Fuzzy Risk Analysis Under Influence of Non-Homogeneous Preferences Elicitation in Fiber Industry

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Abstract. Fuzzy risk analysis plays an important role in mitigating the levels of harm of a risk. In real world scenarios, it is a big challenge for risk analysts to make a proper and comprehensive decision when coping with risks that are incomplete, vague and fuzzy. Many established fuzzy risk analysis approaches do not have the flexibility to deal with knowledge in the form of preferences elicitation which lead to incorrect risk decision. The inefficiency is reflected when they consider only risk analyst preferences elicitation that is partially known. Nonetheless, the preferences elicited by the risk analyst are often non-homogeneous in nature such that they can be completely known, completely unknown, partially known and partially unknown. In this case, established fuzzy risk analysis methods are considered as inefficient in handling risk, hence an appropriate fuzzy risk analysis method that can deal with the non-homogeneous nature of risk analyst's preferences elicitation is worth developing. Therefore, this paper proposes a novel fuzzy risk analysis method that is capable to deal with the non-homogeneous risk analyst's preferences elicitation based on grey numbers. The proposed method aims at resolving the uncertain interactions between homogeneous and non-homogeneous natures of risk analyst's preferences elicitation by using a novel consensus reaching approach that involves transformation of grey numbers into grey parametric fuzzy numbers. Later on, a novel fuzzy risk assessment score approach is presented to correctly evaluate and distinguish the levels of harm of the risks faced, such that these evaluations are consistent with preferences elicitation of the risk analyst. A real world risk analysis problem in fiber industry is then carried out to demonstrate the novelty, validity and feasibility of the proposed method.

Keywords: Fuzzy risk analysis; Grey numbers; Non-homogeneous preferences elicitation; Fiber industry.

1 Introduction

Incomplete and vague real-world information has often been characterised by decision makers' preferences elicitation (or behaviour) over a specific decision making situation (Niwas & Garg, 2018). In decision making on fuzzy risk analysis, the risk analysts' preferences elicitation are usually described in the form of linear or nonlinear membership function values (Garg, 2016; Garg, 2018). The membership function value plays a crucial role where it explicitly and effectively represents subjective and fuzzy preferences elicited by the risk analysts, so that more informed and better decisions are reached.

In order to define a formal basis with risk analysts' preferences elicitation using membership function value, several established concepts that are concerned with this matter are introduced. Among others are type-1 fuzzy sets, rough sets and type-2 fuzzy sets. Type-1 fuzzy sets represent the risk faced using membership function that is monotonic increasing and decreasing (Derelli, 2011). Rough sets on the other hand, express the membership function value of the risk faced using rough membership function (Du & Hu, 2017). While, type-2 fuzzy sets (Jana & Ghosh, 2018) define the membership function value of the risk faced using another fuzzy set which includes the Footprint of Uncertainty (Wallsten & Budescu, 1995; Yaakob et al., 2015). Unfortunately, the aforementioned established concepts have weaknesses. The type-1 fuzzy sets are insufficient to model perception as they are unable to cope with the increasing level of imprecision when preferences elicitation is used on a decision situation (John & Coupland, 2009). Meanwhile, the rough set representations are said to be incomplete because there are some well-defined values that belong to the considered risk situations are missing and not defined. At the same time, type-2 fuzzy sets are unable to clarify the incorporation of one fuzzy set with another fuzzy set (Yang & John, 2012) in fuzzy risk analysis due to the fact that the membership value of the considered risk needs a representation that can express both possible values of type-2 fuzzy sets. More importantly, this value is a single value as defined in fuzzy sets.

Apart from the aforementioned limitations, there is another major weakness of the established concepts that is worth noting. It is the flexibility towards risk analysts' preferences elicitation. As far as the literature on fuzzy risk analysis problems is concerned, the preferences elicited by the risk analysts are not restricted to be homogeneous or similar in nature. This is because the preferences elicited by risk analysts can also be non-homogeneous. This matter has been thoroughly discussed in Yang & John (2012), where the nature of preferences elicited by decision makers can be homogeneous, non-homogeneous and both simultaneously. As in the real world risk analysis scenarios, the preferences elicited by a risk analyst are often depending on the actual knowledge of the risk analyst, whether the risk is completely known, completely unknown, partially known or partially unknown. The variations of knowledge expressed by the risk analyst indicate that preferences elicited by the risk analyst are actually non-homogeneous in nature. Nonetheless, this matter has not received high attention and consideration by the established fuzzy risk analysis methods when they only take into account the partially known risk analyst's preferences elicitation in their risk evaluations. This situation indicates that established fuzzy risk analysis methods are in need of better incorporation of flexible methodology into modelling complex decision making in fuzzy risk analysis processes. Furthermore, the presence of non-homogeneous preferences elicitation along with the homogeneous ones makes the established fuzzy risk analysis methods to be having low transparency level, and therefore unable to track the performance of a risk problem. The inefficiencies mentioned above justify the motivation for this study.

In the literature, there is a concept called grey number. This concept has successfully served as an alternative methodology that complements the uncertainty in systems with partial information (Deng, 1989; Liu et al., 2000; Lin et al., 2004; Liu & Lin, 2006). Grey number aims at redefining the membership or characteristic function value that is unclear in traditional crisp sets and fuzzy sets (Liu et al., 2000; Deng, 1982). A grey number is defined as a number with an unknown position within clear lower and upper boundaries (Liu et al., 2000; Deng, 1982). Based on the wide applications of grey numbers, such as (Haq & Kannan, 2007) in supply chain management model, forecasting (Lin & Lee, 2007), software effort estimation model (Huang et al., 2008), grey-TOPSIS in subcontractor selection (Lin et al., 2008) and contractor's selection (Zavadskas et al., 2009), grey numbers can be expressed in various forms namely grey numbers, white numbers and black numbers. The multiple forms of grey numbers as indicated in the aforementioned applications signify that grey numbers are actually non-homogeneous in nature. However, the actual capability of grey numbers that acknowledge the presence of non-homogeneous nature in decision makers' preferences elicitation have not thoroughly been discussed in the literature. Most of the case studies covered in the literature focus only on one form of grey numbers.

The main reason grey number receives limited attentions from the researchers is due to the fact that grey numbers and intervals shared some common aspects (Deschrijver & Kerre, 2003). In fact, interval-valued fuzzy sets conceptually solve the issue related to decision makers' preferences elicitation in the case of fuzzy sets. Nevertheless, this understanding is a misconception, as grey numbers have special features which intervals do not have. In addition, the interval-valued fuzzy sets concept is inconsistent with respect to the epistemic uncertainty of an interval representation. Furthermore, grey sets provide better coverage when dealing with partial information than interval-valued fuzzy sets (Yang & John, 2012). Thus, the capability of grey numbers to describe non-homogeneous preferences elicited by decision makers is different from the interval-valued fuzzy sets and worth investigating.

