaheuristics [11]. In effect, the debate that focuses on cardinality, quantity and pre-assignment constraints has gained momentum in the literature. The cardinality constraint determines the minimum and the maximum number of assets to be selected in a portfolio. While the minimum number of assets aims at portfolio diversification, the maximum number accounts for the fact that marginal benefits of diversification diminish after a certain threshold [111], which increases managerial efforts and transaction costs. The quantity constraint sets boundaries for the weights of included assets. While the lower limit ensures a minimum investment as smaller investments may be prohibitively costly due to transaction costs [90], the upper limit prevents excessive exposure to a particular asset. Finally, the pre-assignment constraint implies that certain assets may be included (or pre-assigned) in a portfolio due to the investor's individual preferences and are independent from their risk-return characteristics. In the following subsections, the optimisation approaches are, on the one hand, categorised by whether they optimise a single or multiple objective functions and, on the other hand, investment strategies and risk management approaches are scrutinised (see Table 4).

4.1.1. Single-objective portfolio optimisation

The classical POP has a single objective, in which the investor *(i)* minimises the portfolio risk subject to a minimum expected portfolio return; or *(ii)* maximises the portfolio expected return subject to a minimum desired portfolio risk. In the first variant, the objective function is computed by adding up the co-variances of individual asset returns and is then minimised [21]:

$$Min \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}. \tag{1}$$

Given a minimum expected portfolio return, the constraint that forces the weights to add up to unity, and the constraint that restrains all asset weights to take on values between zero and one, inclusively, thus eliminating short selling as a measure of preventing investors from excessive risk-taking by restricting them to the available budget. In formal terms:

$$\sum_{i=1}^{N} w_i \mu_i = R^*, \tag{2}$$

$$0 \le w_i \le 1, \quad \forall\ i = 1, 2, \ldots, N \tag{3}$$

where N stands for the number of available assets, μ_i represents the expected return on an ith asset, R^* represents the minimum expected portfolio return, w_i is the weight of an ith asset in the portfolio, and σ_{ij} is the co-variance between returns of two assets i and j.

Chang et al. [21] solved the classical POP outlined by means of Eqs. (1)–(3) using three distinct metaheuristic techniques (GA, SA, and TS) in order to yield a cardinality-constrained efficient frontier. Since no individual heuristic was found to be uniformly dominant across all data sets, they advocated pooling the results from the aforementioned three different techniques. In an effort to simplify the mathematical model, Vijayalakshmi Pai and Michel [172] employ a k-means cluster analysis to eliminate the cardinality constraint. Subsequently, they use a simplified evolutionary optimisation heuristic to diversify the risk of investment subject to the remaining constraints (bound and class constraints). The cardinality-constrained portfolio was also at the centre of the work of Streichert et al. [155] who employed evolutionary algorithms with further extensions for specific constraint-handling and performance improvements. As further constraints, Soleimani et al. [153] introduced sector capitalisation and minimum transaction lots. They found that, for their instances and problem definition, GA showed superior performance than TS and SA. Following Chang et al. [21], Woodside-Oriakhi et al. [185] sought to solve the POP by pooling GA, TS, and SA. Their findings indicate that, for their data sets, pooling GA and TS outperforms single metaheuristics at the cost of longer computational times, and that SA adds little to the performance of pooling approaches.

As for the application of strict single metaheuristic methodologies, PSO was found to be competitive with all three of the previously employed algorithms (GA, TS, and SA) for the cardinality-constrained POP, which comprised low-risk portfolios [34]. To evaluate the performance of PSO for even more realistic instances, Golmakani and Fazel [73] combined cardinality constraints with minimum transaction lots, limits on holdings, and sector capitalisation. These authors combined binary PSO and improved PSO (CBIPSO) and found that CBIPSO in their problem definition outperforms GA. Specifically, it yields better solutions with shorter computational times, especially for the large-scale problems they investigated. Vijayalakshmi Pai and Michel [176] performed an integrated optimisation of long-short portfolios with the 130-30-strategy-based constraint. To this end, the authors proposed two metaheuristics, namely, evolution strategy with hall of fame (ES-HOF) and differential evolution (rand/1/bin) with hall of fame (DE-HOF) to solve the problem. Experimental results of the study confirm the consistency of performance of the proposed metaheuristics for the data sets and the investment strategies.

As constraints become increasingly complex, the question of constraint-handling in determining feasible solutions arises. In this context, differential evolution (DE) has been extensively employed to solve a complex constrained POP. In this regard, Vijayalakshmi Pai and Michel

Table 4
Financial classification of portfolio optimisation problems

Work	Optimisation	Investment perspective	Risk management perspective
Chang et al. [21]	Single-objective	Mean-variance strategy	Diversification strategy
Schaerf [148]	Single-objective	Mean-variance strategy	Diversification strategy
Maringer and Kellerer [109]	Single-objective	Mean-variance strategy	Diversification strategy
Streichert et al. [155]	Multi-objective	Mean-variance strategy	Diversification strategy
Cura [34]	Single-objective	Mean-variance strategy	Diversification strategy
Vijayalakshmi Pai and Michel [172]	Single-objective	Mean-variance strategy for a small stock portfolio	Diversification strategy combined with robust portfolios
Soleimani et al. [153]	Single-objective	Mean-variance strategy	Diversification strategy with sector capitalisation as additional constraint to reduce investment risk
Golmakani and Fazel [73]	Single-objective	Mean-variance strategy for large-scale problems	Diversification strategy with sector capitalisation as additional constraint to reduce investment risk
Woodside-Oriakhi et al. [185]	Single-objective	Mean-variance strategy	Diversification strategy
Gaspero et al. [63]	Single-objective	Mean-variance strategy	Diversification strategy
Zhu et al. [191]	Hybrid single- and multi-objective using weights	Mean-variance strategy	Diversification strategy with Sharpe ratio as a risk-adjusted measure of portfolio performance
Krink and Paterlini [92]	Multi-objective	Mean-variance strategy	Diversification strategy with leptokurtic distribution of returns combined with non-parametric risk measures, such as value-at-risk and expected shortfall
Vijayalakshmi Pai and Michel [176]	Single-objective	Long-short 130-30 investment strategy	Management of larger expected losses given the leverage of short-selling
Vijayalakshmi Pai and Michel [175]	Single-objective	Long-short portfolio strategy combined with equity market neutral (EMNP) portfolio	EMNP specific constraints of zero net market exposure, close-to-zero portfolio beta and zero financial leveraging
Ma et al. [104]	Single-objective	Mean-variance strategy	Diversification strategy with value-at-risk as risk portfolio risk measure
Deng et al. [41]	Multi-objective	Mean-variance strategy	Diversification strategy
He and Huang [80]	Multi-objective	Mean-variance strategy for four different POPs	Diversification strategy with four different risk measures
Cesarone et al. [20]	Single-objective	Mean-variance strategy	Diversification strategy
Tuba and Bacanin [166]	Multi-objective	Mean-variance strategy	Diversification strategy
Tuba and Bacanin [165]	Multi-objective	Mean-variance strategy	Diversification strategy
Vijayalakshmi Pai and Michel [177]	Multi-objective	Long-short investment strategy in futures portfolio	Diversification strategy with risk budgets for asset classes to effectively portray the risk tolerance of individual investor
Lwin et al. [103]	Multi-objective	Mean-variance strategy	Diversification strategy
Bacanin and Tuba [8]	Multi-objective	Mean-variance strategy	Diversification strategy
Reid and Malan [142]	Single-objective	Mean-variance strategy	Diversification strategy with value-at-risk as portfolio risk measure
Reid and Malan [142]	Single-objective	Mean-variance strategy	Diversification strategy
Babaei et al. [7]	Multi-objective	Mean-variance strategy	Diversification strategy with leptokurtic distribution of returns combined with non-parametric risk measures, such as value-at-risk
Adebiyi and Ayo [2]	Multi-objective	Mean-variance strategy combining historical data and expert opinions	Diversification strategy
Strumberger et al. [157]	Single-objective	Mean-variance strategy	Diversification strategy
Suthiwong and Sodanil [160]	Multi-objective	Mean-variance strategy	Diversification strategy
Pouya et al. [137]	Hybrid single- and multi-objective by using weights and fuzzy normalisation	Mean-variance strategy combining historical data and expert opinions	Diversification strategy
Ni et al. [126]	Single-objective	Mean-variance strategy	Diversification strategy
Liu [101]	Multi-objective	Mean-variance strategy for large-scale investments	Diversification strategy
Kumar and Mishra [94]	Multi-objective	Mean-variance strategy	Diversification strategy combined with a regulatory 5-10-40 constraint to reduce heavyweight risk exposure
Liu and Yin [100]	Single-objective	Mean-variance strategy with irrational consumer behaviour	Diversification strategy with irrational consumer behaviour is modelled
Strumberger et al. [158]	Single-objective	Mean-variance strategy	Diversification strategy
Liagkouras and Metaxiotis [98]	Multi-objective	Mean-variance strategy	Diversification strategy
Wang et al. [178]	Multi-objective	Mean-variance strategy	Diversification strategy with Fuzzy Sharpe ratio as a risk-adjusted measure of financial performance as well as fuzzy value-at-risk as risk measure that captures non-statistical uncertainties
Salehpoor and Molla-Alizadeh-Zavardehi [145]	Single-objective	Mean-variance strategy	Diversification strategy

[175] investigated this research line and developed a DE-HOF metaheuristic to compute a complex constrained equity market neutral portfolio (EMNP). Computational results of the study validate the consistency of performance of DE-HOF across all its runs. Furthermore, the authors applied these methodologies to a global asset allocation portfolio, comprising equities, currencies and commodities of global markets [174]. Global asset allocation portfolios have further been subject to optimisation and weight repair strategies with Multi-objective Differential Evolution algorithms [168]. In a similar vein, hybrid differential evolution algorithm was implemented by Ma et al. [104] to solve the POP, in which the minimum of value-at-risk is used as an objective function. The numerical results show that the algorithm is efficient in solving the cardinality constrained POP.

