5th March 2007
hep-th/0703040

Ekpyrotic collapse with multiple fields

Kazuya Koyama [1] and David Wands [2]

Institute of Cosmology and Gravitation, Mercantile House, University of Portsmouth, Portsmouth PO1 2EG, United Kingdom

Abstract

A scale invariant spectrum of isocurvature perturbations is generated during collapse in the scaling solution in models where two or more fields have steep negative exponential potentials. The scale invariance of the spectrum is realised by a tachyonic instability in the isocurvature field. We show that this instability is due to the fact that the scaling solution is a saddle point in the phase space. The late time attractor is identified with a single field dominated ekpyrotic collapse in which a steep blue spectrum for isocurvature perturbations is found. Although quantum fluctuations do not necessarily to disrupt the classical solution, an additional preceding stage is required to establish classical homogeneity.

1 Introduction

Given the success of simple inflation models in explaining the primordial homogeneity and almost scale-invariant spectrum of curvature perturbations on large scales, it is important to consider whether there is any other model for the early universe that might provide an alternative explanation. One contender is the idea of an early collapse phase rather than an accelerated expansion. This invokes the same basic mechanism as inflation to generate large-scale perturbations. A shrinking comoving Hubble scale can take quantum vacuum fluctuations of scalar fields on small scales and evolve them into overdamped perturbations on super-Hubble scales. But the approach to solving the homogeneity problem is quite different.

In pre big bang model of Gasperini and Veneziano [1] the collapsing universe (in the Einstein frame [2]) is dominated by the kinetic energy of massless fields in the low energy string effective action. Such a solution is an attractor in a collapsing universe for a wide range of interaction potentials so offers a solution to the homogeneity problem, though this kinetic-dominated collapse is only marginally stable with respect to anisotropic shear [3]. By contrast the ekpyrotic scenario proposed an ultra-stiff collapse phase dominated by a steep negative potential [4] (see also [5, 6]). This is a late time attractor and is stable with respect to anisotropy.

[1] E-mail: Kazuya.Koyama@port.ac.uk
[2] E-mail: David.Wands@port.ac.uk

On the other hand both models produce a steep blue spectrum of perturbations (in the spatially flat gauge) in the fields driving the collapse and hence in the comoving curvature perturbation [7, 8, 9, 10]. A collapse phase dominated by a field with a pressureless equation of state does yield a scale-invariant spectrum of comoving curvature perturbations [11, 12, 13] but it is not a late time attractor [14, 15].

It was shown in [16] that axion fields in the pre big bang can acquire a scale-invariant spectrum of isocurvature field perturbations in a pre big bang phase and that this could in principle be converted to curvature perturbations at a bounce [17] or some time after the bounce via the curvaton mechanism [18, 19]. The idea of producing a scale-invariant spectrum of isocurvature perturbations in axion fields was also investigated in the ekpyrotic scenario [20, 21].

In the ekpyrotic scenario it is also possible to generate a scale-invariant spectrum of isocurvature perturbations in models where two or more fields have steep exponential potentials. In this case there exists a scaling solution [22, 23] where the energy densities of the fields grow at the same rate during collapse, analogous to assisted inflation solutions in an expanding universe [24]. Several authors [25, 26, 27] have recently proposed new ekpyrotic models based on this scaling solution.

If a scale-invariant spectrum of comoving curvature perturbations can be generated during the collapse phase then there is still the "graceful exit problem" of turning collapse to expansion. In the pre big bang scenario it was envisaged that such a bounce could be achieved by higher-order corrections to string effective action which violate the null energy condition [28, 29, 30]. A bounce due to higher-order kinetic terms in a ghost condensate model has recently been considered in the new ekpyrotic scenarios [26, 27] (see also [31, 32]).

In this paper we discuss the tachyonic instability of long wavelength perturbations about the new ekpyrotic scaling solution which leads to the scale-invariant spectrum [22, 25, 26, 27]. We show that the solution is not an attractor at early or late times, but rather a saddle point in the phase-space. The late time attractor is the old ekpyrotic collapse dominated by a single field and the instability drives the scaling solution to this late time attractor. By contrast we find steep blue spectra for isocurvature field perturbations about the kinetic dominated early time attractors, or single-field dominated ekpyrotic late time attractors. We present the phase space analysis in an Appendix.

