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Reconstructing the temporal evolution of the speed of light in a flat FRW Universe

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Abstract We present a novel method to reconstruct the temporal evolution of the speed of light $c(z)$ in a flat FRW Universe using astronomical observations. After validating our pipeline using mock datasets, we apply our method to the latest BAO and supernovae observations, and reconstruct $c(z)$ in the redshift range of $z \in [0, 1.5]$. We find no evidence of a varying speed of light, although we see some interesting features of $\Delta c(z)$, the fractional difference between $c(z)$ and c_0 (the speed of light in SI), *e.g.*, $\Delta c(z) < 0$ and $\Delta c(z) > 0$ at $0.2 \lesssim z \lesssim 0.5$ and $0.8 \lesssim z \lesssim 1.3$, respectively, although the significance of these features is currently far below statistical importance.

Key words: cosmology: cosmology — dark energy: cosmology — large scale structure

1 INTRODUCTION

In this era of precision cosmology, we are fortunately equipped with high quality observational data of various kinds, which makes it possible to test fundamental laws of the Universe. Physical laws of the Universe are built with a few "constants", including the gravitational constant G, the electron charge e, the speed of light c, and so on. Actually, assuming the constancy of these "constants" at all cosmic times and scales is a significant extrapolation of our knowledge on a rather limited temporal and spacial scales, which is subject to observational scrutiny.

Theorists including Dirac started thinking about building models for a varying G or e, well before any observational tests became feasible (Dirac 1937, Bekenstein, 1982). However, the constancy of c was much more sacred (Barrow 1999), as it is the pillar of special relativity, built by Einstein in 1905. Making c vary is much more destructive to the structure of formalisms in modern physics, than varying other constants may do. Nevertheless, varying speed of light (VSL) may solve the cosmological constant problem and build a new framework of cosmic structure formation, as an alternative to inflation. This is sufficiently attractive for theorists to develop viable VSL theories (see Magueijo 2003 for a review of recent developments in VSL). On the other hand, techniques for testing VSL observationally have been developed in parallel. Recently, a new method to constrain VSL at a single redshift using baryonic acoustic oscillations (BAO) was developed (Salzano et al. 2015), and has been applied to cosmological observations (Cao et al. 2017, Cai et al. 2016).

The method developed in (Salzano et al. 2015) is briefly summerised as follows. Given the relation between the angular diameter distance $D_A(z)$ and the Hubble function $H(z)$, the speed of light c at a specific redshift $z = z_{\rm M}$ is just a product of $D_A(z_{\rm M})$ and $H(z_{\rm M})$, *i.e.*,

$$c(z_{\rm M}) = D_A(z_{\rm M})H(z_{\rm M}) \tag{1}$$

where $z_{\rm M}$ is the peak location of $D_A(z)$, *i.e.*, $\partial D_A/\partial z|_{z=z_{\rm M}} = 0$. Note that the exact value of $z_{\rm M}$ is model-dependent, and $z_{\rm M} \sim 1.7$ for a flat ΛCDM model that is consistent with the Planck *2018* observations (Planck Collaboration et al. 2018).

This method has pros and cons. On the positive side,

- It is model-independent, simply because it does not require any theoretical modeling at all in the first place. As shown in Eq (1), it only needs the peak redshift $z_{\rm M}$, and the product of D_A and H measured at $z_{\rm M}$, which can be derived from the BAO data alone;
- It is free from degeneracy with other cosmological parameters such the fractional matter density, the equation of state of dark energy *etc.*, since what we need is essentially a product of D_A and H, which is directly observable.

However, this method has drawbacks, including,

- It is difficult to determine $z_{\rm M}$ accurately, as $D_A(z)$ is rather flat around its peak;
- For a wide range of cosmologies, $z_{\rm M}$ is as large as ~ 1.7, making it unaccessible for most current BAO observations. Even for future deep BAO surveys including Dark Energy Spectroscopic Instrument (DESI) (DESI Collaboration et al. 2016), it is challenging to get decent D_A or H measurements at such high redshifts;
- This method only constrains c at one single redshift, which is far from sufficient for tests against various VSL models.