Since, the relationship between non-homogeneous grey numbers' forms and non-homogeneous risk analyst's preferences elicitation looks significant, hence this paper proposes for the first time a novel fuzzy risk analysis method that has the flexibility to deal with the non-homogeneous risk analyst's preferences elicitation based on grey numbers. The proposed method aims at resolving the uncertain interactions between homogeneous and non-homogeneous natures of the risk analyst's preferences elicitation by using a novel consensus reaching approach that involves transformation of grey numbers into grey parametric fuzzy numbers. Later on, a novel fuzzy risk assessment score approach is presented to correctly evaluate and distinguish the levels of harm of the risks faced, such that these evaluations are consistent with human intuition. Then, a validation of the proposed method is presented along with real world risk analysis problem in Fiber industry to demonstrate the novelty, validity and feasibility of the proposed methodology.

The rest of the paper is structured as follows. Section 2 introduces the theoretical preliminaries related to this study. Section 3 presents the proposed fuzzy risk analysis method. Section 4 covers the theoretical validation of the proposed fuzzy risk analysis method. Section 5 shows the application of the proposed fuzzy risk analysis method on real world risk analysis problem in Fiber industry. Finally, a conclusion is given in Section 6.

2 Theoretical Preliminaries

In this section, basic concepts of grey numbers, degree of greyness and parametric fuzzy numbers are reviewed. These concepts are adopted to develop the proposed fuzzy risk analysis method that is later presented in Section 3.

2.1 Grey Number

***Definition 1*:** (Deng, 1982) A grey number, G_A , is a number with clear upper and lower boundaries but has an unknown position within the boundaries. Mathematically, a grey number for the system is expressed as

$$G_A \in \left[g^-, g^+\right] = \left\{g^- \le t \le g^+\right\} \tag{1}$$

where t is information about $g^{\pm}$ while g^- and g^+ are the upper and lower limits of information t respectively.

***Definition 2*:** (Yang & John, 2012) For a set $A \subseteq U$, if its characteristic function value of each x with respect to A, $g_A^{\pm}(x)$, can be expressed with a grey number, $g_A^{\pm}(x) \in \bigcup_{i=1}^{n}\left[a_i^-, a_i^+\right] \in D[0,1]^{\pm}$, then A is a grey set, where $D[0,1]^{\pm}$ is the set of all grey numbers within the interval $[0,1]$.

In the literature on grey numbers, if the value of the characteristic function is completely known or completely unknown, then it is called as the white number or black number respectively. In other words, characteristic function value 1 refers to the element is a white numbers and 0 is a black number. Likewise, any values in $[0,1]$ are considered as the grey numbers. Without loss of generality of Yang & John (2012), the white sets, black sets and grey sets are defined as follows.

***Definition 3*:** (White Sets).

For a set $A \subseteq U$, if its characteristic function value of each x with respect to A, $g_{A_i}^{\pm}(x), i = 1, 2, ..., n$, can be expressed with a white number, then A is a white set.

***Definition 4*:** (Black Sets)

For a set $A \subseteq U$, if its characteristic function value of each x with respect to A, $g_{A_i}^{\pm}(x), i = 1, 2, ..., n$, can be expressed with a black number, then A is a black set.

***Definition 5*:** (Grey Sets)

For a set $A \subseteq U$, if its characteristic function value of each x with respect to A, $g_{A_i}^{\pm}(x), i = 1, 2, ..., n$, can be expressed with a grey number, then A is a grey set.

The following Table 1 presents comparison between white number, black number and grey number.

Table 1. Comparison between white number, black number and grey number.

Number	Description	Value Form
0	Black Number	Numerical
[0 , 1]	Grey Number	Interval
1	White Number	Numerical

2.2 Degree of Greyness

In Yang & John (2012), methods to determine the degree of greyness of an element and a set are introduced. These methods are crucial to measure the significance of interval to the unknown number represented by a grey number.

***Definition 6*:** (Degree of greyness of element) (Yang & John, 2012)

Let U be the finite universe of discourse, x be an element and $x \in U$. For a grey set $A \subseteq U$, the characteristic function value of x with respect to A is $g_A^{\pm}(x) \in D[0,1]^{\pm}$.

Then, the degree of greyness, $g_A^{o}(x)$, of element x for set A can be expressed as

$$g_A^{o}(x) = \left| g^{+} - g^{-} \right| \tag{2}$$

***Definition 7*:** (Degree of greyness of a set) (Yang & John, 2012)

Let U be the finite universe of discourse, A be a grey set and $A \subseteq U$. x_i is element relevant to A and $x_i \in U$ $i = 1, 2, .., n$ and n is the cardinality of U.

Then, the degree of greyness of set A, g_A^{*}, can be defined as

$$g_A^{*} = \frac{\sum_{i=1}^{n} g_A^{o}(x_i)}{n} \tag{3}$$

It is worth pointing out here that equation (3) can be expressed in term of fuzzy set expression (Yang & John, 2012), given by

$$A = g_A^{\pm}(x_1)/x_1 + g_A^{\pm}(x_2)/x_2 + \ldots + g_A^{\pm}(x_n)/x_n \tag{4}$$

2.3 Parametric Fuzzy Number

The parametric fuzzy number is introduced as an extension of fuzzy number (Ma et al., 1999). It represents information in the combined-form of left fuzziness and right fuzziness that can be defined as the following triangular and trapezoidal parametric fuzzy numbers.

***Definition 8*:** (Ma et al., 1999) A trapezoidal parametric fuzzy number A is represented by the following equation (5) given by

$$\mu_A(x) = (x_A, y_A, \sigma_A, \beta_A) = \begin{cases} \dfrac{x - x_A + \sigma_A}{\sigma_A} & if \quad x_A - \sigma_A \le x \le \sigma_A \\ 1 & if \quad x_A \le x \le y_A \\ \dfrac{y_A + \beta_A - x}{\beta_A} & if \quad y_A \le x \le y_A + \beta_A \\ 0 & otherwise \end{cases} \tag{5}$$

where $A = [x_A , y_A]$ with σ_A and β_A represent the left fuzziness and right fuzziness respectively.

If $x_A = y_A$, then A is a triangular parametric fuzzy number given as the following definition.