2 Homogeneous field dynamics

During the ekpyrotic collapse the expansion of the universe is assumed to be described by a 4D Friedmann equation in the Einstein frame with scalar fields with negative exponential potentials

$$3H^2 = V + \frac{1}{2}\dot{\phi}_i^2 \,, \tag{1}$$

where

$$V = -\sum_i V_i e^{-c_i \phi_i} \,, \tag{2}$$

and we take $V_i > 0$ and set $8\pi G$ equal to unity.

The authors of [25] found a scaling solution (previously studied in [22, 23]) in which both fields roll down their potential as the universe approaches a big crunch singularity, analogous to the assisted inflation dynamics found by [24] for positive potentials in an expanding universe. In this assisted ekpyrotic collapse we find power-law solution for the scale factor

$$a \propto (-t)^p \,, \quad \text{where } p = \sum_i \frac{2}{c_i^2} \,, \tag{3}$$

where

$$\frac{\dot{\phi}_i^2}{\dot{\phi}_j^2} = \frac{-V_i e^{-c_i \phi_i}}{-V_j e^{-c_j \phi_j}} = \frac{c_j^2}{c_i^2} \,. \tag{4}$$

As in the case of assisted inflation, this behaviour is easily understood after a rotation in field space [22, 34] into so-called adiabatic field, σ, along the background trajectory (4), and the orthogonal, entropy field directions, χ_i, [33]. The potential (2) can then be simply re-written as

$$V = -U(\chi_i)\, e^{-c\sigma} \,, \tag{5}$$

where

$$\frac{1}{c^2} = \sum_i \frac{1}{c_i^2} \,. \tag{6}$$

To simplify the analysis we restrict ourselves to the case of two scalar fields, though the general discussion applies to an arbitrary number of fields. In this case we have

$$\sigma = \frac{c_2\phi_1 + c_1\phi_2}{\sqrt{c_1^2 + c_2^2}} \,, \quad \chi = \frac{c_1\phi_1 - c_2\phi_2}{\sqrt{c_1^2 + c_2^2}} \,, \tag{7}$$

and we can write the explicit dependence of the potential (5) on the orthogonal field [34, 22]

$$U(\chi) = V_1\, e^{-(c_1/c_2)c\chi} + V_2\, e^{(c_2/c_1)c\chi} \,. \tag{8}$$

This is clearly a positive function (given $V_1 > 0$ and $V_2 > 0$) and bounded from below. Close to its minimum we can expand this as [34, 22]

$$U(\chi) = U_0 \left[1 + \frac{c^2}{2}(\chi - \chi_0)^2 + \ldots \right] . \tag{9}$$

Thus we can confirm that there is a classical trajectory for the two fields in which χ remains fixed, $\chi = \chi_0$, while the adiabatic field σ rolls down a steep exponential potential (5), which reduces to

$$V|_{\chi=\chi_0} = -U_0\, e^{-c\sigma} \,, \tag{10}$$

and we identify the scaling solution (3) with the usual power law solution for a single field solution for a negative exponential potential which exists for $c^2 > 6$ [14, 15].

However we also see that the entropy field, χ, has a negative mass-squared

$$m_\chi^2 = c^2 V < 0 \,. \tag{11}$$

Note this effective mass-squared is a function only of the same parameter c that appears in the exponential potential for σ and not any other combination of the individual values c_1 and c_2.

Relative to the Hubble scale we have

$$\eta_\chi \equiv \frac{m_\chi^2}{3H^2} = -\frac{2}{3}\epsilon^2 + 2\epsilon \,, \tag{12}$$

where we define the dimensionless parameter

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \,, \tag{13}$$

and we have $\epsilon = 1/p$ for a power-law solution, and $\epsilon \gg 1$ during ekpyrotic collapse.

Thus there is a strong tachyonic instability associated with the ekpyrotic scaling solution (4).