In this work, we generalise (Salzano et al. 2015) to propose a new method for testing VSL at multiple redshifts in the framework of a flat Friedmann-Robertson-Walker (FRW) Universe. After presenting the methodology in Sec. 2, we perform validation tests using mock datasets, before applying to observational data and presenting our main result in Sec. 4. We conclude and discuss our results in Sec. 5.

2 METHODOLOGY

We start from the relation between the angular diameter distance $D_A(z)$, the Hubble function $H(z)$, and the general speed of light function $c(z)$ in a flat FRW Universe [1], *i.e.*,

$$D_A(z) = \frac{1}{1+z}\int_0^z \frac{c(z')}{H(z')}\mathrm{d}z', \tag{2}$$

Differentiating both sides with respect to redshift z yields,

$$c(z) = \chi'(z)H(z), \tag{3}$$

where χ is the comoving distance so that $\chi(z) = (1+z)D_A(z)$. Our aim is to reconstruct the entire evolution history of $c(z)$, thus we parametrise the redshift-dependence of $\chi(z)$ and $H(z)$ as follows,

$$\frac{\chi(z)}{\chi_{\rm fid}(z)} = \alpha_0\left(1+\alpha_1 x + \frac{1}{2}\alpha_2 x^2 + \frac{1}{6}\alpha_3 x^3\right), \tag{4}$$

$$\frac{H_{\rm fid}(z)}{H(z)} = \beta_0 + \beta_1 x + \frac{1}{2}\beta_2 x^2 + \frac{1}{6}\beta_3 x^3. \tag{5}$$

The variable $x \equiv \chi_{\rm fid}(z)/\chi_{\rm fid}(z_{\rm p}) - 1$ where the subscript $_{\rm fid}$ denotes the fiducial cosmology used, and $z_{\rm p}$ is the pivot redshift at which we apply the Taylor expansion. In this work, the fiducial cosmology is chosen to be a flat ΛCDM model with $\Omega_{\rm M,fid} = 0.31$.

To test against the constancy of the speed of light, we define a deviation function of c as,

$$\Delta c(z) \equiv \frac{c(z)}{c_0} - 1, \tag{6}$$

where c_0 is the speed of light in the International System of Units (SI), *i.e.*, $c_0 = 299,792,458~{\rm m~s^{-1}}$.

Combining Eqs (3), (4), (5) and (6), we have,

$$\Delta c(z) = \frac{\alpha_0\left[1+\alpha_1+(2\alpha_1+\alpha_2)\,x+\left(\frac{3}{2}\alpha_2+\frac{1}{2}\alpha_3\right)x^2\right]}{\beta_0+\beta_1 x+\frac{1}{2}\beta_2 x^2.} - 1. \tag{7}$$

Specially, at $z_{\rm p}$ where x vanishes,

$$\Delta c(z_{\rm p}) = \frac{\alpha_0(1+\alpha_1)}{\beta_0} - 1. \tag{8}$$

Although Δc does not explicitly depends on other cosmological parameters, its measurement does implicitly rely on how well we are able to model D_A and H using Eqs (4), (5) for general cosmologies. The expansion Eq (4) was actually proposed by (Zhu et al. 2015) for implementing the optimal redshift weighting method for BAO analyses, and it was shown that expansions up to the quadratic order in x can precisely recover a wide range of cosmologies at the sub-percent level in the redshift range of $z \in [0.5, 1.5]$. Note that in (Zhu et al. 2015), the speed of light was assumed to be a constant, thus $H(z)$ was derived from $\chi(z)$. In our case, as $c(z)$ is promoted to a general function of z, we have to double the number of the expansion coefficients for $H(z)$. Also, we are more ambitious on the valid redshift range for these expansions, since we plan to use all the available observations probing the background expansion of the Universe from $z=0$ to $z=2$. All these motivated us to use expansions to higher order to achieve the precision we need.

[1] Note that in non-flat Universes, the Friedmann equations get modified by the time derivative of $c(z)$ so Eq (2) does not hold (Barrow, & Magueijo 1999). In this work, we consider a flat Universe for simplicity.

