Ghosts in the self-accelerating universe

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The self-accelerating universe realizes the accelerated expansion of the universe at late times by large-distance modification of general relativity without a cosmological constant. The Dvali-Gabadadze-Porrati (DGP) braneworld model provides an explicit example of the self-accelerating universe. Recently, the DGP model becomes very popular to study the observational consequences of the modified gravity models as an alternative to dark energy models in GR. However, it has been shown that the self-accelerating universe in the DGP model contains a ghost at the linearized level. The ghost carries negative energy densities and it leads to the instability of the spacetime. In this article, we review the origin of the ghost in the self-accelerating universe and explore the physical implication of the existence of the ghost.

I. INTRODUCTION

The acceleration of the late-time universe, as implied by observations of Supernovae redshifts [1, 2], cosmic microwave background anisotropies [3] and the large-scale structure [4], poses one of the deepest theoretical problems facing cosmology. Within the framework of general relativity (GR), the acceleration must originate from a dark energy field with effectively negative pressure, such as vacuum energy or a slow-rolling scalar field ("quintessence"). So far, none of the available models has a natural explanation. For example, in the simplest option of vacuum energy, leading to the "standard" LCDM model, the incredibly small,

$$\rho_{\Lambda,\text{obs}} = \frac{\Lambda}{8\pi G} \sim H_0^2 M_P^2 \ll \rho_{\Lambda,\text{theory}}, \tag{1}$$

and incredibly fine-tuned,

$$\Omega_\Lambda \sim \Omega_m|_{\text{today}}, \tag{2}$$

value of the cosmological constant cannot be explained by current particle physics.

An alternative to dark energy plus GR is provided by models where the acceleration is due to modifications of gravity on very large scales, $r \gtrsim H_0^{-1}$ (see [5, 6] and references therein). One of the simplest covariant models is based on the Dvali-Gabadadze-Porrati (DGP) brane-world model [7], in which gravity leaks off the 4D Minkowski brane into the 5D "bulk" Minkowski spacetime at large scales. The 5D action describing the DGP model is given by

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} R + \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-\gamma} \, ^{(4)}R - \int d^4x \sqrt{-\gamma} \mathcal{L}_m. \tag{3}$$

The important ingredient of the model is the induced Einstein-Hilbert term on the brane (see [8] for an early attempt). The existence of the brane imposes the junction condition for the metric at the position of the brane:

$$K_{\mu\nu} - K\eta_{\mu\nu} = -\frac{\kappa^2}{2}(T_{\mu\nu} - \kappa_4^{-2}G_{\mu\nu}), \tag{4}$$

where $K_{\mu\nu}$ is the extrinsic curvature and we assume the reflection (Z_2) symmetry across the brane. Due to the induced Einstein-Hilbert action, the 4D Einstein tensor appears in the junction condition.

On small scales, gravity is effectively bound to the brane and 4D Newtonian dynamics is recovered to a good approximation. The transition from 4- to 5D behaviour is governed by a crossover scale r_c [7]

$$r_c = \frac{\kappa^2}{2\kappa_4^2}. \tag{5}$$

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The weak-field gravitational potential behaves as [7]

$$\Psi \sim \begin{cases} r^{-1} & \text{for } r < r_c, \\ r^{-2} & \text{for } r > r_c. \end{cases} \tag{6}$$

The DGP model was generalized by Deffayet to a Friedman-Robertson-Walker brane in a Minkowski bulk [9, 10]; the gravity leakage at late times initiates acceleration – not due to any negative pressure field, but due to the weakening of gravity on the brane. The energy conservation equation remains the same as in GR, but the Friedman equation is modified:

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{7}$$

$$H^2 - \frac{H}{r_c} = \frac{8\pi G}{3}\rho. \tag{8}$$

The modified Friedman equation (8) shows that at late times in a CDM universe, with $\rho \propto a^{-3} \to 0$, we have

$$H \to H_\infty = \frac{1}{r_c}. \tag{9}$$

Since $H_0 > H_\infty$, in order to achieve acceleration at late times, we require $r_c \gtrsim H_0^{-1}$, and this is confirmed by fitting SN observations [11]. Like the LCDM model, the DGP model is simple, with a single parameter r_c to control the late-time acceleration although the DGP model does not provide a natural solution to the late-acceleration problem; similarly to the LCDM model, where Λ must be fine-tuned, the DGP parameter r_c must be fine-tuned to match observation.

The most interesting aspect of the DGP model is that there is a possibility to distinguish the model from dark energy models in GR. This is because the recovery of the 4D GR on small scales is very subtle. Although the weak-field gravitational potential behaves as 4D on scales smaller than r_c, the linearized gravity is *not* described by GR. This is because there is no normalized zero-mode in this model and 4D gravity is recovered as a resonance of the massive KK gravitons. The massive graviton contains 5 degrees of freedom compared with 2 degrees of freedom in a massless graviton. One of them is a helicity-0 polarization. Due to this scalar degree of freedom, linearized gravity is described by Brans-Dicke (BD) gravity with vanishing BD parameter in the case of Minkowski spacetime. Thus this model would be excluded by solar system experiments. However, the non-linear interactions of the scalar mode becomes important on larger scales than expected [12–19]. Let us consider a static source with mass M. Gravity becomes non-linear near the Schwarzshild radius $r_g = 2GM$. However, the scalar mode becomes non-linear at $r_* = (r_g r_c^2)^{1/3}$ (the Vainstein radius) which is much larger than r_g if $r_c \sim H_0^{-1}$. In fact, for the Sun r_* is much larger than the size of the solar system. A remarkable finding is at once the scalar mode becomes non-linear, GR is recovered. This non-linear shielding of the scalar mode is crucial to escape from the tight solar system constraints. Fig. 1 summarizes the behaviour of gravity in the DGP model (see [20] for a review on the DGP model).

The fact that the linearized gravity is not described by GR offers an exciting possibility to distinguish this model from the dark energy models in GR. In cosmology, the modification of linear perturbations affects the growth rate of the structure in the Universe. Thus structure formation in the DGP model is different from that in GR even if the expansion history of the Universe is exactly the same. Then combining the various observations, it is possible to distinguish the DGP model from the dark energy models in GR. Due to this possibility, the DGP model has been a very popular model for modified gravity alternative to dark energy (for a review, see Ref. [6]).

However, these interesting features of gravity could be signatures of a pathology of the model. The non-linear interaction of the scalar mode becomes important on very large scales compared with usual 4D gravity. If we consider a Planck mass particle, the Vainstein scale becomes $r_* = (\ell_{pl} r_c^2)^{1/3} \sim 1000$km where ℓ_{pl} is a Planck length and $r_c \sim H_0^{-1}$. This implies that quantum gravity corrections cannot be neglected below 1000km [21–23]. Even if we focus on the linearized behaviour of the scalar mode, there appears a problem of a ghost instability [22–28]. In the case where the brane is described by de Sitter spacetime, it has been proved that the scalar mode becomes a ghost.

In this article, we focus on the problem of the ghost that appears in the self-accelerating universe. In section II, we study the spectrum of linearized perturbations about a de Sitter brane. Then we identify the origin of the ghost in section III. For the positive tension brane, the ghost is originated from a massive spin-2
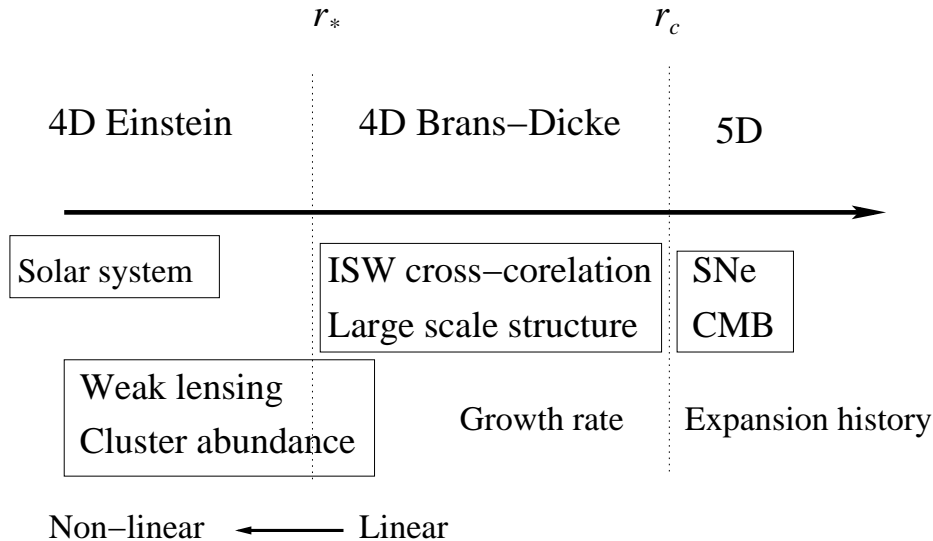
FIG. 1: Summary of the behaviour of gravity in the DGP model. At large scales $r > r_c$, the theory is 5D. On small scales $r < r_c$, gravity becomes 4D but the linearized theory is described by a Brans-Dicke theory. This affects the large scale structure (LSS) and the Integrated Sachs-Wolfe (ISW) effect and its cross-correlation to LSS. Below the Vainstein radius $r < r_*$, the theory approaches GR. This transition can be probed by weak lensing and cluster abundance as the non-linear dynamics is important for these measures. The solar system tests also provide constraints on the model in the 4D Einstein phase. From [6].

graviton with mass $0 < m^2 < 2H^2$. For the negative tension brane, the spin-0 perturbation associated from a fluctuation of the brane becomes a ghost. For a self-accelerating brane without tension, the ghost appears from the mixing between the spin-0 and spin-2 perturbations. In section IV, the spectrum with matter source on the brane is studied. We highlight a difference between the DGP model and the massive gravity model that makes it difficult to remove the ghost by a simple modification of the model such as two-brane model. This ghost can be identified with the brane bending mode on small scales which is a mix of helicity-0 components of massive spin-2 perturbations and the spin-0 perturbation. In section V, the effective action for the brane bending mode is discussed. The leading order non-linear interaction is identified. It defines the Vainstein length below which the linearized analysis cannot be trusted. We confirm again the existence of ghost at the linearized level. In section VI, fully non-linear solutions are discussed. These solutions indicate that the self-accelerating universe may suffer from instabilities even at non-perturbative level. Section VII is devoted to conclusions and discussions.