***Definition 9*:** (Ma et al., 1999) A triangular parametric fuzzy number A is represented by the following equation (6) given by

$$\mu_A(x) = (x_A, \sigma_A, \beta_A) = \begin{cases} \dfrac{x - x_A + \sigma_A}{\sigma_A} & if \quad x_A - \sigma_A \le x \le \sigma_A \\ \dfrac{x_A + \beta_A - x}{\beta_A} & if \quad x_A \le x \le x_A + \beta_A \\ 0 & otherwise \end{cases} \tag{6}$$

3 Method Formulation

As mentioned in the introduction section, the presence of non-homogeneous nature of risk analyst's preferences elicitation in fuzzy risk analysis is crucial and worth acknowledging. This study aims at dealing with the complex interaction of the non-homogeneous nature of risk analysts' preferences elicitation in fuzzy risk analysis using grey numbers. Grey numbers are more flexible than type-1 fuzzy sets, rough sets and interval-valued fuzzy sets in managing the non-homogeneous nature of risk analyst's preferences elicitation. This is because their value forms (i.e. numerical value form and interval value form) allow the non-homogeneous risk analyst's preferences elicitation that are completely known, completely unknown, partially known and partially unknown, to be consistently represented. Therefore, a novel fuzzy risk analysis method that is developed from the grey number perspective and structure of fuzzy risk analysis (Jana & Ghosh, 2018; Du & Hu, 2017; Sen et al., 2016) is proposed for the first time here. The proposed method first resolves the uncertain interactions between homogeneous and non-homogeneous natures of risk analyst's preferences elicitation by using a novel consensus reaching approach that transforms grey number forms into grey parametric fuzzy numbers. Later on, a novel fuzzy risk assessment score approach is presented to correctly evaluate and distinguish the levels of harm of the risks faced, such that these evaluations are consistent with human intuition. It is worth mentioning that the structure of fuzzy risk analysis is given as Figure 1. Since, the grey parametric fuzzy number is introduced for the first time here and in the literature, its definition is given as follows.

Definition 10: A grey parametric fuzzy number, A_g, given as $A_g = (x_{A_g}, y_{A_g}, \sigma_{A_g}, \beta_{A_g}; h_{A_g})$ with $(A_g)_1 = (x_{A_g}, y_{A_g})$, σ_{A_g} as the left fuzziness, β_{A_g} as the right fuzziness and $h_{A_g} \in [0,1]$ as the height of grey parametric fuzzy number. The representation of A_g is given as follows.

$$\mu_{A_g}(x) = (x_{A_g}, y_{A_g}, \sigma_{A_g}, \beta_{A_g}; h_{A_g}) = \begin{cases} \dfrac{x - x_{A_g} + \sigma_{A_g}}{\sigma_{A_g}} & if \quad x_{A_g} - \sigma_{A_g} \le x \le \sigma_{A_g} \\ h_{A_g} & if \quad x_{A_g} \le x \le y_{A_g} \\ \dfrac{y_{A_g} + \beta_{A_g} - x}{\beta_{A_g}} & if \quad y_{A_g} \le x \le y_{A_g} + \beta_{A_g} \\ 0 & otherwise \end{cases}$$

It is worth noting here that, the representation of grey parametric fuzzy number given above is consistent with Definitions 8-9. The introduction of height, $h_{A_g} \in [0,1]$ in Definition 10, complements the confidence level of the risk analyst on the risk faced (Bakar & Gegov, 2014; Bakar & Gegov, 2015; Chutia & Gogoi, 2017).

Without loss of generality of the mentioned structure and grey numbers' forms, details on the proposed fuzzy risk analysis method are given as follows.

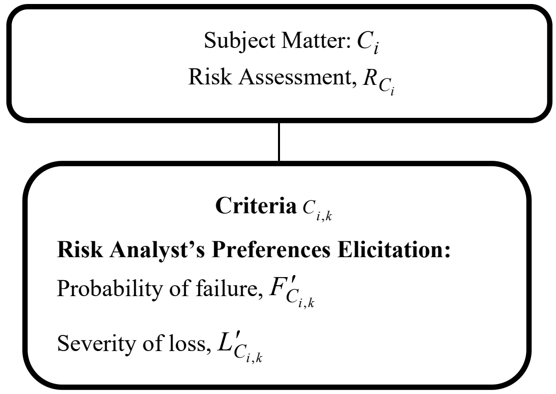

Fig. 1: Structure of Fuzzy Risk Analysis in the presence of grey numbers.

Step 1: Transform all preferences elicited by risk analyst for $F'_{C_{i,k}}$ and $L'_{C_{i,k}}$ (in the form of grey numbers), into grey parametric fuzzy numbers, $F^*_{C_{i,k}}$ and $L^*_{C_{i,k}}$ respectively.

Based on Definition 3-5 and Table 1, the following procedures are applied.

1) If $F'_{C_{i,k}} \in [0,1]$ and $L'_{C_{i,k}} \in [0,1]$ are numerical values, then $F'_{C_{i,k}}$ and $L'_{C_{i,k}}$ are transformed into grey parametric fuzzy numbers, $F^*_{C_{i,k}}$ and $L^*_{C_{i,k}}$ respectively using the transformation function, Q_m , $m = F'_{C_{i,k}}$, $L'_{C_{i,k}}$ given as follows.

$$Q_{F'_{C_{i,k}}} : F'_{C_{i,k}} \to F^*_{C_{i,k}}$$

and

$$Q_{L'_{C_{i,k}}} : L'_{C_{i,k}} \to L^*_{C_{i,k}}$$

2) If $F'_{C_{i,k}} \in [0,1]$ and $L'_{C_{i,k}} \in [0,1]$ are interval values, then $F'_{C_{i,k}}$ and $L'_{C_{i,k}}$ are transformed into grey parametric fuzzy numbers, $F^*_{C_{i,k}}$ and $L^*_{C_{i,k}}$ respectively using the transformation function, Q_n , $n = F'_{C_{i,k}}$, $L'_{C_{i,k}}$, given as follows.

$$Q_{F'_{C_{i,k}}} : [0,1] \to F^*_{C_{i,k}}$$

and

$$Q_{L'_{C_{i,k}}} : [0,1] \to L^*_{C_{i,k}}$$

where $i = 1, 2, ..., n$, $k = 1, 2, ..., n$.

Hence, both $F^*_{C_{i,k}}$ and $L^*_{C_{i,k}}$ in Step 1 can be defined in the form of triangular grey parametric fuzzy numbers as $F^*_{C_{i,k}} = \left(x_{F^*_{C_{i,k}}}, \sigma_{F^*_{C_{i,k}}}, \beta_{F^*_{C_{i,k}}}; h_{F^*_{C_{i,k}}} \right)$, $L^*_{C_{i,k}} = \left(x_{L^*_{C_{i,k}}}, \sigma_{L^*_{C_{i,k}}}, \beta_{L^*_{C_{i,k}}}; h_{F^*_{C_{i,k}}} \right)$ and trapezoidal grey parametric fuzzy numbers as $F^*_{C_{i,k}} = \left(x_{F^*_{C_{i,k}}}, y_{F^*_{C_{i,k}}}, \sigma_{F^*_{C_{i,k}}}, \beta_{F^*_{C_{i,k}}}; h_{F^*_{C_{i,k}}} \right)$, $L^*_{C_{i,k}} = \left(x_{L^*_{C_{i,k}}}, y_{L^*_{C_{i,k}}}, \sigma_{L^*_{C_{i,k}}}, \beta_{L^*_{C_{i,k}}}; h_{L^*_{C_{i,k}}} \right)$ respectively.

Step 2: Compute the consensus reaching score for each subject matter C_i as

$$C_i = \frac{\sum_{k=1}^{n} \left(F^*_{C_{i,k}} \times L^*_{C_{i,k}} \right)}{\sum_{k=1}^{n} \left(L^*_{C_{i,k}} \right)} \tag{7}$$

In Step 2, equation (7) is an often used established aggregation method to assess the degree of risk. Nonetheless, since the subject matter C_i is in the form of grey parametric fuzzy number, hence the following steps are proposed and carried out.