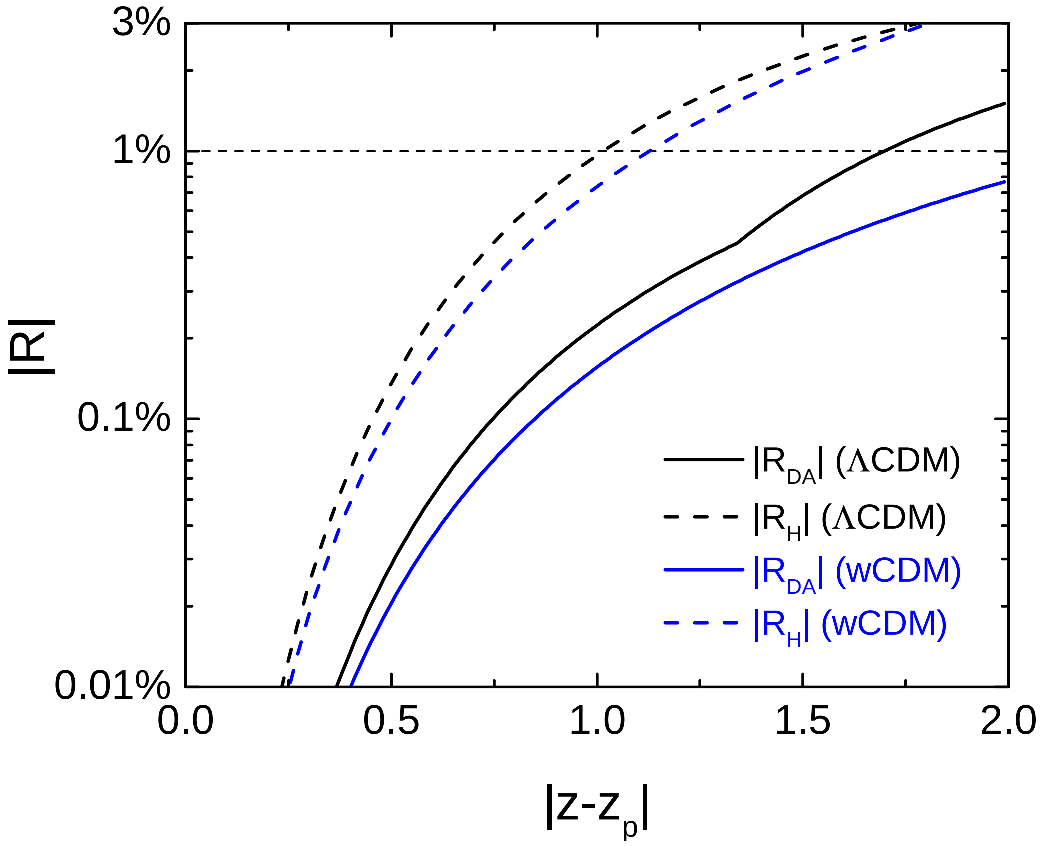

Fig. 1 The residual R defined in Eq (11) as a function of $|z - z_{\rm p}|$. As shown, the residuals are below the 1% level for $|z - z_{\rm p}| < 1$, although $H(z)$ is less accurately modeled than $D_A(z)$. For larger $|z - z_{\rm p}|$, say, $1 < |z - z_{\rm p}| < 2$, R for $D_A(z)$ can still be controlled at the per cent level, while the residual for H approaches the 4% level.

To validate our parametrisations for $\chi(z)$ and $H(z)$, we attempt to fit two cosmologies which sufficiently deviate from the fiducial cosmology we use for the expansion. Specifically,

$$\text{Cosmo. Model I}: \ \text{a flat } \Lambda\text{CDM model with } \Omega_{\rm M} = 0.2; \tag{9}$$

$$\text{Cosmo. Model II}: \ \text{a flat } w\text{CDM model with } \Omega_{\rm M} = 0.31; \ w = -0.8. \tag{10}$$

Both models are excluded by Planck *2018* observations, making them representative for "extreme" models that may be later sampled in the parameter space. For each model, we first choose a pivot redshift $z_{\rm p}$, and then tune the coefficients α's and β's to minimise the difference between our model prediction for D_A and H computed using Eqs (4), (5), and the exact quantities calculated using models I and II. Then we compute the residual R for $D_A(z)$ and $H(z)$ respectively, *i.e.*,

$$R_o(z, z_{\rm p}) \equiv 100 \times \left[\frac{O^{\rm model}(z, z_{\rm p})}{O^{\rm exact}(z, z_{\rm p})} - 1\right] \%, \tag{11}$$

where O stands for D_A or H.