II. PERTURBATIONS ABOUT A DE SITTER BRANE

A. Background solution

Let us consider a situation where the brane is de Sitter spacetime. The bulk spacetime is a 5D Minkowski spacetime and the metric is given by

$$ds^2 = dy^2 + N(y)^2 \gamma_{\mu\nu} dx^\mu dx^\nu, \quad N(y) = 1 \pm H|y|, \tag{10}$$

where $\gamma_{\mu\nu}$ is the metric for the de Sitter spacetime and the brane is located at $y = 0$. The Z_2 symmetry across the brane is imposed. The junction condition at the brane gives the modified Friedmann equation

$$\pm \frac{H}{r_c} = H^2 - \frac{\kappa_4^2}{3}\sigma, \tag{11}$$

where σ is the tension of the brane. There are two branches of bulk solutions. The solution with $-$ sign is called the normal branch whereas the solution with $+$ sign is called the self-accelerating solution. These

two different solutions correspond to different embeddings of the brane. The 4D de Sitter spacetime is a hyperboloid in a 5D Minkowski spacetime. In the self-accelerating universe, we take the outside of the hyperboloid as the bulk spacetime. On the other hand, in the normal branch, we take the inside of the hyperboloid.
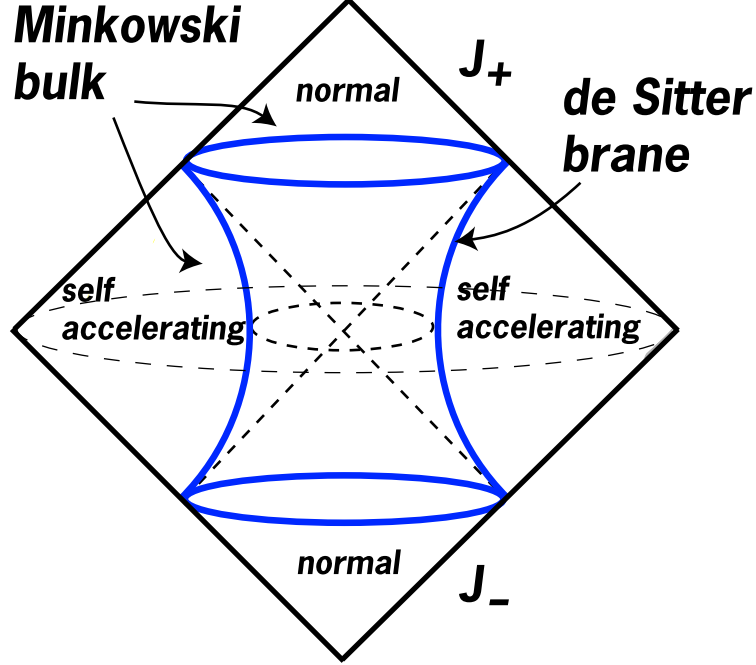

FIG. 2: Embedding of a de Sitter brane in a flat 5D bulk. The brane world volume is the hyperboloid in the Minkowski bulk. The normal branch corresponds to keeping the interior of the hyperboloid, and its mirror image around the brane. In contrast, for the self-accelerating branch, we keep the exterior, and its reflection. From [26].

B.   Spectrum of perturbations

We study the linear perturbations

$$ds^2 = dy^2 + (N(y)^2 \gamma_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu, \tag{12}$$

about the background de Sitter spacetime. In addition to the gravitational perturbations $h_{\mu\nu}$, we must take into account a perturbation of the position of the brane $y = \varphi(x)$ [30]. Using the transverse-traceless gauge $\nabla^\mu h_{\mu\nu} = h = 0$, the perturbed junction condition is given by [24]

$$k_{\mu\nu} \; - \; \mathcal{H}h_{\mu\nu} - r_c \left[X_{\mu\nu}(h) - \kappa_4^2 \left(T_{\mu\nu} - \frac{1}{3}\gamma_{\mu\nu}T \right) \right]$$
$$= -(1 - 2Hr_c)\left(\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu} \right) \varphi, \tag{13}$$

where $\mathcal{H} = N'/N|_{y=0} = \pm H$, ∇_μ is a covariant derivative with respect to $\gamma_{\mu\nu}$, $k_{\mu\nu} = (1/2)\partial_y h_{\mu\nu}$ on the brane is perturbations of the extrinsic curvature $K_{\mu\nu}$ and $X_{\mu\nu}$ is given by

$$X_{\mu\nu} \; = \; \delta^{(4)}G_{\mu\nu} + 3H^2 h_{\mu\nu}$$
$$= -\frac{1}{2}\left(\Box_4 h_{\mu\nu} - \nabla_\mu \nabla_\alpha h^\alpha_\nu - \nabla_\nu \nabla_\alpha h^\alpha_\mu + \nabla_\mu \nabla_\nu h \right)$$
$$- \frac{1}{2}\gamma_{\mu\nu}(\nabla_\alpha \nabla_\beta h^{\alpha\beta} - \Box_4 h) + H^2 \left(h_{\mu\nu} + \frac{1}{2}\gamma_{\mu\nu}h \right). \tag{14}$$

The equation of motion for φ is obtained from the traceless condition $h = 0$;

$$(1 - 2\mathcal{H}r_c)(\Box_4 + 4H^2)\varphi = \frac{\kappa^2 T}{6}, \tag{15}$$

where $\Box_4 = \nabla^\mu\nabla_\mu$.

Let us find solutions for the vacuum brane $T_{\mu\nu} = 0$. Using the separation of variables $h_{\mu\nu} = \int dm\, e_{\mu\nu}(x)u_m(y)$, the equation of motion in the bulk is written as

$$u_m'' + \frac{1}{N^2}(m^2 - 2H^2)u_m = 0, \quad (\Box_4 - m^2 - 2H^2)e_{\mu\nu}(x) = 0, \tag{16}$$

where prime denotes a derivative with respect to y. There are two types of solutions to Eq. (13). One type of the solution is a homogeneous solution with $\varphi = 0$, which is called the spin-2 perturbation. The spin-2 perturbations $\chi_{\mu\nu}$ satisfy the junction condition without φ

$$\chi_{\mu\nu}' - 2\mathcal{H}\chi_{\mu\nu} = -m^2 r_c \chi_{\mu\nu}. \tag{17}$$

We find a tower of continuous Kaluza-Klein (KK) modes starting from $m^2 = (9/4)H^2$ as well as a normalizable discrete mode. In the self-accelerating branch, the solution for the discrete mode is given by

$$u_{m_d} \propto N(y)^{-1+1/Hr_c}, \quad \frac{m_d^2}{H^2} = \frac{1}{(Hr_c)^2}(3Hr_c - 1), \tag{18}$$

for $Hr_c > 2/3$ [29]. For $Hr_c > 1$, the mass is in the range $0 < m_d^2 \leq 2H^2$ where $m_d^2 = 2H^2$ for the self-accelerating universe $Hr_c = 1$ and $m_d^2 \to 0$ for $Hr_c \to \infty$. In the normal branch, the discrete mode is a zero mode $m_d^2 = 0$.

The other type of solution to Eq. (13) is an inhomogeneous solution sourced by the scalar mode φ. We call this solution the spin-0 perturbation. In the self-accelerating branch, there is a normalizable solution given by [24]

$$h_{\mu\nu} = \frac{1 - 2Hr_c}{H(1 - Hr_c)}(\nabla_\mu\nabla_\nu + H^2\gamma_{\mu\nu})\varphi. \tag{19}$$

This is a solution with $m^2 = 2H^2$. In the normal branch, there is no normalisable spin-0 perturbation. Then, the normal branch is ghost-free. Eq. (19) is singular at $Hr_c = 1$ and we will deal with this case separately in section III.C.

III. GHOST IN DE SITTER SPACETIME

A. Effective action

We can construct the 2nd order action for $h_{\mu\nu}$ and φ from the 5D action (3). The result is given by [24]

$$\delta_2 S = -\frac{1}{4\kappa^2}\int d^5x\sqrt{-g}N^{-4}h^{\mu\nu}\delta^{(5)}G_{\mu\nu} + \frac{1}{\kappa^2}\int d^4x\sqrt{-\gamma}\mathcal{L}_B, \tag{20}$$

where $\delta^{(5)}G_{\mu\nu}$ is the 5D perturbed Einstein tensor and

$$\begin{aligned}
\mathcal{L}_B &= k^{\mu\nu}h_{\mu\nu} - kh + \frac{1}{2}\mathcal{H}(h^2 - h^{\mu\nu}h_{\mu\nu}) \\
&+ (1 - 2\mathcal{H}r_c)\left(h_{\mu\nu}\nabla^\mu\nabla^\nu\varphi - h\nabla^\rho\nabla_\rho\varphi - 3H^2h\varphi\right) \\
&- 3\mathcal{H}\left(-(1 - 2\mathcal{H}r_c)\varphi(\Box_4 + 4H^2)\varphi + \frac{\kappa^2}{3}T\varphi\right) \\
&+ \frac{1}{2}\kappa^2 h^{\mu\nu}T_{\mu\nu} - \frac{r_c}{2}h^{\mu\nu}X_{\mu\nu}(h).
\end{aligned} \tag{21}$$

This action gives the correct equation of motion and the junction condition for $h_{\mu\nu}$ and the equation of motion for φ.