Step 3: Calculate the horizontal component value for C_i as

$$H_{C_i} = \frac{1}{3}\left[\left(x_{C_i} - \sigma_{C_i}\right) + x_{C_i} + y_{C_i} + \left(y_{C_i} + \beta_{C_i}\right) - \frac{y_{C_i}\left(y_{C_i} + \beta_{C_i}\right) - x_{C_i}\left(x_{C_i} - \sigma_{C_i}\right)}{y_{C_i} + \left(y_{C_i} + \beta_{C_i}\right) - x_{C_i} + \left(x_{C_i} - \sigma_{C_i}\right)} \right]$$

and the vertical component value for C_i as

$$V_{C_i} = \frac{w_{C_i}}{3}\left[1 + \frac{y_{C_i}\left(y_{C_i} + \beta_{C_i}\right) - x_{C_i}\left(x_{C_i} - \sigma_{C_i}\right)}{y_{C_i} + \left(y_{C_i} + \beta_{C_i}\right) - x_{C_i} + \left(x_{C_i} - \sigma_{C_i}\right)} \right]$$

where $H_{C_i} \in [0,1]$ and $V_{C_i} \in [0,1]$.

In this step, both values of $H_{C_i} \in [0,1]$ and $V_{C_i} \in [0,1]$ represent the a centroid point (center of gravity) for each C_i under consideration. Note that, the centroid point plays the role as the mean for C_i.

Step 4: Obtain the spread value for C_i as

$$S_{C_i} = \left| \left(y_{C_i} + \beta_{C_i}\right) - \left(x_{C_i} - \sigma_{C_i}\right) \right| \times V_{C_i}$$

For this step, the spread value, S_{C_i}, represents the standard deviation for each C_i under consideration.

Step 5: Evaluate the risk assessment score value for all C_i under consideration as

$$R_{C_i} = H_{C_i} \times V_{C_i} \times \left(1 - S_{C_i}\right). \tag{8}$$

Risk assessment score value descriptions:

If $R_{C_i} > R_{C_j}$, then $C_i(x) \succ C_j(x)$
If $R_{C_i} = R_{C_j}$, then $C_i(x) \approx C_j(x)$
If $R_{C_i} < R_{C_j}$, then $C_i(x) \prec C_j(x)$

The main reason horizontal, vertical and spread components are utilised in this risk analysis methodology is because these components are often used to determine the consistency and dispersion of risk in the literature. In this case, any C_i with greater centroid point and smaller spread, is classified as more consistent and less disperse than those with smaller centroid point and higher spread.

4 Method Validation

This section theoretically validates the proposed fuzzy risk assessment method in accordance to steps involved as given in Section 3. Since, the validation is assessed from the theoretical perspective, hence both grey numbers and grey parametric fuzzy numbers utilised in this section are defined to be more generic than those in Section 3. Thus, without loss of generality of Section 3, the following validation applies.

Step 1

As transformation of grey sets into grey parametric fuzzy sets is one of the novel contributions of this study, the following property is presented. It has to be noted that, the property is consistent with Yang & John (2012).

Let U be the finite universe of discourse, A be a grey set and $A \subseteq U$. x is an element and $x \in U, g_A^{\pm}(x)$ is the characteristic function value of x with respect to A, $g_A^o(x)$ is the degree of greyness of $g_A^{\pm}(x)$ and g_A^* is the degree of greyness for A.

Property*:* A is a grey parametric fuzzy set if and only if $g_A^* = 0$ and $g_A^{\pm}(x) \in [0,1]$ for any $x \in U$

Proof 1: If A is a grey parametric fuzzy set, then $g_A^* = 0$ and $g_A^{\pm}(x) \in [0,1]$ for any $x \in U$

Let A be a grey parametric fuzzy set expressed as

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n \quad (9)$$

where $\mu_A(x)$ is the membership value for A with $\mu_A(x) \in [0,1]$.
When $\mu_A(x) = g_A^{\pm}(x) \in [0,1]$, then the following is obtained based on equation (3).

$$g_A^* = \frac{\left|(\mu_A(x_1) - \mu_A(x_1)) + (\mu_A(x_2) - \mu_A(x_2)) + \ldots + (\mu_A(x_n) - \mu_A(x_n))\right|}{n} = 0 \quad (10)$$

where $\mu_A(x) = g_A^{\pm}(x) \in [0,1]$ for any $x \in U$ (proven).

Proof 2: If $g_A^* = 0$ and $g_A^{\pm}(x) \in [0,1]$ for any $x \in U$, then A is a grey parametric fuzzy set.

Let A be a grey set expressed as

$$A = g_A^{\pm}(x_1)/x_1 + g_A^{\pm}(x_2)/x_2 + \ldots + g_A^{\pm}(x_n)/x_n$$

Based on Definition (2), $g_A^{\pm}(x_i) \in [0,1]$ where $i = 1, 2, \ldots, n$, is a single grey number. Thus, the following can be proven as

$$g_A^* = \frac{\left|\left(g_A^\pm(x_1) - g_A^\pm(x_1)\right) + \left(g_A^\pm(x_2) - g_A^\pm(x_2)\right) + \ldots + \left(g_A^\pm(x_n) - g_A^\pm(x_n)\right)\right|}{n} = 0 \,. \quad (11)$$

If $\mu_A(x) = g_A^\pm(x) \in [0,1]$, then equation (11) is defined as

$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n$ (proven).

With respect to grey numbers and grey parametric fuzzy numbers, the validation on both numerical forms are given as follows.

Let G_A and μ_A be the grey number and membership value for grey parametric fuzzy number *A* respectively, where $G_A \in D[0\,,1]^\pm$ and $\mu_A \in [0\,,1]$.

1) $g_A^\pm \in [0,1]$ is a numerical value.

Property 1: If $G_A = \mu_A$, then $\mu_A : U \to D[0\,,1]^\pm$.

Proof: $G_A = \mu_A$, implies that $G_A = \mu_A \in D[0\,,1]^\pm$
hence, $\mu_A : U \to D[0\,,1]^\pm$ (proven). (12)

It is worth noting here that, equation (12) is consistent with equation (8) to equation (10).

2) $g_A^\pm \in [0,1]$ is an interval value

Property 2: If membership interval, $t = \left[g^-, g^+\right]$, then $\mu : U \to D[0\,,1]^\pm$.

Proof: $t = \left[g_A^-, g_A^+\right]$ implies that $t \in D[0\,,1]^\pm$
For continuous grey numbers, $G_A \in t$, any unknown value of G_A within t indicates that $G_A \in D[0\,,1]^\pm$. Thus, when $G_A = \mu_A$

then $\mu_A : U \to D[0\,,1]^\pm$ (proven).

<u>Step 2</u>

No validation is required for this step as the aggregation technique is adopted from the literature.

