Multiple field inflation

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Summary. Inflation offers a simple model for very early evolution of our Universe and the origin of primordial perturbations on large scales. Over the last 25 years we have become familiar with the predictions of single-field models, but inflation with more than one light scalar field can alter preconceptions about the inflationary dynamics and our predictions for the primordial perturbations. I will discuss how future observational data could distinguish between inflation driven by one field, or many fields. As an example, I briefly review the curvaton as an alternative to the inflaton scenario for the origin of structure.[1]

1 Introduction

Inflation provides an attractively simple model for the early evolution of our Universe, which can produce a large, spatially flat and largely homogeneous observable universe. It also provides a source for small primordial perturbations which are the origin of the large-scale structure in our Universe today. The vacuum fluctuations of any light scalar field present during inflation can be swept up by the inflationary expansion to scales much larger than the Hubble scale.

Inflation is most commonly discussed in terms of a potential energy which is a function of a single, slowly rolling, scalar field. Single field, slow-roll inflation produces an almost Gaussian distribution of adiabatic density perturbations on super-Hubble scales with an almost scale-invariant spectrum. But supersymmetric field theories can contain many scalar fields that could play a role during inflation and string theory, and other higher-dimensional theories, yield four-dimensional effective actions with many moduli fields describing the higher dimensional degrees of freedom. One should be aware of the different possibilities that open up in particle physics models containing more than one light scalar field during inflation.

The presence of multiple fields during inflation can lead to quite different inflationary dynamics, that might appear unnatural in a single field model, and to spectra of primordial perturbations that would actually be impossible in single field models.

[1] Based on a talk given at the 22nd IAP Colloquium, "Inflation +25", Paris, June 2006

The presence of multiple light fields during inflation leads to the generation of non-adiabatic field perturbations during inflation. This can alter the evolution of the overall curvature perturbation, for instance leading to detectable non-Gaussianity, and may leave residual isocurvature fluctuations in the primordial density perturbation on large scales after inflation, which can be correlated with the curvature perturbation. Such alternative models are interesting not only as theoretical possibilities, but because they could be distinguished by increasingly precise observations in the near future.

In this short review I will discuss some of the distinctive observational predictions of inflation in presence of more than one scalar field. For a more comprehensive review of inflationary dynamics and reheating with multiple fields see Ref. [1].

2 Homogeneous scalar field dynamics

The time-evolution of a single, spatially homogeneous scalar field is governed by the Klein-Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi}\,, \tag{1}$$

where the Hubble expansion rate is given by the Friedmann constraint

$$3H^2 = 8\pi G\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]\,. \tag{2}$$

Multiple scalar fields obey the Klein-Gordon equation

$$\ddot{\varphi}_I + 3H\dot{\varphi}_I = -\frac{\partial}{\partial\varphi_I}\left(\sum_J U_J\right)\,, \tag{3}$$

where one must allow for the possibility that the potential energy is given by a sum over many terms

$$V = \sum_J U_J\,. \tag{4}$$

The wider range of interaction potentials possible in multiple field models leads to possibilities such as hybrid inflation.

In hybrid inflation models [2, 3] the *inflaton* field, φ_1, can roll towards the non-zero minimum of its potential, $U_1 = V_0 + m_1^2\varphi_1^2/2$, which would lead to eternal inflation into the future in a single field model. But in a hybrid model there is a second *waterfall* scalar field trapped during inflation in a local minimum, $\varphi_2 = 0$, with a potential, e.g., $U_2 = (g^2\varphi_1^2 - m_2^2)\varphi_2^2/2$ which becomes unstable below a critical value of the φ_1 field, triggering an instability of the vacuum energy driving inflation, rapidly bringing inflation to an end.

Another more subtle change enters through the Friedmann constraint:

$$3H^2 = 8\pi G\left(V + \sum_I \frac{1}{2}\dot{\varphi}_I^2\right)\,. \tag{5}$$

Even in the absence of explicit interactions in the scalar field Lagrangian, the fields will still be coupled gravitationally. In particular the Hubble expansion rate that enters the Klein-Gordon equation (3) is due to the sum over all fields in Eq. (5) and this can also alter the field dynamics even if the potential for each individual field is left unchanged. The additional Hubble damping present due to multiple fields can be used to drive slow-roll inflation in assisted inflation models [4] where the individual potentials would be too steep to drive inflation on their own.

The original assisted inflation model [4] considered n scalar fields with steep exponential potentials

$$V = \sum_I U_{I0} \exp\left(-\lambda_I \varphi_I / m_p\right) \tag{6}$$

where I have used the reduced Planck mass $m_p^2 = (8\pi G)^{-1}$. Each scalar field potential is too steep to drive inflation on its own if $\lambda_I^2 > 2$, but the additional damping effect due to the presence of the other scalar fields leads to a particular power-law inflation solution, $a \propto t^p$ where $p = 2/\lambda^2$ where the combined fields have an effective potential $V \propto \exp(-\lambda\sigma/m_p)$ with

$$\frac{1}{\lambda^2} = \sum_I \frac{1}{\lambda_I^2}\,. \tag{7}$$

Thus $\lambda \to 0$ for many fields as $n \to \infty$ and we can have slow-roll inflation even when each $\lambda_I^2 > 2$.

Even though the background dynamics can be reduced to an equivalent single field with a specified potential [5], there is an important qualitative difference between the inflationary dynamics in multiple field inflation with respect to the single field case. The Hubble damping during inflation drives a single scalar field to a unique attractor solution during slow-roll inflation where the Hubble rate, field time-derivative and all local variables are a function of the local field value: $H(\phi)$, $\dot\phi(\phi)$, etc. This means that the evolution rapidly becomes independent of the initial conditions.

In multiple field models we may have a family of trajectories in phase space where, for example, the Hubble rate at a particular value of φ_1 is also dependent upon the value of φ_2. In this case the inflationary dynamics, and hence observational predictions, may be dependent upon the trajectory in phase space and thus the initial field values. It is this that allows non-adiabatic perturbations to survive on super-Hubble scales in multiple field inflation.

It is important to distinguish here between models, such as most hybrid models, with multiple fields but only one light direction in field space with small effective mass $\partial^2 V/\partial\varphi^2 \ll H^2$ during inflation, and models with many light fields, such as assisted inflation models. Only models with multiple *light* fields can have multiple slow-roll trajectories.

2.1 Inflaton field direction during inflation

It is convenient to identify the *inflaton* field direction as the direction in field space corresponding to the evolution of the background (spatially homogeneous) fields during inflation [6] (see also [7, 8]). Thus for n scalar fields φ_I, where I runs from 1 to n, we have

$$\sigma = \int \sum_I \hat\sigma_I \dot\varphi_I \mathrm{d}t\,, \tag{8}$$

where the inflaton direction is defined by

$$\hat{\sigma}_I \equiv \frac{\dot{\varphi}_I}{\sqrt{\sum_J \dot{\varphi}_J^2}} \,. \tag{9}$$

The n evolution equations for the homogeneous scalar fields (3) can then be written as the evolution for a single inflaton field (1)

$$\ddot{\sigma} + 3H\dot{\sigma} + V_\sigma = 0 \,, \tag{10}$$

where the potential gradient in the direction of the inflaton is

$$V_\sigma \equiv \frac{\partial V}{\partial \sigma} = \sum_I \hat{\sigma}_I \frac{\partial V}{\partial \varphi_I} \,. \tag{11}$$

The total energy density and pressure are then given by the usual single field results for the inflaton.

2.2 An example: Nflation

A topical example of multiple field inflation is Nflation. Dimopoulos et al [9] proposed this model based on the very large number of axion fields predicted in low energy effective theories derived from string theory. Near the minimum of the effective potential the fields have a potential energy

$$V = \frac{1}{2} \sum_I m_I^2 \varphi_I^2 \,. \tag{12}$$

This form of potential with a large number of massive fields was also previously studied by Kanti and Olive [10] and Kaloper and Liddle [11].

With a single scalar field the quadratic potential yields the familiar chaotic inflation model with a massive field $V = m^2\phi^2/2$. But to obtain inflation with a single massive field the initial value of the scalar field must be several times the Planck mass and there is a worry that we have no control over corrections to the potential at super-Planckian values in the effective field theory [12]. But with many scalar fields the collective dynamics can yield inflation even for sub-Planckian values if there are a sufficiently large number of fields. Kim and Liddle [13] have found that for random initial conditions, $-m_p < \varphi_I(0) < m_p$, the total number of e-folds is given by $n/12$, where n is the total number of fields. Thus we require $n > 600$ for sufficient inflation if none of the fields is allowed to exceed the Planck scale. This may seem to be a large number, but Dimopoulos et al [9] cite string theory models with of order 10^5 axion fields.