Turning to the application of metaheuristics for complex constrained POPs, PSO has been applied extensively by researchers. Reid and Malan [142] developed a portfolio repair constraint handling technique applied to portfolio optimisation. Employing this technique, they were able to further improve the performance of the metaheuristic, again particularly for large instances. Along similar lines, Ni et al. [126] implemented a PSO algorithm based on random population topology to solve the POP and compared its performance with the constriction factor PSO (CPSO). According to their respective instances, PSO outperforms CPSO. PSO with factor scaling was proposed by Liu and Yin [100] in order to avoid premature convergence, which might be a problem of standard PSO. Empirical results of the study confirmed the effectiveness of the proposed algorithm. For an extensive review of the implementation of swarm intelligence metaheuristics including PSO for solving the POP, the reader is referred to [51].

Di Tollo and Roli [45] provided a survey concerned with the early applications of metaheuristics to the POP and some of the proposed constraints explicitly highlighting the potential use of hybrid approaches. Likewise, such a hybrid method was proposed by Maringer and Kellerer [109], who employed a hybrid local search algorithm combining principles of SA and evolutionary algorithms (EA) to optimise a cardinality-constrained portfolio. By combining exact mathematical programming and metaheuristic methods, Woodside-Oriakhi et al. [185] further hybridised and created different matheuristics. This option was also investigated by Schaerf [148] and Gaspero et al. [63] who respectively combined TS and first descent (FD) and steepest descent (SD) local search metaheuristics with quadratic programming to optimise a portfolio while accounting for cardinality constraints, lower and upper boundaries for the quantity of an included asset, and pre-assignment constraints. According to their results, the developed solver finds the optimal solution in several instances and is at least comparable to other state-of-the-art methods for the others. Concerning optimality, Cesarone et al. [20] were able to develop an exact increasing set algorithm that, for small instances, solves the POP with quantity and cardinality constraints optimally and can be extended into a heuristic procedure to account for larger instances. It outperforms the metaheuristics employed by Gaspero et al. [63] and Schaerf [148] in the investigated instances. Among recent studies, Strumberger et al. [157] adopted a hybridisation between bat and artificial bee colony metaheuristics to solve the portfolio optimisation problem. Comparison of hybridised metaheuristics with GA, FA, ABC, and krill herd (KH) algorithms verified that the suggested algorithm has satisfactory potential in solving the POP. Likewise, Strumberger et al. [158] hybridised artificial bee colony and genetic algorithm (GI-ABC) and applied the proposed algorithm to the cardinality-constrained portfolio optimisation problem. The study compared the GI-ABC with TS, GA, SA, PSO, ABC-FS, and modified firefly algorithm (mFA) and found it to be superior to these metaheuristics for their data sets. By hybridising PSO, GA, SA, genetic network programming (GNP), and an electromagnetism-like algorithm (EM), Salehpoor and Molla-Alizadeh-Zavardehi [145] analysed a boundary- and cardinality-constrained POP. The results, measured by diversification metric (DM) and mean ideal distance (MID), reveal the effectiveness of the suggested algorithm.

4.1.2. Multi-objective portfolio optimisation

While single-objective optimisation methods consider either a minimal risk for a given expected return or a maximum return for a given expected level of risk, multi-objective optimisation methods combine two objective measures into a single one through linear scalarisation that is to be optimised [21,120,171–173] or, more often, find a set of Pareto solutions while balancing two or more objective functions simultaneously. With respect to single-objective optimisation methods that require the *ex-ante* definition of an acceptable degree of profitability, multi-objective optimisation requires no previous knowledge about the investor's degree of risk aversion and is thus a more general approach transferable to different decision-makers. The approach of combining risk and return characteristics into a single objective function is taken by Zhu et al. [191]. They introduced the Sharpe ratio as a simultaneous measure and, since GA and PSO have been found to be competitively successful in solving the single-objective version, performed a comparison of these metaheuristics in solving the non-linear constrained portfolio optimisation problem. As previously established, they also argue that PSO outperforms GAs, especially in many large instances. While they did not include realistic constraints other than a total portfolio weight equal to one in addition to portfolio assets restricted to positive weights, in which the short selling of the portfolio's underlying assets is prohibited, the authors also investigated unrestricted portfolios. The solution portfolios obtained with the PSO solver outperformed those constructed using GA for all test problems in terms of Sharpe ratio, and the established efficient frontier was above that of GA portfolios in all but one instance.

According to Streichert et al. [155], the multi-objective POP can be formulated employing two simultaneous objective functions as follows. For a multi-objective optimisation it becomes necessary to minimise the portfolio risk expressed by the portfolio variance:

$$Min \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}, \tag{4}$$

while maximising the return of the portfolio, i.e.:

$$Max \sum_{i=1}^{N} w_i \mu_i, \tag{5}$$

subject to:

$$\sum_{i=1}^{N} w_i = 1, \tag{6}$$

$$0 \le w_i \le 1, \quad \forall\ i = 1, 2, \ldots, N. \tag{7}$$

Alternatively, Eqs. (4) and (5) can be combined into a single one by incorporating objective weights as follows [120]:

$$Min\ \lambda \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} - (1-\lambda) \sum_{i=1}^{N} w_i \mu_i, \tag{8}$$

subject to the aforementioned constraints. In this case, the weights as determined by the parameter λ represent the risk aversion of the investor. By varying this parameter and running repeatedly, a Pareto efficient frontier can be established. Because of the high performance of PSO in solving the single-objective POP, enhanced PSO algorithms for solving the multi-objective POP have been proposed by Deng et al. [41] and He and Huang [80]. Cardinality and bounding constraints were incorporated by Deng et al. [41] who find that their algorithm mostly outperforms GA, SA, and TS algorithms as well as previous PSO approaches, especially in the case of low-risk portfolios. It can be concluded that different findings unanimously favour PSO in situations when low-risk investment is demanded in addition to a larger-scale potential asset pool. Similarly, He and Huang [80] proposed a modified PSO (MPSO) algorithm that outperforms regular PSO for their four optimisation sets. More recently, they also developed a new PSO to deal with discontinuous modelling of the POP and find that it generally outperforms PSO and also performs better than MPSO in larger search spaces [81]. Other population-based algorithms applied in optimising cardinality-constrained portfolios include firefly algorithms (FA) [8,166] and artificial bee colony (ABC) algorithms [94,160,165]. However, because the results were satisfactory at most even after modifications, the authors hybridised FA and ABC by incorporating the FA search strategy into ABC to enhance exploitation and found that their data suggested superiority of the methodology compared to GA, SA, TS, and PSO [165]. Streichert et al. [155] accounted for further constraints: buy-in thresholds (acquisition prices) and round lots (smallest volume of an asset that can be purchased). They employed two multi-objective evolutionary algorithms (MOEA): GA and an EA enhanced through the integration of a local search that applies Lamarckism, thus allowing the individual improvements to be passed on to the offspring. They found that this enhancement greatly improved the reliability of the results, especially with respect to the additional constraints. Unfortunately, these approaches are hardly reproducible due to their complexity, reinforcing the need for a less metaphorical and more scientifically reproducible approach.

Population-based metaheuristics have been employed for solving the multi-objective POP. Hence, Pouya et al. [137] proposed PSO, IWO, and reduced gradient method (RGM) to solve the multi-objective portfolio selection model. According to their results, IWO outperforms PSO in terms of solving time as well as solution performance. Using MOEA, Liagkouras and Metaxiotis [98] solved a multi-constrained portfolio optimisation problem (MCPOP). The authors introduced reparation operators, round-lots, and pre-assignment constraints into the MCPOP. Results of the study confirmed, for the instances at hand, that the suggested algorithm outperforms NSGAII and MOEA. Similarly, Wang et al. [178] proposed Sharpe and value-at-risk ratios in fuzzy environments. These authors employed a fuzzy simulation-based multi-objective PSO algorithm.