<u>Step 3-5</u>

As Step 3-5 are interrelated, the validation for these steps are conducted concurrently. This validation consists of properties that distinguish a grey parametric fuzzy number with other grey parametric fuzzy numbers under consideration. Thus, without loss of generality of Wang & Kerre (2001a, b) and Bakar & Gegov (2014, 2015), the validation process is as follows.
Let G_A and G_B be any grey parametric fuzzy numbers.

Property 1: If $G_A \succcurlyeq G_B$ and $G_B \succcurlyeq G_A$, then $G_A \approx G_B$.

Proof:

$G_A \succcurlyeq G_B$ implies that $R_{G_A} \geq R_{G_B}$ and $G_B \succcurlyeq G_A$ implies that $R_{G_B} \geq R_{G_A}$, thus $R_{G_B} = R_{G_A}$ which is $G_A \approx G_B$.

Property 2: If $G_A \succcurlyeq G_B$ and $G_B \succcurlyeq G_C$, then $G_A \succcurlyeq G_C$.

Proof:

$G_A \succcurlyeq G_B$ implies that $R_{G_A} \geq R_{G_B}$ and $G_B \succcurlyeq G_C$ implies that $R_{G_B} \geq R_{G_C}$, thus $R_{G_A} = R_{G_C}$ which is $G_A \succcurlyeq G_C$.

Property 3: If $G_A \cap G_B = \phi$ and G_A is on the right side of G_B, then $G_A \succcurlyeq G_B$.

Proof:

$G_A \cap G_B = \phi$ and G_A is on the right side of G_B implies that $R_{G_A} \geq R_{G_B}$, thus $G_A \succcurlyeq G_B$.

Property 4: The order of G_A and G_B are not affected by other grey parametric fuzzy numbers under comparison.

Proof:

The ordering of G_A and G_B are completely determined by R_{G_A} and R_{G_B} respectively, thus the ordering of G_A and G_B are not affected by other grey parametric fuzzy numbers under comparison.

5 Fuzzy Risk Analysis in Fiber Industry

In order to illustrate the applicability and validity of the proposed fuzzy risk analysis method in a realistic scenario, this study experiments the initial step in the process of vehicles' dashboard production, which is the risk assessment on the fibers' mechanical properties. In this step, the risk analyst is responsible in ensuring that the fibers used are not risky in nature such that the fibers used complement the vehicles' dashboards production so that high level of durability and quality of dashboards are produced (Montignies et al., 2010; Mantovani et al., 2017).

In the automotive sector, the durability and quality elements of a dashboard are important for a vehicle because they concern with the drivers and passengers safety. This purpose makes the risk assessment on fibers' mechanical properties in the vehicles' dashboard production is significant and challenging. This is because fibers comprise of three mechanical properties namely density, elongation at break and tensile strength (Trabelsi et al., 2018; Sarfarazi et al., 2018), where all of them contribute significantly towards the vehicles' dashboards production through the following explanations.

1. Density – low density fibers indicate that the vehicles' dashboards produced are not heavy, thus low processing costs are incurred.
2. Elongation at break – high elongation at break fibers indicate that the vehicles' dashboards produced are flexible.
3. Tensile strength – fibers with high tensile strength, high tensile modulus and high elongation at break indicate that the vehicles' dashboards produced are good and tough.

Based on the details given, the structure of risk assessment on fibers' mechanical properties in the vehicles' dashboard production is illustrated as Figure 2. It is worth mentioning here that fibers, C_i, under consideration for the risk assessment are Asbestos (mineral-based fiber) as C_1, Mohair (animal-based fiber) as C_2 and Hemp (plant-based fiber) as C_3. All of these fibers are examined on their mechanical properties based on two risk assessment criteria namely probability of failure and severity of loss (Jana & Ghosh, 2018; Du & Hu, 2017; Sen et al., 2016; Liang et al., 2019).

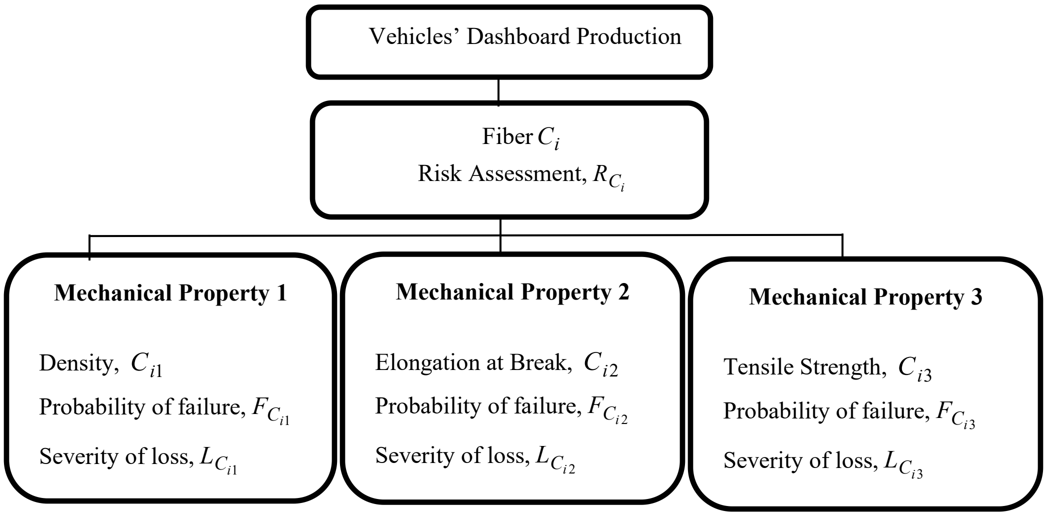

Fig. 2. Structure of risk assessment on fibers' mechanical properties in the vehicles' dashboard production process.

Although, the criteria considered in this risk assessment are consistent with established research works, the examinations are not easy to experiment as descriptions in Figure 2 are conveyed in the form of preferences elicited by the risk analyst as tabulated in Table 3. It is worth mentioning here that all information given in Table 3 is the actual risk analyst's preferences elicitation based on the nature of each fiber under consideration from the material-based perspective, given as Table 2.

Table 2. Actual numerical value on each fiber mechanical properties under consideration.

Fiber	C_{i1} (gcm^{-3})	C_{i2} (%)	C_{i3} (MPa)
C_1	1.45	1.43	620-850
C_2	1.49	0.35	70-230
C_3	1.48	1.60	550-900

Table 3. Risk analyst's preferences elicitation on fibers' mechanical properties (actual).