There is a simple explanation for why the scaling solution is unstable. This represents evolution along an effective exponential potential (5) which is less steep than any of the individual potentials (2) from which it was constructed:

$$c^2 < c_i^2 \ \forall\ i \,. \tag{14}$$

This is what makes assisted inflation solutions stable in an expanding universe as the field direction with the flatest potential tends to win out at late times. In a collapsing universe, by contrast, the field direction with the steepest (negative) gradient and hence the stiffest equation of state tends comes to dominate at late times.

A full phase space analysis is presented in an Appendix. In addition to the scaling solution we have fixed points corresponding to any one of the original fields ϕ_i dominating the energy density where the other fields have negligible energy density. These correspond to power-law solutions where

$$p = \frac{2}{c_i^2} \,, \tag{15}$$

for $c_i^2 > 6$. We find that any of these single dominant field solutions is a stable local attractor at late times during collapse. We present the phase-space analysis in Appendix A.

Finally we note that the scaling solution (4) is also not the early time attractor. Restricting our attention to solutions along the adiabatic field direction, where the entropy fields remain in the minimum value of $U(\chi_i)$, we can use the phase-plane analysis for a single scalar field with a steep negative potential [14]. These tend to be dominated at early times in a collapsing universe by the kinetic energy of the scalar fields and the potential is negligible. This corresponds to a power law solution this time with

$$p = \frac{1}{3} \,. \tag{16}$$

This is the original pre big bang proposal [1] for an early collapse phase dominated by the kinetic energy of effectively massless moduli fields (see also [17, 35]).

In summary, we confirm that there exists a scaling solution corresponding to the combined evolution of two or more scalar fields rolling down their exponential potentials. However this solution is a saddle point in the classical phase-space of the system (see Fig. 1 in Appendix A), representing either a set of measure zero in terms of initial conditions, or a transient evolution between the early- and late-time attractors that we have identified.

3 Perturbations

For a collapse model where $\ddot{a} < 0$, small-scale fluctuations that start far inside the Hubble length, H^{-1}, contract less slowly than the Hubble length and may enter an overdamped long-wavelength regime on super-Hubble scales, just as sub-Hubble modes are streched up to super-Hubble scales during an inflationary expansion. Thus initial vacuum fluctuations on small scales can provide the initial state for large (super-Hubble) scale perturbations during collapse.

We will consider the evolution of isocurvature modes in each of the power-law background solutions in the previous section.

Linear perturbations in fields orthogonal to the background trajectory are decoupled from first-order metric perturbations so long as the trajectory remains straight in field-space. They obey the wave equation for a massive field in an unperturbed FRW metric

$$\ddot{\delta\chi} + 3H\dot{\delta\chi} + \left(\frac{k^2}{a^2} + m_\chi^2\right)\delta\chi = 0\,. \tag{17}$$

Note that such isocurvature perturbations are automatically gauge-invariant. Introducing the rescaled field $v = a\delta\chi$, and writing the wave equation in terms of conformal time $\tau = \int dt/a$, we have

$$v'' + \left[k^2 - \frac{a''}{a} + m_\chi^2 a^2\right] v = 0\,, \tag{18}$$

where a prime denotes derivatives with respect to τ. For any power-law solution $a \propto |t|^p$ we have

$$aH = \frac{1}{(\epsilon - 1)\tau}\,, \tag{19}$$

and

$$\frac{a''}{a} = -(\epsilon - 2)a^2H^2\,. \tag{20}$$

Thus for any isocurvature field we can write

$$v'' + \left[k^2 + \frac{\epsilon + 3\eta_\chi - 2}{(\epsilon - 1)^2\tau^2}\right] v = 0\,, \tag{21}$$

where we have used the dimensionless fast (or slow) roll parameters $\epsilon = 1/p$ and η_χ, defined in Eq. (12).

