

# More on ghosts in DGP model

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It is shown by an explicit calculation that the excitations about the self-accelerating cosmological solution of the Dvali–Gabadaze–Porrati model contain a ghost mode. This raises serious doubts about viability of this solution. Our analysis reveals the similarity between the quadratic theory for the perturbations around the self-accelerating Universe and an Abelian gauge model with two Stückelberg fields.

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## I. INTRODUCTION AND SUMMARY

Accelerated expansion of the Universe [1] is one of the most important discoveries in cosmology. Usually, it is explained by the existence of tiny cosmological constant or some scalar fields in the framework of the Einstein’s general relativity. However, it is worth exploring an alternative possibility that general relativity breaks down on cosmological scales and the accelerated expansion of the Universe results from modification of gravitational laws themselves. Many attempts have been made in this direction, but, it has been recognized that it is extremely difficult to construct a consistent theory. One of the problems often encountered by the attempts to modify gravity in the infrared is appearance of ghost modes in the spectrum of the theory. These are modes with the wrong sign in front of the kinetic term. They have negative kinetic energy and lead to vacuum instability with respect to creation of ghost particles together with ordinary matter particles, the interaction between ordinary matter and ghost being mediated (at least) by gravity. The rate of the development of such instability diverges due to infinite phase volume unless one introduces a Lorentz-violating cutoff, see e.g. [2].

An interesting model incorporating modification of gravitational laws at large distances was proposed by Dvali, Gabadadze and Porrati (DGP) in [3]. The model describes a brane with four-dimensional worldvolume, embedded into flat five-dimensional bulk. Ordinary matter is supposed to be localized on the brane, while gravity can propagate in the bulk. A crucial ingredient of the model is the induced Einstein–Hilbert action on the brane. The action of the model is given by

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} R + \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-\gamma} {}^{(4)}R + \int d^4x \sqrt{-\gamma} \mathcal{L}_{mat}, \quad (1)$$

where  $R$  is the five-dimensional Ricci curvature,  ${}^{(4)}R$  is the intrinsic curvature of the brane computed using the induced metric  $\gamma_{\mu\nu}$ , and  $\mathcal{L}_{mat}$  is the Lagrangian of the matter on the brane. Throughout the paper we use

$(-, +, +, \dots)$  convention for the signature of the metric. The induced gravity term is responsible for the recovery of four-dimensional Einstein gravity at moderate scales<sup>1</sup>, while at distances larger than

$$r_c \equiv \frac{\kappa^2}{2\kappa_4^2} \quad (2)$$

gravity is five-dimensional. Cosmology in the DGP model is governed by the following modification of the Friedman equation [6],

$$H^2 = \frac{\kappa_4^2 \rho_{mat}}{3} \mp \frac{H}{r_c}, \quad (3)$$

where  $H$  is the Hubble parameter and  $\rho_{mat}$  is the matter density on the brane. Two possible choices of sign in (3) give two branches of the cosmological evolution. The upper sign corresponds to the Universe whose expansion, in the absence of the cosmological constant on the brane, decelerates at late times, the Hubble parameter tending to zero as the matter on the brane dissolves. We call this branch of solutions the Friedman–Robertson–Walker (FRW) branch. On the other hand, the choice of the lower sign in (3) makes possible de Sitter expansion of the Universe with the Hubble parameter  $H = 1/r_c$  even in the absence of matter. Thus, the latter branch contains the self-accelerating solution, where the accelerated expansion of the Universe is realized without introduction of the cosmological constant on the brane. Below, we will refer to this branch of solutions as the self-accelerating branch.

However, the self-accelerating branch of solutions turns out to be plagued by the ghost instability<sup>2</sup>. This was first demonstrated in [5, 7] using the boundary effective action formalism. In [8] this result was confirmed by explicit calculation of the spectrum of linear perturbations in the five-dimensional framework. The analysis was performed in the case when the role of matter on the brane

<sup>1</sup> Let us mention that the mechanism of this recovery of four-dimensional gravity is rather non-trivial [4, 5].

<sup>2</sup> This is not the case for the FRW branch.

is played by non-zero brane tension  $\sigma \neq 0$ . At  $\sigma < 0$  the ghost mode was identified with the scalar field describing the brane bending, while at  $\sigma > 0$  it was found that the ghost degree of freedom coincides with the helicity-0 component of the graviton, which turns out to be massive. By continuity, one expects the ghost to be present also in the case  $\sigma = 0$  which corresponds precisely to the self-accelerating cosmological evolution.

The purpose of this paper is to show the presence of ghost in the self-accelerating Universe with  $\sigma = 0$  by an explicit calculation. The analysis is subtler in this case than for  $\sigma \neq 0$ . The reason is that at  $\sigma = 0$  the masses of the graviton and the brane bending mode coincide and the two modes mix. As a consequence, it is impossible to diagonalize the quadratic Lagrangian for these modes and single out the Lagrangian for the ghost mode. Instead, to demonstrate the existence of ghost, we use the Hamiltonian approach [9, 10]. We construct the Hamiltonian for the helicity-0 excitations and find that it is unbounded from below. For modes of high momentum,  $k \gg H$ , the mixing terms in the Hamiltonian can be neglected and it decomposes into a sum of Hamiltonians for a positive energy mode and ghost.