Nevertheless, apart from the neglect of realistic non-linear constraints, there is a second point of criticism to the original Markowitz model: its assumption of normal financial returns, which are characterised by a leptokurtic distribution [92], making it necessary to consider non-parametric risk measures. Such a measure is the value-at-risk, as employed by Babaei et al. [7], who developed two multi-objective algorithms based on PSO to solve a cardinality- and quantity-constrained POP. Through splitting the whole swarm into sub-swarms that are then evolved distinctly, their methodology outperformed similar benchmark metaheuristics. In order to optimise a non-parametric value-at-risk and to include further constraints –including lower and upper bounds for the weights of included assets–, a threshold for asset weight changes, lower and upper bounds for the weights of one asset class, and a turnover rate that determines the maximum asset allocation changes possible at once, Krink and Paterlini [92] developed the differential evolution (DE) for multi-objective portfolio optimisation (DEMPO) algorithm. Along similar lines, Vijayalakshmi Pai and Michel [177] employed a multi-objective evolution strategy and a multi-objective differential evolution strategy to solve a POP with optimal allocation to futures contracts constrained by risk budgets, asset class, and bounding constraints. Computational experiments show that both metaheuristics yielded consistent results. Further, Vijayalakshmi Pai [170] employs DE to tackle a topic that is less researched in traditional portfolio optimisation, but even more so for enhanced index tracking (Section 4.2.2): active re-balancing of the portfolio. When assets are sold and bought in order to re-establish risk and return expectations, the problem becomes even more complex given that the original portfolio must already fulfil a certain number of constraints. An extended version of a generalised DE metaheuristic was also employed in optimising a highly constrained POP by Adebiyi and Ayo [2]. The included constraints consist of bounds on holdings, cardinality, minimum transaction lots, and expert opinion. An expert can form an opinion based on indicators beyond the scope of the analysed data and influence whether or not an asset should be included. For their data sets, the methodology showed improved performance when compared to GA, TS, SA, and PSO. Lwin et al. [103] considered cardinality, quantity, pre-assignment and round lot constraints. They developed a multi-objective evolutionary algorithm that is improved through a learning-guided solution generation strategy, which promotes efficient convergence (learning-guided multi-objective evolutionary algorithm with external archive, MODEwAwL). The study repaired each constructed portfolio during the population sampling. It was shown that the developed algorithm outperformed four benchmark state-of-the-art multi-objective evolutionary algorithms in that its efficient frontier was superior. More recently, by adapting optimal computing budget allocation (OCBA) in DE, Liu [101] analysed a large-scale portfolio optimisation problem. According to the results of the study, the integration of OCBA remarkably improves the performance in stochastic simulation optimisation.

An extensive review of the application of evolutionary algorithms to the POP is provided by Metaxiotis and Liagkouras [117]. For an extensive review of different portfolio optimisation problems, including single- and multi-objective optimisation, the reader is referred to Mansini et al. [106]. Likewise, the application of metaheuristics to different investment strategies, such as risk budgeting, the 130-30 strategy, and equity market neutral portfolios are elucidated in Vijayalakshmi Pai [169].

4.1.3. Financial classification of portfolio optimisation

The reviewed papers are organised chronologically in Table 4. The classification of POPs from two different perspectives – methodological and conceptual – are summarised in this table. The methodological perspective distinguishes between single- or multi-objective POPs and provides a detailed account of the wealth of constraints that bind these problems (column “Optimisation”). The conceptual/financial perspective is aligned with a financial investor’s point of view. In column “Investment perespective”, we delve into the examination of specific investment strategies. More specifically, if this column features the term “mean-variance strategy” –also coined as the sample-based mean-variance strategy by DeMiguel et al. [40]–, then this paper deals with a variant of the Markowitz’s POP underpinned by a wealth of constraints, but not any additional investor-specific strategies. In column “Risk management perspective”, it is further specified whether the authors considered measures to explicitly reduce the risk of investment. In particular, if the column features the term “diversification strategy”, the work refers to portfolio variance as a risk measure. By pooling a relatively large number of assets with imperfectly correlated returns into a portfolio, the investor is able to diversify away idiosyncratic risks of those assets. This risk management strategy is effectively founded on the “insurance principle”, according to which assets in the portfolio with positive price changes offset (or insure against) those assets whose returns turn out to be low or negative. In this light, it once again becomes obvious how closely intertwined portfolio optimisation and risk management problems are when considering the intricacies of real-life financial decision-making.

4.2. Passive investment

Closely related to portfolio optimisation as an active portfolio management strategy, passive investment strategies have received less attention in the optimisation literature. These strategies are characterised by limited on-going buying and selling, as well as by ensuing limited maintenance. Based on the traditional capital market theory stating that market portfolios offer the greatest return per unit of risk, passive investment strategies have been shown to outperform actively managed funds and thus gained popularity [4].

4.2.1. Index tracking

The index tracking problem (ITP) is a passive portfolio management strategy in that investors aim at mimicking a market or sector index. This is done by either replicating the index or by selecting a portfolio that follows the index behaviour as closely as possible without including all the stocks that make up the original index. In the case of perfect replication, there are transaction costs associated with updating the portfolio to continuously accurately depict the index, which thus have to be deducted when evaluating the performance. Therefore, the ITP is largely concerned with the latter, partial replication. There are thus two stages in index tracking, the common goal of which is to minimise the resulting tracking error (the distance between the portfolio and benchmark returns). The first consists of selecting the assets to include in the portfolio and the second relates to determining the weights. Thus, it consists of a combinatorial and a continuous numerical problem, which both have to be addressed simultaneously [93]. Once similar constraints as in portfolio optimisation are introduced (e.g., floor and ceiling constraints, cardinality constraints, pre-assignments, or class constraints), minimising the objective function of the tracking error becomes extraordinarily difficult to solve with exact methods. The optimisation problem can thus be addressed with the following formulation [12]. Minimise the tracking error:

$$Min\ E = \frac{[\sum_{t\in S} |r_t - R_t|^{\alpha}]^{\left(\frac{1}{\alpha}\right)}}{T}, \tag{9}$$

where $S = 1, 2, ..., T$ are the time periods considered during which the portfolio return was below that of the tracked index, r_t is the tracking portfolio return, R_t is the return of the tracked index itself, and α is the penalisation power that is applied to the difference between the realised return and the benchmark return. If we set $\alpha = 2$, the tracking error is defined as the root mean square error (RMSE). In the case of a perfect reproduction of an index, the tracking error would naturally be equal to zero. In the most basic formulation, the following constraints have to be considered:

$$\sum_{i=1}^{N} z_i = K, \tag{10}$$

which represents the cardinality constraint and ensures that any new tracking portfolio contains K stocks, as z_i takes on the value of one if a stock is included in the replication portfolio and zero otherwise. As in portfolio optimisation, the weights have to be limited:

$$0 < w_i \leq 1,\ z_i = 1,\ \ \forall\ i = 1, 2, ..., N. \tag{11}$$

This limits the weights of the included stocks to be larger than zero and equal to or below one. The non-included stocks must naturally dispose of a weighting of zero:

$$w_i = 0,\ z_i = 0,\ \ \forall\ i = 1, 2, ..., N \tag{12}$$

Maringer and Oyewumi [110] investigated partial replication and introduced cardinality constraints concerning upper and lower weight limits and integer constraints in the ITP employing a DE methodology. Their findings suggest that partial replication is indeed sufficient in replicating the benchmark index. This is due to the fact that only a decreasing marginal improvement is reached by increasing the cardinality.

Scozzari et al. [149] were able to develop a mixed integer quadratic programming formulation to solve the ITP including hard constraints set by the European Union on ceilings of asset inclusion weights as well as low turnover rates and resulting low transaction costs in small instances. Likewise, Xu et al. [186] suggested a non-monotone projected gradient (NPG) method for solving a cardinally-constrained ITP model. A comparative analysis was also conducted with a hybrid evolutionary algorithm and the hybrid half thresholding algorithm for the ITP. According to their results, the proposed algorithm produces sparse portfolios with high consistency between in and out of sample tracking errors.

However, the introduction of realistic constraints generally makes it difficult to use exact methods in solving large ITP instances. Early research by Beasley et al. [12] introduced a population-based evolutionary metaheuristics to solve the partial reproduction ITP with regard to stock indices including constraints on transaction costs –as well as a ceiling for the total inclusion of stocks. Derigs and Nickel [43] developed a two-stage SA metaheuristic, in which they controlled for cardinality constraints and transaction costs through turnover volume restrictions.