As remarked earlier, in the presence of more than one light field, the trajectory in field space at late times, and hence the observable predictions, may be dependent upon the initial conditions for the different fields. But Kim and Liddle [13] found evidence for what they called a "thermodynamic" regime where the predicted spectral index, $n_{\mathcal{R}}$, for the primordial curvature perturbations that arise from quantum fluctuations of the scalar fields, became independent of the precise initial conditions for a sufficiently large number of fields. In fact inflation with an arbitrary number of massive fields always yields a robust prediction for the tensor-to-scalar ratio r in terms of the number of e-foldings, N, from the end of inflation [12]

$$r = 8/N\,, \tag{13}$$

completely independently of the initial conditions. Thus Nflation seems to be an example of a multiple field model of inflation which makes observable predictions which need not depend upon the specific trajectory in field space.

In the limit where the masses become degenerate, $m_I^2 \to m$, the Nflation dynamics becomes particularly simple. The fields evolve radially towards the origin and the potential (12) reduces to that for a single field

$$V \to \frac{1}{2}m^2\sigma^2\,, \tag{14}$$

where σ is the inflaton field (8). Thus in this limit Nflation reproduces the single field prediction for the tensor-scalar ratio $r = 0.16$ and the spectral index $n_{\mathcal{R}} = 0.96$. However the presence of n light fields during Nflation also leads to $n-1$ isocurvature modes during inflation and these have an exactly scale-invariant spectrum (up to first order in the slow-roll parameters) in the limit of degenerate masses [14]. In the following sections I will describe some of the distinctive predictions that can arise due to the existence of such non-adiabatic perturbations during inflation.

3 Primordial perturbations from inflation

I have so far only presented equations for the dynamics of homogeneous scalar fields driving inflation. But to test theoretical predictions against cosmological observations we need to consider inhomogeneous perturbations. It is the primordial perturbations produced during inflation that offer the possibility of determining the physical processes that drove the dynamical evolution of the very early universe. In the standard Hot Big Bang model there seems to be no way to explain the existence of primordial perturbations, during the radiation dominated era, on scales much larger than the causal horizon, or equivalently the Hubble scale. But inflation takes perturbations on small, sub-Hubble, scales and can stretch them up to arbitrarily large scales.

3.1 Scalar field perturbations, without interactions

Consider an inhomogeneous perturbation, $\varphi_I \to \varphi_I(t) + \delta\varphi_I(t,\mathbf{x})$, of the Klein-Gordon equation (3) for a non-interacting scalar field in an unperturbed FRW universe. (I will include perturbations of the metric and other fields later, but for simplicity I will neglect this complication for the moment.)

$$\ddot{\delta\varphi}_I + 3H\dot{\delta\varphi}_I + \left(m_I^2 - \nabla^2\right)\delta\varphi_I = 0\,, \tag{15}$$

where the effective mass-squared of the field is $m_I^2 = \partial^2 V/\partial\varphi_I^2$, and ∇^2 is the spatial Laplacian. Decomposing an arbitrary field perturbation into eigenmodes of the spatial Laplacian (Fourier modes in flat space) $\nabla^2\delta\varphi_I = -(k^2/a^2)\delta\varphi_I$, where k is the comoving wavenumber and a the FRW scale factor, we find that small-scale fluctuations in scalar fields on sub-Hubble scales (with comoving wavenumber $k > aH$) undergo under-damped oscillations, and on sufficiently small scales are essentially freely oscillating. Normalising the initial amplitude of these small-scale fluctuations to the zero-point fluctuations of a free field in flat spacetime we have [39]

$$\delta\varphi_I \simeq \frac{e^{-ikt/a}}{a\sqrt{2k}} \,. \tag{16}$$

During an accelerated expansion $\dot{a} = aH$ increases and modes that start on sub-Hubble scales ($k > aH$) are stretched up to super-Hubble scales ($k < aH$). Perturbations in light fields (with effective mass-squared $m^2 < 9H^2/4$) become over-damped (or "frozen-in") and Eq. (16) evaluated when $k \simeq aH$ gives the power spectrum for scalar field fluctuations at "Hubble-exit"

$$\mathcal{P}_{\delta\varphi_I} \equiv \frac{4\pi k^3}{(2\pi)^3} \left|\delta\varphi_I^2\right| \simeq \left(\frac{H}{2\pi}\right)^2 \,. \tag{17}$$

Heavy fields with $m^2 > 9H^2/4$ remain under-damped and have essentially no perturbations on super-Hubble scales. But light fields become over-damped and can be treated as essentially classical perturbations with a Gaussian distribution on super-Hubble scales.

Thus inflation generates approximately scale-invariant perturbation spectra on super-Hubble scales in any light field (for $m^2 \ll H^2$ and $|\dot{H}| \ll H^2$).

3.2 Scalar field and metric perturbations, with interactions

The simplified discussion in the preceding subsection gives a good approximation to the scalar field perturbations generated around the time of Hubble-exit during slow-roll inflation, where field interactions and metric backreaction are small. However to accurately track the evolution of perturbations through to the end of inflation and into the radiation dominated era we need to include interactions between fields and, even in the absence of explicit interactions, we need to include gravitational backreaction.

For an inhomogeneous matter distribution the Einstein equations imply that we must also consider inhomogeneous metric perturbations about the spatially flat FRW metric. The perturbed FRW spacetime is described by the line element [34, 1]

$$\begin{aligned} ds^2 = &-(1+2A)dt^2 + 2a\partial_i B dx^i dt \\ &+ a^2\left[(1-2\psi)\delta_{ij} + 2\partial_{ij}E + h_{ij}\right]dx^i dx^j \,, \end{aligned} \tag{18}$$

where ∂_i denotes the spatial partial derivative $\partial/\partial x^i$. We will use lower case latin indices to run over the 3 spatial coordinates.

The metric perturbations have been split into scalar and tensor parts according to their transformation properties on the spatial hypersurfaces. The field equations for the scalar and tensor parts then decouple to linear order. Vector metric perturbations are automatically zero at first order if the matter content during inflation is described solely by scalar fields.

The tensor perturbations, h_{ij}, are transverse ($\partial^i h_{ij} = 0$) and trace-free ($\delta^{ij} h_{ij} = 0$). They are automatically independent of coordinate gauge transformations. These describe gravitational waves as they are the free part of the gravitational field and evolve independently of linear matter perturbations.

We can decompose arbitrary tensor perturbations into eigenmodes of the spatial Laplacian, $\nabla^2 e_{ij} = -(k^2/a^2)e_{ij}$, with comoving wavenumber k, and scalar amplitude $h(t)$:

$$h_{ij} = h(t) e_{ij}^{(+,\times)}(x)\,, \tag{19}$$

with two possible polarisation states, $+$ and $\times$. The Einstein equations yield a wave equation for the amplitude of the tensor metric perturbations

$$\ddot{h} + 3H\dot{h} + \frac{k^2}{a^2} h = 0\,, \tag{20}$$

This is the same as the wave equation (15) for a massless scalar field in an unperturbed FRW metric.

The four scalar metric perturbations A, $\partial_i B$, $\psi\delta_{ij}$ and $\partial_{ij}E$ are constructed from 3-scalars, their derivatives, and the background spatial metric. The intrinsic Ricci scalar curvature of constant time hypersurfaces is given by

$$^{(3)}R = \frac{4}{a^2}\nabla^2\psi\,. \tag{21}$$

Hence we refer to ψ as the curvature perturbation.

First-order scalar field perturbations in a first-order perturbed FRW universe obey the wave equation [1]

$$\begin{aligned} &\ddot{\delta\varphi}_I + 3H\dot{\delta\varphi}_I + \frac{k^2}{a^2}\delta\varphi_I + \sum_J V_{IJ}\delta\varphi_J \\ &= -2V_I A + \dot{\varphi}_I\left[\dot{A} + 3\dot{\psi} + \frac{k^2}{a^2}(a^2\dot{E} - aB)\right]. \end{aligned} \tag{22}$$

where the mass-matrix $V_{IJ} \equiv \partial^2 V/\partial\varphi_I\partial\varphi_J$. The Einstein equations relate the scalar metric perturbations to matter perturbations via the energy and momentum constraints [34]

$$3H\left(\dot{\psi} + HA\right) + \frac{k^2}{a^2}\left[\psi + H(a^2\dot{E} - aB)\right] = -4\pi G\delta\rho\,, \tag{23}$$

$$\dot{\psi} + HA = -4\pi G\delta q\,, \tag{24}$$

where the energy and pressure perturbations and momentum for n scalar fields are given by [1]

$$\delta\rho = \sum_I\left[\dot{\varphi}_I\left(\dot{\delta\varphi}_I - \dot{\varphi}_I A\right) + V_I\delta\varphi_I\right]\,, \tag{25}$$

$$\delta P = \sum_I\left[\dot{\varphi}_I\left(\dot{\delta\varphi}_I - \dot{\varphi}_I A\right) - V_I\delta\varphi_I\right]\,, \tag{26}$$

$$\delta q_{,i} = -\sum_I \dot{\varphi}_I\delta\varphi_{I,i}\,, \tag{27}$$

where $V_I \equiv \partial V/\partial\varphi_I$.