Using the usual Bunch-Davies vacuum state to normalise the amplitude of fluctuations at early times, we obtain the

$$v = \frac{\sqrt{\pi}}{2}\frac{e^{i(\nu+1/2)\frac{\pi}{2}}}{k^{1/2}}(-k\tau)^{1/2}H_\nu^{(1)}(-k\tau)\,, \tag{22}$$

where the order of the Hankel function is given by

$$\nu^2 = \frac{9}{4} - \frac{2\epsilon^2 - 3\epsilon + 3\eta_\chi}{(\epsilon - 1)^2}\,, \tag{23}$$

and we take $\nu \geq 0$ without loss of generality. At late times, $-k\tau \gg \nu$, this yields [13]

$$\mathcal{P}_{\delta\chi} \equiv \frac{k^3}{2\pi^2}|\delta\chi^2| = C_\nu^2 \frac{k^2}{a^2}(-k\tau)^{1-2\nu}\,, \tag{24}$$

where $C_\nu = 2^{\nu-3/2}\Gamma(\nu)/\pi^{3/2}$, and hence we obtain a spectrum of isocurvature perturbations on large, super-Hubble, scales with spectral tilt

$$\Delta n_{\delta\chi} \equiv \frac{d\ln \mathcal{P}_{\delta\chi}}{d\ln k} = 3 - 2\nu\,. \tag{25}$$

3.1 Scaling solution

We have seen that the isocurvature field has a tachyonic instability with a negative effective mass-squared, given in Eq. (12). Substituting this into the wave equation (21) we obtain

$$\nu^2 = \frac{9}{4} - \frac{3\epsilon}{(\epsilon-1)^2}\,. \tag{26}$$

Thus from Eq. (25) we obtain a scale-invariant spectrum for the isocurvature perturbations either as $\epsilon \to 0$, corresponding to slow-roll inflation, or as $\epsilon \to \infty$, corresponding to ekpyrotic collapse. To leading order in a fast-roll expansion ($\epsilon \gg 1$) we obtain

$$\Delta n_{\delta\chi} \simeq \frac{2}{\epsilon}\,. \tag{27}$$

Thus we obtain a slightly blue spectrum for a steep exponential potential [25, 26, 27], but becoming scale-invariant as $\epsilon \to \infty$. Any deviations from an exponential potential for the adiabatic field, and hence the exact power-law collapse, introduces corrections into the tilt [25, 26], but we note that such corrections could also alter the effective mass η_χ and the tilt becomes model-dependent.

3.2 Single-field dominated solution

As remarked earlier, the scaling solution during collapse (3) is unstable with respect to collapse dominated by any single field, ϕ_i. In this case fluctuations in any orthogonal field, ϕ_j, correspond to isocurvature perturbations decoupled from ϕ_i and first-order metric perturbations, so we can again use Eq. (17) where we identify $\delta\chi = \delta\phi_j$. However in this case the effective mass of the isocurvature field,

$$m_\chi^2 = -c_j^2 V_j e^{-c_j\phi_j}\,, \tag{28}$$

is much less than the Hubble rate, H^2. Substituting the dimensionless mass parameter, $\eta_\chi \simeq 0$, into Eq. (21), we find

$$\nu^2 \simeq \frac{1}{4}\,, \tag{29}$$

and thus the spectral tilt

$$\Delta n_2 = 2\,. \tag{30}$$

We recover the same steep blue spectrum for isocurvature perturbations during (stable) ekpyrotic collapse dominated by a single-field, as is found for perturbations in the dominant, adiabatic field [7].

3.3 Kinetic-dominated solution

We have seen that an early time attractor is the kinetic dominated evolution with $p = 1/3$ proposed in the pre-big bang scenario. In this limit all the scalar field potentials are negligible and all field perturbations, adiabatic and isocurvature, acquire steep blue spectra with $\Delta n = 3$, if they have canonical kinetic terms [17].

On the other hand there are pseudo-scalar axion fields with non-minimal kinetic terms. Such isocurvature fields with a kinetic coupling to dilaton-type fields can acquire a scale-invariant spectrum of perturbations during kinetic dominated collapse as first noted in Ref. [16] (see also [17]).

4 Discussion

We have seen that in this model we are only able to generate a scale-invariant spectrum of isocurvature field perturbations during collapse in the scaling solution (3) which is a saddle point in the phase space of the system [23]. Thus it is neither the early, nor late time attractor of the system.