For larger instances, especially in multi-period analysis, Scozzari et al. [149] proposed hybridising metaheuristics with exact methods. This has been done by Krink et al. [93] who addressed the two sub-tasks of ITP simultaneously and applied a metaheuristic approach based on DE combined with a combinatorial search operator. Although their developed methodology initially failed to find acceptable solutions, they showed that extending DE with a search operator by selecting the assets with highest weights in the benchmark improved the results greatly in comparison with GA, SA, and PSO. Ruiz-Torrubiano and Suárez [144] employed a GA hybridised with quadratic programming. Likewise, Ni and Wang [125] also tackled the ITP employing a hybridised GA with increased learning ability that is enabled through goal programming. The authors included cardinality and integer constraints, as well as proportion constraints for individual portfolio assets. While both methodologies yielded successful solutions, the models neglect transaction costs, which are however indicated by the authors as a variable important to investigate in future research. The trade-off between transaction costs and tracking performance was then studied by Chiam et al. [28], who developed a multi-objective evolutionary index tracking platform that considers multiple periods and simultaneously optimises tracking performance and transaction costs while considering round lots and non-negativity constraints –as well as floor constraints as buy-in threshold to prevent unnecessary transaction costs and capital injections. GA has also been employed in recent studies for ITP. Thus, Sant'Anna et al. [146] combined mathematical programming with a genetic algorithm to solve the ITP for the S&P 100, FTSE 100, DAX, and Ibovespa indexes. The study included cardinality, non-negativity, floor, and ceiling constraints and it found satisfactory results in terms of performance and computational time of the proposed algorithm. Strub and Trautmann [156] proposed a GA for the ITP. The computational results of the study confirmed effectiveness of the metaheuristic in real data for large ITP instances. Furthermore, García et al. [59] compared the performance of GA and TS for a cardinality-constrained ITP, and found TS superior to GA for their respective computational experiments.

Although different metaheuristic approaches have been chosen to cope with the realistic constraints of the ITP, Affolter et al. [3] found that due to the missing measure to define the distance between portfolios with respect to their assets and weights, invasive weed optimisation (IWO) did not lead to satisfactory optimisation results. Di Tollo and Maringer [44] created a framework for classifying the metaheuristics applied to the ITP and present a review of the literature.

4.2.2. Enhanced index tracking

Beasley et al. [12] defined an objective function that accounts for a trade-off between the tracking error and excess returns above those of the benchmark index. This enhanced index tracking allows the manager discretion in pursuing risk-limited active strategies to enhance return. Considering that investors might see a trade-off between the trading error and excess returns above the index has led to the enhanced index tracking problem (EITP), in which investors aim at beating the benchmark index. This can either be done through active selection of the included assets and weights or through a passive extension of the methodology by incorporating the excess return as a further optimisation objective. The EITP then becomes a multi-objective optimisation problem, in which the tracking error is minimised while maximising the degree of beating the benchmark index –so that a solution dominates another if the excess return is higher given the same level of trading inaccuracy or if the trading accuracy for the same level of excess return exceeds that of the other solution. This can be formulated by including a second objective function that defines the excess return between r_t and R_t:

$$Min\ E = \frac{[\sum_{t\in S} |r_t - R_t|^{\alpha}]^{\left(\frac{1}{\alpha}\right)}}{T}, \tag{13}$$

while maximising the excess return r^*:

$$Max\ r^* = \sum_{t=1}^{T} \frac{r_t - R_t}{T}, \tag{14}$$

subject to the aforementioned constraints:

$$\sum_{i=1}^{N} z_i = K, \tag{15}$$

$$0 \leq w_i \leq 1,\ z_i = 1,\quad \forall\ i = 1, 2, \ldots, N, \tag{16}$$

$$w_i = 0,\ z_i = 0,\quad \forall\ i = 1, 2, \ldots, N. \tag{17}$$

Canakgoz and Beasley [18] solved the ITP as well as the EITP including transaction costs, an upper limit on the total number of stocks purchased, and a limit on the incurred transaction costs using exact methods (mixed-integer linear programming formulations). However, Li and Bao [97] showed they could mostly outperform the methodology employed by Canakgoz and Beasley [18] by implementing an immunity-based optimisation algorithm. It is an EA based on the clonal selection of an immune system, or the immune response to antigens [38]. Including further constraints, Li and Bao [97] also employed an immunity-based multi-objective optimisation algorithm with non-negativity and floor and ceiling buy-in thresholds. They concluded that the inclusion of optimisation of the tracking process in addition to optimising tracking error and excess return is valuable as the optimisation of the tracking process improves results in most instances. A perfectly enhanced tracking portfolio would outperform the index by a low-frequency trend such as steady excess return while negative returns should be trendless and characterised by high frequency variation. Thus, the tracking process can be enhanced by considering different frequencies for tracking error and excess returns when the former is minimised and the latter maximised [97]. Optimisation of the tracking process is expected to increase in importance for multi-period assessment. The authors, however, leave this for further research. The question of multi-periodicity was investigated by Andriosopoulos et al. [5], who addressed the EITP employing both DE and GA. They could show that the so-constructed mimicking portfolios inhibit less risk compared to the underlying benchmark index, while proficiently replicating their performance. Nevertheless, they concluded that the GA version outperforms DE in terms of minimum tracking errors, as well as maximum mean excess returns for the investigated instances. As they explicitly considered different time horizons for re-balancing the portfolio, these authors reinforced the idea that there exists a trade-off between transaction costs, which decrease with longer re-balancing periods, and Sharpe ratios (as a measure of the tracking performance and profitability), which is negatively impacted by decreased re-balancing frequency as investigated by Chiam et al. [28] for the ITP. Guastaroba et al. [77] further proposed Omega ratio for the large EITP instances including cardinality and buy-in threshold constraints under two different scenarios (OR and EOR models). Computational experiments of the study verified superiority of EOR to OR regarding to out-of-sample performance.

Gnägi and Strub [71] proposed a mixed-integer quadratic programming (MIQP) and a mixed-integer linear programming (MILP) formulation of the EITP using the tracking error variance and the mean-absolute deviation as objectives, respectively. The study also employed local branching and iterated greedy heuristics and found superior solutions in terms of out-of-sample tracking error to minimise the tracking error variance. [187] employed hybridised GA for an enhanced indexation model, which includes chance and cardinality constraints. The proposed model was tested on four indices from China and four major indices. According to the test results, the suggested algorithm effectively solves the enhanced indexation problem with high out-of-sample excess return.

An alternative approach was pursued by Guastaroba and Speranza [76], who applied a kernel search framework to both the ITP and the EITP. They argued that error measurements should be undertaken as absolute values and introduced the possibility that an investor already holds a portfolio as a further constraint to consider in addition to transaction costs. However, they treated the EITP as a single-objective optimisation by outperforming the market index, while keeping the tracking error below a given threshold. Compared to a general-purpose solver, the performance of the kernel search model was superior. Filippi et al. [58] extend the kernel search algorithm given in Guastaroba and Speranza [76] for the ITP to the bi-objective EITP. The results indicate that the suggested approach compute an accurate approximation of the trade-off curve between the objective functions. Further including metaheuristics into the optimisation, Thomaidis [163] considered an EITP problem with restrictions on the maximum of tradable assets, and employed fuzzy set theory to consider non-standard investment objectives –such as the probability of under-performing. The resulting cardinality-constrained problem was solved using nature-inspired optimisation techniques: SA, GA, and PSO.

Lastly, while some authors declare active and passive portfolio management as mutually exclusive concepts, the close connection between index tracking and portfolio optimisation could be illustrated by the approach taken by Di Tollo et al. [46], who combined the two methods in a multi-criteria optimisation problem. They employed a hybrid metaheuristic consisting of local search metaheuristics (FD, SD, and TS) and quadratic programming to estimate the efficient frontier. Combining the concepts of risk and return with tracking error led to a three-dimensional objective function and Pareto frontiers. The developed methodology was found competitive in performance with other metaheuristics.

4.3. Project portfolio selection

Unlike banks and institutional investors, non-financial companies as well as governments are faced with a different type of portfolio choice. As a method to determine which proposals to pursue and the corresponding budget allocation, investment or project appraisal is related to portfolio optimisation in its goal of maximising a benefit figure. This figure can be monetary, but also related to knowledge gain in the case of research projects. Usually, decisions cannot be altered or adjusted during the course of the projects, or at least not without incurring considerable financial losses. Thus, investment appraisal determines a strategic organisational path for the medium and long term. This problem becomes NP-hard due to its sheer complexity [57]. It is by its very nature a multi-period problem and the budget-allocating entity usually pursues several conflicting objectives, some of which can be of qualitative nature. For that matter, Doerner et al. [47] proposed a two-stage procedure. During the first phase, the Pareto frontier is constructed. Then, in the second phase, it is interactively explored by the decision-makers to account for personal preferences. The optimisation process is carried out in the first phase. A formal description of this problem, based on the one presented in Doerner et al. [47], is included next. The benefit function $b_{l,t}(x)$ that comprises the value of the l different benefit groups, such as generated funds, cash flows, patents or other beneficial outcomes of the selected projects is to be maximised over all considered time periods t for all included projects, i.e.:

$$b_{l,t}(x) = \sum_{i=1}^{N} b_{i,l,t} x_i, \tag{18}$$

where x_i is a binary variable that takes on the value of one for included projects and zero otherwise, subject to constraints concerning resource limitations $R_{q,t}$ (which apply to all resource categories r_q, such as budget, capacity, or manpower) as well as minimum benefit requirements $B_{l,t}$ (which define a threshold, below which the decision-maker is uninterested in the implementation of projects):

$$r_{q,t}(x) \leq R_{q,t};\ q = 1, \ldots, R \text{ and } t = 1, 2, \ldots, T, \tag{19}$$

$$b_{l,t}(x) \leq B_{l,t};\ l = 1, \ldots, B \text{ and } t = 1, 2, \ldots, T. \tag{20}$$