In the following discussions, we focus on the self-accelerating branch. We can derive an effective action for the brane fluctuation φ by substituting the 5D solution for $h_{\mu\nu}$ given by φ (19) into the 5D action (20) and get the off-shell action for φ by integrating out only with respect to the extra coordinate y [31]. This yields the action for φ as

$$S_\varphi = \frac{3H}{2\kappa^2}\left(\frac{1-2Hr_c}{1-Hr_c}\right)\int d^4x\sqrt{-\gamma}\varphi(\Box_4 + 4H^2)\varphi. \tag{22}$$

The 4D effective action for the spin-2 perturbations is also obtained in a similar way. For the discrete mode with m_d^2, we get

$$S_\chi = \frac{r_c(3Hr_c-1)}{4\kappa^2(3Hr_c-2)}\int d^4x\sqrt{-\gamma}\chi^{\mu\nu}(\Box_4 - 2H^2 - m_d^2)\chi_{\mu\nu}, \tag{23}$$

where transverse-traceless gauge fixing conditions $\nabla^\mu\chi_{\mu\nu} = \chi^\mu_\mu = 0$ are imposed . This is exactly the same action for the spin-2 perturbations in the 4D massive gravity theory where the Pauli-Fierz (PF) mass term is added to the Einstein-Hilbert action by hand [32]

$$S_M = -\frac{M^2}{8\kappa_4^2}\int d^4x\sqrt{-\gamma}(h^{\mu\nu}h_{\mu\nu} - h^2). \tag{24}$$

B. Ghost in de Sitter spacetime with a tension

Ref. [24] studied the existence of the ghost based on the above effective action. In the limit $Hr_c \to \infty$, the action (23) approaches the one for massless spin-2 perturbations. However, there is a discontinuity between the massless perturbations and the massive perturbations that is known as the van Dam-Veltman-Zakharov discontinuity [33]. Due to the lack of gauge symmetry, the massive spin-2 perturbations contain a helicity-0 excitation. Moreover, it has been shown that this helicity-0 excitation becomes a ghost if $0 < M^2 < 2H^2$ [34, 35] (see also [26]). This is exactly the same mass range for the discrete mode in the self-accelerating branch for $Hr_c > 1$. Thus we identify the ghost as the helicity-0 mode of the discrete mode of the spin-2 perturbations. On the other hand, for $Hr_c < 1$, the spin-2 perturbations become healthy as the mass for the spin-2 perturbations is larger than $2H^2$. However, in this case, the coefficient in front of the effective action for the spin-0 perturbations becomes negative and the spin-0 perturbation becomes a ghost.
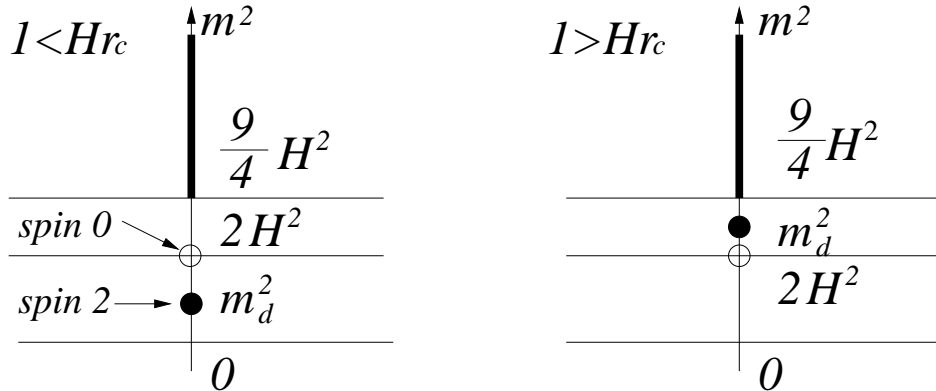

FIG. 3: Summary of the mass spectrum of the scalar perturbations in + branch. Spin-2 perturbation has continuous modes with $m^2 \geq (9/4)H^2$ and a discrete mode with $m^2 = m_d^2$ while spin-0 perturbation has $m^2 = 2H^2$. In the limit $Hr_c \to 1$, both the helicity-0 excitation of spin-2 perturbation and the spin-0 perturbation have mass $m^2 = 2H^2$ and there is a resonance.
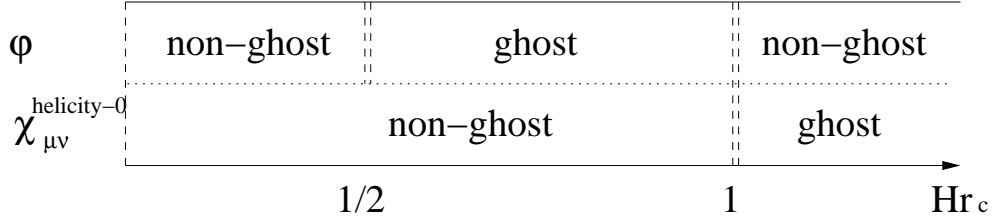
FIG. 4: Summary of the existence of the ghost. From [24].

C. Ghost in self-accelerating universe

In the self-accelerating universe $Hr_c = 1$, the mass of the discrete mode of the spin-2 perturbations becomes $2H^2$. This is a special mass in the massive gravity as the action is invariant under the transformation

$$\chi_{\mu\nu} \rightarrow \chi_{\mu\nu} + (\nabla_\mu \nabla_\nu - H^2 \gamma_{\mu\nu})X, \tag{25}$$

where X is any solution of the equation $(\Box_4 + 4H^2)X = 0$. This is so called enhanced symmetry and the helicity-0 mode can be eliminated by this symmetry [35]. Then there are only four polaization when the mass is $m^2 = 2H^2$ and the ghost disappears.

However, this does not happen in the self-accelerating universe as there is an additional spin-0 perturbations with the same mass [25]. In fact in the self-accelerating universe, $Hr_c = 1$, the bulk wave function for the discrete mode is given by $u_{m_d} \propto N(y)$ and it is the same as the spin-0 perturbation. Then the discrete mode of the spin-2 perturbations and the spin-0 perturbation degenerate and they can mix. This comes from the fact that at $m^2 = 2H^2$, the spin-0 and spin-2 perturbations degenerate. In fact we can make the spin-2 perturbations from a scalar

$$h_{\mu\nu}^{(2H^2)} = (\nabla_\mu \nabla_\nu - H^2 \gamma_{\mu\nu})X, \quad (\Box_4 + 4H^2)X = 0. \tag{26}$$

This is a scalar mode with mass squared $-4H^2$. At the same time $h_{\mu\nu}$ is a transverse-traceless perturbations and from the identity,

$$(\Box_4 - 4H^2)h_{\mu\nu}^{(2H^2)} = \left(\nabla_\mu \nabla_\nu - H^2 \gamma_{\mu\nu}\right)(\Box_4 + 4H^2)X = 0,$$

it is identified to have a mass given by $m^2 = 2H^2$.

Let us investigate the limit $Hr_c \rightarrow 1$ carefully. The solutions for the metric perturbations are given by

$$h_{\mu\nu} = \chi_{\mu\nu}^{m_d}(x)u_{m_d}(y) + \frac{1 - 2Hr_c}{H(1 - Hr_c)}(\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu})\varphi, \tag{27}$$

where u_{m_d} is given by Eq. (18) and we only consider the localized modes. The limit $Hr_c \rightarrow 1$ looks singular, but we can perform a field re-definition

$$\chi_{\mu\nu}^{m_d}(x) = A_{\mu\nu} - \frac{1 - 2Hr_c}{H(1 - Hr_c)}(\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu})\varphi. \tag{28}$$

The metric perturbations read

$$h_{\mu\nu}(x,y) = A_{\mu\nu}u_{m_d}(y) + \frac{1 - 2Hr_c}{H(1 - Hr_c)}(\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu})\varphi(1 - u_{m_d}(y)). \tag{29}$$

Then taking the limit $Hr_c \rightarrow 1$, we get [25]

$$h_{\mu\nu} = A_{\mu\nu}(x) + \frac{1}{H}(\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu})\varphi(x)\log(1 + Hy) . \tag{30}$$

Substituting this expressions into the bulk equation and the junction condition, we get an equation for $A_{\mu\nu}$

$$\Box_4 A_{\mu\nu} - 4H^2 A_{\mu\nu} = H\left(\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu}\right)\varphi. \tag{31}$$

The effective action for $A_{\mu\nu}$ and φ is obtained by substituting the solutions for $A_{\mu\nu}$ and φ into the 5D action and integrating out the extra-dimension as

$$S_{eff} = \frac{1}{\kappa^2 H} \int d^4x \sqrt{-\gamma} \bigg\{ - A^{\mu\nu} X_{\mu\nu}(A) - H^2 A^{\mu\nu} A_{\mu\nu} + H^2 A^2$$
$$- H(A^{\mu\nu}\nabla_\mu\nabla_\nu\varphi - A\Box_4\varphi - 3H^2 A\varphi) - \frac{9H^2}{4}\varphi(\Box_4 + 4H^2)\varphi \bigg\} , \tag{32}$$

where we introduced the notation $A \equiv A^\mu_\mu$. The first line in Eq. (32) coincides with the quadratic Lagrangian for the Pauli-Fierz theory of massive gravity with $m^2 = 2H^2$. Thus the action could have the enhanced symmetry (25) if there were no mixing to φ. However, the mixing between φ and $A_{\nu\mu}$ breaks this symmetry explicitly.

In order to study the existence of the ghost, it is necessary to write the effective action only in terms of physical degrees of freedom. Ref. [25] performed the Hamiltonian analysis to derive the reduced Hamiltonian written only in terms of physical degrees of freedom. There are two dynamical degrees of freedom, namely the spin-0 mode and the helicity-0 excitation of spin-2 perturbations. It is found that, in general, the Hamiltonian cannot be diagonalized. However, on small scales under horizon, it is possible to diagonalize the Hamiltonian and we find a ghost from a mixing between the spin-0 perturbation and the helicity-0 excitation of spin-2 perturbations.