As a result it does not solve the classical homogeneity problem. Large scale inhomogeneities in the initial spatial distribution of the fields will grow during the collapse phase, in contrast to the original single-field ekpyrotic collapse.

Even if some preceding phase drives the classical background solution to this unstable fixed point throughout space, one might worry that quantum fluctuations could destabilise this classical evolution. We see from Eq. (24) that for long wavelength perturbations we have $\delta\chi \propto \tau^{1-2\nu}/a^2$ and in the ekpyrotic scaling solution where $\nu \sim 3/2$ this gives $\delta\chi \sim H$.

Indeed it is easy to see that we always require an instability of this form in a canonical scalar field in order to generate a scale invariant spectrum during collapse, as the amplitude of field perturbations at Hubble exit are also of order H and thus we require the super-Hubble perturbations to grow at the same rate to maintain a scale-invariant spectrum. Precisely the same form of instability appears in the power-law collapse model with $p = 2/3$ which produces a scale-invariant spectrum of comoving curvature perturbations from quantum fluctuations [11, 12, 15, 13].

As H is rapidly growing in the ekpyrotic scenario ($\epsilon \gg 1$) this corresponds to a rapid tachyonic growth of the entropy field. On the other hand one should remember that the minimum duration of such a phase is also set by the rate of change of H. The change in the Hubble rate H determiness the range of comoving scales that exit the Hubble scale (as the scale factor a changes only slowly). Thus it might be sufficient for $|H|$ and thus $\delta\chi$ to grow by a factor of $k_{\rm max}/k_{\rm min} \sim 10^5$ while observable scales exit the Hubble scale.

The actual density of long wavelength quantum fluctuations (per logarithmic range in wavelength) either from the kinetic energy or potential energy density of the entropy field fluctuations, can be estimated relative to the total energy density to be

$$\Omega_\chi \sim \epsilon H^2 \,. \tag{31}$$

Thus the fractional density of the entropy modes remains small and does not threaten to destabilise the classical evolution so long as the Hubble rate remains well below the Planck scale (here set to unity).

In any case we do not necessarily expect the scaling solution, or any classical solution to hold arbitrarily close to the Planck scale, especially if we want to appeal to high-energy corrections to produce a bounce in the scale-factor, as envisaged in pre big bang models [1, 35] and now proposed in ekpyrotic models [26, 27].

We note that the existence of a natural turning point in the field-space trajectory due to the instability of the scaling solution offers the possibility of converting the scale-invariant spectrum of isocurvature field perturbations into a scale-invariant spectrum of curvature perturbations before the singularity or bounce without introducing any additional mechansim [25, 26, 27].

Thus the ekpyrotic collapse with multiple fields offers an alternative scenario for the origin of structure in our Universe to contrast with inflationary models. However to do so we seem to require at least four distinct phases in the evolution: (1) an initial phase to establish spatial homogeneity, (2) a scaling collapse phase to produce a scale-invariant spectrum of field perturbations, (3) a phase to turn isocurvature field perturbations into curvature perturbations, and (4) a bounce phase to turn collapse into expansion. Note that phases (3) and (4) could happen in the reverse order, as in the curvaton mechanism. By comparison, the simplest inflationary models require only a single quasi-de Sitter phase to establish spatial homogeneity plus a scale-invariant spectrum of curvature perturbations.

A Appendix

In this appendix, we present the phase space analysis for mutli-fields. Note that a similar analysis was done in Ref. [23] for the case $c_1 = c_2$ in an expanding universe. The field equations and an acceleration equation are

$$\phi_i'' + 2h\phi_i' + a^2 c_i V_i e^{-c_i\phi_i} = 0, \tag{32}$$

$$h' = \frac{1}{3}\sum_i(-\phi_i'^2 - a^2 V_i e^{-c_i\phi_i}), \tag{33}$$

where prime denotes a derivative with respect to a conformal time and $h = a'/a$. The Friedman equation is given by

$$h^2 = \frac{1}{3}\sum_i\left(\frac{1}{2}\phi_i'^2 - a^2 V_i e^{-c_i\phi_i}\right). \tag{34}$$