Fiber	Mechanical Property	Preferences Elicitation (Actual)	
		Severity of Loss	**Probability of Failure**
	C_{11}	$L_{C_{11}} = [0.10, 0.18]$	$F_{C_{11}} = 0.18$
C_1 (Asbestos)	C_{12}	$L_{C_{12}} = [0.63, 0.80]$	$F_{C_{12}} = [0.41, 0.58]$
	C_{13}	$L_{C_{13}} = 0.07$	$F_{C_{13}} = [0.63, 0.80]$
	C_{21}	$L_{C_{21}} = 0.10$	$F_{C_{21}} = 0.98$
C_2 (Mohair)	C_{22}	$L_{C_{22}} = [0.63, 0.80]$	$F_{C_{22}} = [0.63, 0.80]$
	C_{23}	$L_{C_{23}} = 0.02$	$F_{C_{23}} = [0.58, 0.65]$
	C_{31}	$L_{C_{31}} = [0.10, 0.18]$	$F_{C_{31}} = [0.17, 0.22]$
C_3 (Hemp)	C_{32}	$L_{C_{32}} = 0.63$	$F_{C_{32}} = [0.78, 0.92]$
	C_{33}	$L_{C_{33}} = 0.07$	$F_{C_{33}} = [0.63, 0.80]$

In order to apply the proposed fuzzy risk analysis method, all information related to risk analyst's preferences elicitation in Table 3 is expressed in the form of grey numbers, as given in the following Table 4.

Table 4. Risk analyst's preferences elicitation on fibers' mechanical properties in the form of grey numbers.

Fiber	Mechanical Property	Preferences Elicitation (Grey Number)	
		Severity of Loss	**Probability of Failure**
	C_{11}	$L'_{C_{11}} = [0.10, 0.18]$	$F'_{C_{11}} = 0.18$
C_1 (Asbestos)	C_{12}	$L'_{C_{12}} = [0.63, 0.80]$	$F'_{C_{12}} = [\ 0.41, 0.58]$
	C_{13}	$L'_{C_{13}} = 0.07$	$F'_{C_{13}} = [0.63, 0.80]$
	C_{21}	$L'_{C_{21}} = 0.10$	$F'_{C_{21}} = 0.98$
C_2 (Mohair)	C_{22}	$L'_{C_{22}} = [0.63, 0.80]$	$F'_{C_{22}} = [0.63, 0.80]$
	C_{23}	$L'_{C_{23}} = 0.02$	$F'_{C_{23}} = [0.58, 0.65]$
	C_{31}	$L'_{C_{31}} = [0.10, 0.18]$	$F'_{C_{31}} = [0.17, 0.22]$
C_3 (Hemp)	C_{32}	$L'_{C_{32}} = 0.63$	$F'_{C_{32}} = [0.78, 0.92]$
	C_{33}	$L'_{C_{33}} = 0.07$	$F'_{C_{33}} = [0.63, 0.80]$

It is worth noting here that, all preferences elicited by the risk analyst in Table 2 on both severity of loss, $L_{C_{ij}}$ and probability of failure, $F_{C_{ij}}$, $i, j = 1, 2, 3$, are expressed in the form of grey numbers, $L'_{C_{ij}}$ and $F'_{C_{ij}}$, $i, j = 1, 2, 3$ respectively as in Table 3. The indifference in terms of the actual preferences elicited by the risk analyst in Table 3 and their respective grey numbers' representations in Table 4, indicates that grey numbers are capable to give correct representations to the risk assessment criteria such that the representations are consistent with the actual preferences elicited by the risk analyst. Thus, without loss of generality of the proposed fuzzy risk analysis method, the risk assessments on fibers' mechanical properties in the vehicles' dashboards production are as follows.

Step 1:

The transformation of each criterion for the risk assessment into grey parametric fuzzy number, defined as Definition 10, is given in Table 5, so that a consensus form for the risk analyst's preferences elicitation in Table 4 are achieved.

Table 5. Descriptions of risk analyst's preferences elicitation on fibers in the form of grey parametric fuzzy numbers.

Fiber	Mechanical Property	Preferences Elicitation (Grey Parametric Fuzzy Number)	
		Severity of Loss	**Probability of Failure**
	C_{11}	$L^*_{C_{11}} = (0.10, 0.18, 0.06, 0.05; 1.0)$	$F^*_{C_{11}} = (0.10, 0.18, 0.06, 0.05; 0.9)$
C_1	C_{12}	$L^*_{C_{12}} = (0.63, 0.80, 0.05, 0.06; 1.0)$	$F^*_{C_{12}} = (0.41, 0.58, 0.09, 0.07; 0.7)$
	C_{13}	$L^*_{C_{13}} = (0.0, 0.02, 0.00, 0.05; 1.0)$	$F^*_{C_{13}} = (0.63, 0.80, 0.05, 0.06; 0.8)$
	C_{21}	$L^*_{C_{21}} = (0.10, 0.18, 0.06, 0.051.0)$	$F^*_{C_{21}} = (0.98, 1.00, 0.05, 0; 0.85)$
C_2	C_{22}	$L^*_{C_{22}} = (0.63, 0.80, 0.05, 0.06; 1.0)$	$F^*_{C_{22}} = (0.63, 0.80, 0.05, 0.06; 0.95)$
	C_{23}	$L^*_{C_{23}} = (0.0, 0.02, 0.00, 0.05; 1.0)$	$F^*_{C_{23}} =(0.41, 0.58, 0.09, 0.07; 0.9)$
	C_{31}	$L^*_{C_{31}} = (0.10, 0.18, 0.06, 0.05; 1.0)$	$F^*_{C_{31}} = (0.22, 0.36, 0.05, 0.06; 0.95)$
C_3	C_{32}	$L^*_{C_{32}} = (0.63, 0.80, 0.05, 0.06; 1.0)$	$F^*_{C_{32}} = (0.78, 0.92, 0.06, 0.05; 0.8)$
	C_{33}	$L^*_{C_{33}} = (0.0, 0.02, 0.00, 0.05; 1.0)$	$F^*_{C_{33}} = (0.63, 0.80, 0.05, 0.06; 1.0)$

Step 2:

The consensus reaching scores for all fibers in the form of grey parametric numbers are calculated and tabulated in Table 6. The score represents the aggregated risk assessment evaluation for each respective fiber under consideration.

Table 6. Consensus reaching score on risk assessment for each fiber under consideration.

Fiber	Risk Assessment Evaluation
C_1	$C_1 = (0.17, 0.46, 0.07, 0.25; 0.7)$
C_2	$C_2 = (0.30, 0.70, 0.10, 0.30; 0.85)$
C_3	$C_3 = (0.31, 0.68, 0.09, 0.20; 0.8)$

Step 3-4:

The horizontal, vertical and spread components for all fibers are computed and presented in Table 7.

Table 7. The horizontal, vertical and spread components for all fibers under consideration.

Fiber	Component		
	Horizontal	Vertical	Spread
C_1	$H_{C_1} = 0.3975$	$V_{C_1} = 0.3135$	$S_{C_1} = 0.1592$
C_2	$H_{C_2} = 0.7266$	$V_{C_2} = 0.3810$	$S_{C_2} = 0.3357$
C_3	$H_{C_3} = 0.7029$	$V_{C_3} = 0.3498$	$S_{C_3} = 0.3811$

Step 5:

The risk assessment score for all fibers are calculated and given in Table 8.

Table 8. The risk assessment score for all fibers under consideration.