D. Two-brane model

Ref. [28] tried to remove the ghost by adding the second brane in the bulk. In previous section, we saw that the origin of the ghost for the positive tension brane is coming from the fact the discrete mode for the spin-2 perturbations has a mass in the range $0 < m^2 < 2H^2$. It is possible to eliminate this ghost by introducing the second brane in the bulk. By making the distance between the two branes short, the mass increases as in the usual Kalzua-Klein compactification. Once the mass becomes larger than $2H^2$, the spin-2 perturbations do not contain a ghost. In fact for $Hr_c = 1$, once the second brane is put and it cuts off the bulk spacetime, the mass becomes $m^2 > 2H^2$ regardless of the distance between the two branes. Note that even in the presence of the second brane, the Friedman equation on the visible brane is unchanged and the brane can self-accelerate.

However, as soon as the mass of the discrete mode of the spin-2 perturbations exceeds $m^2 = 2H^2$, it is proven that the spin-0 perturbation becomes a ghost [28]. At the critical length between the two branes where the mass of the discrete mode of the spin-2 perturbations is given by $m^2 = 2H^2$, the spin-0 perturbations mix with the helicity-0 component of the spin-2 perturbations exactly in the same way as the self-accelerating universe in one brane model and there appears a ghost.

The spin-0 perturbation is the radion that describes the distance between two branes. Then one would try to remove this ghost by eliminating the radion by stabilizing the distance between two branes. Once we stabilize the radion, the brane fluctuation mode with mass $m^2 = 2H^2$ becomes non-physical. The simplest way to achieve the stabilization is to introduce a scalar field in the bulk. Then instead of the brane fluctuation, there appears scalar field perturbations which can have a discrete mode and an infinite ladder of massive modes. In general, the mass of the discrete mode is different from $-4H^2$ which corresponds to $m^2 = 2H^2$ in the spin-2 perturbations language. However, it turns out that if the mass of the discrete mode of the spin-2 perturbations becomes $m^2 = 2H^2$, the discrete mode of the scalar field perturbations has a mass $m^2 = -4H^2$ which is the special case where it can mix with the spin-2 perturbations. Then completely the same phenomena happens as in the one brane model. Once the mass of the discrete mode of spin-2 perturbations exceeds $2H^2$, the helicity-0 mode of the spin-2 perturbation becomes non-ghost but the scalar field perturbation becomes a ghost! Then it is impossible to remove the ghost. In the next section, we will explain why it is impossible to remove the spin-2 ghost and spin-0 ghost simultaneously.

IV. SPECTRUM WITH MATTER SOURCE

In the previous section, we identify the origin of the ghost by the analysis of the spectrum without matter source. The most interesting finding is that it is impossible to remove the spin-2 ghost and the spin-0 ghost

simultaneously [26, 28]. We can clarify the reason for this by studying the spectrum with matter source. We also derive the effective theory for perturbations on small scales.

A. Amplitude

We introduce matter source on the brane [26–28]. The junction condition for the transverse-traceless modes including matter perturbations is given by Eq. (13). Using the equation of motion for the brane bending Eq. (15), Eq. (13) is given by

$$(\partial_y - 2H) h_{\mu\nu}^{(TT)} = -\kappa^2 \Sigma_{\mu\nu} - r_c(\Box_4 - 2H^2)h_{\mu\nu}^{(TT)}, \tag{33}$$

where

$$\Sigma_{\mu\nu} = T_{\mu\nu} - \frac{1}{3}H^2\gamma_{\mu\nu}T + \frac{1}{3}(\nabla_\mu\nabla_\nu + H^2\gamma_{\mu\nu})(\Box_4 + 4H^2)^{-1}T. \tag{34}$$

The solution for transverse-traceless perturbations can be obtained by the Green's function which can be constructed by solutions without source obtained in section II. The solution is written as

$$h_{\mu\nu}^{(TT)} = -2\kappa^2 \sum_i \frac{u_i(0)^2}{\Box_4 - 2H^2 - m_i^2}\Sigma_{\mu\nu}. \tag{35}$$

Here the solution for the properly normalized mode functions $u_i(0)^2$ are given by

$$
\begin{aligned}
u_d^2(0) &= \frac{1}{2r_c}\frac{3Hr_c - 2}{3Hr_c - 1}, \quad m_d^2 = \frac{3Hr_c - 1}{r_c^2}, \\
u_m^2(0) &= \frac{H}{\pi}\frac{k^2}{\left(\frac{m^2 r_c}{H} - \frac{3}{2}\right)^2 + k^2}, \quad m^2 > \frac{9H^2}{4}, \quad k = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}.
\end{aligned}
\tag{36}
$$

We should bear in mind that in the TT gauge, the brane is not located at $y = 0$ and the brane bending must be taken into account. The induced metric on the brane is given by

$$h_{\mu\nu}^{induced}(0) = h_{\mu\nu}^{TT} - 2H\gamma_{\mu\nu}\varphi. \tag{37}$$

The one particle exchange amplitude is defined as

$$\mathcal{A} \equiv \frac{1}{2}h_{\mu\nu}^{induced}(0)T^{\mu\nu} = \frac{1}{2}h_{\mu\nu}^{(TT)}T^{\mu\nu} - H\varphi T. \tag{38}$$

Using Eq. (35) and Eq. (15), $\mathcal{A}$ is calculated as

$$
\begin{aligned}
\mathcal{A} = -\kappa^2 \sum_i u_i(0)^2 &\left[T_{\mu\nu}\frac{1}{\Box_4 - 2H^2 - m_i^2}T^{\mu\nu} - \frac{1}{3}T\frac{1}{\Box_4 + 6H^2 - m_i^2}T \right. \\
&\left. + \frac{1}{3}H^2 T\frac{1}{(\Box_4 + 6H^2 - m_i^2)(\Box_4 + 4H^2)}T \right] \\
&- \frac{H}{1 - 2Hr_c}\frac{\kappa^2}{6}T\frac{1}{\Box_4 + 4H^2}T.
\end{aligned}
\tag{39}
$$

The second line becomes a double pole when $m_i^2 = 2H^2$ which occurs in the self-accelerating universe $Hr_c = 1$ [27]. Usually, a double pole can be recast into difference of two simple poles, giving rise to a ghost. In fact, we can rewrite the second line as

$$T\frac{1}{(\Box_4 + 6H^2 - m_i^2)(\Box_4 + 4H^2)}T = \frac{1}{m_i^2 - 2H^2}T\left(\frac{1}{\Box_4 + 6H^2 - m_i^2} - \frac{1}{\Box_4 + 4H^2}\right)T. \tag{40}$$

The first term has a pole at $\Box_4 = m_i^2 - 6H^2$ which corresponds to the spin-2 contributions. The second term has a pole at $\Box_4 = -4H^2$ which corresponds to the spin-0 perturbation. Around $m_i^2 = 2H^2$, these terms become dominant in the amplitude and determine the existence of the ghost. A crucial point is that the contribution of the spin-0 perturbation is always *opposite* to that of the spin-2 massive perturbations [28]. We have already seen that the massive spin-2 perturbations contain the helicity-0 mode that is a ghost if $0 < m_i^2 < 2H^2$. In fact we see that the sign of the amplitude changes at $m_i^2 = 2H^2$. Then for $0 < m_i^2 < 2H^2$, the spin-2 perturbation mediates the repulsive force. This means that spin-0 perturbation is not a ghost as it must mediate the *opposite* force compare to massive spin-2 perturbations. On the other hand, for $2H^2 < m_i^2$, the spin-2 perturbations do not carry a ghost and mediate a normal force. Then the spin-0 perturbations should mediate a repulsive force and it should be the ghost. This explains the reason why the spin-0 ghost appears as soon as the spin-2 ghost disappears.

In the self-accelerating universe, there appears a double pole [27]. This manifests the fact that the spin-2 and spin-0 perturbations degenerate. In fact if we try to separate the poles for the spin-0 and spin-2 perturbations, we encounter a divergence. Then we need a careful treatment for this special case.

It should be emphasized that the existence of spin-0 contribution is crucial to have a non-singular amplitude. It is instructive to compare the amplitude (39) with that in massive Pauli-Fierz theory [36]

$$\mathcal{A}_{massive} = -\kappa_4^2 \left[T_{\mu\nu} \frac{1}{\Box_4 - 2H^2 - M^2} T^{\mu\nu} - \frac{1}{3} \frac{M^2 - 3H^2}{M^2 - 2H^2} T \frac{1}{\Box_4 + 6H^2 - M^2} T \right]. \tag{41}$$

We can check that the spin-2 contributions in (39) are the summation of these massive spin-2 perturbations. The amplitude is singular for $M^2 = 2H^2$. This comes from the fact that in Pauli-Fierz theory, we cannot couple gravity to matter with non-vanishing trace of energy momentum tensor. This is because the equation of motion is given by [36]

$$(2H^2 - M^2)(\Box_4 + 4H^2)h = \frac{8H^2\kappa_4^2}{3}T, \tag{42}$$

where h is the trace of perturbations and T must vanish for $M^2 = 2H^2$. This is the origin of the singularity in the amplitude and this is a pathology of massive gravity theory in de Sitter spacetime. In the DGP model, it is the brane bending mode that couples to the trace of energy momentum tensor and there is no such a restriction even if the discrete mode has a mass $m_i^2 = 2H^2$. In fact the singularity at $m_i^2 = 2H^2$ is exactly canceled by the spin-0 perturbations as can be seen in Eq. (40). Thus in the DGP, we do not have the pathology and we can couple gravity to matter perturbations with non-zero trace of energy-momentum tensor. However, we have seen that it is this mechanism that makes the spin-0 perturbation a ghost when the spin-2 perturbation becomes non-ghost: in order to cancel the singularity in the spin-2 sector at $m_i^2 = 2H^2$, the spin-0 contribution must be *opposite* to that of the spin-2 perturbations.