We define the dimensionless phase-space variables

$$x_i = \frac{\phi_i'}{\sqrt{6}h}, \tag{35}$$

$$y_i = \frac{a\sqrt{V_i e^{-c_i\phi_i}}}{\sqrt{3}h}. \tag{36}$$

As we are interested in a contracting universe, $y_i < 0$. The field equations and the evolution equation for a give the first order evolution equations for the phase space variables

$$\frac{dx_i}{dN} = -3x_i(1 - \sum_j x_j^2) - c_i\sqrt{\frac{3}{2}}y_i^2, \tag{37}$$

$$\frac{dy_i}{dN} = y_i \left(3 \sum x_j^2 - c_i \sqrt{\frac{3}{2}} x_i \right), \tag{38}$$

$$\tag{39}$$

where $N = \log a$. The Friedmann equation gives a constraint

$$\sum_j x_j^2 - \sum_j y_j^2 = 1. \tag{40}$$

There are $n+2$ fixed points of the system where $dx_i/dN = dy_i/dN = 0$.

$$A: \qquad \sum_j x_j^2 = 1, \quad y_j = 0. \tag{41}$$

$$B_i: \qquad x_i = \frac{c_i}{\sqrt{6}}, \quad y_i = -\sqrt{\frac{c_i^2}{6} - 1}, \quad x_j = y_j = 0, \ (\text{for} \quad j \neq i), \tag{42}$$

$$B: \qquad x_j = \frac{\sqrt{6}}{3p} \frac{1}{c_j}, \quad y_j = \sqrt{\frac{2}{c_j^2 p} \left(\frac{1}{3p} - 1 \right)}, \tag{43}$$

where

$$p = \sum_j \frac{2}{c_j^2}. \tag{44}$$

A is a series of kinetic dominated solutions where the potential energy can be neglected. B_i is a single field scaling solution where one of the fields is dominating the energy density. B is the scaling solution which is realized by the combined evolution of multiple fields. Note that at the fixed point B, the solution for the scale factor is given by

$$a = (-t)^p = (-\tau)^{-p/(p-1)}. \tag{45}$$

To study the stability of the solutions, the linear perturbations around each fixed point have to be analyzed. For simplicity we restrict ourselves to the case of two fields. We can eliminate y_2 using the constraint $y_2^2 = -1 + x_1^2 + x_2^2 - y_1^2$. Then the phase space becomes three-dimensional space spanned by (x_1, x_2, y_1). There are four fixed points A, B_1, B_2 and B. The solution for the linearized perturbations are characterized by three eigenfunctions:

$$\delta x = u_1 \exp(m_1 N) + u_2 \exp(m_2 N) + u_3 \exp(m_3 N). \tag{46}$$

Stability during collapse ($N \to -\infty$) requires that the real parts of all the eigenvalues m_i are positive. The eigenvalues are given by

$$A: \quad m_1 = 0, \ m_2 = 3 - \frac{\sqrt{6}}{2} c_1 x_1, \ m_3 = 6 - \sqrt{6} c_2 x_2, \tag{47}$$

$$B_1: \quad m_1 = c_1^2, \ m_2 = -3 + \frac{c_1^2}{2}, \ m_3 = -3 + \frac{c_1^2}{2}, \tag{48}$$

$$B_2: \quad m_1 = \frac{c_2^2}{2}, \ m_2 = -3 + \frac{c_2^2}{2}, \ m_3 = -3 + \frac{c_2^2}{2}, \tag{49}$$

$$B: \quad m_1 = -\frac{1}{p}(3p - 1), \ m_2 = m_+, \ m_3 = m_-, \tag{50}$$

where

$$m_\pm = \frac{1}{2p}(3p-1)\left(-1 \mp \sqrt{\frac{3(p-3)}{3p-1}}\right). \tag{51}$$