The computed residuals are shown in Fig. 1 for various $|z - z_{\rm p}|$ values, and we find that our model with third-order expansions are sufficiently accurate for our analysis: the residuals in most cases are below 1%

level, although $H(z)$ is less accurately modeled than $D_A(z)$. Larger residuals appear when $|z - z_{\rm p}|$ gets larger, which is naturally expected as the accuracy of the Taylor expansion decays with the distance to the expansion point. Quantitatively, $|R| \lesssim 1\%$ for $|z - z_{\rm p}| \lesssim 1$ for both models, and $|R|$ can approach 4% for H in the worst case, *e.g.*, $|z - z_{\rm p}| = 2$, but as we will explain later, this does not affect our result, if we only take the reconstructed values at $z_{\rm p}$ (so that $|z - z_{\rm p}| = 0$).

Before applying our method to actual observations, there is another step missing: an actual mock test for various VSL models, which is presented in the next section.

3 MOCK TESTS

This section is devoted to a robustness test of our pipeline, before applying to actual observations presented in the next section.

To begin with, we choose four phenomenological VSL models for $\Delta c(z)$ to cover various possible features, including a constant, a linear function, quadratic function and an oscillatory function, *i.e.*,

- VSL model 1: $\Delta c(z) = 0.05$;
- VSL model 2: $\Delta c(z) = 0.05z$;
- VSL model 3: $\Delta c(z) = 0.05z + 0.01z^2$;
- VSL model 4: $\Delta c(z) = 0.05\sin(z\pi)$.

These VSL models are then combined with two cosmological models, respectively, as shown in Eqs (9) and (10) to make eight toy models for the mock test (see Table 1 for the labelling of the toy models).

For each toy model, we can first generate mock $H(z)$ data points using cosmological models shown in Eqs (9) and (10), and then produce mock $D_A(z)$ datasets by combining $H(z)$ and $\Delta c(z)$ using Eq (2).

In practice, we produce two kinds of mock datasets, a combined BAO data sample assuming the sensitivity of the Euclid mission (Font-Ribera et al. 2014) and DESI survey (DESI Collaboration et al. 2016), and a combined supernovae sample forecasted for LSST (LSST Science Collaboration et al. 2009) and Euclid (Astier et al. 2014).

For the BAO sample, we follow (Font-Ribera et al. 2014) and (DESI Collaboration et al. 2016) to produce D_A and H pairs at 15 and 18 effective redshifts for Euclid and DESI, respectively, so that our combined BAO sample consists of 33 pairs of D_A and H covering the redshift range of $z \in (0, 2.1)$. For the supernovae sample, we produce luminosity distances assuming the sensitivity of the Deep-Drilling Fields and low-redshift sample to be observed by LSST, and of the DESIRE supernovae survey of Euclid. The Deep-Drilling Fields will observe 8800 supernovae at $z \in [0.15, 0.95]$, and additional 8000 supernovae below redshift 0.35. This sample is complemented by the high-z sample to be collected by the DESIRE survey, which will provide 1740 supernovae at $z \in [0.75, 1.55]$.

For a given $z_{\rm p}$, these datasets can be fitted with theoretical models Eqs (4) and (5) for parameters α's and β's, using a modified version of `CosmoMC` (Lewis, & Bridle 2002). We could then use the resultant α's and β's to reconstruct $\Delta c(z)$ using Eq (7). However, this result may be subject to systematic errors in $\Delta c(z)$ at redshifts that are far away from $z_{\rm p}$, as we have seen in Fig. 1. It is true that the residual $|R|$ is as low as 3% in the worst case as discussed in Sec. 2, but it can be further improved.

Toy model	VSL model	Cosmological model
1	VSL model 1	ΛCDM; Eq (9)
2	VSL model 2	ΛCDM; Eq (9)
3	VSL model 3	ΛCDM; Eq (9)
4	VSL model 4	ΛCDM; Eq (9)
5	VSL model 1	wCDM; Eq (10)
6	VSL model 2	wCDM; Eq (10)
7	VSL model 3	wCDM; Eq (10)
8	VSL model 4	wCDM; Eq (10)

Table 1 Toy models used for the mock test.