B. Effective theory on small scales

On small scales, it is possible to derive the 4D effective theory for perturbations. Let us consider the limit

$$\Box_4 \gg H^2, m_i^2. \tag{43}$$

Then the amplitude is approximated as

$$\mathcal{A} = -\kappa^2 \sum_i u_i(0)^2 \left[T_{\mu\nu} \frac{1}{\Box_4} T^{\mu\nu} - \frac{1}{3} T \frac{1}{\Box_4} T \right] - \frac{\kappa^2}{6} \frac{H}{1 - 2Hr_c} T \frac{1}{\Box_4} T. \tag{44}$$

Note that the self-accelerating universe $Hr_c = 1$ is not a special case anymore. Now using the solutions for the mode functions we get

$$\begin{aligned}
\sum_i u_i(0)^2 &= u_d(0)^2 + \sum u_m(0)^2 \\
&= \frac{1}{2r_c} \frac{3Hr_c - 2}{3Hr_c - 1} + \frac{H}{\pi(Hr_c)^2} \int_0^\infty dk \frac{k^2}{\left(k^2 + \frac{9}{4}\right)\left(k^2 + \frac{9}{4} - m_d^2\right)} \\
&= \frac{1}{2r_c}.
\end{aligned} \tag{45}$$

The effective gravitational coupling is read as

$$\kappa^2 \sum_i u_i(0)^2 = \kappa_4^2. \tag{46}$$

Thus the 4D gravity is recovered by the summation of massive gravitons. Then the amplitude is calculated as

$$\mathcal{A} = -\kappa_4^2 \left[T_{\mu\nu} \frac{1}{\Box_4} T^{\mu\nu} - \frac{1}{3} T \frac{1}{\Box_4} T \right] - \frac{\kappa_4^2}{3} \frac{Hr_c}{1 - 2Hr_c} T \frac{1}{\Box_4} T. \tag{47}$$

The first terms is exactly the same as the amplitude of massive gravity in the Minkowski spacetime. Due to the scalar polarisation, the coefficient in front of $T\Box_4^{-1}T$ is $1/3$ not $1/2$. The last term represents the effect of the curvature of the brane. Then finally we get

$$\mathcal{A} = -\kappa_4^2 \left[T_{\mu\nu} \frac{1}{\Box_4} T^{\mu\nu} - \frac{1}{3} \frac{1 - 3Hr_c}{1 - 2Hr_c} T \frac{1}{\Box_4} T \right]. \tag{48}$$

This result can be compared with the 4D Brans-Dicke (BD) theory. In the BD theory with BD parameter ω, the amplitude is given by

$$\mathcal{A} = -\kappa_4^2 \left[T_{\mu\nu} \frac{1}{\Box_4} T^{\mu\nu} - \frac{1}{3} \frac{1 + \omega}{1 + \frac{2\omega}{3}} T \frac{1}{\Box_4} T \right]. \tag{49}$$

Then the BD parameter is determined as

$$\omega = -3Hr_c. \tag{50}$$

This agrees with the results obtained in Refs. [16, 17, 37, 38]. It is known that the BD theory contains a ghost if

$$\omega < -\frac{3}{2}. \tag{51}$$

This means that there is a ghost if $Hr_c > 1/2$, which agrees with the spectrum analysis.

C. Boundary effective action

The analysis in the previous section indicates that it is possible to derive the 4D effective action on small scales. Let us again consider the limit

$$\Box_4 \gg r_c^{-2}, \quad H^2 \tag{52}$$

Using this approximation, we can only keep the 4D terms in the 5D second order action (20) and (21). Then the 4D boundary effective action is obtained as

$$\begin{aligned} S_B &= \frac{1}{\kappa^2} \int d^4x \sqrt{-\gamma} \Big[(1 - 2\mathcal{H}r_c) \left(h_{\mu\nu}\nabla^\mu\nabla^\nu\varphi - h\nabla^\rho\nabla_\rho\varphi - 3H^2 h\varphi \right) \\ &\quad -3\mathcal{H} \left(-(1 - 2\mathcal{H}r_c)\varphi(\Box_4 + 4H^2)\varphi + \frac{\kappa^2}{3}T\varphi \right) \\ &\quad + \frac{1}{2}\kappa^2 h^{\mu\nu}T_{\mu\nu} - \frac{r_c}{2}h^{\mu\nu}X_{\mu\nu}(h) \Big], \end{aligned} \tag{53}$$

where $\mathcal{H} = H$ in the self-accelerating branch. It is possible to check that this action consistently reproduce the amplitude (48). We can diagonalise the action by defining

$$h_{\mu\nu} = \chi_{\mu\nu} - r_c^{-1}(1 - 2\mathcal{H}r_c)\gamma_{\mu\nu}\varphi. \tag{54}$$

The resultant action is

$$S_B = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-\gamma} \left[-\chi_{\mu\nu}X^{\mu\nu}(\chi) + \kappa_4^2\chi_{\mu\nu}T^{\mu\nu} + \frac{3}{2r_c^2}\left\{ (1 - 2Hr_c)\varphi(\Box_4 + 4H^2)\varphi - \frac{\kappa^2}{3}\varphi T \right\} \right]. \tag{55}$$

The sign of the kinetic term φ changes at $Hr_c = 1/2$ and we find that the brane bending mode φ becomes a ghost for $Hr_c > 1/2$.

V. BOUNDARY EFFECTIVE ACTION

In the previous section, we show that it is possible to derive the 4D effective action. Refs.[22] and [23] derived the 4D effective action for the brane bending mode including the non-linear interactions and identified the scale at which the linearized analysis cannot be trusted.

A. The effective action for the brane bending

Refs. [22] and [23] derived the boundary effective action for perturbations on Minkowski background, $g_{MN} = \eta_{MN} + h_{MN}$. They integrated out the bulk to obtain an effective action for the 4D field living on the boundary. The final result at the quadratic order is

$$S_b = \frac{1}{4\kappa_4^2}\left(\frac{1}{2}\chi_{\mu\nu}\Box_4\chi^{\mu\nu} - \frac{1}{4}\chi\Box_4\chi - r_c^{-1}n_\mu\triangle n^\mu + 3r_c^{-2}\pi\Box_4\pi\right), \tag{56}$$

where $\triangle$ is a non-local differential operator, $\triangle = \sqrt{-\Box_4}$ and the kinetic terms have been diagonalized by defining

$$h_{yy} = -2\triangle\pi, \quad n_\mu = N_\mu - \partial_\mu\pi, \quad \chi_{\mu\nu} = h_{\mu\nu} - r_c^{-1}\pi\eta_{\mu\nu}, \tag{57}$$

and N_μ is a shift which is (y,μ)-component of the 5D metric, $g_{y\mu}$. In this gauge, π plays the same role as the brane bending φ. Note that this result is consistent with the previous effective action (55) for tensor and scalar parts if one takes $H = 0$.

By taking into account bulk interaction with higher powers of h_{MN}, one finds that the leading order boundary interaction terms is cubic in π, and involves four derivatives,

$$S^{(3)} = -\frac{1}{2\kappa^2}\int d^4x(\partial\pi)^2\Box_4\pi. \tag{58}$$

In order to extract the non-trivial non-linear interactions, let us consider the case where the flat approximation is good $\chi_{\mu\nu} \ll 1$. However we want to preserve the self-coupling of the π field. In terms of canonically normalized field $\hat{\pi}$ defined by $\hat{\pi} = \pi/(2\kappa_4 r_c)$, the cubic self-coupling is unchanged if we keep

$$\Lambda = \left(\frac{2M_5^6}{M_4}\right)^{1/3} = \left(\frac{M_4}{2r_c^2}\right)^{1/3}, \quad \kappa^2 = \frac{1}{M_5^3}, \quad \kappa_4^2 = \frac{1}{M_4^2}, \tag{59}$$

fixed (see below). We also preserve the interaction between π and the energy momentum tensor $T_{\mu\nu}$. The interaction between $h_{\mu\nu}$ and $T_{\mu\nu}$ is given by $(1/2)h_{\mu\nu}T^{\mu\nu}$. From the definition of π, $\hat{\pi}$ interacts with matter via $(1/M_4)\hat{\pi}T$. Therefore, if we take the formal limit

$$M_4 \to \infty, \quad r_c \to \infty, \quad T_{\mu\nu} \to \infty, \quad \Lambda = const, \quad \frac{T_{\mu\nu}}{M_4} = const. \tag{60}$$

we can decouple 4D gravity while keeping the full Lagrangian for $\hat{\pi}$. It is possible to check that all further interactions other than the cubic interactions vanish. Then the full action for the $\hat{\pi}$ field in the flat spacetime is given by

$$S = \int d^4x\left[-3(\partial\hat{\pi})^2 - \frac{1}{\Lambda^3}(\partial\hat{\pi})^2\Box_4\hat{\pi} + \frac{1}{M_4}\hat{\pi}T\right]. \tag{61}$$

From this action, the equation of motion for π is derived as

$$3\Box_4\hat{\pi} - \frac{1}{\Lambda^3}(\partial_\mu\partial_\nu\hat{\pi})^2 + \frac{1}{\Lambda^3}(\Box_4\hat{\pi})^2 = -\frac{T}{2M_4}. \tag{62}$$

It is possible to understand this equation from a geometric point of view. The Gauss-Codazzi equation in the bulk is given by

$$^{(4)}R + K_{\mu\nu}K^{\mu\nu} - K^2 = 0. \tag{63}$$

The junction condition at the brane is given by

$$K_{\mu\nu} - K\eta_{\mu\nu} = -\frac{\kappa^2}{2}(T_{\mu\nu} - \kappa_4^{-2}G_{\mu\nu}). \tag{64}$$

Then combining these equations we get

$$\frac{3K}{r_c} + K_{\mu\nu}K^{\mu\nu} - K^2 = -\frac{T}{M_4^2}. \tag{65}$$

Note that this equation is exact as this is a combination of a geometric identity (63) and the junction condition on the brane (64). From Eq. (57), the extrinsic curvature is calculated as

$$K_{\mu\nu} = -\frac{1}{r_c\Lambda^3}\partial_\mu\partial_\nu\hat{\pi}, \tag{66}$$

in the limit (60). Then Eq. (65) gives the equation of Eq. (62).