We can construct a variety of gauge-invariant combinations of the scalar metric perturbations. The longitudinal gauge corresponds to a specific gauge-transformation to a (zero-shear) frame such that $E = B = 0$, leaving the gauge-invariant variables

$$\Phi \equiv A - \frac{\mathrm{d}}{\mathrm{d}t}\left[a^2(\dot{E} - B/a)\right]\,, \tag{28}$$

$$\Psi \equiv \psi + a^2 H(\dot{E} - B/a)\,. \tag{29}$$

Another variable commonly used to describe scalar perturbations during inflation is the field perturbation in the spatially flat gauge (where $\psi = 0$). This has the gauge-invariant definition [15, 16]:

$$\delta\varphi_{I\psi} \equiv \delta\varphi_I + \frac{\dot{\varphi}}{H}\psi\,. \tag{30}$$

It is possible to use the Einstein equations to eliminate the metric perturbations from the perturbed Klein-Gordon equation (22), and write a wave equation solely in terms of the field perturbations in the spatially flat gauge [25]

$$\ddot{\delta\varphi}_{I\psi} + 3H\dot{\delta\varphi}_{I\psi} + \frac{k^2}{a^2}\delta\varphi_{I\psi} + \sum_J \left[V_{IJ} - \frac{8\pi G}{a^3}\frac{d}{dt}\left(\frac{a^3}{H}\dot{\varphi}_I\dot{\varphi}_J\right)\right]\delta\varphi_{J\psi} = 0\,. \tag{31}$$

Only at lowest order in the slow-roll expansion can the interaction terms be neglected and we recover the simplified wave equation (15) for a massless field in an unperturbed FRW universe.

3.3 Adiabatic and entropy perturbations

There are two more gauge-invariant scalars which are commonly used to describe the overall curvature perturbation. The curvature perturbation on uniform-density hypersurfaces is given by

$$-\zeta \equiv \psi + \frac{H}{\dot{\rho}}\delta\rho\,, \tag{32}$$

first introduced by Bardeen, Steinhardt and Turner [17] (see also Refs. [18, 19, 20]). The comoving curvature perturbation (strictly speaking the curvature perturbation on hypersurfaces orthogonal to comoving worldlines)

$$\mathcal{R} \equiv \psi - \frac{H}{\rho + P}\delta q\,, \tag{33}$$

where the scalar part of the 3-momentum is given by $\partial_i \delta q$. $\mathcal{R}$ has been used by Lukash [21], Lyth [22] and many others. For single field inflation we have $\delta q = -\dot{\phi}\delta\phi$ and hence

$$\mathcal{R} = \psi + \frac{H}{\dot{\phi}}\delta\phi\,. \tag{34}$$

The difference between the two curvature perturbations $-\zeta$ and $\mathcal{R}$,

$$-\zeta - \mathcal{R} = \frac{H}{\dot{\rho}}\delta\rho_m\,, \tag{35}$$

is proportional to the comoving density perturbation [23]

$$\delta\rho_m \equiv \delta\rho - 3H\delta q\,. \tag{36}$$

The energy and momentum constraints (23) and (24) can be combined to give a generalisation of the Poisson equation

$$\frac{\delta\rho_m}{\rho} = -\frac{2}{3}\left(\frac{k}{aH}\right)^2 \Psi\,, \tag{37}$$

relating the longitudinal gauge metric perturbation (29) to the comoving density perturbation (36). Thus the two curvature perturbations, $\mathcal{R}$ and $-\zeta$, coincide on large scales ($k/aH \ll 1$) so long as the longitudinal gauge metric perturbation, Ψ, remains finite - which is generally true during slow-roll inflation.

The energy conservation equation can be written in terms of the curvature perturbation on uniform-density hypersurfaces, defined in (32), to obtain the first-order evolution equation [20, 1]

$$\dot{\zeta} = -H\frac{\delta P_{\rm nad}}{\rho + P} - \Sigma\,, \tag{38}$$

where $\delta P_{\rm nad}$ is the non-adiabatic pressure perturbation,

$$\delta P_{\rm nad} = \delta P - \frac{\dot{P}}{\dot{\rho}}\delta\rho\,, \tag{39}$$

and Σ is the scalar shear along comoving worldlines [24], which can be given relative to the Hubble rate as

$$\begin{aligned}\frac{\Sigma}{H} &\equiv -\frac{k^2}{3H}\left\{\dot{E} - (B/a) + \frac{\delta q}{a^2(\rho+P)}\right\} \\ &= -\frac{k^2}{3a^2H^2}\zeta - \frac{k^2\Psi}{3a^2H^2}\left[1 - \frac{2\rho}{9(\rho+P)}\frac{k^2}{a^2H^2}\right]\,. \end{aligned}\tag{40}$$

Thus ζ and $\mathcal{R}$ are constant (and equal upto a sign difference) for adiabatic perturbations on super-Hubble scales ($k/aH \ll 1$), so long as Ψ remains finite, in which case the shear of comoving worldlines can be neglected.

More generally we can define adiabatic perturbations to be perturbations which lie along the background trajectory in the phase space of spatially homogeneous fields [20, 6]. That is, we generalise Eq. (39) so that for adiabatic linear perturbations of any two variables x and y we require

$$\frac{\delta x}{\dot{x}} = \frac{\delta y}{\dot{y}} \;:\quad \text{adiabatic} \tag{41}$$

Thus adiabatic perturbations in a multiple field inflation can be characterised by a unique shift along the background trajectory $\delta N = -H\delta x/\dot{x} = -H\delta y/\dot{y}$. For example, for adiabatic perturbations of the primordial plasma we require that the baryon-photon ratio, n_B/n_γ, remains unperturbed, and hence

$$\frac{\delta(n_B/n_\gamma)}{n_B/n_\gamma} = -3H\left(\frac{\delta n_B}{\dot{n}_B} - \frac{\delta n_\gamma}{\dot{n}_\gamma}\right) = 0\,, \tag{42}$$

where we have used $\dot{n}_x = -3Hn_x$ for baryon number density, and photon number density below about 1 MeV.

For a single scalar field the non-adiabatic pressure (39) can be related to the comoving density perturbation (36) [6]

$$\delta P_{\rm nad} = -\frac{2V_{,\varphi}}{3H\dot{\varphi}}\delta\rho_m\,. \tag{43}$$

From the Einstein constraint (37) this will vanish on large scales ($k/aH \to 0$) if Ψ remains finite, and hence single scalar field perturbations become adiabatic in this large-scale limit. In particular, we have from Eq. (38) that ζ becomes constant for adiabatic perturbations in this large scale limit, and hence the curvature perturbation can be calculated shortly after Hubble-exit in single field inflation and equated directly with the primordial curvature perturbation, independently of the details of reheating, etc, at the end of inflation. But in the presence of more than one light field the vacuum fluctuations stretched to super-Hubble scales will inevitably include non-adiabatic perturbations due to the presence of multiple trajectories in the phase space.

We define entropy perturbations to be fluctuations orthogonal to the background trajectory

$$S_{xy} \propto \frac{\delta x}{\dot{x}} - \frac{\delta y}{\dot{y}} \ : \quad \text{entropy} \tag{44}$$

For example in the primordial era we can have entropy perturbations in the primordial plasma

$$S_B = \frac{\delta(n_B/n_\gamma)}{n_B/n_\gamma} = -3H\left(\frac{\delta n_B}{\dot{n}_B} - \frac{\delta n_\gamma}{\dot{n}_\gamma}\right) . \tag{45}$$

which are also referred to as baryon isocurvature perturbations.

In the radiation-dominated era we can define a gauge-invariant primordial curvature perturbations associated with each of the component fluids [20, 26] in analogy with the total curvature perturbation (32)

$$\zeta_I \equiv -\psi - H\frac{\delta\rho_I}{\dot{\rho}_I} , \tag{46}$$

and these will be constant in the large-scale limit for non-interacting fluids with barotropic equation of state $P_i(\rho_i)$ (and hence vanishing non-adiabatic pressure perturbations for each fluid) [20]. We can identify isocurvature perturbations, such as the baryon isocurvature perturbation (45), with the difference between each ζ_I and, by convention, ζ_γ for the photons

$$S_I \equiv 3\left(\zeta_I - \zeta_\gamma\right) , \tag{47}$$

and the total curvature perturbation (32) is given by the weighted sum

$$\zeta = \sum_I \frac{\dot{\rho}_I}{\dot{\rho}}\zeta_I . \tag{48}$$

Scale-invariant spectra of primordial isocurvature perturbations give rise to distinctive power spectra of CMB temperature anisotropies and polarisation [27] and are thus tightly constrained by observations [28, 29, 30], although there may be hints of isocurvature modes in current CMB data [31].

Arbitrary field perturbations in multiple field inflation can be decomposed into adiabatic perturbations along the inflaton trajectory and $n-1$ entropy perturbations orthogonal to the inflaton direction (9) in field space:

$$\delta\sigma = \sum_I \hat{\sigma}_I \delta\varphi_I , \tag{49}$$

$$\delta s_I = \sum_J \hat{s}_{IJ} \delta\varphi_J , \tag{50}$$

where $\sum_I \hat{s}_{JI}\hat{\sigma}_I = 0$. Without loss of generality I will assume that the entropy fields are also mutually orthogonal in field space. Note that I have assumed that the fields have canonical kinetic terms, that is, the field space metric is flat. See Refs. [7, 32, 33] for the generalisation to non-canonical kinetic terms.