The kinetic-dominated solutions A are unstable for $c_1 x_1 > \sqrt{6}$ and $c_2 x_2 > \sqrt{6}$. The single field scaling solutions, B_i, exist for $c_i > \sqrt{6}$ and it is always stable. The scaling solution B exists for $p < 1/3$ and there is always one negative eigenvalue, which means that the scaling solution is a saddle point. The instability mode is given by

$$\delta x \propto e^{m_- N} \propto (-t)^{p m_-}. \tag{52}$$

For small $p \ll 1$, $m_- = -1/p$, thus $\delta x \propto (-t)^{-1} \propto H$. This is exactly the instability that the entropy field exhibits around the scaling solution. In fact, in terms of $\epsilon = 1/p$, m_- is written as

$$m_- = -\frac{1}{2}(3-\epsilon) + \frac{1}{2}\sqrt{3(3\epsilon-1)(\epsilon-3)}. \tag{53}$$

Then we can show that the solutions for the entropy field Eq. (22) is given by

$$\delta\chi \propto e^{N m_-}. \tag{54}$$

Figure 1 shows the phase space trajectories around B. We see that all solutions go to the stable fixed points B_1 or B_2 at late times. It is clear that B is a saddle point and any solutions near B go to B_1 or B_2 at late times. This is the origin of the instability of the scaling solution.

References

[1] M. Gasperini and G. Veneziano, Astropart. Phys. **1**, 317 (1993) [arXiv:hep-th/9211021].

[2] M. Gasperini and G. Veneziano, Mod. Phys. Lett. A **8**, 3701 (1993) [arXiv:hep-th/9309023].

[3] K. E. Kunze and R. Durrer, Class. Quant. Grav. **17**, 2597 (2000) [arXiv:gr-qc/9912081].

[4] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D **64**, 123522 (2001) [arXiv:hep-th/0103239].

[5] R. Kallosh, L. Kofman and A. D. Linde, Phys. Rev. D **64**, 123523 (2001) [arXiv:hep-th/0104073].

[6] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, arXiv:hep-th/0105212.

[7] D. H. Lyth, Phys. Lett. B **524**, 1 (2002) [arXiv:hep-ph/0106153].

[8] R. Brandenberger and F. Finelli, JHEP **0111**, 056 (2001) [arXiv:hep-th/0109004].

[9] J. c. Hwang, Phys. Rev. D **65**, 063514 (2002) [arXiv:astro-ph/0109045].

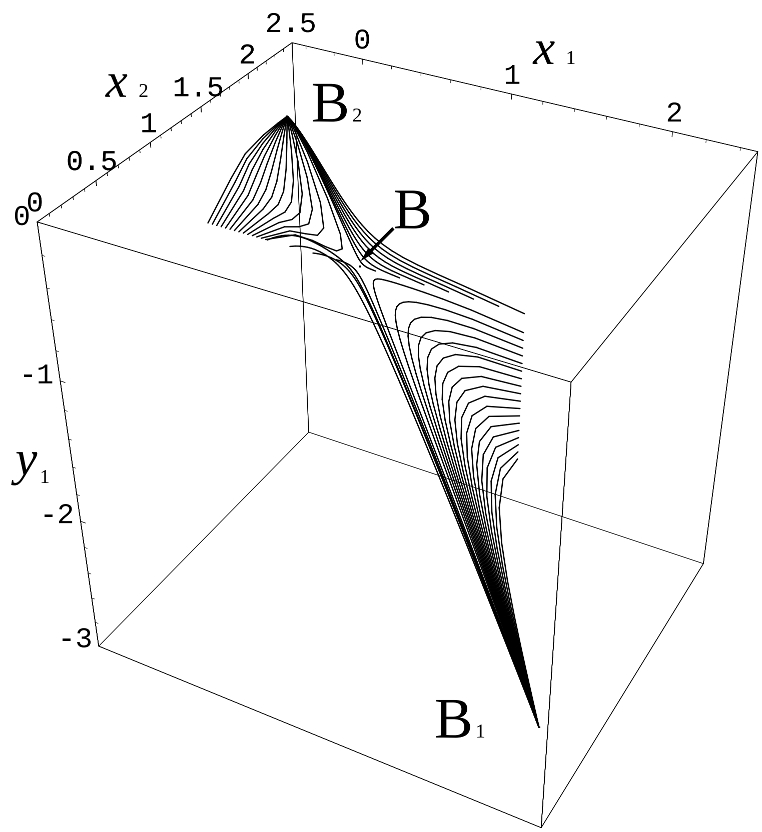

Figure 1: Phase space trajectories around the fixed point B in three dimensional phase space spanned by (x_1, x_2, y_1). We see that the single field fixed points B_1 and B_2 are late time attractors and the scaling solution B is a saddle point. We take $c_1 = 6$ and $c_2 = 4$.Then the fixed points are given by $B_1 : (2.45, 0, -2.24)$, $B_2 : (0, 1.63, 0)$ and $B : (0.75, 1.13, -0.51)$.