Because of the modelled similarities, the methodological approaches employed are inspired by the research on traditional portfolio optimisation. Early work Ghasemzadeh and Archer [66] conducted optimisation after the construction of a weighted objective function and constraints concerning budget and man-hours in an integer linear programming approach. However, test instances were very limited because the authors aspired a comparison between manually computed portfolios and those constructed employing their decision support system. For their metaheuristic two-stage approach Doerner et al. [47] employed Pareto ACO (P-ACO). As there are possible synergies between projects that should be evaluated in order to accurately estimate the benefits of a project portfolio, the authors made an attempt at incorporating these considerations into their methodology. They pointed out that, unlike GA, SA, and TS –which are adaptive metaheuristics–, P-ACO specifically constructs project portfolios through pheromone vectors. This has two advantages. Firstly, infeasible solutions are avoided; and secondly, project interactions can more naturally be considered in the construction of solutions. They further took into account floor and ceiling constraints for inclusion of projects from any given subset, as well as resource limitations and minimum benefit requirements for individual projects. Compared to Pareto SA and a non-dominated sorting GA (NSGA), P-ACO yielded the most efficient results for their instances. This approach was then further enhanced by Stummer and Sun [159], who compared the performance of a P-ACO procedure enhanced through adding a neighbourhood search routine, a TS procedure, and a variable neighbourhood procedure. Their findings suggested that the improved P-ACO model performs better in their experiments than TS with many objective functions and a large set of efficient solutions. Hence, it is specifically suitable for real-life problems. Furthermore, Doerner et al. [48] concluded that including both a learning and a two step integer linear pre-processing procedure to initialise several initial efficient project portfolios improves the performance of the P-ACO algorithm.

Research has also drawn on findings from other areas, such as scheduling: Gutjahr et al. [78], [79]] also took employee competencies and the evolution of their knowledge scores over time through learning or depreciation into account. Their earlier work optimised a weighted average objective function using ACO and GA metaheuristic procedures, and found the GA to be superior when the search space is not highly constrained. Later, the authors developed a multi-objective optimisation model, which simultaneously optimises the objectives of maximum economic gains and aggregated competence increase. They also divided the problem into master and slave sub-problems, the first of which is concerned with the project selection, while the slave problem optimises the allocation of personnel to the projects over time. Although the slave problem can be solved using exact methods, the master problem was solved using the NSGA-II and P-ACO metaheuristics. While both performed reasonably well, NSGA-II outperformed P-ACO in synthetic test instances, while P-ACO outperformed NSGA-II for the investigated real-life instances. Carazo et al. [19] further investigated this research line and included scheduling as a continuative concept following the project selection. Their developed metaheuristics approach is based on scatter search (SS) for project portfolio selection (SS-PPS). As previous work, they also considered interdependencies between different projects and can show that their model outperformed other heuristic approaches based on EA (SPEA). Also using NSGA-II as well as SPEA2 and a preference-inspired co-evolutionary algorithm with goal vectors (PICEA-g), Brester et al. [16] reduced the PPSP to a $0-1$ knapsack-constrained multi-objective optimisation problem. According to their results, the island model-based multi-objective algorithms are efficient and promising in solving the NP-hard problem. In their model formulation, Kumar et al. [95] focused on simultaneous selection and scheduling of the PPSP using teaching-learning-based optimisation (TLBO), TS, and TS-TLBO metaheuristics. Mutual exclusiveness and complementariness were considered as interdependencies between the projects and the performance of the hybrid TS-TLBO was found superior to TLBO and TS with regard to quality and convergence. Similar to Rabbani et al. [139], who presented a multi-objective PSO metaheuristic and found it to be competitive with respect to SPEA II, Urli and Terrien [167] formulated the project portfolio selection problem as a multi-objective non-linear integer program, which they solved using the SSPMO metaheuristic [122]. In a first phase, they generated an initial set of efficient solutions through TS, and then combined these via SS. While this approach solved small and medium instances in satisfactory computation time, the determination of all non-dominated project portfolios still remains difficult when considering large, but realistically relevant instances (100 projects or more). While this might not be relevant in most firm investment decisions, it is a significant drawback for governments or bodies awarding funding for projects.

Another issue that has only recently been addressed is project divisibility. While business projects are at least partially indivisible, research projects funded by governments can often also be executed with partial funding and it is thus a further question how much of the sought after funding is awarded, introducing further constraints to the budget allocation. Hence, more recent research increasingly focused on large-scale instances and partial allocation. Cruz et al. [33] used ACO in solving a stationary project portfolio optimisation problem, in which partial support of the requested budget was allowed. They developed a non-outranked ACO approach, incorporating a fuzzy outranking preference model. Unlike previous research, they assumed that the preferences of the decision-maker are to some extent known. Outranking was employed in an *a priori* preference system in order to model that decision-makers will have preferences towards different portfolios on the efficient frontier based on their personal goals concerning the achievement of objectives. Incorporating these preferences allows identifying those portfolios that lie on the efficient frontier and simultaneously are not outranked by another portfolio. They incorporated budgetary constraints in that they defined upper and lower bounds for inclusion of projects from a particular group. Fernandez et al. [57] further enhanced this approach by including integer linear programming methods to generate an initial population and thus hybridising the metaheuristic further. They also included synergies in their optimisation, concluding that their model outperformed state-of-the-art metaheuristics for these instances. It can be asserted that project synergies, project divisibility, the incorporation of multi-periodicity, and outranking are the prominent real-life constraints and trends that specifically increase the complexity of the portfolio selection process and thus distinguish this COP from a classical POP.

More recently, Esfahani et al. [52] implemented a harmony search metaheuristic for solving the PPSP. According to experimental results, HS can solve the PPSP to near optimality in a reasonable amount of time. Focusing on multiperiodicity, Tofighian et al. [164] implemented a GA metaheuristic hybridised with a local search procedure. A comparative analysis of GA with PSO and electromagnetism-like algorithm confirmed the superiority of the proposed algorithm in terms of computational time, accuracy, and robustness. Taking into account that many of the factors in determining an optimal project portfolio are not deterministic in real life, Panadero et al. [133] consider a stochastic and rich variant of the PPSP, which includes pre-assignment, cardinality and quantity constraints. The authors employed a simheuristic approach, which combines simulation methods with an adaptation of a simulation annealing-based variable neighbourhood search (VNS) metaheuristic. The study also compared deterministic and stochastic variants of the PPSP and concluded that a near-optimal solution in the deterministic PPSP is mostly sub-optimal in the stochastic one. Besides, a near-optimal solution found for the stochastic PPSP yields higher net present values (NPV) in a simulated real-life scenario than the corresponding near-optimal solution from the deterministic model, indicating the importance of taking uncertainty in decision-making into consideration as a future line of research.

5. Risk management

Risk management of financial and non-financial companies refers to the evaluation, often in real time, of realistic data concerning the institution's exposure to a certain source of risk and it is further concerned with statistics on trends that will influence that exposure in the future. While quantitative data is relevant and necessary for this, it must be complemented by qualitative information for informed

Table 5
The application of traditional metaheuristics and hybridisation to subproblems of risk management

Optimisation problem	Single-solution search		Population-based search												Hybrid
	SA	TS	GA	ACO	EA	ABC	PSO	SS	HBMO	FA	BA	HS	GWO	GSO	
Credit risk assessment		2	5		1		2		1						7
Bankruptcy prediction			7		1		2						1		8
Optimisation in stock trading	2		5			1	4			1	1	1			10
Optimisation in foreign exchange trading			3		1									1	8

decision-making, both in financial as well as non-financial institutions [30]. Risk management is addressed in terms of optimisation through metaheuristics for credit risk assessment and the resulting bankruptcy prediction. García et al. [60] provide a detailed review of developed systems and, to a less obvious extent, applications to the optimisation of trading rules in the financial markets. As depicted previously for portfolio management, Table 5 presents the metaheuristic methodologies applied to the different subproblems of risk management. As before, the numbers inside each cell refer to the number of articles reviewed for each topic and methodology.

From Table 5, several conclusions can be drawn. Firstly, GA are the preferred metaheuristics in risk management as well. Their popularity is expressed through the use in every single reviewed problem. Furthermore, PSO has also received widespread attention. Contrary to that, more exotic algorithms, such as harmony search, firefly algorithms, or bat algorithms were employed to a lesser extent. Secondly, it can be seen that bankruptcy prediction –as an advancement of credit risk analysis–, as well as optimisation of trading systems for foreign exchange (Forex) markets –as an advancement of optimisation in stock trading–, have received less attention in the literature and have been approached with fewer methodologies. They thus represent interesting future research lines. Thirdly, it becomes evident that hybridisation among metaheuristics or other optimisation methods is far more prevailing in risk management optimisation than in portfolio optimisation. Lastly, it is evident that relatively recently developed metaheuristics, such as IWO and honeybees mating optimisation (HBMO), have not been applied as comprehensively as well-established ones.