Once a solution of Eq. (62) or Eq. (65) is found, we can perturb $\hat{\pi}$ and expand the action up to quadratic order in the perturbations ρ. We find

$$\delta S = \int d^4x \left[-3(\partial\rho)^2 - 2(\tilde{K}_{\mu\nu} - \eta_{\mu\nu}\tilde{K})\partial^\mu\rho\partial^\nu\rho \right]. \tag{67}$$

where $\tilde{K}_{\mu\nu} = r_c K_{\mu\nu}$ and it satisfies

$$3\tilde{K} + \tilde{K}_{\mu\nu}\tilde{K}^{\mu\nu} - \tilde{K}^2 = -\frac{T}{2\Lambda^3 M_4}. \tag{68}$$

B. Self-accelerating universe

Let us consider a de Sitter background solution with tension σ. The equation for $\tilde{K}_{\mu\nu}$ admits two branches of solutions:

$$\tilde{K}_{0\ \nu}^{\mu} = \mathcal{H}r_c\delta_\nu^\mu, \quad \mathcal{H} \equiv \frac{1 \pm \sqrt{1 + 4\,r_c^2\sigma/3M_4^2}}{2r_c}\delta_\nu^\mu. \tag{69}$$

This corresponds to de Sitter solution where the Hubble parameter H is given by

$$H = |\mathcal{H}| \tag{70}$$

The self-accelerating branch corresponds to $+$ sign in Eq. (69). It is also possible to derive a corresponding solution for $\hat{\pi}$. The de Sitter invariant solutions are

$$\hat{\pi}_0 = -\frac{\mathcal{H}r_c}{2}\Lambda^3 x^\mu x_\mu. \tag{71}$$

Now let us consider a small fluctuations around the de Sitter spacetime. The action for the fluctuations ρ (67) is given by

$$\delta S = -\int d^4x \left[3\left(1 - 2\mathcal{H}r_c\right)(\partial\rho)^2 \right]. \tag{72}$$

This agrees with the result for perturbations around de Sitter solution in the self-accelerating branch in the limit $\Box_4 \gg H^2$. In particular, the fluctuation ρ becomes a ghost when $Hr_c > 1/2$ in the self-accelerating universe. In fact this was the first discovery of the ghost in the self-accelerating universe in literatures. Note that for large $\mathcal{H}r_c$, the kinetic term becomes large and the scalar mode becomes non-dynamical. In the BD theory, this corresponds to the limits where the BD parameter $\omega = -3\mathcal{H}r_c$ becomes infinite. Then we recover GR at the linearized level [39]. In this limit, the non-linear interactions are also suppressed.

C. Non-linearity interactions

Next, we study the effect of the non-linear interactions of $\hat{\pi}$. Let is consider a static localized source of mass M. We look for a static spherically symmetric solution $\hat{\pi}(r)$ [22, 23]. The equation for $\hat{\pi}$ (62) is then given by [18, 19]

$$\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right)(3\beta\hat{\pi} + \Xi) = \frac{\rho}{2M_4},\tag{73}$$

where $\beta = 1 - 2\mathcal{H}r_c$ and

$$\Xi = \frac{2}{\Lambda^3}\int \frac{1}{r}\left(\frac{d\hat{\pi}}{dr}\right)^2 dr.\tag{74}$$

Then it is possible to integrate the equation to get

$$3\beta\hat{\pi} + \Xi + \frac{M_4}{2}\frac{r_g}{r} = 0,\tag{75}$$

where

$$r_g = \frac{1}{M_4^2}\int_0^r dr\, r^2\rho,\tag{76}$$

is the Schwarzschild radius of the source. Hereafter, we assume $r_g = $ const, for simplicity. Taking the r derivative of Eq. (75) gives an algebraic equation for $d\hat{\pi}/dr$. Then we get a solution for $d\hat{\pi}/dr$ as

$$\frac{d\hat{\pi}}{dr} = \frac{3}{4}\beta\Lambda^3 r\left(\sqrt{1 + \left(\frac{r_*}{r}\right)^3} - 1\right),\tag{77}$$

where

$$r_* = \left(\frac{8r_c^2 r_g}{9\beta^2}\right)^{1/3},\tag{78}$$

which is the Vainshtein radius for a source. On scales larger than r_*, the non-linear interactions can be neglected. On the other hand, on scales smaller than r_* the non-linear interactions cannot be neglected and the linearized analysis cannot be trusted. Note that r_* is much larger than r_g if $r_c \sim H_0$ where H_0 is the present-day horizon scale.

D. Spherically symmetric solution

Let us study the behaviour of the spherically symmetric solution [14, 16–19]. The metric perturbations are given by

$$h_{\mu\nu} = \chi_{\mu\nu} + \frac{2}{M_4}\hat{\pi}\eta_{\mu\nu},\tag{79}$$

where $\chi_{\mu\nu}$ satisfies the 4D GR equations. We take the scalar perturbations on a brane as

$$ds^2 = -(1 + 2\Psi)dt^2 + e^{2Ht}(1 + 2\Phi)\delta_{ij}dx^i dx^j.\tag{80}$$

Then the solutions for the metric perturbations can be obtained as

$$\Phi = \frac{r_g}{2r} + \frac{1}{M_4}\hat{\pi},\tag{81}$$

$$\Psi = -\frac{r_g}{2r} + \frac{1}{M_4}\hat{\pi}.\tag{82}$$

On scales larger than the Vainshtein radius $r > r_*$, the solutions are given by

$$\Phi = \frac{r_g}{2r}\left(1 - \frac{1}{3\beta}\right), \tag{83}$$

$$\Psi = -\frac{r_g}{2r}\left(1 + \frac{1}{3\beta}\right). \tag{84}$$

These solutions can be described by BD theory with BD parameter $\omega = 3(1 - \beta)/2 = -3\mathcal{H}r_c$. This agrees with the analysis in the previous section. On scales smaller than the Vainshtein radius, $r < r_*$, the cubic interaction cannot be neglected. Then the linearized analysis cannot be trusted. In this case, the solutions for Ψ and Φ are obtained as

$$\Phi = \frac{r_g}{2r} + \text{sign}(\beta)\sqrt{\frac{r_g r}{2r_c^2}}, \tag{85}$$

$$\Psi = -\frac{r_g}{2r} + \text{sign}(\beta)\sqrt{\frac{r_g r}{2r_c^2}}. \tag{86}$$

In this region, the corrections to the solution in 4D GR are suppressed for $r < r_*$ so that Einstein gravity is recovered. From Eq. (73), we can see that Ξ dominates over the linear term in this region. This indicates that once $\hat{\pi}$ becomes non-linear, the solutions for the metric approach those in 4D GR. We should note that β is negative in the self-accelerating solution while β is positive in the normal branch solution. Then the corrections to 4D GR solutions have opposite signs in these solutions, as was first pointed out in Ref [16]. This means that the metric perturbations on small scales $r \ll H^{-1}$ is sensitive to the cosmological background solutions. We should note that Ref. [40] claimed that these solutions do not satisfy the full set of the non-linear equations. However, Ref. [19] showed that these solution are fully consistent with the 5D non-linear equations as long as we consider scales larger than r_g.

VI.   NON-PERTURBATIVE SOLUTIONS

In the previous section, we saw that the non-linear interaction of the brane bending mode is important below Vainstein length r_*. This also means that as long as we consider length scales larger than Vainstein radius, the linearized analysis can be trusted and we find a ghost. However, in massive gravity theory, it is known that the non-linear interactions are very subtle. As in the DGP model, the perturbative approach that takes into account non-linear interactions of the scalar mode (helicity-0 excitations) shows that the solution approaches GR near the source [41, 42]. However numerical attempts to find fully non-perturbative solutions have failed in massive gravity theory [43]. Then it is required to study fully non-linear solutions carefully to check the validity of the linear perturbations [44].

A.   Schwarzschild solution

The attempt to find fully non-perturbative Schwarzschild solution was made in Ref. [45]. They assume that the 5D metric takes the form

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\Omega^2 + 2\gamma dy dr + e^\sigma dy^2, \tag{87}$$

where y is the extra-dimensional coordinate. In general, we should solve 5D Einstein equations and impose the junction conditions to find solutions. Instead of solving the bulk, they assumed $\nu = -\lambda$ to close the equations without solving the bulk equations. Then, remarkably, they can manage to find analytic solutions for λ. In the normal branch, below the Vainshtein radius, the solution is given by [45, 46]

$$-\lambda = -\frac{r_g}{r} - 0.84\left(\frac{r}{r_c}\right)^2\left(\frac{r_*}{r}\right)^{2(\sqrt{3}-1)}, \tag{88}$$

which is similar to the solution obtained in the previous section. However, on scales larger than Vainstein scale, they find

$$-\lambda = -\frac{\tilde{r}_g^2}{r^2}, \quad \tilde{r}_g \sim r_g\left(\frac{r_c}{r_g}\right)^{1/3}, \tag{89}$$

which is the 5D solution and quite different from the previous 4D BD solution. Especially the mass of the source for observers located at $r > r_*$ is 'screened'. In a naive perturbative approach, we would expect that the large scale Newton potential behaves as

$$-\lambda = -\frac{r_{g,5}^2}{r^2}, \quad r_{g,5}^2 = 2G_5M = 2r_cr_g. \tag{90}$$

In the solution (89) the 5D gravitatinal length is smaller $\tilde{r}_g^2 \ll r_{g,5}^2$. This indicates that the effective mass $\tilde{M}$ measured at $r > r_*$ is screened by $\tilde{M} = M(r_g/r_c)^{1/3}$.

If this solution is a true non-perturbative solution in the DGP model, this means that the linearized perturbations cannot be trusted even at $r > r_*$. However, we should bear in mind that the solution is obtained by closing the equations by an *ad hoc* metric ansatz. It is far from trivial that the solution obtained in this way satisfies the reasonable boundary conditions in the bulk. In fact it is pointed out that the regularity condition in the bulk is crucial to specify the behaviour of gravity on the brane [38]. It is an open question to check that the solution obtained in Ref. [45] has a regular bulk solution.

However, this solution also shows interesting features in the self-accelerating background that might be related to the ghost. In the self-accelerating universe, the effective mass measured at $r > r_*$ becomes *negative*. This suggests that the self-accelerating background may not be problem-free even in the full non-linear theory.