[10] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D **66**, 046005 (2002) [arXiv:hep-th/0109050].

[11] D. Wands, Phys. Rev. D **60**, 023507 (1999) [arXiv:gr-qc/9809062].

[12] F. Finelli and R. Brandenberger, Phys. Rev. D **65**, 103522 (2002) [arXiv:hep-th/0112249].

[13] L. E. Allen and D. Wands, Phys. Rev. D **70**, 063515 (2004) [arXiv:astro-ph/0404441].

[14] I. P. C. Heard and D. Wands, Class. Quant. Grav. **19**, 5435 (2002) [arXiv:gr-qc/0206085].

[15] S. Gratton, J. Khoury, P. J. Steinhardt and N. Turok, Phys. Rev. D **69**, 103505 (2004) [arXiv:astro-ph/0301395].

[16] E. J. Copeland, R. Easther and D. Wands, Phys. Rev. D **56**, 874 (1997) [arXiv:hep-th/9701082].

[17] J. E. Lidsey, D. Wands and E. J. Copeland, Phys. Rept. **337**, 343 (2000) [arXiv:hep-th/9909061].

[18] K. Enqvist and M. S. Sloth, Nucl. Phys. B **626**, 395 (2002) [arXiv:hep-ph/0109214].

[19] D. H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002) [arXiv:hep-ph/0110002].

[20] A. Notari and A. Riotto, Nucl. Phys. B **644**, 371 (2002) [arXiv:hep-th/0205019].

[21] F. Di Marco, F. Finelli and R. Brandenberger, Phys. Rev. D **67**, 063512 (2003) [arXiv:astro-ph/0211276].

[22] F. Finelli, Phys. Lett. B **545**, 1 (2002) [arXiv:hep-th/0206112].

[23] Z. K. Guo, Y. S. Piao, R. G. Cai and Y. Z. Zhang, Phys. Lett. B **576**, 12 (2003) [arXiv:hep-th/0306245].

[24] A. R. Liddle, A. Mazumdar and F. E. Schunck, Phys. Rev. D **58**, 061301 (1998) [arXiv:astro-ph/9804177].

[25] J. L. Lehners, P. McFadden, N. Turok and P. J. Steinhardt, arXiv:hep-th/0702153.

[26] E. I. Buchbinder, J. Khoury and B. A. Ovrut, arXiv:hep-th/0702154.

[27] P. Creminelli and L. Senatore, arXiv:hep-th/0702165.

[28] M. Gasperini, M. Maggiore and G. Veneziano, Nucl. Phys. B **494**, 315 (1997) [arXiv:hep-th/9611039].

[29] R. Brustein and R. Madden, Phys. Lett. B **410**, 110 (1997) [arXiv:hep-th/9702043].

[30] R. Brustein and R. Madden, Phys. Rev. D **57**, 712 (1998) [arXiv:hep-th/9708046].

[31] P. Creminelli, M. A. Luty, A. Nicolis and L. Senatore, JHEP **0612**, 080 (2006) [arXiv:hep-th/0606090].

[32] S. Tsujikawa, R. Brandenberger and F. Finelli, Phys. Rev. D **66**, 083513 (2002) [arXiv:hep-th/0207228].

[33] C. Gordon, D. Wands, B. A. Bassett and R. Maartens, Phys. Rev. D **63**, 023506 (2001) [arXiv:astro-ph/0009131].

[34] K. A. Malik and D. Wands, Phys. Rev. D **59**, 123501 (1999) [arXiv:astro-ph/9812204].

[35] M. Gasperini and G. Veneziano, Phys. Rept. **373**, 1 (2003) [arXiv:hep-th/0207130].