5.1. Credit risk assessment and optimisation

Credit risk assessment is one of the most researched and recognised topics in the banking industry. There are many different approaches and sophisticated credit risk assessment tools for financial institutions. However, during the last years, non-financial companies have also recognised the need to treat their trade credits to customers with the same caution and scrutiny. Thus, both financial and non-financial analysts have to decide on the granting as well as the extension of loans. While the use of metaheuristics is still scarce in this area of application, they are increasingly used as a pre-processing procedure in order to identify the most relevant predictors of credit risk in the analysis of large data sets of information. Marinakis et al. [108] classified a set of companies into different classes of credit risk level. They propose and compare TS, GA, and ACO for solving the feature selection subset problem, which are then used in determining the appropriate level of credit risk. The employed accuracy measures are determined by whether or not a subject has been classified in the right category. In a simple two-class model, this is based on the four scenarios depicted in Table 6.

Table 6
Definitions of the classified and the misclassified samples

		Actual class	
		1	2
Estimated class	1	T_1	F_2
	2	F_1	T_2

The overall classification accuracy (OCA) can then serve as optimisation objective that is to be maximised:

$$Max\ \ OCA = \frac{T_1 + T_2}{T_1 + F_1 + T_2 + F_2} * 100. \tag{21}$$

This framework can be extended to include as many different credit risk classes as the decision-maker considers.

More recently, Marinaki et al. [107] employed HBMO in determining the relevant features. They were able to show that this metaheuristic reduced the number of used features by more than half and still yielded superior results compared to PSO, ACO, GA, and TS. Oreski et al. [129] employed neural networks (NN) hybridised with GA (GA-NN) to enhance the classification accuracy of the NN classifiers by choosing optimal features. They found that the prediction ability was as accurate as in the case of using all available data features in the analysis. In a subsequent work, Oreski and Oreski [128] further improved the results by employing a hybrid GA. Their results suggested that they hence achieved a higher and less volatile accuracy with on average fewer selected features through a reduction of the search space and an incremental phase of the GA. Chi and Hsu [27] employed GA in selecting relevant variables to combine a bank's internal behavioural scoring model with an external credit bureau scoring model and thus creating a dual scoring model that subsequently outperformed their individual model in credit risk prediction. A survey on the importance of employing the right fitness function in the GA for credit assessment is provided in Kozeny [91]. More recently, Metawa et al. [115] proposed a GA which uses a random search to suggest the best convenient design for optimising bank lending decision. Similar to the previous studies, Metawa et al. [116] employed GA to optimise bank lending decisions with a credit crunch constraint (GAMCC). Test results of the suggested algorithm on real and simulated data showed that the GA boosted the bank's profit. Employing a multi-stage hybrid model which integrates feature selection and classifier selection with an enhanced multi-population niche genetic algorithm (EMPNGA), Zhang et al. [189] found solutions to the credit scoring problem. For their computational experiments, results confirmed that the hybridised EA is superior to single EMPNGA, binary GA, and binary PSO.

Trends in credit risk assessment concern the hybridisation of metaheuristics with other techniques for feature selection. Wang et al. [180] developed a feature selection based on rough set and TS (FSRT). In comparison with non-preselecting models, the savings in computational time and performance accuracy were significant. Similarly, Wang et al. [181] used a rough set and scatter search feature selection (RSFS) that is able to improve results in all three considered base sets, i.e.: neural network model, decision trees, and logistic regression (LR). Lastly, Danenas and Garsva [35] pursued the idea of optimising the classifiers of a linear support vector machine (SVM) using PSO. While their results were comparable to the use of other classifiers (LR and RBF networks), the proposed methodology, however, delivered less stable performance.

5.2. Bankruptcy prediction

Closely related to credit risk evaluation is the prediction of

Table 7
Definitions of the classified and the misclassified samples for bankruptcy prediction

		Actual class	
		Bankrupt	Not bankrupt
Estimated class	Bankrupt	T_1	F_2
	Not bankrupt	F_1	T_2

bankruptcy of firms. Strictly defined, bankruptcy occurs when debtors are unable to repay outstanding debts. While bankruptcy prediction constitutes part of the credit risk evaluation process, it is vital for banks and companies to constantly monitor their credit risk exposure. Because of the two-class framework (firms that go bankrupt and firms that do not), the basic optimisation framework is similar as suggested for credit risk assessment. The difficulty and difference to credit risk assessment lies however in the relatively longer aspired forecasting period and the difficulty in predicting the exact time of bankruptcy.

$$Max\ \ OCA = \frac{T_1 + T_2}{T_1 + F_1 + T_2 + F_2}*100. \tag{22}$$

Especially in practical bankruptcy prediction, it is worth considering to differently value the two classes of mistakes that occur (Table 7). While falsely classifying a subject as bankruptcy candidate (type II misclassification) merely leads to missed revenues, a false classification as healthy company (type I misclassification) usually leads to at least partial failure on a payment and thus has greater consequences for profitability. This is thus an improvement to the methodology open for further research.

Early research conducted by Back et al. [9] highlighted the contribution of GA in predicting bankruptcy when hybridised with artificial neural networks (ANN). Shin and Lee [151] introduced the prediction of corporate bankruptcy using GA and historical financial data. Kim and Han [85] further employed GA to extract decision rules based on qualitative expert decisions. For the particular instances computed, their approach was superior to the use of neural networks or inductive learning methods. An extensive survey on the early research in this knowledge area can be found in Kumar and Ravi [96], who reviewed both statistical and computing methods. Their evaluation concluded that statistical methods are outperformed by the most popular neural network methodology: back propagation neural networks. They further highlighted the prediction accuracy of SVM and pushed for further research in the area of hybridising different soft computing approaches. More recently, Kirkos [88] presented the literature on artificial intelligence and machine learning techniques employed in bankruptcy prediction.

Hybridisation of machine learning techniques with metaheuristics has also been a trend in the literature on bankruptcy prediction. In this respect, Wang et al. [182] combined kernel extreme learning machine (KELM) with a PSO-based on differential evolution (EPSO-KELM) to model bankruptcy prediction. Likewise, Wang et al. [183] further proposed a KELM algorithm, which uses grey wolf optimisation (GWO) to model bankruptcy prediction. The proposed model was found dominant to GWO, PSO, GA, PSO-KELM, GA-KELM, and GS-KELM. Likewise, Zhao et al. [190] employed KELM including a two-step grid search strategy. A comparison with SVM, extreme learning machine, random forest, PSO-enhanced fuzzy *k*-nearest neighbour, and logit model confirmed superiority of the proposed approach regarding classification accuracy, type I and type II error, as well as AUC.

Following the conclusions of Kumar and Ravi [96], another research line attempts to optimise SVM with metaheuristics. Min et al. [119] improved the performance of SVM with regards to optimising the feature subset and parameters simultaneously. They showed that selecting an appropriate feature subject has implications for the kernel, and that their integration improved bankruptcy prediction accuracy. Chen [23] highlighted that, while intelligent techniques provide higher prediction accuracy for smaller data sets and are adversely affected by increasing data sets, statistical methods perform more accurately when the data set is large. But the author also indicated that a hybrid between PSO and SVM could yield a good balance between short- and long-term prediction accuracy. This was consequently done by Lu et al. [102], who combined switching PSO (SPSO) and SVM. The SPSO was employed in searching the optimal parameter values of radial basis function (RBF) kernel of the SVM. The authors showed that this hybridisation yielded superior results to GA-SVM and PSO, respectively. These findings were supported by Chen [24], [25]], who also employed PSO-SVM and showed high accuracy with a significantly reduced number of parameters. Furthermore, Gaspar-Cunha et al. [62] proposed an evolutionary multi-objective approach that simultaneously minimises the number of features and maximises the accuracy of the classifier in SVM, so that the algorithm is self-adaptive. The general advantage of multi-objective optimisation lies in the attainment of a set of efficient solutions from which the decision-maker can perform a trade-off based on personal preferences. The power of SVM has been subject to many recent studies. Santoso and Wibowo [147] investigated the bankruptcy prediction problem and compared the performances of hybrid stepwise SVM and linear discriminant analysis (LDA) to model the problem. Comparative analysis demonstrated that SVM outperformed LDA in terms of prediction accuracy.

However, the performance of SVM in predicting bankruptcy was not found superior in other studies. Cleofas-Sánchez et al. [31] compared the performance of the hybrid associative classifier with translation (HACT) with those of the multi-layer perceptron (MLP), the probabilistic or Bayesian network (BN), RBF and the voted perceptron (VP), SVM, and logistic regression. According to their results, HACT was found to be the most successful. Along similar lines, Barboza et al. [10] investigated bankruptcy prediction using machine learning models (SVM, bagging, boosting, and random forest) one year prior to the event. The authors could not find the SVM to lead to higher accuracy rates than other models.