B.   Domain wall

The other example of the exact non-linear solutions is a domain wall. Ref. [47] found an exact domain wall solution. In the 4D spacetime, the domain wall creates a jump in the extrinsic curvature. On the other hand, the domain wall in the 5D spacetime is a co-dimension 2 object like a string. Then the domain wall creates a deficit angle in 5D spacetime. The relation between the deficit angle δ and the tension of the domain wall μ is given by

$$2Hr_c\tan\gamma \pm \gamma = \frac{\sigma}{4M_5^3}, \tag{91}$$

where γ is related to the deficit angle as $\delta = \mp 4\gamma$, where $-$ is for the self-accelerating branch and $+$ is for the normal branch. The first term comes from the jump of the 3D extrinsic curvature in the brane as in 4D GR and the second term is the contribution from the deficit angle in the 5D spacetime.

For small tension $\sigma \ll M_5^3$, the deficit angle is given by

$$\delta = \frac{1}{1 - 2\mathcal{H}r_c}\frac{\sigma}{M_5^3}, \quad \mathcal{H} = \pm H. \tag{92}$$

In the normal branch, there is a screening of the tension. The deficit angle in the 5D spacetime is smaller than expected. On the other hand, in the self-accelerating branch solution, there is 'over-screening'. This indicates that the wall in the self-accelerating universe behaves as with *negative* tension. Then it appears that the stability of the solution is not granted. This may be related to the existence of the ghost. In fact, we see that the factor that determines the screening $1 - 2\mathcal{H}r_c$ is exactly the factor that determines the existence of the ghost.

VII.   CONCLUSIONS AND DISCUSSIONS

In this review, we discuss the ghost problem in the self-accelerating universe. First we studied the spectrum of the perturbations without matter perturbations about a de Sitter brane. For the positive tension brane, the ghost comes from the fact that there is a discrete mode for the spin-2 perturbations with mass $0 < m^2 < 2H^2$. The massive spin-2 perturbations contain a helicity-0 mode that becomes a ghost if the mass is in the range $0 < m^2 < 2H^2$. In the self-accelerating universe without tension, the mass becomes $m^2 = 2H^2$. This is a special mass in Pauli-Fierz massive gravity theory because there exists an 'enhanced symmetry' that eliminates the helicity-0 mode. However, in the DGP model, there is a spin-0 perturbation with the same

mass and this breaks the symmetry, leading to a ghost from the mixing between the spin-0 and spin-2 perturbations. For a negative tension brane, the spin-0 perturbation becomes a ghost if $Hr_c > 1/2$. It is easy to remove the spin-2 ghost by putting a second brane in the bulk and make the distance between the two branes small. Then the mass of the discrete mode of the spin-2 perturbations becomes larger than $2H^2$. However we find that the spin-0 perturbation, the radion, becomes a ghost. If we stabilize the radion, the perturbations of the scalar field that is necessary to stabilize the radion becomes a ghost.

We recovered the same result by studying the spectrum with matter source. The one particle exchange amplitude is the summation of massive spin-2 perturbations and a spin-0 perturbations. The amplitude of the massive spin-2 perturbations diverges if the mass is $m^2 = 2H^2$. In the DGP model, the spin-0 perturbation exactly cancels this singularity at $Hr_c = 1$ where the spin-2 mass becomes $m^2 = 2H^2$. This means that the spin-0 interaction is opposite to that of massive spin-2 perturbations. This is the reason why the spin-0 perturbation becomes a ghost once the massive spin-2 perturbation becomes healthy. At small scales, the effective theory for perturbations is described by the BD theory with BD parameter $\omega = -3Hr_c$. Here the BD scalar is a mix of the spin-0 perturbation and the helicity-0 component of spin-2 perturbations. In the BD theory, the BD scalar mode becomes a ghost if $\omega < -3/2$ and this condition is given by $Hr_c > 1/2$ which agrees with the spectrum analysis. This scalar can be identified as the brane bending mode. It is possible to construct the effective action for the brane bending by keeping the boundary terms in the full 5D action.

The effective action for the brane bending was shown to be a powerful tool to analyze the non-linear interactions of the scalar mode. In the decoupling limits where gravity decouples from the scalar interactions, the cubic interaction is the dominant non-linear interaction. For a local source with the gravitational length r_g, the scalar mode becomes strongly coupled below the Vainstein length $r_* = (r_g r_c^2)^{1/3}$. Then the linearized perturbations cannot be trusted below this length scale. We checked that the effective action for the brane bending reproduces the results obtained in the linearized analysis beyond r_*. Especially, the brane bending mode becomes a ghost in this linear regime if $Hr_c > 1/2$.

This seems to indicate that the linearized analysis makes sense at least at large scales $r > r_*$ and we cannot avoid the ghost. However, in massive gravity theory, fully non-perturbative effects are very subtle and it is possible that the linearized solution cannot be matched to the fully non-perturbed solutions. There are two known fully non-perturbed solutions. One is Schwarzschild solution obtained in Ref. [45]. Although it is an open question that this solution has a physical solution in the bulk, the solution does show that the effective mass measured at $r > r_*$ becomes negative in the self-accelerating universe. This may be related to the ghost in the perturbative solution. The other solution is a domain wall solution. In this case, the exact 5D solution is known. For a small domain wall tension, the domain wall has a negative deficit angle from 5D point of view in the self-accelerating branch solutions. This is yet another evidence that the self-accelerating universe suffers from the ghost instability even at non-perturbative level.

There are many open questions that deserve further studies. We will address some of issues.

Instabilities

The ghost has a wrong sign for its kinetic term. This means that the energy density can be indefinitely negative. This can lead the classical instability of the system. In Ref. [26] it is argued that the self-accelerating universe must be classically unstable. However, the ghost is found at linearized level and so far no classical instability was found at linearized level. We should carefully study the non-linear interactions to address the classical stability of the model and this is still an open question. See Ref. [48] for recent discussions on non-linear instabilities.

Quantum mechanically, the ghost leads to spontaneous pair creation of ghosts and normal particles. Once such a channel opens, Lorentz invariance leads to a divergence of the particle creation rate and the decay rate of the vacuum is infinite. The same problem occurs in so-called phantom cosmology where the ghost scalar field is introduced to explain the dark energy with equation of state smaller than -1. It is argued that we can avoid the rapid decay of the vacuum if there is a Lorentz non-invariant UV cut off of the order MeV in the phantom cosmology [49]. In the DGP model, the situation is more subtle [50]. If we consider the situation in which there is a spin-2 ghost, we need to treat the helicity zero mode in a different way. Otherwise, negative norm states appear. But if we take a different prescription for the quantization for the helicity zero mode, this procedure necessarily breaks de Sitter invariance. When there is a spin-0 ghost, a similar phenomenon happens. In this case, the mass of the ghost is given by $-4H^2$. But we know that there is no de Sitter invariant vacuum state for a scalar field with negative mass squared. Once de Sitter invariance is broken, one may be allowed to consider the possibility that the non-covariant cutoff scale may

arise due to the strong coupling effect. The strong coupling length is very large $\Lambda^{-1} \sim 1000$ km. Then the particle creation could be milder than the usual ghost in the Minkowski background [51]. Clearly, further studies are necessary to verify this.

The fate of instabilities

If the self-accelerating branch solutions have the ghost instability, then we are naturally lead to ask what does this solution decay to. An interesting fact is that the normal branch solutions are ghost-free. Then it is tempting to think that the self-accelerating solutions decay into the normal branch solutions. In fact for a given tension, the Hubble parameter in the self-accelerating universe is larger than that of the normal branch solutions. Then there could be the nucleation of bubbles of the normal branch in the environment of the self-accelerating branch solution. This would resemble a kind of false vacuum decay in de Sitter space. False vacuum decay is described by an instanton which is a classical solution in an Euclidean time connecting initial and final configurations. In our case, we are interested in a solution that interpolates between the self-accelerating and the normal branches. Ref. [50] tried to construct such an instanton solution. It was found that the solution requires the presence of a 2-brane (the bubble wall) which induces the transition. However, this instanton cannot be realized as the thin wall limit of any smooth solution. Once the bubble thickness is resolved, the equations of motion do not allow $O(4)$ symmetric solutions joining the two branches. It was concluded that the thin wall instanton is unphysical, and that one cannot have processes connecting the two branches. This suggests that the self-accelerating branch does not decay into the normal branch by forming normal branch bubbles. Thus it is still unclear what is the end state of the ghost instability.

Possible ways out of the ghost?

In Ref. [27], several ways out of the ghost problem were discussed. In the self-accelerating universe $Hr_c = 1$, there could be the enhanced symmetry that can eliminate the helicity-0 ghost if there were no spin-0 perturbation. Ref. [27] performed a gauge transformation

$$A_{\mu\nu} = B_{\mu\nu} + (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu})X, \quad X = \frac{3}{4H}\varphi. \tag{93}$$

Then the action becomes

$$S_{eff} = \frac{1}{\kappa^2 H} \int d^4 x \sqrt{-\gamma} \bigg\{ -B^{\mu\nu} X_{\mu\nu}(B) - H^2 B^{\mu\nu} B_{\mu\nu} + H^2 B^2 \\ - \varphi H (\nabla_\mu \nabla_\nu - \gamma_{\mu\nu} \Box_4 - 3H^2 \gamma_{\mu\nu}) B^{\mu\nu} \bigg\}. \tag{94}$$

They propose to treat the spin-0 perturbation φ as a Lagrangian multiplier and perform quantization a la Nakanishi-Lautrup way in the QED. Effectively, this removes φ from the spectrum and the enhanced symmetry ensures that there is no helicity-0 mode and no ghost (see Ref. [52] for a criticism on this procedure). However, this method cannot be applied if there is matter fluctuations with non-zero trace of energy-momentum tensor. This is because the trace of energy-momentum tensor breaks the enhanced symmetry and excite φ as we saw in section IV.A.