The paper is organized as follows. We start by considering in Sec. II a simple model illustrating the subtleties encountered by the analysis of the spectrum in the self-accelerating Universe. In Sec. III we find the perturbations about the self-accelerating solution of the DGP model and construct the four-dimensional effective action for the discrete graviton mode and the brane-bending mode. In Sec. IV we construct the Hamiltonian for the helicity-0 excitations and show that it contains a ghost mode. Technical details of the calculation of the Hamiltonian are presented in Appendix.

## II. ABELIAN EXAMPLE

To set the stage for the analysis of the quadratic theory for perturbations about the self-accelerating solution, let us consider a simple toy model whose spectrum exhibits analogous properties. We consider an Abelian gauge theory coupled to two Stückelberg fields<sup>3</sup>. The Lagrangian of the model has the form,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{f_1^2}{2}(\partial_\mu\varphi_1 + eA_\mu)^2 + \frac{f_2^2}{2}(\partial_\mu\varphi_2 + eA_\mu)^2, \quad (4)$$

where  $f_1^2, f_2^2 > 0$ . When the interaction between the vector field and the scalar bosons is switched off by setting  $e = 0$ , the fields  $\varphi_1$  and  $\varphi_2$  are an ordinary scalar field and ghost, respectively. Below we assume  $e \neq 0$ . The system possesses gauge symmetry

$$A_\mu \mapsto A_\mu - \frac{1}{e}\partial_\mu\alpha, \quad \varphi_1 \mapsto \varphi_1 + \alpha, \quad \varphi_2 \mapsto \varphi_2 + \alpha, \quad (5)$$

which can be used to set  $\varphi_2 = 0$ . In the case  $f_1^2 \neq f_2^2$  one can diagonalize the Lagrangian (4) by introducing the vector field

$$B_\mu = A_\mu + \frac{f_1^2\partial_\mu\varphi_1}{(f_1^2 - f_2^2)e}. \quad (6)$$

The result of the diagonalization has the form

$$\mathcal{L} = -\frac{1}{4}B_{\mu\nu}^2 - \frac{(f_1^2 - f_2^2)e^2}{2}B_\mu^2 + \frac{f_1^2f_2^2}{2(f_1^2 - f_2^2)}(\partial_\mu\varphi_1)^2. \quad (7)$$

It describes a massless scalar field and a massive vector with the mass  $m_B^2 = (f_1^2 - f_2^2)e^2$ . If  $f_1^2 - f_2^2 > 0$  the scalar is ghost, while the vector field has positive mass squared. On the other hand, when  $f_1^2 - f_2^2 < 0$  the sign in front of the scalar kinetic term is correct, while the mass squared of the vector becomes negative. This implies that the longitudinal component of the vector is ghost in this case.

At the point  $f_1^2 - f_2^2 = 0$  in the parameter space the vector field becomes massless. Simultaneously, the transformation (6) becomes singular suggesting that the Lagrangian cannot be diagonalized. In terms of fields  $A_\mu, \varphi_1$  the Lagrangian reads

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{f_1^2}{2}(\partial_\mu\varphi_1)^2 - f_1^2eA_\mu\partial^\mu\varphi_1. \quad (8)$$

The last term describes mixing between the vector and the scalar fields. Note that, if this last term were absent, the Lagrangian (8) would possess an Abelian gauge symmetry which would make the helicity-0 component of the vector unphysical, and the theory would be ghost-free. However, the Lagrangian (8) as it stands, is not gauge invariant and the helicity-0 component of the vector field becomes a physical ghost through its mixing with the scalar field.

To demonstrate this we consider the Hamiltonian of the theory (8). One concentrates on the helicity-0 sector as the transverse degrees of freedom are unaffected by the presence of the scalar field, and hence, their energy is positive definite. One performs the Fourier decomposition

$$\begin{aligned} A_0 &= \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} n(\mathbf{k}, t), \\ A_i &= -i \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} \frac{k_i}{|k|} \alpha(\mathbf{k}, t), \\ \varphi_1 &= \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} \phi(\mathbf{k}, t). \end{aligned} \quad (9)$$

Inserting these expressions into (8) and integrating out the variable  $n$  one obtains the Lagrangian for the fields  $\alpha, \phi$ ,

$$L_0 = \int d^3k \left\{ \frac{f_1^2}{2} \left( 1 - \frac{(f_1e)^2}{k^2} \right) |\dot{\phi}|^2 - \frac{f_1^2e}{|k|} \dot{\phi}\dot{\alpha}^* - \frac{f_1^2k^2}{2} |\phi|^2 + f_1^2e|k|\alpha\phi^* \right\}, \quad (10)$$

<sup>3</sup> We thank R. Rattazzi for suggesting this model to us.