Ensemble learning has been applied to the bankruptcy prediction problem (BPP). Kim and Kang [86] proposed hybridising an ensemble with neural networks and showed that it improved prediction accuracy compared to regular neural networks. However, these attempts often suffer from multicollinearity, or high correlation among the individual classifiers, and thus Kim and Kang [87] improved their methodology to include a GA-based coverage optimisation to alleviate multicollinearity through classifier selection. Davalos et al. [36] developed an accurate GA-based ensemble classifier model with heterogeneous instead of individual classifiers that is comprehensible due to its if-then-structure. They showed that the fitness function can be tailored to accommodate further constraints and showed the improved performance of their approach. More recently, García et al. [61] measured performance of classifier ensembles (bagging, AdaBoost, random subspace, DECORATE, rotation forest, random forest, and stochastic gradient boosting) in predicting bankruptcy and found that the performance of the ensemble configuration strongly depends on the data type in the sample.

However, one main disadvantage in real-life scenarios is that the financial ratios employed in the main research lines are unavailable for a large portion of companies. Small and medium-sized enterprises (SMEs) do not dispose of regular audited financial data or market prices and public ratings due to publicly traded equity or debt instruments. Also, it is necessary to include available and relevant indicators for these individual firms. Thus, with special regards to SMEs, Gordini [74] compared the prediction accuracy of GA, SVM, and LR. The author showed that the prediction of GA was superior, especially with regard to type II misclassifications (assuming bankruptcy when this is not the case) and with regard to prediction of bankruptcy for small firms. Furthermore, the reduction of type II misclassification improves business relationships between SMEs and prevents reputational damage, which might lead to credit crunches, especially in small firms.

5.3. Optimisation of decision-support systems for trading

As a result of the above optimisations in predicting markets and prices, the development and optimisation of automated trading systems has become of prevalent importance and special interest for broker investment banks and other institutional investors alike. As for forecasting, a large portion of the literature addresses stock trading, while some researchers have concentrated on the foreign exchange markets.

5.3.1. Stock market trading

Derigs and Nickel [42] developed a decision support system (DSS) for portfolio optimisation and index tracking that can be tailored to meet the constraints and portfolio types of different institutional agents. They stressed the importance of hard (government-imposed and compulsory) and soft (shaped by preferences of the investor) constraints. They implemented a local search guided by SA in order to optimise the DSS with respect to floor and ceiling constraints and transaction costs. These authors have shown for the application to passive tracking of the DAX 30 that their developed system delivered solutions with minimal tracking errors in acceptable computing time.

Focusing on real-time decisions, Chavarnakul and Enke [22] proposed a trading system for the stock market based on volume adjusted moving average (VAMA) that is hybridised with neural networks to decrease the time of receiving trading signals (which is of major importance for the adoption of a real-time trading framework), fuzzy logic to deal with uncertainty, and GA techniques to optimise the trading signals to overall increase efficiency. Depending on the strength and direction of a given signal, the system assumes a buy or sell position. If the signal is not confident enough, a hold position is taken. The so established neuro-fuzzy-based GA (NF-GA) was shown to lead to fewer trades and thus reduced transaction costs, while profitability was increased for the observed data set.

Gorgulho et al. [75] also proposed a system to automatically manage a portfolio of assets and highlighted the necessity of adapting the system to the state of the market to optimise performance. They employed GA and technical analysis rules. The system adapts to the different states of the market and always outperforms the random and at most instances the buy and hold strategy. The system requires the user to input the available budget, the maximum of assets to be included in the portfolio at any time, whether or not short selling is considered and the allowed amount of transaction costs. Then, an initial portfolio is constructed, as it would be in the POP, employing a GA. However, over the course of the investment, the system regularly updates the proposal based on technical trading rules based on closing positions that are either prone to losses or have achieved a profit and can be closed and refilling empty positions. Teixeira and De Oliveira [162] combined technical trading rules with nearest neighbour classification. Their analysis was solely based on historical data of stock closing prices and volume, on the basis of which trading rules were formed. Their proposed system outperforms a buy and hold strategy in most cases in terms of profitability. Because however, the parameters in these functions have to be determined, metaheuristics have been applied in the optimisation. The hybridisation of technical trading rules and metaheuristics is seen as an especially promising research area. Brasileiro et al. [15] further refined the strategy by Teixeira and De Oliveira [162] by searching for the best system parameters (the parameter of the classifier and the values of the stop loss and stop gain) and number of lags with an ABC algorithm. They outperformed the previous trading system as well as the buy and hold strategy in most instances. Nunez-Letamendia [127] had already shown that GA is robust and powerful when applied to optimising technical trading rules. Similarly, Lin et al. [99] applied a GA to improve trading rules for individual stocks, which are then combined with echo state networks to provide suggestions for trading. Their results showed an outperformance of the buy and hold strategy as well as significant profits even in bear market situations. In a more recent work, Wang et al. [179] employed a time-variant PSO (TVPSO) to determine the optimal parameter values of a complex trading system: the performance-based reward strategy (PRS). The PRS combines moving average and breakout trading rules, which are combined based on weights that are determined by previous performance. Considering transaction costs and excluding short selling, the system was able to achieve high profits and the application of the TVPSO significantly optimised the trading system.

As previously applied to bankruptcy prediction and credit risk assessment, hybridisations of metaheuristics and ANN have also recently shown to provide accurate forecasts of stock market prices. While both provide better results than a passive buy and hold strategy, harmony search based models have been shown to outperform GA-based models with regards to forecasting errors [72]. Chiang et al. [29] utilised ANN, PSO, and denoising to develop an adaptive decision support system model for stock trading. Likewise, Ghasemiyeh et al. [65] implemented the hybrid ANN and cuckoo search (CS), improved cuckoo search (ICS), ICS-GA, GA, and PSO. Experimental results stated that, as previously stated by other authors, PSO outperforms other metaheuristics for the conducted experiments.

Hybridisation of data mining and machine learning techniques with metahueristics –e.g. FA, BA, and PSO– has created clustering metaheuristics able to predict patterns in the general movement of stock markets, such as periods of lower and higher return [138]. Chen and Hao [26] hybridised a feature-weighted SVM and a *k*-nearest neighbour to predict the performance of several indexes. Likewise, Ren et al. [143] combined sentiment analysis with SVM to forecast stock market movement direction for the SSE 50 Index. These insights together with automated trading systems could further enhance investment decisions.

5.3.2. Foreign exchange market trading

Foreign exchange market trading can either concern hedging foreign exchange risk or speculation. Only the latter has the objective of making a profit by exploiting market inefficiencies and is thus the central assumption in trading systems. Speculation in the foreign exchange markets requires the application of real-time trading systems to an even greater extent than stock market trading due to the nature of trading profits, which are realised at the time of trading. Myszkowski and Bicz [124] established a trading system based on decision trees that consider technical trading indicators. EA then generates trading strategies. While these were able to achieve a profit, the system is still too fragile to be used in automated trading, but can serve as additional indicator to investors. Mendes et al. [114] proposed employing GA to optimising trading rules and although their developed trading system performs well in terms of computational time because of the small population size employed, it fails to perform well in terms of profitability when faced with transaction costs. Zhang and Ren [188] developed a high-frequency trading system based on the optimisation of technical indicators through GA that was able to produce annualised profits. In addition to intraday prediction, highlighting the importance of real-time, Evans et al. [53] implemented a trading system based on feedforward neural networks with back propagation architecture, whose topology was optimised using GA. In comparison with Zhang and Ren [188], they were able to considerably improve annualised net profits.

For foreign exchange market trading, hybridisation with machine learning techniques has been and still is a main research line. Ozturk et al. [132] hybridised a GA with a greedy search algorithm (GSA). GA and GSA are used in a qualification test for the trading rules using closing prices. On average, 60% of the trades were found profitable. Likewise, Özorhan et al. [131] combined SVM with GA for predicting the magnitude and direction of the exchange rates in the Forex market. According to their results, the proposed algorithm proved useful with regards to profits and directional symmetry. Abreu et al. [1] integrated GA and Naïve Bayes and obtained a significant improvement on the trading system (with an estimated ROI between 0.43% and 10.29%).

Following this idea, [37] hybridised SVM with a dynamic genetic algorithm (DGA) to optimise trading rules in the Forex market. The study implements SVM for identifying and classifying the market and it employs DGM for optimising trading rules.

6. Linkage between portfolio optimisation and risk management

Despite the fact they have been addressed as two independent types of problems in most of the scientific literature, this section highlights the relationship between portfolio selection and risk management. In the first place, portfolio optimisation problems directly consider a risk measure (such as portfolio variance, portfolio semivariance, value-at-risk, alpha and beta, among others). Therefore, they can also be seen as risk management problems. For example, the specification of adequate risk measures that accurately depict return distributions is a well-established area of research of on-going interest within academia and especially financial institutions. It is directly concerned with one-dimensional risk measures of individual assets and multi-dimensional measure to account for interaction of asset portfolios [140], unambiguously linking risk management and portfolio optimisation.