Fiber	Risk Assessment Score
C_1	$R_{C_1} = 0.1048$
C_2	$R_{C_2} = 0.1839$
C_3	$R_{C_3} = 0.1522$

From Table 8, it can be concluded that by using the novel fuzzy risk analysis method, the most risky fiber is C_2, followed by C_1 and C_3. Thus, with respect to the vehicles' dashboard production, the most suitable fiber to be used is the Hemp (plant-based fiber) because it has the lowest risk assessment score as compared to Asbestos (mineral-based fiber) and Mohair (animal-based fiber).

Analysis of Results

As to conform the proposed fuzzy risk analysis method is feasible and valid, previous results obtained under this section are validated with the combined assessments of actual numerical value on fiber mechanical properties and their influence towards vehicles' dashboard production. For this purpose, information tabulated in Table 2 is utilised as the benchmark for this analysis.

Expert Opinion

Based on Table 2, all of the fibers examined are having almost equal density values. Thus, all fibers under consideration are investigated on their durability and quality based on elongation at break and tensile strength values. According to Trabelsi et al. (2018) and Sarfarazi et al. (2018), durable fibers are those having high elongation at break and tensile strength values. In this experiment, C_2 is considered as the least durable fiber because it has the lowest elongation at break and tensile strength values as compared to C_1 and C_3. This situation indicates that C_2 is the most brittle fiber among those fibers under consideration and therefore is the least suitable material for dashboard of a vehicle.

For C_1 and C_3, both are having almost equal tensile strength value but differ in terms of elongation at break value. Based on Table 2, C_3 is having a slightly higher elongation at break value than C_1, thus C_1 is less durable than C_3. Therefore, based on these observations, the correct ranking order for each fiber under consideration in terms of suitability as vehicles' dashboard such that the ranking result is consistent with the durability perspective of a dashboard is $C_2 \prec C_1 \prec C_3$. This also concludes that the correct risk ordering for each fiber under consideration such that the ranking result is consistent with the fibers' mechanical properties and their suitability in dashboard production is $C_2 \succ C_1 \succ C_3$.

Methods Performance

Table 9 (a). Evaluation of risk assessment by risk analysis methods under consideration.

Risk Analysis Method	C_{i1}	C_{i2}	C_{i3}	**Risk Assessment**
Chutia & Gogoi (2017)	0.0512/0.1428	0.1428/0.0997	0.0997/0.0512	$C_2 \succ C_3 \succ C_1$
Liang et al. (2019)	0.2260	0.4094	0.4245	$C_3 \succ C_2 \succ C_1$
The proposed method	0.1048	0.1839	0.1522	$C_2 \succ C_1 \succ C_3$

As clarified in the previous analysis, the correct risk ordering for each fiber under consideration such that the result is consistent with the fibers' mechanical properties and their suitability in dashboard production is $C_2 \succ C_1 \succ C_3$. Based on Table 9 (a) and (b), only the proposed method obtains the correct risk ordering for all fibers under consideration such that the ranking result is consistent with the fibers' mechanical properties and their suitability in dashboard production, i.e. $C_2 \succ C_1 \succ C_3$. This is due to the fact that the proposed method evaluates fiber with the highest combined value of centroid point and spread as the most risky fiber ($C_2 \succ C_1 \succ C_3$).

For Chutia & Gogoi (2017), the method calculates risk for each fiber under consideration as $C_2 \succ C_3 \succ C_1$, where the risks are assessed based on the combination of value and ambiguity. However, the risk evaluations obtained are incorrect for this experiment when C_3 is considered to be greater than C_1, even if C_3 is having a much lower elongation at break value than C_1. In this case, combined value of centroid point and spread approach used by the proposed method is more efficient than the combined value and ambiguity approach by Chutia & Gogoi (2017) as the latter unable to give correct risk evaluation for C_3 and C_1. Thus, risk evaluation by Chutia & Gogoi (2017) method is considered to be incorrect such that the risk result is inconsistent with the fibers' mechanical properties and their suitability in dashboard production.

Meanwhile for Liang et al. (2019), the method evaluates risk for each fiber under consideration based on defuzzified value as $C_3 \succ C_2 \succ C_1$. Unlike Chutia & Gogoi (2017) and the proposed method, this method evaluates C_3 as the most risky fiber than C_2 and C_1, even if C_3 is the most durable fiber because it has the highest elongation at break value and tensile strength. Not only that, this method assesses C_2 as less risky than C_3 even if C_2 is brittle than C_3. In this respect, combined value of centroid point and spread approach used by the proposed method is more efficient than the defuzzified value approach by Liang et al. (2019). Hence, risk evaluation by Liang et al. (2019) method is also considered to be incorrect such that the risk result is inconsistent with the fibers' mechanical properties and their suitability in dashboard production. Thus, based on these empirical evaluations, the proposed method outperforms established fuzzy risk analysis methods under consideration.

Table 9 (b). Consistency Evaluation of risk assessment by risk analysis methods under consideration.

Correct Risk Order: $C_2 \succ C_1 \succ C_3$	
Risk Analysis Method	**Consistency**
Chutia & Gogoi (2017)	Inconsistent
Liang et al. (2019)	Inconsistent
The proposed method	Consistent

6. Conclusion

In this paper, a novel fuzzy risk analysis method based on grey numbers has successfully developed. The main motivation of this study is to deal with the presence of non-homogeneity in the preferences elicited by the risk analyst, which is neglected by the established fuzzy risk analysis methods. To appropriately deal with the non-homogeneous preferences elicitation by the risk analyst, this study first resolves the uncertain interactions in preferences elicited in the form of grey numbers by means of a novel consensus reaching method that transforms grey numbers into grey parametric fuzzy numbers. Later on, the transformed grey parametric fuzzy numbers are aggregated and assessed using the novel consequence steps in Section 3. All novelties presented in this paper are theoretically validated. To ensure the applicability and validity of the proposed fuzzy risk analysis method in a realistic scenario, a real world risk analysis problem in Fiber industry is conducted and resolved. With respect to the performance analysis conducted, the proposed fuzzy risk analysis method outperforms other established fuzzy risk analysis methods. The main advantage of the proposed fuzzy risk analysis method is that it not only has the capability to resolve the uncertain interactions between homogeneous and non-homogeneous natures of risk analyst's preferences elicitation, but is also efficient in giving correct risk evaluation in real world decision making problems such that the evaluation results are consistent with the preference elicitation of the risk analyst.

For future research, further investigations on computation of grey numbers from the perspective of the intuitionistics fuzzy sets (Kaur & Garg, 2018; Garg & Rani, 2018; Garg & Singh, 2018; Singh & Garg, 2018) are to be carried out. These efforts will address the incomplete and vague real-world information in a more flexible and accurate way.