The total momentum and pressure perturbation (27) and (26) for n scalar field perturbations can be written in the same form as for a single inflaton field

$$\delta q = -\dot{\sigma}\delta\sigma\,, \tag{51}$$
$$\delta P = \dot{\sigma}(\dot{\delta\sigma} - \dot{\sigma}A) - V_\sigma \delta\sigma\,. \tag{52}$$

However the density perturbation (25) is given by

$$\delta\rho = \dot{\sigma}(\dot{\delta\sigma} - \dot{\sigma}A) + V_\sigma\delta\sigma + 2\delta_s V\,, \tag{53}$$

where the deviation from the single field result arises due to the non-adiabatic perturbation of the potential orthogonal to the inflaton trajectory:

$$\delta_s V \equiv \sum_I V_I \delta\varphi_I - V_\sigma\delta\sigma\,. \tag{54}$$

The non-adiabatic pressure perturbation (39) is written as [6, 1]

$$\delta P_{\rm nad} = -\frac{2V_\sigma}{3H\dot{\sigma}}\delta\rho_m - 2\delta_s V\,, \tag{55}$$

where the comoving density perturbation, $\delta\rho_m$, is given by Eq. (36). Although the constraint Eq. (37) requires the comoving density perturbation to become small on large scales, as in the single field case, there is now an additional contribution to the non-adiabatic pressure due to non-adiabatic perturbations of the potential which need not be small on large scales.

It is important to emphasise that the presence of entropy perturbations during inflation does not mean that the "primordial" density perturbation (at the epoch of primordial nucleosynthesis) will contain isocurvature modes. In particular, if the universe undergoes conventional reheating at the end of inflation and all particle species are driven towards thermal equilibrium with their abundances determined by a single temperature (with no non-zero chemical potentials) then the primordial perturbations must be adiabatic [36]. It is these primordial perturbations that set the initial conditions for the evolution of the radiation-matter fluid that determines the anisotropies in the cosmic microwave background and large-scale structure in our Universe, and thus are directly constrained by observations. We will see that while the existence of non-adiabatic perturbations after inflation requires the existence of non-adiabatic perturbations during inflation [35], it is not true that non-adiabatic modes during inflation necessarily give primordial isocurvature modes [36].

4 Perturbations from two-field inflation

In this section I will consider the specific example of the coupled evolution of two canonical scalar fields, ϕ and χ, during inflation and how this can give rise to correlated curvature and entropy perturbations on large scales after inflation [37]. I will use the local rotation in field space defined by Eq. (49) and (50) to describe the instantaneous adiabatic and entropy field perturbations.

The inflaton field perturbation (49) is gauge-dependent, but we can choose to work with the inflaton perturbation in the spatially flat ($\psi = 0$) gauge:

$$\delta\sigma_\psi \equiv \delta\sigma + \frac{\dot\sigma}{H}\psi \,. \tag{56}$$

On the other hand, the orthogonal entropy perturbation (50) is automatically gauge-invariant.

The generalisation to two fields of the evolution equation for the inflaton field perturbation in the spatially flat gauge, obtained from the perturbed Klein-Gordon equations (22), is [6]

$$\begin{aligned} &\ddot{\delta\sigma}_\psi + 3H\dot{\delta\sigma}_\psi + \left[\frac{k^2}{a^2} + V_{\sigma\sigma} - \dot\theta^2 - \frac{8\pi G}{a^3}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a^3\dot\sigma^2}{H}\right)\right]\delta\sigma_\psi \\ &= \quad 2\frac{\mathrm{d}}{\mathrm{d}t}(\dot\theta\delta s) - 2\left(\frac{V_\sigma}{\dot\sigma} + \frac{\dot H}{H}\right)\dot\theta\delta s \,, \end{aligned} \tag{57}$$

and the entropy perturbation obeys

$$\ddot{\delta s} + 3H\dot{\delta s} + \left(\frac{k^2}{a^2} + V_{ss} + 3\dot\theta^2\right)\delta s = \frac{\dot\theta}{\dot\sigma}\frac{k^2}{2\pi G a^2}\Psi \,, \tag{58}$$

where $\tan\theta = \dot\chi/\dot\phi$ and

$$V_{\sigma\sigma} \equiv (\cos^2\theta)V_{\phi\phi} + (\sin 2\theta)V_{\phi\chi} + (\sin^2\theta)V_{\chi\chi}, \tag{59}$$

$$V_{ss} \equiv (\sin^2\theta)V_{\phi\phi} - (\sin 2\theta)V_{\phi\chi} + (\cos^2\theta)V_{\chi\chi}. \tag{60}$$

We can identify a purely adiabatic mode where $\delta s = 0$ on large scales. However a non-zero entropy perturbation does appear as a source term in the perturbed inflaton equation whenever the inflaton trajectory is curved in field space, i.e., $\dot\theta \neq 0$. We note that $\dot\theta$ is given by [6]

$$\dot\theta = -\frac{V_s}{\dot\sigma} \,, \tag{61}$$

where V_s is the potential gradient orthogonal to the inflaton trajectory in field space.

The entropy perturbation evolves independently of the curvature perturbation on large-scales. It couples to the curvature perturbation only through the gradient of the longitudinal gauge metric potential, Ψ. Thus entropy perturbations are also described as "isocurvature" perturbations on large scales. Eq. (57) shows that the entropy perturbation δs works as a source term for the adiabatic perturbation. This is in fact clearly seen if we take the time derivative of the curvature perturbation [6]:

$$\dot{\mathcal{R}} = \frac{H}{\dot H}\frac{k^2}{a^2}\Psi + \frac{2H}{\dot\sigma}\dot\theta\delta s \,. \tag{62}$$

Therefore $\mathcal{R}$ (or ζ) is not conserved even in the large-scale limit in the presence of the entropy perturbation δs with a non-straight trajectory in field space ($\dot\theta \neq 0$).

Analogous to the single field case we can introduce slow-roll parameters for light, weakly coupled fields [38]. At first-order in a slow-roll expansion, the inflaton rolls directly down the potential slope, that is $V_s \simeq 0$. Thus we have only one slope parameter

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{1}{16\pi G}\left(\frac{V_\sigma}{V}\right)^2 , \tag{63}$$

but three parameters, $\eta_{\sigma\sigma}$, $\eta_{\sigma s}$ and η_{ss}, describing the curvature of the potential, where

$$\eta_{IJ} \equiv \frac{1}{8\pi G}\frac{V_{IJ}}{V} . \tag{64}$$

The background slow-roll solution is described in terms of the slow-roll parameters by

$$\dot{\sigma}^2 \simeq \frac{2}{3}\epsilon V , \quad H^{-1}\dot{\theta} \simeq -\eta_{\sigma s} , \tag{65}$$

while the perturbations obey

$$\begin{aligned} H^{-1}\dot{\delta\sigma}_\psi &\simeq (2\epsilon - \eta_{\sigma\sigma})\,\delta\sigma_\psi - 2\eta_{\sigma s}\delta s , \\ H^{-1}\dot{\delta s} &\simeq -\eta_{ss}\delta s , \end{aligned} \tag{66}$$

on large scales, where we neglect spatial gradients. Although $V_s \simeq 0$ at lowest order in slow-roll, this does not mean that the inflaton and entropy perturbations decouple. $\dot{\theta}$ given by Eq. (65) is in general non-zero at first-order in slow-roll and large-scale entropy perturbations do affect the evolution of the adiabatic perturbations when $\eta_{\sigma s} \neq 0$.

While the general solution to the two second-order perturbation equations (57) and (58) has four independent modes, the two first-order slow-roll equations (66) give the approximate form of the squeezed state on large scales. This has only two modes which we can describe in terms of dimensionless curvature and isocurvature perturbations:

$$\mathcal{R} \equiv \frac{H}{\dot{\sigma}}\delta\sigma_\psi , \quad \mathcal{S} \equiv \frac{H}{\dot{\sigma}}\delta s . \tag{67}$$

The normalisation of $\mathcal{R}$ coincides with the standard definition of the comoving curvature perturbation, Eq. (33). The normalisation of the dimensionless entropy during inflation, $\mathcal{S}$, is chosen here coincide with Ref. [38]. It can be related to the non-adiabatic pressure perturbation (39) on large scales

$$\delta P_{\rm nad} \simeq -\epsilon\eta_{\sigma s}\frac{H^2}{2\pi G}\mathcal{S} . \tag{68}$$

The slow-roll approximation can provide a useful approximation to the instantaneous evolution of the fields and their perturbations on large scales during slow-roll inflation, but is not expected to remain accurate when integrated over many Hubble times, where inaccuracies can accumulate. In single-field inflation the constancy of the comoving curvature perturbation after Hubble exit, which does not rely on the slow-roll approximation, is crucial in order to make accurate predictions of the primordial perturbations using the slow-roll approximation only around Hubble crossing. In a two-field model we must describe the evolution after Hubble exit in terms of a general transfer matrix:

$$\begin{pmatrix} \mathcal{R} \\ \mathcal{S} \end{pmatrix} = \begin{pmatrix} 1 & T_{\mathcal{RS}} \\ 0 & T_{\mathcal{SS}} \end{pmatrix} \begin{pmatrix} \mathcal{R} \\ \mathcal{S} \end{pmatrix}_* . \tag{69}$$