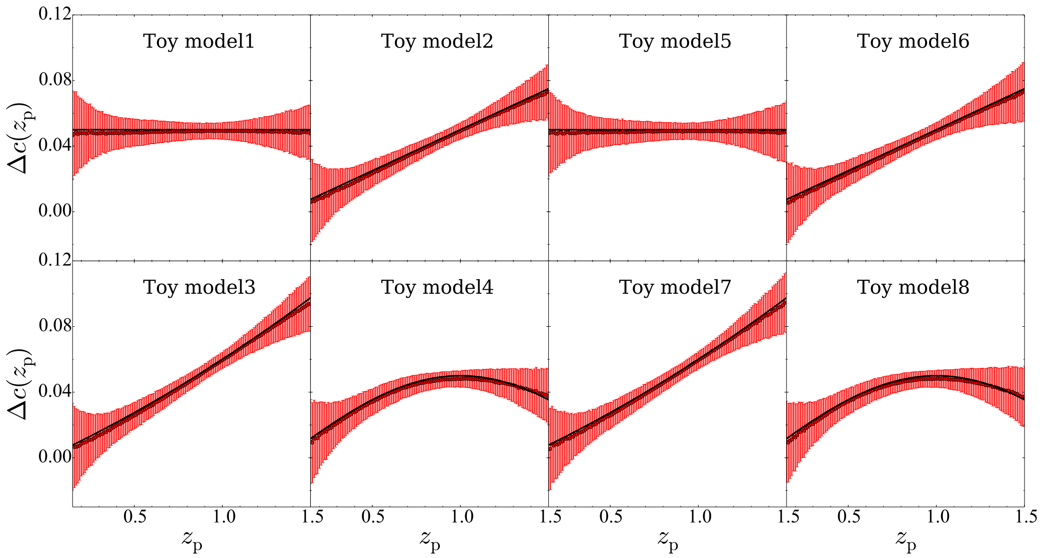

Fig. 2 The result of the mock test for eight toy models shown in Table 1. The black solid lines and the red dots show the input and reconstructed values of $\Delta c(z_{\rm p})$, respectively, and the error bars illustrate the 68% CL uncertainty.

The way out is to abandon the reconstructed $\Delta c(z)$ at all redshifts except for $z = z_{\rm p}$, where the Taylor expansion is error free, and repeat the fitting process for every $z_{\rm p}$ (with equal space of $\Delta z = 0.01$ in z) running in the entire redshift range. This is computationally expensive, and the error of $\Delta c(z)$ at different redshifts are highly correlated, but it is much more accurate.

The result of this mock test is summerised in Fig. 2. As shown, the input VSL models are reconstructed perfectly for all toy models, with negligible bias compared to the statistical error budget.

4 IMPLICATION ON THE LATEST OBSERVATIONAL DATASETS

Now it is time to apply our pipeline for measuring $\Delta c(z)$ to actual observational data. We shall first introduce datasets we use, and then present the result.

4.1 The observational dataset

The datasets used in this analysis include,

- **The BAO measurements.** As we are interested in reconstructing the time evolution of Δc, we use the tomographic BAO measurements from the Baryonic Oscillation Spectroscopic Survey (BOSS) Data Release (DR) 12 sample, which provides measurement of $D_A/r_{\rm d}$ and $Hr_{\rm d}$ pairs at nine effective redshifts in the redshift range of $z \in [0.31, 0.64]$ (Zhao et al. 2017). To approach the high-z end, we also use the tomographic BAO measurement from the extended BOSS (eBOSS) DR14 quasar sample, which offered four additional $D_A/r_{\rm d}$ and $Hr_{\rm d}$ pairs at redshifts $z \in [0.98, 1.94]$ (Zhao et al. 2019). Note that as Eq (3) shows, the speed of light is a product between H and $\mathrm{d}\chi/\mathrm{d}z$, $r_{\rm d}$ cancels out.
- **The observational $H(z)$ data (OHD).** The OHD, as 'cosmic chronometers', are measured using the ages of passively evolving galaxies, and we use a compilation of 30 data points shown listed in Table A.1.
- **The supernovae data.** We use the 'joint light-curve analysis (JLA)' sample (Betoule et al. 2014), which is a re-anaylsis of 740 data points consisting of samples from the three year SDSS-II supernovae survey (Sako et al. 2018) and the SNLS samples presented in (Conley et al. 2011).