In the second place, most risk management models related to optimisation problems can be seen as rich variants of portfolio optimisation problems. For instance, stock market trading is in the essence of the problems concerned with constructing an initial portfolio and updating it over time to reflect current macro- and microeconomic developments. Likewise, while credit risk and bankruptcy risk are only estimated in the overwhelming majority of the risk management literature, it is the ultimate goal to build low-risk portfolios (e.g., loan or customer portfolios) by including preferably those assets with a lower credit risk and excluding other assets expected to go bankrupt. Using a multi-objective EA, Moreno-Paredes et al. [123] explicitly acknowledge the linkage between credit risk management and portfolio optimisation by treating the loan decision among a set of customers as a credit portfolio optimisation problem (CPOP). More generally, implicit in a portfolio optimisation problem is pooling assets with imperfectly correlated returns that leads to a diversification of idiosyncratic sources of risk and a reduction in the overall risk of portfolio investment.

Fig. 5 depicts the relationship between risk management and portfolio optimisation problems reviewed in this paper. They have been divided according to whether single- or multi-objective optimisation approaches have been taken in solving them. The extension of the ovals representing risk management problems and portfolio optimisation problems respectively signifies the extension of possible solving approaches beyond traditional optimisation techniques. The intersecting set comprises the CPOP, as well as stock trading. The risk management problems of bankruptcy and credit risk prediction are located directly on the border to portfolio optimisation, as the predictions are generally employed in a following portfolio selection process. Foreign exchange trading, unlike stock trading, is considered a sole risk management problem. While stock trading consists of the establishment and maintaining of a portfolio, speculative profits in foreign exchange trading are generated through simultaneous buying and selling and not the establishment of a portfolio.

Concerning the subproblems of portfolio optimisation, the ITP and EITP do not directly consider risk measures. Unlike that, the POP is directly concerned with risk minimisation and thus closely related to risk management problems.

7. Emerging trends in the literature

From the previous discussion, one clear trend that we see in the literature is the transfer of methodological knowledge from portfolio optimisation to risk management optimisation problems. In effect, since very efficient metaheuristics have been developed to solve single- and multi-objective POPs, and since most optimisation problems in the risk management arena can be seen as enriched variants of POPs, it is reasonable to assume that these metaheuristics will constitute the base for developing new solving approaches in the risk management field. Another trend is related to the increasing complexity of the problems being addressed, which makes even more evident the need for faster (or parallelisable) metaheuristic approaches. These 'fast metaheuristics' will be needed as the models introduce further constraints to account for real-life circumstances, and as the real-time factor that is required in most of the decision-making processes will become even more relevant.

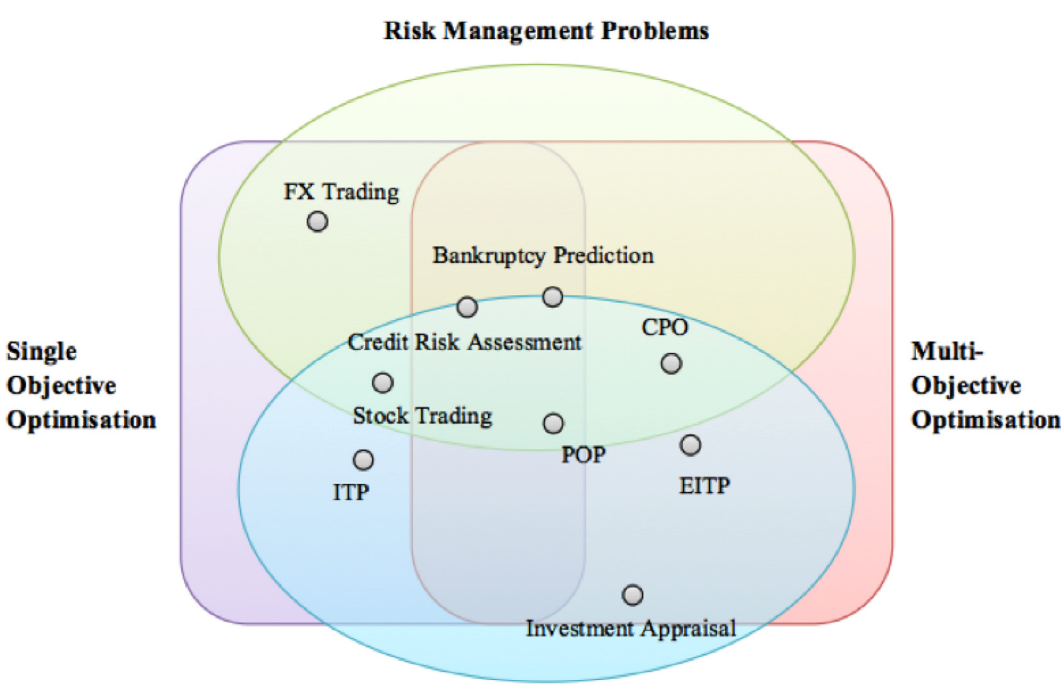

Fig. 5. A unified classification of portfolio optimisation and risk management

Strongly related to this point, distributed and parallel computing techniques can be employed to accelerate the 'wall-clock times' necessary to obtain near-optimal solutions to large-scale problems [83]. Furthermore, the fact that two or more objectives have to be considered simultaneously to account for the complexity has shown to increase the employment of multi-objective optimisation techniques.

Another clear trend that can be derived from the previous analysis of the literature is the predominance in the use of population-based metaheuristics over single-point metaheuristics. It is our opinion that no family of metaheuristics are shown to be superior in performance (regarding both quality of solutions as well as computing times) to another. At least, we have not found any scientific evidence that supports that claim. Thus, a lot of research can be done yet regarding the use of single-point metaheuristics (other than SA and TS) in solving both rich variants of portfolio optimisation problems and risk management optimisation problems. Related to this, it is possible to observe in the reviewed literature a clear trend to develop hybrid algorithms, which combine different types of metaheuristics as well as metaheuristics with machine learning and statistical techniques. While hybridisation can be an effective strategy to solve complex problems, it might also add some degree of additional complexity to the solving algorithms. This, in turn, might make them more difficult to be clearly understood, correctly implemented, and applied in practical scenarios. Adding complexity to algorithms –e.g., in the form of additional parameters that require fine tuning– also makes it difficult to reproduce their experimental results by independent researchers. For those reasons, it is our opinion that only in cases in which significant improvements in performance are obtained (both in solutions quality and computing times), is the hybridisation of algorithms justified. As it has happened before in other application fields, we expect the emergence of relatively easy to implement and understand metaheuristics (either single-solution or population-based ones with a few number of parameters) that can be competitive and flexible (i.e., adaptable to different variants of the problem without much effort).

With regards to data, recent research has shown a trend to employ different risk measures to more accurately depict the characteristics of the underlying data. This is also a particularly interesting research area as further stakeholders of financial optimisations (e.g. SMEs) do not provide traditional optimisation inputs (financial data) and thus alternative measurements of risk are promising. Hybridisations of simulation, machine learning, and optimisation have recently been developed and gained popularity in the application to stochastic and dynamic combinatorial optimisation problems in different application areas [17,84]. Sill, the above finance-related problems have not yet been extensively addressed by simheuristics and learnheuristics, even though financial data is characterised by macro- as well as firm-level uncertainty and dynamism. It can thus be expected that this research line is promising, with respect to both, the design of new problems at the interface of the two main research areas that have been treated separately in the literature but are fairly interrelated and the application of simheuristics to established COPs that previously neglected stochastic uncertainty on the one hand and the newly formulated COPs on the other.

8. Conclusions

In this paper we have reviewed the state of the art regarding the use of metaheuristics in the fields of portfolio optimisation problems and risk management problems. From this review, several conclusions can be extracted: *(i)* the number of related publications has been increasing during the last decade; *(ii)* population-based metaheuristics, and in particular GA and PSO, have been the predominant solving methodologies; *(iii)* regarding single-solution metaheuristics, TS and SA have been extensively applied too; *(iv)* there is not a 'single winner' approach, meaning that different metaheuristic implementations have provided results of comparable quality to different problems, which is consistent with the work of Wolpert et al. [184]; *(v)* there is a clear trend in promoting the development of hybrid algorithms, either by combining different metaheuristics or by combining metaheuristics with machine/statistical learning techniques; *(vi)* most portfolio optimisation problems include some kind of risk management and, in the other direction, most risk management problems considering optimisation issues can be modelled as enriched variants of portfolio optimisation problems; and *(vii)* there is a lack of work considering stochastic versions of the optimisation problems (i.e., random variables modelling inputs such as return rates, or risk measurements, but also uncertainty in the objective function, or constraints).

From this analysis of the state of the art, we foresee different open research lines that need to be fully explored by researchers and practitioners, among others: *(i)* methodological knowledge transfer between the more studied portfolio optimisation problem and risk management optimisation problems; *(ii)* development of easy-to-implement and fast metaheuristics (e.g., variable neighbourhood search, iterated local search, etc.) that require few parameters and perform similar to more complex ones; *(iii)* hybridisation of metaheuristics with machine/statistical learning methods ('learnheuristics') to account for dynamic behaviour in time-evolving systems; *(iv)* hybridisation of metaheuristics with simulation ('simheuristics') to account for uncertainty; and *(v)* use of distributed and parallel computing paradigms to allow for 'real-time' solutions in complex decision-making processes.

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