On large scales the comoving curvature perturbation still remains constant for the purely adiabatic mode, corresponding to $\mathcal{S} = 0$, and adiabatic perturbations remain adiabatic. These general results are enough to fix two of the coefficients in the transfer matrix, but $T_{\mathcal{RS}}$ and $T_{\mathcal{SS}}$ remain to be determined either within a given theoretical model, or from observations, or ideally by both. The scale-dependence of the transfer functions depends upon the inflaton-entropy coupling at Hubble exit during inflation and can be given in terms of the slow-roll parameters as [38]

$$\begin{aligned} \frac{\partial}{\partial \ln k} T_{\mathcal{RS}} &= 2\eta_{\sigma s} + (2\epsilon - \eta_{\sigma\sigma} + \eta_{ss}) T_{\mathcal{RS}} \,, \\ \frac{\partial}{\partial \ln k} T_{\mathcal{SS}} &= (2\epsilon - \eta_{\sigma\sigma} + \eta_{ss}) T_{\mathcal{SS}} \,. \end{aligned} \tag{70}$$

4.1 Initial power spectra

For weakly-coupled, light fields we can neglect interactions on wavelengths below the Hubble scale, so that vacuum fluctuations give rise to a spectrum of uncorrelated field fluctuations on the Hubble scale ($k = aH$) during inflation given by Eq. (17):

$$\mathcal{P}_{\delta\phi} \simeq \mathcal{P}_{\delta\chi} \simeq \left(\frac{H}{2\pi}\right)_*^2 , \tag{71}$$

where we use a $*$ to denote quantities evaluated at Hubble-exit. If a field has a mass comparable to the Hubble scale or larger then the vacuum fluctuations on wavelengths greater than the effective Compton wavelength are suppressed. In addition fluctuations in strongly interacting fields may develop correlations before Hubble exit. But during slow-roll inflation the correlation between vacuum fluctuations in weakly coupled, light fields at Hubble-exit is suppressed by slow-roll parameters. This remains true under a local rotation in fields space to another orthogonal basis such as the instantaneous inflaton and entropy directions (49) and (50) in field space.

The curvature and isocurvature power spectra at Hubble-exit are given by

$$\mathcal{P}_{\mathcal{R}}\big|_* \simeq \mathcal{P}_{\mathcal{S}}\big|_* \simeq \left(\frac{H^2}{2\pi\dot\sigma}\right)_*^2 \simeq \frac{8}{3}\left(\frac{V}{\epsilon M_{\rm Pl}^4}\right)_* , \tag{72}$$

while the cross-correlation is first-order in slow-roll [49, 50],

$$\mathcal{C}_{\mathcal{RS}}\big|_* \simeq -2C\eta_{\sigma s}\, \mathcal{P}_{\mathcal{R}}\big|_* \,, \tag{73}$$

where $C = 2 - \ln 2 - \gamma \approx 0.73$ and γ is the Euler number. The normalisation chosen for the dimensionless entropy perturbation in Eq. (67) ensures that the curvature and isocurvature fluctuations have the same power at horizon exit [38]. The spectral tilts at horizon-exit are also the same and are given by

$$\Delta n_{\mathcal{R}}|_* \simeq \Delta n_{\mathcal{S}}|_* \simeq -6\epsilon + 2\eta_{\sigma\sigma} \,. \tag{74}$$

where $\Delta n_X \equiv d\ln \mathcal{P}_X / d\ln k$.

The tensor perturbations (20) are decoupled from scalar metric perturbations at first-order and hence the power spectrum has the same form as in single field inflation. Thus the power spectrum of gravitational waves on super-Hubble scales during inflation is given by

$$\mathcal{P}_{\rm T}|_* \simeq \frac{16H^2}{\pi M_{\rm Pl}^2} \simeq \frac{128}{3}\frac{V_*}{M_{\rm Pl}^4}\,, \tag{75}$$

and the spectral tilt is

$$\Delta n_{\rm T}|_* \simeq -2\epsilon\,. \tag{76}$$

4.2 Primordial power spectra

The resulting primordial power spectra on large scales can be obtained simply by applying the general transfer matrix (69) to the initial scalar perturbations. The scalar power spectra probed by astronomical observations are thus given by [38]

$$\mathcal{P}_{\mathcal{R}} = (1+T_{\mathcal{RS}}^2)\mathcal{P}_{\mathcal{R}}|_* \tag{77}$$
$$\mathcal{P}_{\mathcal{S}} = T_{\mathcal{SS}}^2\mathcal{P}_{\mathcal{R}}|_* \tag{78}$$
$$\mathcal{C}_{\mathcal{RS}} = T_{\mathcal{RS}}T_{\mathcal{SS}}\mathcal{P}_{\mathcal{R}}|_*\,. \tag{79}$$

The cross-correlation can be given in terms of a dimensionless correlation angle:

$$\cos\Theta \equiv \frac{\mathcal{C}_{\mathcal{RS}}}{\sqrt{\mathcal{P}_{\mathcal{R}}\mathcal{P}_{\mathcal{S}}}} = \frac{T_{\mathcal{RS}}}{\sqrt{1+T_{\mathcal{RS}}^2}}\,. \tag{80}$$

We see that if we can determine the dimensionless correlation angle, Θ, from observations, then this determines the off-diagonal term in the transfer matrix

$$T_{\mathcal{RS}} = \cot\Theta\,, \tag{81}$$

and we can in effect measure the contribution of the entropy perturbation during two-field inflation to the resultant curvature primordial perturbation. In particular this allows us in principle to deduce from observations the power spectrum of the curvature perturbation at Hubble-exit during two-field slow-roll inflation [38]:

$$\mathcal{P}_{\mathcal{R}}|_* = \mathcal{P}_{\mathcal{R}}\sin^2\Theta\,. \tag{82}$$

The scale-dependence of the resulting scalar power spectra depends both upon the scale-dependence of the initial power spectra and of the transfer coefficients. The spectral tilts are given from Eqs. (77–79) by

$$\begin{aligned}
\Delta n_{\mathcal{R}} &= \Delta n_{\mathcal{R}}|_* + H_*^{-1}(\partial T_{\mathcal{RS}}/\partial t_*)\sin 2\Theta\,,\\
\Delta n_{\mathcal{S}} &= \Delta n_{\mathcal{R}}|_* + 2H_*^{-1}(\partial \ln T_{\mathcal{SS}}/\partial t_*)\,,\\
\Delta n_{\mathcal{C}} &= \Delta n_{\mathcal{R}}|_* + H_*^{-1}\left[(\partial T_{\mathcal{RS}}/\partial t_*)\tan\Theta + (\partial \ln T_{\mathcal{SS}}/\partial t_*)\right]\,,
\end{aligned} \tag{83}$$

where we have used Eq. (81) to eliminate $T_{\mathcal{RS}}$ in favour of the observable correlation angle Θ. Substituting Eq. (74) for the tilt at Hubble-exit, and Eqs. (70) for the scale-dependence of the transfer functions, we obtain [38]

$$\begin{aligned}
\Delta n_{\mathcal{R}} &\simeq -(6-4\cos^2\Theta)\epsilon\\
&\qquad +2\left(\eta_{\sigma\sigma}\sin^2\Theta + 2\eta_{\sigma s}\sin\Theta\cos\Theta + \eta_{ss}\cos^2\Theta\right)\,,\\
\Delta n_{\mathcal{S}} &\simeq -2\epsilon + 2\eta_{ss}\,,\\
\Delta n_{\mathcal{C}} &\simeq -2\epsilon + 2\eta_{ss} + 2\eta_{\sigma s}\tan\Theta\,.
\end{aligned} \tag{84}$$

Although the overall amplitude of the transfer functions are dependent upon the evolution after Hubble-exit and through reheating into the radiation era, the spectral tilts can be expressed solely in terms of the slow-roll parameters at Hubble-exit during inflation and the correlation angle, Θ, which can in principle be observed.