4.2 The final result

We apply our pipeline to the actual observational data, in the same way as we performed on the mock data, and present the result in Fig. 3. Again, this is a compilation of $\Delta c(z_{\rm p})$ for numerous $z_{\rm p}$'s with equally spaced in z with $\Delta z = 0.01$.

As shown, the uncertainty of $\Delta c(z_{\rm p})$ gets minimised at $z \sim 0.2$, where the Universe is best probed by current supernovae and BAO experiments. The constraints on the low-z and high-z ends are looser, because of the small volume at low-z and the sparsity of galaxy distributions at high-z, which dilutes the BAO constraints. Furthermore, the current supernovae surveys can barely access the Universe at $z \gtrsim 1$.

The result is consistent with $\Delta c(z) = 0$ given the level of uncertainty, but interesting features show up at $0.2 \lesssim z \lesssim 0.5$ and $0.8 \lesssim z \lesssim 1.3$ where $\Delta c(z) < 0$ and $\Delta c(z) > 0$, respectively, which awaits further investigation using future observations.

We are interested in quantifying the signal-to-noise ratio of $\Delta c(z) \neq 0$, yet it is not straightforward because the error bars are highly correlated, but a covariance matrix is unavailable as we performed the parameter constraints at various $z_{\rm p}$ individually.

However, for a given $z_{\rm p}$, we are able to quantify the deviation of $\Delta c(z)$ from zero with the reconstructed $\Delta c(z)$ using Eq (7), as the covariance for the α and β parameters is available. Before proceeding, we show the reconstructed $\Delta c(z)$ in Fig. 4 using three values of $z_{\rm p}$ at $0.05, 0.2$ and 1.5, which covers a reasonable choice of $z_{\rm p}$ (0.2) and two extreme values (0.05 and 1.5). As shown, these results agree reasonably well with that shown in Fig. 3, although the result using $z_{\rm p} = 1.5$ over-estimates the uncertainty (but the central value remains accurate).

We then quantify the difference between $\Delta c(z)$ and zero using the α's and β's derived using a specific $z_{\rm p}$. Note that $\Delta c(z) = 0$ means that the numerator is identical to the denominator of the fraction in Eq (7), which translates into,