The other way to remove the ghost is to take into account the non-normalizable modes in the spectrum [27]. Then it is possible to eliminate the double pole from the amplitude that was the origin of the ghost. Instead, the amplitude has a single pole corresponding to the non-normalizable massless mode. However, it is found that this massless mode has an opposite sign for the amplitude compared with continuous massive states. This indicates that this massless mode becomes ghost-like. A very similar phenomena was found in the analysis of the shock wave analysis where the non-normalizable modes contribute a repulsive potential [26, 53].

Modifying the model

Finally, in order to remove the ghost from the theory, we may have to modify the starting action. There are several attempts to extend the model [28, 54–57]. As we explained in section III.D, it is impossible to remove the ghost in two-brane model [28]. It was also shown that the introduction of Gauss-Bonnet term in

the bulk does not help [54]. Recently, a new model was proposed where the bulk is the solution in the normal branch but the junction condition is the one in the self-accelerating branch [56]. In order to achieve this, the 5D Einstein -Hilbert action with a opposite sign to the conventional one is assumed to be localized near the brane. It remains to be seen whether this model can evade the ghost or not by studying the perturbations. The other approach would be consider higher co-dimensional branes [58] or intersecting branes [59]. It still remains to be seen whether there exists the self-accelerating universe and there is no ghost in these models. It is also important to study whether it is possible to embed the DGP model in string theory [60]. A UV completion of the model is necessary to study the fate of the ghost instability and it can guide us to make some simple modification of the model that will ameliorate the problems discussed in this review [48].

[1] B. P. Schmidt *et al.* [Supernova Search Team Collaboration], Astrophys. J. **507**, 46 (1998) [arXiv:astro-ph/9805200]. A. G. Riess *et al.* [Supernova Search Team Collaboration], Astron. J. **116**, 1009 (1998) [arXiv:astro-ph/9805201]; S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], Astrophys. J. **517** 565 (1999) [arXiv:astro-ph/9812133].
[2] P. Ruiz-Lapuente, Class. Quantum. Grav. **24** R91 (2007) [arXiv:0704.1058 [gr-qc]].
[3] D. N. Spergel *et al.* [WMAP Collaboration], arXiv:astro-ph/0603449.
[4] M. Tegmark *et al.* [SDSS Collaboration], Phys. Rev. D **69**, 103501 (2004) [arXiv:astro-ph/0310723].
[5] R. Maartens, arXiv:astro-ph/0602415.
[6] K. Koyama, arXiv:0706.1557 [astro-ph].
[7] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485**, 208 (2000) [arXiv:hep-th/0005016].
[8] K. Akama, Prog. Theor. Phys. **60**, 1900 (1978).
[9] C. Deffayet, Phys. Lett. B **502**, 199 (2001) [arXiv:hep-th/0010186].
[10] C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D **65** (2002) 044023 [arXiv:astro-ph/0105068].
[11] C. Deffayet, S. J. Landau, J. Raux, M. Zaldarriaga and P. Astier, Phys. Rev. D **66**, 024019 (2002) [arXiv:astro-ph/0201164].
[12] C. Deffayet, G. R. Dvali, G. Gabadadze and A. I. Vainshtein, Phys. Rev. D **65** (2002) 044026 [arXiv:hep-th/0106001];
[13] A. Lue, Phys. Rev. D **66** (2002) 043509 [arXiv:hep-th/0111168].
[14] A. Gruzinov, New Astron. **10** (2005) 311 [arXiv:astro-ph/0112246];
[15] T. Tanaka, Phys. Rev. D **69** (2004) 024001 [arXiv:gr-qc/0305031].
[16] A. Lue and G. Starkman, Phys. Rev. D **67** (2003) 064002 [arXiv:astro-ph/0212083].
[17] A. Lue, R. Scoccimarro and G. D. Starkman, Phys. Rev. D **69**, 124015 (2004) [arXiv:astro-ph/0401515].
[18] C. Middleton and G. Siopsis, Mod. Phys. Lett. A **19**, 2259 (2004) [arXiv:hep-th/0311070].
[19] K. Koyama and F. P. Silva, Phys. Rev. D **75**, 084040 (2007) [arXiv:hep-th/0702169].
[20] A. Lue, Phys. Rept. **423**, 1 (2006) [arXiv:astro-ph/0510068].
[21] V. A. Rubakov, arXiv:hep-th/0303125;
[22] M. A. Luty, M. Porrati and R. Rattazzi, JHEP **0309** (2003) 029 [arXiv:hep-th/0303116];
[23] A. Nicolis and R. Rattazzi, JHEP **0406** (2004) 059 [arXiv:hep-th/0404159];
[24] K. Koyama, Phys. Rev. D **72** (2005) 123511 [arXiv:hep-th/0503191].
[25] D. Gorbunov, K. Koyama and S. Sibiryakov, Phys. Rev. D **73** (2006) 044016 [arXiv:hep-th/0512097].
[26] C. Charmousis, R. Gregory, N. Kaloper and A. Padilla, JHEP **0610** (2006) 066 [arXiv:hep-th/0604086].
[27] C. Deffayet, G. Gabadadze and A. Iglesias, JCAP **0608** (2006) 012 [arXiv:hep-th/0607099].
[28] K. Izumi, K. Koyama and T. Tanaka, [arXiv:hep-th/0610282].
[29] K. Koyama and K. Koyama, Phys. Rev. D **72** (2005) 043511 [arXiv:hep-th/0501232].
[30] J. Garriga and T. Tanaka, Phys. Rev. Lett. **84** (2000) 2778; T. Tanaka, Phys. Rev. **D69** (2004) 024001.
[31] A. Padilla, Class. Quant. Grav. **21** (2004) 2899.
[32] M. Fierz and W. Pauli, Proc. Roy. Soc. **173** (1939) 211.
[33] H. van Dam and M. Veltman, Nucl. Phys. **B22** (1970) 397; V. I. Zakharov, JETP Lett. **12** (1970) 312.
[34] A. Higuchi, Nucl. Phys. **B282** (1987) 397; S. Deser and R. I. Nepomechie, Ann. Phys. **154** (1984) 396.
[35] S. Deser and A Waldron, Phys. Lett. **B508** (2001) 347.
[36] M. Porrati, Phys. Lett. B **498** (2001) 92 [arXiv:hep-th/0011152].
[37] K. Koyama and R. Maartens, JCAP **0601** (2006) 016 [arXiv:astro-ph/0511634].
[38] K. Koyama and S. Mizuno, JCAP **0607**, 013 (2006) [arXiv:gr-qc/0606056].
[39] C. Deffayet, Phys. Rev. D **71**, 023520 (2005) [arXiv:hep-th/0409302].
[40] G. Gabadadze and A. Iglesias, Phys. Lett. B **639**, 88 (2006) [arXiv:hep-th/0603199].
[41] A. I. Vainshtein, Phys. Lett. B **39**, 393 (1972).
[42] N. Arkani-Hamed, H. Georgi and M. D. Schwartz, Annals Phys. **305**, 96 (2003) [arXiv:hep-th/0210184].

[43] T. Damour, I. I. Kogan and A. Papazoglou, Phys. Rev. D **67**, 064009 (2003) [arXiv:hep-th/0212155].

[44] G. Dvali, New J. Phys. **8** (2006) 326 [arXiv:hep-th/0610013].

[45] G. Gabadadze and A. Iglesias, Phys. Rev. D **72** (2005) 084024 [arXiv:hep-th/0407049].

[46] G. Gabadadze and A. Iglesias, Phys. Lett. B **639**, 88 (2006) [arXiv:hep-th/0603199].

[47] G. Dvali, G. Gabadadze, O. Pujolas and R. Rahman, [arXiv:hep-th/0612016].

[48] R. Gregory, N. Kaloper, R. C. Myers and A. Padilla, arXiv:0707.2666 [hep-th].

[49] J. M. Cline, S. Jeon and G. D. Moore, Phys. Rev. D **70**, 043543 (2004) [arXiv:hep-ph/0311312].

[50] K. Izumi, K. Koyama, O. Pujolas and T. Tanaka, arXiv:0706.1980 [hep-th].

[51] K. Izumi and T. Tanaka, arXiv:0709.0199 [gr-qc].

[52] A. Padilla, arXiv:hep-th/0610093.

[53] N. Kaloper, Phys. Rev. Lett. **94**, 181601 (2005) [Erratum-ibid. **95**, 059901 (2005)] [arXiv:hep-th/0501028]; N. Kaloper, Phys. Rev. D **71**, 086003 (2005) [Erratum-ibid. D **71**, 129905 (2005)] [arXiv:hep-th/0502035].

[54] C. de Rham and A. J. Tolley, JCAP **0607**, 004 (2006) [arXiv:hep-th/0605122].

[55] M. Carena, J. Lykken, M. Park and J. Santiago, Phys. Rev. D **75**, 026009 (2007) [arXiv:hep-th/0611157].

[56] G. Gabadadze, arXiv:hep-th/0612213; G. Gabadadze, arXiv:0705.1929 [hep-th].

[57] A. S. Koshelev and T. N. Tomaras, arXiv:0706.3393 [hep-th].

[58] G. R. Dvali and G. Gabadadze, Phys. Rev. D **63**, 065007 (2001) [arXiv:hep-th/0008054]; S. L. Dubovsky and V. A. Rubakov, Phys. Rev. D **67**, 104014 (2003) [arXiv:hep-th/0212222]; M. Kolanovic, M. Porrati and J. W. Rombouts, Phys. Rev. D **68**, 064018 (2003) [arXiv:hep-th/0304148]; G. Gabadadze and M. Shifman, Phys. Rev. D **69**, 124032 (2004) [arXiv:hep-th/0312289]; N. Kaloper and D. Kiley, arXiv:hep-th/0703190.

[59] O. Corradini, K. Koyama and G. Tasinato, to appear.

[60] E. Kiritsis, N. Tetradis and T. N. Tomaras, JHEP **0108**, 012 (2001) [arXiv:hep-th/0106050]; I. Antoniadis, R. Minasian and P. Vanhove, Nucl. Phys. B **648**, 69 (2003) [arXiv:hep-th/0209030].