If the primordial curvature perturbation results solely from the adiabatic inflaton field fluctuations during inflation then we have $T_{\mathcal{RS}} = 0$ in Eq. (77) and hence $\cos\Theta = 0$ in Eqs. (84), which yields the standard single field result

$$\Delta n_{\mathcal{R}} \simeq -6\epsilon + 2\eta_{\sigma\sigma} \,. \tag{85}$$

Any residual isocurvature perturbations must be uncorrelated with the adiabatic curvature perturbation (at first-order in slow-roll) with spectral index

$$\Delta n_{\mathcal{S}} \simeq -2\epsilon + 2\eta_{ss} \,. \tag{86}$$

On the other hand, if the observed primordial curvature perturbation is produced due to some entropy field fluctuations during inflation, we have $T_{\mathcal{RS}} \gg 1$ and $\sin\Theta \simeq 0$. In a two-field inflation model any residual primordial isocurvature perturbations will then be completely correlated (or anti-correlated) with the primordial curvature perturbation and we have

$$\Delta n_{\mathcal{R}} \simeq \Delta n_{\mathcal{C}} \simeq \Delta n_{\mathcal{S}} \simeq -2\epsilon + 2\eta_{ss} \,. \tag{87}$$

The gravitational wave power spectrum is frozen-in on large scales, independent of the scalar perturbations, and hence

$$\mathcal{P}_{\rm T} = \mathcal{P}_{\rm T}|_* \,. \tag{88}$$

Thus we can derive a modified consistency relation [39] between observables applicable in the case of two-field slow-roll inflation:

$$r = \frac{\mathcal{P}_{\rm T}}{\mathcal{P}_{\mathcal{R}}} \simeq -8\Delta n_{\rm T} \sin^2\Theta \,. \tag{89}$$

This relation was first obtained in Ref. [40] at the end of two-field inflation, and verified in Ref. [41] for slow-roll models. But it was realised in Ref. [38] that this relation also applies to the primordial perturbation spectra in the radiation era long after two-field slow-roll inflation has ended and hence may be tested observationally.

More generally, if there is any additional source of the scalar curvature perturbation, such as additional scalar fields during inflation, then this could give an additional contribution to the primordial scalar curvature spectrum without affecting the gravitational waves, and hence the more general result is the inequality [42]:

$$r \leq -8\Delta n_{\rm T} \sin^2\Theta \,. \tag{90}$$

This leads to a fundamental difference when interpretting the observational constraints on the amplitude of primordial tensor perturbations in multiple inflation models. In single field inflation, observations directly constrain $r = [\mathcal{P}_{\rm T}/\mathcal{P}_{\mathcal{R}}]_*$ and hence, from Eqs. (72) and (75), the slow-roll parameter ϵ. However in multiple field inflation, non-adiabatic perturbations can enhance the power of scalar perturbations after Hubble exit and hence observational constraints on the amplitude of primordial tensor perturbations do not directly constrain the slow-roll parameter ϵ.

Current CMB data alone require $r < 0.55$ (assuming power-law primordial spectra) [43] which in single-field models is interpretted as requiring $\epsilon < 0.04$. But in multiple field models ϵ could be larger if the primordial density perturbation comes from non-adiabatic perturbations during inflation.

5 Non-Gaussianity

A powerful technique to calculate the primordial curvature perturbation resulting from many inflation models, including multi-field models, is to note that the curvature perturbation ζ defined in Eq. (32) can be interpreted as a perturbation in the local expansion [44, 42, 46]

$$\zeta = \delta N \,, \tag{91}$$

where δN is the perturbed expansion to uniform-density hypersurfaces with respect to spatially flat hypersurfaces, which is given to first-order by

$$\zeta = -H \frac{\delta\rho_\psi}{\dot\rho} \,, \tag{92}$$

where $\delta\rho_\psi$ must be evaluated on spatially flat ($\psi = 0$) hypersurfaces.

An important simplification arises on large scales where anisotropy and spatial gradients can be neglected, and the local density, expansion, etc, obeys the same evolution equations as in a homogeneous FRW universe [42, 45, 20, 24, 47, 46]. Thus we can use the homogeneous FRW solutions to describe the local evolution, which is known as the "separate universe" approach [42, 45, 20, 47]. In particular we can evaluate the perturbed expansion in different parts of the universe resulting from different initial values for the fields during inflation using the homogeneous background solutions [42]. The integrated expansion from some initial spatially flat hypersurface up to a late-time fixed density hypersurface, say at the epoch of primordial nucleosynthesis, is some function of the field values on the initial hypersurface, $N(\varphi_I|_\psi)$. The resulting primordial curvature perturbation on the uniform-density hypersurface is then

$$\zeta = \sum_I N_I \delta\varphi_{I\psi} \,, \tag{93}$$

where $N_I \equiv \partial N / \partial \varphi_I$ and $\delta\varphi_{I\psi}$ is the field perturbation on some initial spatially-flat hypersurfaces during inflation (30). In particular the power spectrum for the primordial density perturbation in a multi-field inflation can be written (at leading order) in terms of the field perturbations after Hubble-exit as

$$\mathcal{P}_\zeta = \sum_I N_I^2 \mathcal{P}_{\delta\varphi_{I\psi}} \,. \tag{94}$$

This approach is readily extended to estimate the non-linear effect of field perturbations on the metric perturbations [45, 24, 46]. We can take Eq. (91) as our definition of the non-linear primordial curvature perturbation, ζ, so that in the radiation dominated era the non-linear extension of Eq. (92) is given by [46]

$$\zeta = \frac{1}{4} \ln \left(\frac{\tilde\rho}{\rho} \right)_\psi \,, \tag{95}$$

where $\tilde\rho(t, \mathbf{x})$ is the perturbed (inhomogeneous) density evaluated on a spatially flat hypersurface and $\rho(t)$ is the background (homogeneous) density. This non-linear curvature perturbation as a function of the initial field fluctuations can simply be expanded as a Taylor expansion [57, 55, 48, 52]

$$\zeta \simeq \sum_I N_I \delta\varphi_{I\psi} + \frac{1}{2} \sum_{I,J} N_{IJ} \delta\varphi_{I\psi} \delta\varphi_{J\psi} + \frac{1}{6} \sum_{I,J,K} N_{IJK} \delta\varphi_{I\psi} \delta\varphi_{J\psi} \delta\varphi_{K\psi} + \dots \,. \tag{96}$$

where we now identify (93) as the leading-order term.

We expect the field perturbations at Hubble-exit to be close to Gaussian for weakly coupled scalar fields during inflation [53, 54, 56, 55, 51]. In this case the bispectrum of the primordial curvature perturbation at leading (fourth) order, can be written using the δN-formalism, as [75, 57]

$$B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \frac{6}{5} f_{NL} \left[P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1) \right] . \quad (97)$$

where $P_\zeta(k) = 2\pi^2 \mathcal{P}_\zeta(k)/k^3$, and the dimensionless non-linearity parameter is given by [57]

$$f_{NL} = \frac{5}{6} \frac{N_A N_B N^{AB}}{(N_C N^C)^2} . \quad (98)$$

Similarly, the connected part of the trispectrum in this case can be written as [52, 48]

$$\begin{aligned} T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = {} & \tau_{NL} \left[P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|) P_\zeta(k_3) P_\zeta(k_4) + (11 \text{ perms}) \right] \\ & + \frac{54}{25} g_{NL} \left[P_\zeta(k_2)P_\zeta(k_3)P_\zeta(k_4) + (3 \text{ perms}) \right] . \end{aligned} \quad (99)$$

where

$$\tau_{NL} = \frac{N_{AB} N^{AC} N^B N_C}{(N_D N^D)^3} , \quad (100)$$

$$g_{NL} = \frac{25}{54} \frac{N_{ABC} N^A N^B N^C}{(N_D N^D)^3} . \quad (101)$$

The expression for τ_{NL} was first given in [58]. Note that we have factored out products in the trispectrum with different k dependence in order to define the two k independent non-linearity parameters τ_{NL} and g_{NL}. This gives the possibility that observations may be able to distinguish between the two parameters [59].

In many cases there is single direction in field-space, χ, which is responsible for perturbing the local expansion, $N(\chi)$, and hence generating the primordial curvature perturbation (96). For example this would be the inflaton field in single field models of inflation, or it could be the late-decaying scalar field in the curvaton scenario [61, 62, 63] as will be discussed in the next section. In this case the curvature perturbation (96) is given by

$$\zeta \simeq N' \delta\chi_\psi + \frac{1}{2} N'' \delta\chi_\psi^2 + \frac{1}{6} N''' \delta\chi_\psi^3 + \dots , \quad (102)$$

and the non-Gaussianity of the primordial perturbation has the simplest "local" form

$$\zeta = \zeta_1 + \frac{3}{5} f_{NL} \zeta_1^2 + \frac{9}{25} g_{NL} \zeta_1^3 + \dots \quad (103)$$

where $\zeta_1 = N' \delta\chi_\psi$ is the leading-order Gaussian curvature perturbation and the non-linearity parameters f_{NL} and g_{NL}, are given by [57, 64]

$$f_{NL} = \frac{5}{6} \frac{N''}{(N')^2} , \quad (104)$$

$$g_{NL} = \frac{25}{54} \frac{N'''}{(N')^3} , \quad (105)$$

The primordial bispectrum and trispectrum are then given by Eqs. (97) and (99), where the non-linearity parameters f_{NL} and g_{NL}, given in Eqs. (98) and (101), reduce to Eqs. (104) and (105) respectively, and τ_{NL} given in Eq.(100) reduces to

$$\tau_{NL} = \frac{(N'')^2}{(N')^4} = \frac{36}{25} f_{NL}^2 \,. \tag{106}$$

Thus τ_{NL} is proportional to f_{NL}^2 (first shown in [59] using the Bardeen potential, and in [57] using this notation). However the trispectrum could be large even when the bispectrum is small because of the g_{NL} term [59, 64].