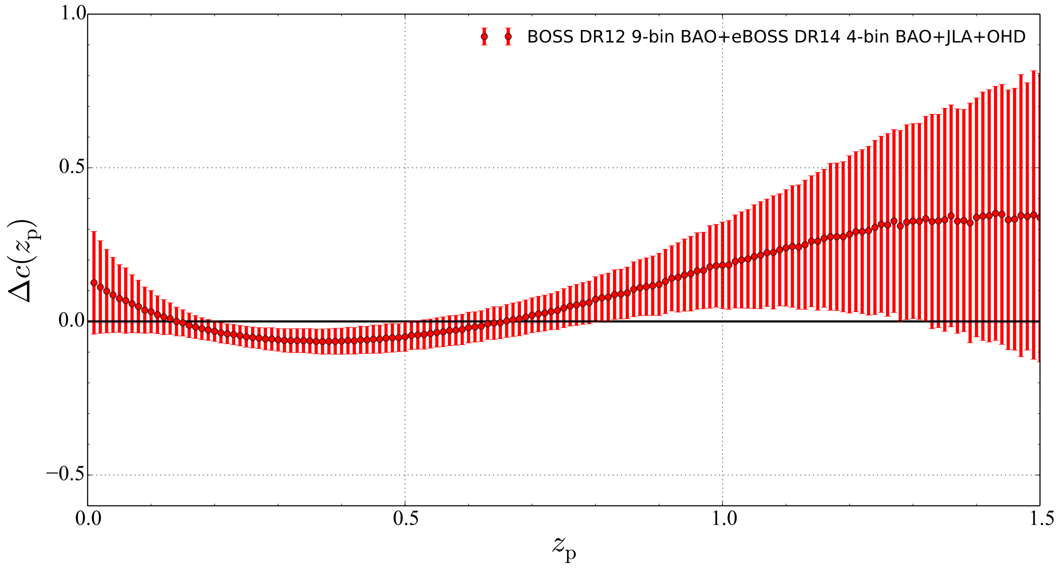

Fig. 3 A reconstruction of $\Delta c(z_\mathrm{p})$ derived from a compilation of current observations described in Sec. 4.1. The black horizontal line shows $\Delta c(z_\mathrm{p}) = 0$ to guide eyes.

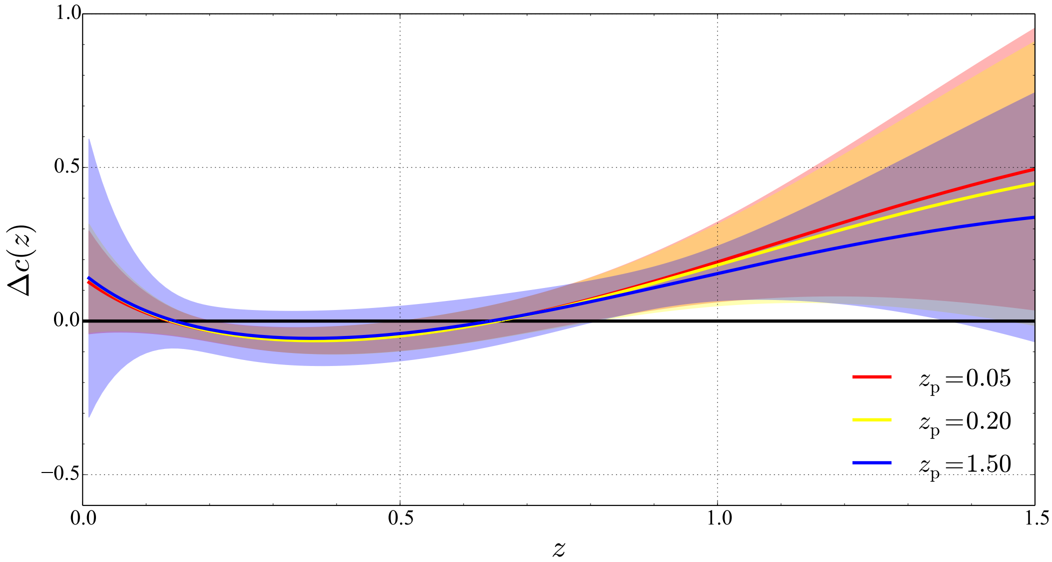

Fig. 4 The reconstructed $\Delta c(z)$ using three values of z_p, as illustrated in the legend. The shaded bands show the 68% uncertainty.

$$
\begin{aligned}
\Delta_0 &\equiv \beta_0 - \alpha_0(1+\alpha_1) = 0, \\
\Delta_1 &\equiv \beta_1 - \alpha_0(2\alpha_1+\alpha_2) = 0, \\
\Delta_2 &\equiv \beta_2 - \alpha_0(3\alpha_2+\alpha_3) = 0.
\end{aligned} \tag{12}
$$

Given the measured α's and β's and the corresponding covariance matrix, we can easily evaluate the following χ^2,

$$\chi^2_{z_{\rm p}} = \Delta^T C_\Delta \Delta \tag{13}$$

where $\Delta \equiv \{\Delta_0, \Delta_1, \Delta_2\}^T$ and C_Δ is the derived covariance matrix for vector Δ. We average over the χ^2 for all the $z_{\rm p}$'s and compute the mean and variance, which gives,

$$\chi^2 = 3.2 \pm 1.1. \tag{14}$$

which is statistically expected for fitting a fixed value, 0, to three data points ($\Delta_i, \ i = 1, 2, 3$). The error of χ^2 quantifies the fluctuation of the result using various $z_{\rm p}$, thus it can be viewed as systematics. So the conclusion is, the constant speed of light measured in SI is consistent with current cosmological observations.