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References

1. Bakar ASA, Gegov A (2014) Ranking of fuzzy numbers based on centroid point and spread. Journal of Intelligent and Fuzzy System 27(3):1179–1186.
2. Bakar ASA, Gegov A (2015) Multi-Layer Decision Making Methodology for Ranking Z-Numbers, International Journal of Computational Intelligent Systems 8(2):395–406.
3. Chutia R, Gogoi MK (2017) A new method of ranking parametric form of fuzzy numbers using value and ambiguity. Applied Soft Computing 52:1154–1168.
4. Deng J (1982) The control problems of grey systems. Systems and Control Letters.
5. Deng J (1989) Introduction to grey system theory. Journal of Grey Systems 1(1):1–24.
6. Dereli T, Baykasoglu A, Altun K, Durmusoglu A, Turksen IB (2011) Industrial application of type-2 fuzzy sets and systems: A concise review. Computers in Industry 62(2):125–137.
7. Deschrijver G, Kerre E (2003) On the relationship between some extensions of fuzzy set theory. Fuzzy Sets and Systems 133(2):227–235.
8. Du WS, Hu BQ (2017) Dominance-based rough fuzzy set approach and its application to rule induction. European Journal of Operational Research 261(2):690–703.
9. Garg H (2016) A novel approach for analyzing the reliability of series-parallel system using credibility theory and different types of intuitionistics fuzzy numbers. Journal of the Brazilian Society of Mechanical Sciences and Engineering 38(3):1441–1451.

10. Garg H (2018) Some arithmetic operations on the generalized sigmoidal fuzzy numbers and its application. Granular Computing 3(1):9–25.
11. Garg H, Rani D (2018) Some Generalized Complex Intuitionistic Fuzzy Aggregation Operators and Their Application to Multicriteria Decision-Making Process. Arabian Journal for Science and Engineering:1–20.
12. Garg H, Singh S (2018) A novel triangular interval type-2 intuitionistic fuzzy set and their aggregation operators, Iranian Journal of Fuzzy Systems, 15(5):69–93.
13. Haq N, Kannan G (2007) A hybrid normalised multi criteria decision making for the vendor selection in a supply chain model. International Journal Management and Decision Making 8(5/6):601– 622.
14. Huang SJ, Chiu NH, Chen LW (2008) Integration of the grey relational analysis with genetic algorithm for software effort estimation. European Journal of Operational Research 188(3):898–909.
15. Jana DK, Ghosh R (2018) Novel interval type-2 fuzzy logic controller for improving risk assessment model of cyber security. Journal of Information Security and Applications 40:173-182.
16. John R, Coupland S (2009) Type-2 Fuzzy Logic and the Modelling of Uncertainty in Applications. In: Bargiela A., Pedrycz W. (eds) Human-Centric Information Processing Through Granular Modelling. Studies in Computational Intelligence, vol 182. Springer, Berlin, Heidelberg.
17. Kaur G, Garg H (2018) Generalized Cubic Intuitionistic Fuzzy Aggregation Operators Using t-Norm Operations and Their Applications to Group Decision-Making Process. Arabian Journal for Science And Engineering:1–20.
18. Khuman AS, Yang Y, John R (2015) A significance measure for r-fuzzy sets. 2015 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE).
19. Khuman AS, Yang Y, John R, Liu S (2016) R-fuzzy sets and grey system theory. 2016 IEEE International Conference on Systems, Man, and Cybernetics (SMC 2016).
20. Korshidi HA, Nikfalazar S (2017) An improved similarity measure for generalized fuzzy numbers and its application to fuzzy risk analysis. Applied Soft Computing 52:478–486.
21. Liang W, Zhao G, Wu H, Dai B (2019) Risk assessment of rockburst via an extended MABAC method under fuzzy environment. Tunnelling and Underground Space Technology 83:533–544.
22. Lin Y, Chen M, Liu S (2004) Theory of grey systems: Capturing uncertainties of grey information. Kybernetes: The International Journal of Systems and Cybernetics 33:196–218.
23. Lin YH, Lee PC (2007) Novel high-precision grey forecasting model. Automation in Construction 16(6):771–777.
24. Lin YH, Lee PC, Chang TP, Ting HI (2008) Multi-attribute group decision making model under the condition of uncertain information. Automation in Construction 17(6):792–797.
25. Liu S, Gao T, Dang Y (2000) Grey systems theory and its applications. The Science Press of China, Beijing.
26. Liu S, Lin Y (2006) Grey Information Theory and Practical Applications. Springer.
27. Ma M, Friedman M, Kandal A (1999) A new fuzzy arithmetic, Fuzzy Sets and Systems 108:83–90.
28. Mantovani S, Presti IL, Cavazzoni L, Baldini A (2017) Influence of Manufacturing Constraints on the Topology Optimization of an Automotive Dashboard. Procedia Manufacturing 11:1700–1708.
29. Montignies F, Nosulenko V, Parizet E (2010) Empirical identification of perceptual criteria for customer-centred design. Focus on the sound of tapping on the dashboard when exploring a car. International Journal of Industrial Ergonomics 40(5):592–603.
30. Niwas R, Garg H (2018) An approach for analyzing the reliability and profit of an industrial system based on the cost free warranty policy. Journal of the Brazilian Society of Mechanical Sciences and Engineering 40:265.
31. Roca E, Verdaguer AJ, Villares M, Casals MR (2018) Applying network analysis to assess coastal risk planning. Ocean & Coastal Management 162:127–136.
32. Sarfarazi V, Haeri H, Ebneabbasi P, Shemirani AB, Hedayat A (2018) Determination of tensile strength of concrete using a novel apparatus. Construction and Building Materials 166:817–832.
33. Sen S, Patra K, Mondal SK (2016) Fuzzy risk analysis in familial breast cancer using a similarity measure of interval-valued fuzzy numbers. Pacific Science Review A: Natural Science and Engineering18(3):203–221.
34. Singh S, Garg H (2018) Symmetric triangular interval type-2 intuitionistic fuzzy sets with their applications in multi criteria decision making, Symmetry 10(9):401
35. Trabelsi H, Romero E, Jamei M (2018) Tensile strength during drying of remoulded and compacted clay: The role of fabric and water retention. Applied Clay Science 162:57–68.
36. Wallsten TS, Budescu DV (1995) A review of human linguistic probability processing: general principles and empirical evidence. The Knowledge Engineering Review 10(1):43–62.
37. Wang X, Kerre EE (2001a) Reasonable Properties for the Ordering of Fuzzy Quantities (I). Fuzzy Sets and Systems 118:375–385.

38. Wang X, Kerre EE (2001b) Reasonable Properties for the Ordering of Fuzzy Quantities (II). Fuzzy Sets and Systems 118:387– 405.
39. Yaakob AM., Serguieva A, Gegov A (2015) FN-TOPSIS: Fuzzy Network for Ranking Traded Equities. IEEE Transaction on Fuzzy Systems 25(2):315–332.
40. Yan F, Xu K (2018) A set pair analysis based layer of protection analysis and its application in quantitative risk assessment. Journal of Loss Prevention in the Process Industries 55:313–319.
41. Yang Y, John R (2012) Grey sets and greyness. Information Sciences 185(1):249–264.
42. Zavadskas EK, Kaklauskas A, Turskis Z, Tamošaitiene J (2009) Multi-Attribute Decision Making model by applying grey numbers. Informatica 20(2):305–320.