In the case of where the primordial curvature perturbation is generated solely by adiabatic fluctuations in the inflaton field, σ, the curvature perturbation is non-linearly conserved on large scales [76, 46, 60] and we can calculate N', N'', N''', etc, at Hubble-exit. In terms of the slow-roll parameters, we find

$$N' = \frac{H}{\dot{\varphi}} \simeq \frac{1}{\sqrt{2}} \frac{1}{m_p} \frac{1}{\sqrt{\epsilon}} \sim \mathcal{O}\left(\epsilon^{-\frac{1}{2}}\right) \,, \tag{107}$$

$$N'' \simeq -\frac{1}{2} \frac{1}{m_p^2} \frac{1}{\epsilon} (\eta_{\sigma\sigma} - 2\epsilon) \sim \mathcal{O}\left(1\right) \,, \tag{108}$$

$$N''' \simeq \frac{1}{\sqrt{2}} \frac{1}{m_p^3} \frac{1}{\epsilon\sqrt{\epsilon}} \left(\epsilon\eta_{\sigma\sigma} - \eta_{\sigma\sigma}^2 + \frac{1}{2}\xi_\sigma^2\right) \sim \mathcal{O}(\epsilon^{\frac{1}{2}}) \,, \tag{109}$$

where we have used the reduced Planck mass $m_p^2 = (8\pi G)^{-1}$ and introduced the second-order slow-roll parameter $\xi_\sigma^2 = m_p^4 V_\sigma V_{\sigma\sigma\sigma}/V^2$. Hence the non-linearity parameters for single field inflation, (104) and (105), are given by

$$f_{NL} = \frac{5}{6}(\eta_{\sigma\sigma} - 2\epsilon) \,, \tag{110}$$

$$g_{NL} = \frac{25}{54}\left(2\epsilon\eta_{\sigma\sigma} - 2\eta_{\sigma\sigma}^2 + \xi_\sigma^2\right) \,. \tag{111}$$

with τ_{NL} given by Eq. (106). Although there are additional contributions to the primordial bispectrum and trispectrum coming from the intrinsic non-Gaussianity of the field perturbations at Hubble-exit, these are also suppressed by slow-roll parameters in slow-roll inflation. Thus the primordial non-Gaussianity is likely to be too small to ever be observed in the conventional inflaton scenario of single-field slow-roll inflation. Indeed any detection of primordial non-Gaussianity $f_{NL} > 1$ would appear to rule out this inflaton scenario.

However significant non-Gaussianity can be generated due to non-adiabatic field fluctuations. Thus far it has proved difficult to generate significant non-Gaussianity in the curvature perturbation during slow-roll inflation, even in multiple field models. But detectable non-Gaussianity can be produced when the curvature perturbation is generated from isocurvature field perturbations at the end of inflation [65, 12], during inhomogeneous reheating [66, 67, 14], or after inflation in the curvaton model, which I will discuss next.

6 Curvaton scenario

Consider a light, weakly-coupled scalar field, χ, that decays some time after inflation has ended. There are many such scalar degrees of freedom in supersymmetric theories and if they are too weakly coupled, and their lifetime is too long, this leads to the "Polonyi problem" [71]. Assuming the field is displaced from the minimum of its effective potential at the end of inflation, the field evolves little until the Hubble rate drops below its effective mass. Then it oscillates, with a time-averaged equation of state of a pressureless fluid, $P_\chi = 0$, (or, equivalently, a collection of non-relativistic particles) and will eventually come to dominate the energy density of the Universe. To avoid disrupting the standard "hot big bang" model and in particular to preserve the successful radiation-dominated model of primordial nucleosythesis, we require that such fields decay into radiation before $t \sim 1$ second. For a weakly-coupled field that decays with only gravitational strength, $\Gamma \sim m_\chi^3/M_P^2$, this requires $m_\chi >$ 100 TeV.

But there is a further important feature of late-decaying scalar fields that has only recently received serious consideration. If the field is inhomogeneous then it could lead to an inhomogeneous radiation density after it decays [72, 73]. This is the basis of the curvaton scenario [61, 62, 63].

If this field is light ($m < H$) during inflation then small-scale quantum fluctuations will lead to a spectrum of large-scale perturbations, whose initial amplitude at Hubble-exit is given (at leading order in slow-roll) by Eq. (17). When the Hubble rate drops and the field begins oscillating after inflation, this leads to a first-order density perturbation in the χ-field:

$$\zeta_\chi = -\psi + \frac{\delta\rho_\chi}{3\rho_\chi} \,. \tag{112}$$

where $\rho_\chi = m_\chi^2\chi^2/2$. ζ_χ remains constant for the oscillating curvaton field on large scales, so long as we can neglect its energy loss due to decay. Using Eq. (17) for the field fluctuations at Hubble-exit and neglecting any non-linear evolution of the χ-field after inflation (consistent with our assumption that the field is weakly coupled), we have

$$\mathcal{P}_{\zeta_\chi} \simeq \left(\frac{H}{6\pi\chi}\right)^2_{k=aH} \,. \tag{113}$$

The total density perturbation (32), considering radiation, γ, and the curvaton, χ, is given by [62]

$$\zeta = \frac{4\rho_\gamma\zeta_\gamma + 3\rho_\chi\zeta_\chi}{4\rho_\gamma + 3\rho_\chi} \,. \tag{114}$$

Thus if the radiation generated by the decay of the inflaton at the end of inflation is unperturbed ($\mathcal{P}_{\zeta_\gamma}^{1/2} \ll 10^{-5}$) the total curvature perturbation grows as the density of the χ-field grows relative to the radiation: $\zeta \sim \Omega_\chi\zeta_\chi$.

Ultimately the χ-field must decay (when $H \sim \Gamma$) and transfer its energy density and, crucially, its perturbation to the radiation and/or other matter fields. In the simplest case that the non-relativistic χ-field decays directly to radiation a full analysis [70, 74] of the coupled evolution equation gives the primordial radiation perturbation (after the decay)

$$\zeta = r_\chi(p)\zeta_\chi \,, \tag{115}$$

where $p \equiv [\Omega_\chi/(\Gamma/H)^{1/2}]_{\rm initial}$ is a dimensionless parameter which determines the maximum value of Ω_χ before it decays, and empirically we find [74]

$$r_\chi(p) \simeq 1 - \left(1 + \frac{0.924}{1.24}p\right)^{-1.24} \,. \tag{116}$$

For $p \gg 1$ the χ-field dominates the total energy density before it decays and $r_\chi \sim 1$, while for $p \ll 1$ we have $r_\chi \sim 0.924p \ll 1$.

Finally combining Eqs. (113) and (115) we have

$$\mathcal{P}_\zeta \simeq r_\chi^2(p) \left(\frac{H}{6\pi\chi} \right)^2_{k=aH} . \tag{117}$$

Note that the primordial curvature perturbation tends to have less power on small scales due to the decreasing Hubble rate at Hubble exit in Eq. (117), but can also have more power on small scales due to the decreasing χ, for a positive effective mass-squared, during inflation. In terms of slow-roll parameters the actual tilt is given by Eq. (84) when $\sin\Theta = 0$

$$\Delta n_{\mathcal{R}} \simeq -2\epsilon + 2\eta_{\chi\chi} . \tag{118}$$

In the extreme slow-roll limit the spectrum becomes scale-invariant, as in the inflaton scenario.

In contrast to the inflaton scenario the final density perturbation in the curvaton scenario is a very much dependent upon the physics after the field perturbation exited the Hubble scale during inflation. For instance, if the curvaton lifetime is too short then it will decay before it can significantly perturb the total energy density and $\mathcal{P}_{\zeta_\gamma}^{1/2} \ll 10^{-5}$. The observational constraint on the amplitude of the primordial perturbations gives a single constraint upon both the initial fluctuations during inflation and the post-inflationary decay time. This is in contrast to the inflaton scenario where the primordial perturbation gives a direct window onto the dynamics of inflation, independently of the physics at lower energies. In the curvaton scenario there is the possibility of connecting the generation of primordial perturbations to other aspects of cosmological physics. For instance, it may be possible to identify the curvaton with fields whose late-decay is responsible for the origin of the baryon asymmetry in the universe, in particular with sneutrino models of leptogenesis (in which an initial lepton asymmetry is converted into a baryon asymmetry at the electroweak transition) [68].

The curvaton scenario has re-invigorated attempts to embed models of inflation in the very early universe within minimal supersymmetric models of particle physics constrained by experiment [69]. It may be possible that the inflaton field driving inflation can be completely decoupled from visible matter if the dominant radiation in the universe today comes from the curvaton decay rather than reheating at the end of inflation. Indeed the universe need not be radiation-dominated at all until the curvaton decays if instead the inflaton fast-rolls at the end of inflation.

The curvaton offers a new range of theoretical possibilities, but ultimately we will require observational and/or experimental predictions to decide whether the curvaton, inflaton or some othe field generated the primordial perturbation. I will discuss observational predictions of the curvaton scenario in the following subsection.