5 CONCLUSION AND DISCUSSIONS

As a competitor of inflation, viable VSL theories deserve serious investigation both theoretically and observationally. The speed of light, under the assumption that c is a universal constant, was extremely well measured using the laser interferometry in 1972 (Evenson et al. 1972), but it is a significant extrapolation to assume that c takes the same value across cosmic time scales [2]. With the accumulation of high quality cosmological datasets, it is time to develop theoretical and numerical tools to put constraints on the VSL scenario.

In this work, we propose a new method to constrain the speed of light using astronomical observations for the background expansion history of the Universe, including BAO, OHD and supernovae. Our method enables an accurate reconstruction of the temporal evolution of the speed of light $c(z)$ in a flat FRW universe, with no dependence on other cosmological parameters. Note that the time-evolution of $c(z)$ can be further constrained by observations probing the structure growth of the Universe including the cosmic microwave background (CMB) surveys, as a variation in c means a variation in the fine-structure constant α, which leaves imprint on the CMB. We shall expand our work in the near future to combine with the CMB constraint.

After validating our method and pipeline using mock datasets of BAO and supernovae, we apply our method to the latest astronomical observations including the anisotropic BAO measurements from BOSS (DR12) and eBOSS (DR14 quasar), and the JLA supernovae sample, and reconstruct $c(z)$ in the redshift range of $z \in [0, 1.5]$. We find no evidence of a varying speed of light, although we see some interesting features of $\Delta c(z)$, the fractional difference between $c(z)$ and c_0 (the speed of light in SI), *e.g.*, $\Delta c(z) < 0$ and $\Delta c(z) > 0$ at $0.2 \lesssim z \lesssim 0.5$ and $0.8 \lesssim z \lesssim 1.3$, respectively, which are primarily driven by the BAO and supernovae data. Note that the significance of these features is far below statistical importance, and it may be subject to unknown observational systematics. Future observations including DESI (DESI Collaboration et al. 2016), LSST (LSST Science Collaboration et al. 2009) and Euclid (Font-Ribera et al. 2014) should be able to confirm or falsify the result we show in this work, up to deeper redshifts. [3]

[2] We focus on probing the temporal evolution of c in this work, *i.e.*, we assume that c does not depend on distance scales. In general, one could probe the spatial variation of c as well using astronomical observations, but it is beyond the scope of this paper.

[3] We can hardly reconstruct c beyond $z = 1.5$ using current data due to the sparsity of data points at higher redshifts.

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Appendix A: THE OHD DATA

We summerise the OHD data used in this work in the following table.

z	$H(z)$	$\sigma_{H(z)}$	Reference
0.07	69.0	19.6	Zhang et al. 2014
0.09	69	12	Simon et al. 2005
0.12	68.6	26.2	Zhang et al. 2014
0.017	83	8	Simon et al. 2005
0.179	75	4	Moresco et al. 2012
0.199	75	5	Moresco et al. 2012
0.20	72.9	29.6	Zhang et al. 2014
0.27	77	14	Simon et al. 2005
0.28	88.8	36.6	Zhang et al. 2014
0.352	83	14	Moresco et al. 2012
0.3802	83	13.5	Moresco et al. 2016
0.4	95	17	Simon et al. 2005
0.4004	77	10.2	Moresco et al. 2016
0.4247	87.1	11.2	Moresco et al. 2016
0.44497	92.8	12.0	Moresco et al. 2016
0.4783	80.9	9	Moresco et al. 2016
0.48	97	62	Stern et al. 2010
0.583	104	13	Moresco et al. 2012
0.68	92	8	Moresco et al. 2012
0.781	105	12	Moresco et al. 2012
0.875	125	17	Moresco et al. 2012
0.88	90	40	Stern et al. 2010
0.9	117	23	Simon et al. 2005
1.037	154	20	Moresco et al. 2012
1.3	168	17	Simon et al. 2005
1.363	160	33.6	Moresco 2015
1.43	177	18	Simon et al. 2005
1.53	140	14	Simon et al. 2005
1.75	202	40	Simon et al. 2005
1.965	186.5	50.4	Moresco 2015

Table A.1 $H(z)$ measurements (in unit of [$\mathrm{km\ s^{-1}\ Mpc^{-1}}$]) used in this work.