6.1 Non-Gaussianity

The best way to distinguish between different scenarios for the origin of structure could be the statistical properties of the primordial density perturbation. Primordial density perturbations in the curvaton scenario originate from the small-scale vacuum fluctuations of the weakly interacting curvaton field during inflation, which can be described on super-Hubble scales by a Gaussian random field. Thus deviations from Gaussianity in the primordial bispectrum and connected trispectrum can be parameterised by the dimensionless parameters f_{NL} and g_{NL} defined in Eqs. (104) and (105).

When the curvaton field begins oscillating about a quadratic minimum of its potential we have $\rho_\chi = m_\chi^2\chi^2/2$, and the time-averaged equation of state becomes, $P_\chi = 0$. The non-linear generalisation of the primordial curvature perturbation (46) on hypersurfaces of uniform-curvaton density is then

$$\zeta_\chi = \frac{1}{3}\ln\left(\frac{\tilde{\rho}_\chi}{\rho_\chi}\right)_\psi , \tag{119}$$

where we distinguish here between the inhomogeneous density $\tilde{\rho}_\chi$ on spatially flat hypersurfaces and the average density ρ_χ. Given that $\rho_\chi \propto \chi^2$ we thus have

$$\zeta_\chi = \frac{1}{3}\ln\left(1+\frac{2\chi\delta\chi+\delta\chi^2}{\chi^2}\right)_\psi . \tag{120}$$

This gives the full probability distribution function for ζ_χ for Gaussian field perturbations $\delta\chi$. Expanding to first-order we obtain $\zeta_{\chi 1} = 2\delta\chi_\psi/3\chi$, and then to second- and third-order we obtain by analogy with Eq. (103) the non-linearity parameters for the curvaton density perturbation

$$\zeta_\chi \simeq \zeta_{\chi 1} + \frac{3}{5}f_{NL}^\chi \zeta_{\chi 1}^2 + \frac{9}{25}g_{NL}^\chi \zeta_{\chi 1}^3 + \dots , \tag{121}$$

where [64]

$$f_{NL}^\chi = -\frac{5}{4} , \tag{122}$$

$$g_{NL}^\chi = \frac{25}{12} . \tag{123}$$

If the curvaton dominates the total energy density before it decays into radiation, then this is the curvature perturbation, and specifically the non-linearity parameters, inherited by the primordial radiation density. Although not suppressed by slow-roll parameters, this non-Gaussianity is still smaller than the best upper limits expected from the Planck satellite [75].

On the other hand if the curvaton decays before it dominates over the energy density of the existing radiation, so the transfer function $r_\chi(p) \ll 1$ in Eq. (115), then the curvaton may lead to a large and detectable non-Gaussianity in the radiation density after it decays. Assuming the sudden decay of the curvaton on the $H = \Gamma$ uniform-density hypersurface leads to a non-linear relation between the local curvaton density and the radiation density before and after the decay [64]

$$\rho_\gamma e^{-4\zeta} + \rho_\chi e^{3(\zeta_\chi - \zeta)} = \rho_\gamma + \rho_\chi . \tag{124}$$

Expanding this term by term yields [77, 57, 64]

$$f_{NL} = \frac{5}{4r_\chi} - \frac{5}{3} - \frac{5r_\chi}{6} , \tag{125}$$

$$g_{NL} = -\frac{25}{6r_\chi} + \frac{25}{108} + \frac{125r_\chi}{27} + \frac{25r_\chi^2}{18} . \tag{126}$$

These reduce to Eqs. (122) and (123) as $r_\chi \to 1$, but become large for $r_\chi \ll 1$. These analytic results rely on the sudden decay approximation but have been tested against numerical solutions [78, 64] and give an excellent approximation for both $r_\chi \ll 1$ and $r_\chi \simeq 1$.

More generally one can use Eqs. (119) and (124), or use Eq. (91) and solve the non-linear, but homogeneous equations of motion to determine $N(\chi)$ to give the full probability distribution function for the primordial curvature perturbation ζ [64].

Current bounds from the WMAP satellite require $-54 < f_{NL} < 114$ at the 95% confidence limit [43], and hence require $r_\chi > 0.011$. But future cosmic microwave background (CMB) experiments such as Planck could detect f_{NL} as small as around 5 [75], and it has even been suggested that it might one day be possible to constrain $f_{NL} \sim 0.01$ [79].

6.2 Residual isocurvature perturbations

In the curvaton scenario the initial curvaton perturbation is a non-adiabatic perturbation and hence can in principle leave behind a residual non-adiabatic component. Perturbations in this one field, would be responsible for both the total primordial density perturbation and any isocurvature mode and hence there is the clear prediction that the two should be completely correlated, corresponding to $\cos\Theta = \pm 1$ in Eq. (80) and $\Delta n_{\mathcal{R}} - 1 = \Delta n_{\mathcal{S}} = \Delta n_{\mathcal{C}}$ in Eqs. (84).

Using ζ_I defined in Eq. (46) for different matter components it is easy to see how the curvaton could leave residual isocurvature perturbations after the curvaton decays. If any fluid has decoupled before the curvaton contributes significantly to the total energy density that fluid remains unperturbed with $\zeta_I \simeq 0$, whereas after the curvaton decays into radiation the photons perturbation is given by (115). Thus a residual isocurvature perturbation (47) is left

$$S_I = -3\zeta \,, \tag{127}$$

which remains constant for decoupled perfect fluids on large scales.

The observational bound on isocurvature matter perturbations completely correlated with the photon perturbation, is [80]

$$-0.42 < \frac{S_B + (\rho_{\rm cdm}/\rho_B)S_{\rm cdm}}{\zeta_\gamma} < 0.25 \,. \tag{128}$$

In particular if the baryon asymmetry is generated while the total density perturbation is still negligible then the residual baryon isocurvature perturbation, $S_B = -3\zeta_\gamma$ would be much larger than the observational bound and such models are thus ruled out. The observational bound on CDM isocurvature perturbations are stronger by the factor $\rho_{\rm cdm}/\rho_B$ [81] although CDM is usually assumed to decouple relatively late.

An interesting amplitude of residual isocurvature perturbations might be realised if the decay of the curvaton itself is the non-equilibrium event that generates the baryon asymmetry. In this case the net baryon number density directly inherits the perturbation $\zeta_B = \zeta_\chi$ while the photon perturbation $\zeta_\gamma \leq \zeta_\chi$ may be diluted by pre-existing radiation and is given by Eq. (115). Note that so long as the net baryon number is locally conserved it defines a conserved perturbation on large scales, even though it may still be interacting with other fluids and fields [76]. Hence the primordial baryon isocurvature perturbation (45) in this case is given by

$$S_B = 3(1 - r_\chi)\zeta_\chi = \frac{3(1 - r_\chi)}{r_\chi}\zeta_\gamma \,. \tag{129}$$

Thus the observational bound (128) requires $r_\chi > 0.92$ if the baryon asymmetry is generated by the curvaton decay.

There is no lower bound on the predicted amplitude of residual non-adiabatic modes and, although the detection of completely correlated isocurvature perturbations would give strong support to the curvaton scenario, the non-detection of primordial isocurvature density perturbations cannot be used to rule out the curvaton scenario. In particular, if the curvaton decays at sufficiently high temperature and all the particles produced relax to a thermal equilibrium abundance, characterised by a common temperature (and vanishing chemical potentials) then no residual isocurvature perturbations survive. In full thermal equilibrium there is a unique attractor trajectory in phase-space and only adiabatic perturbations (along this trajectory) survive on large scales.

7 Conclusions

Inflation offers a beautifully simple origin for structure in our Universe. The zero-point fluctuations of the quantum vacuum state on sub-atomic scales are swept up by the accelerated expansion to astronomical scales, seeding an almost Gaussian distribution of primordial density perturbations. The large scale structure of our Universe can then form simply due to the gravitational instability of overdense regions.

Astronomical observations over recent years have given strong support to this simple picture. But increasingly precise astronomical data will increasingly allow us to probe not only the parameters of what has become the standard cosmological model, but also to probe the nature of the primordial perturbations from which structure formed. Any evidence of primordial gravitational waves, primordial isocurvature fluctuations, and/or non-Gaussianity of the primordial perturbations could provide valuable information about the inflationary dynamics that preceded the hot big bang.

Single-field slow-roll inflation predicts adiabatic density perturbations with negligible non-Gaussianity, but could produce a gravitational wave background which could be detected by upcoming CMB experiments.

On the other hand multiple field inflation can lead to a wider range of possibilities which could be distinguished by observations. A spectrum of non-adiabatic field fluctuations on large scales during inflation could leave residual isocurvature perturbations after inflation, which can be correlated with the primordial curvature perturbation, and can give rise to a detectable level of non-Gaussianity. In simple models, such as the curvaton scenario, where the primordial curvature perturbations originate from almost Gaussian fluctuations in a single scalar field, any residual isocurvature perturbations are expected to be completely correlated with the curvature perturbation and the non-Gaussianity is of a specific “local” form.

After 25 years studying inflation, we may for the first time have evidence of a weak scale-dependence of the power spectrum of the primordial curvature perturbation [43] which would begin to reveal the slow-roll dynamics during inflation. Primordial perturbations may have much more to tell us about the physics of inflation in the future.

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