where dot denotes derivative with respect to time. In writing (10) we made use of the relations  $\alpha(-\mathbf{k}, t) = \alpha^*(\mathbf{k}, t)$ , etc. Introducing the canonical momenta

$$\pi_\alpha = -\frac{f_1^2 e}{|k|} \dot{\phi}, \quad \pi_\phi = -\frac{f_1^2 e}{|k|} \dot{\alpha} + f_1^2 \left(1 - \frac{(f_1 e)^2}{k^2}\right) \dot{\phi},$$

we obtain the Hamiltonian,

$$\mathcal{H}_0 = \int d^3k \left\{ -\frac{k}{f_1^2 e} \pi_\phi \pi_\alpha^* + \frac{-k^2 + (f_1 e)^2}{2f_1^2 e^2} |\pi_\alpha|^2 + \frac{f_1^2 k^2}{2} |\phi|^2 - f_1^2 e k \alpha \phi^* \right\}. \quad (11)$$

After the canonical transformation

$$\begin{aligned} \tilde{\alpha} &= f_1 e \alpha, & \tilde{\phi} &= -f_1 e \alpha + f_1 k \phi, \\ \pi_\alpha &= f_1 e (\tilde{\pi}_\alpha - \tilde{\pi}_\phi), & \pi_\phi &= f_1 k \tilde{\pi}_\phi, \end{aligned} \quad (12)$$

it takes the following form

$$\mathcal{H}_0 = \int d^3k \left\{ \frac{k^2 + (f_1 e)^2}{2} |\tilde{\pi}_\phi|^2 - (f_1 e)^2 \tilde{\pi}_\phi \tilde{\pi}_\alpha^* + \frac{-k^2 + (f_1 e)^2}{2} |\tilde{\pi}_\alpha|^2 + \frac{|\tilde{\phi}|^2}{2} - \frac{|\tilde{\alpha}|^2}{2} \right\}. \quad (13)$$

Clearly, this Hamiltonian is unbounded from below, negative energies being associated with the field  $\tilde{\alpha}$ . For the modes of high spatial momentum,  $k^2 \gg (f_1 e)^2$ , one can neglect the mixing terms in (13); then, it is clear that the mode  $\tilde{\alpha}$  is ghost.

At generic values of the momentum  $k$ , the Hamiltonian (13) represents one of the normal forms of quadratic Hamiltonians. It cannot be diagonalized by a canonical transformation (see, e.g. [11]). This fact is a manifestation of the *resonance* between the modes  $\tilde{\alpha}$  and  $\tilde{\phi}$ . The solution of the equations of motion following from (13) has the form

$$\begin{pmatrix} \tilde{\alpha} \\ \tilde{\phi} \end{pmatrix} = a^\pm \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{\pm i|k|t} + b^\pm \left[ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \mp i \frac{(f_1 e)^2}{|k|} t \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right] e^{\pm i|k|t}$$

It contains a linearly growing part due to the resonance. Again, this part can be neglected for high frequency modes and at short time scales  $t \sim \frac{1}{|k|} \ll \frac{|k|}{(f_1 e)^2}$ . The solution in this regime becomes the sum of two purely oscillatory modes.

### III. PERTURBATIONS IN SELF-ACCELERATING UNIVERSE

We now proceed to apply the Hamiltonian analysis, analogous to that of the previous section, to the DGP model. In this section we study the linear perturbations about the self-accelerating cosmological solution, and construct quadratic four-dimensional action for the localized modes.

The five dimensional background metric corresponding to the self-accelerated branch has the form [6],

$$ds^2 = dy^2 + N^2(y) \gamma_{\mu\nu}(x) dx^\mu dx^\nu, \quad (14)$$

where  $N(y) = 1 + H|y|$ , and  $\gamma_{\mu\nu}$  is the four-dimensional de Sitter (dS) metric with the Hubble parameter  $H$ . The brane is located at  $y = 0$ , and  $Z_2$  symmetry across the brane is imposed. The case of vanishing brane tension, which is of primary interest to us, corresponds to

$$H = \frac{1}{r_c}.$$

However, we will not use this relation for some while in order to be able to compare the cases of vanishing and non-zero brane tensions.

Let us now consider perturbations of the metric. We impose the gauge

$$\delta g_{55} = \delta g_{5\mu} = 0, \quad (15)$$

and write  $g_{\mu\nu} = N^2 \gamma_{\mu\nu} + h_{\mu\nu}(x, y)$ . For perturbations obeying equations of motion it is possible to impose in addition to (15) the transverse-traceless (TT) gauge  $h_\mu^\mu = 0$ ,  $\nabla_\mu h_\nu^\mu = 0$ , where  $\nabla_\mu$  denotes covariant derivative with respect to the metric  $\gamma_{\mu\nu}$ , and indices are raised using the metric  $\gamma^{\mu\nu}$ . In this gauge the Einstein's equations reduce to

$$h''_{\mu\nu} + \frac{1}{N^2} (\square h_{\mu\nu} - 4H^2 h_{\mu\nu}) = 0, \quad (16)$$

where  $\square \equiv \nabla_\lambda \nabla^\lambda$ . We will call the coordinate frame where the metric is TT the bulk frame. In this frame the brane is displaced from the origin. We parametrize the perturbation of its position by  $y = \varphi(x)$ . With the account for the brane bending one obtains the following junction condition at the brane in the TT gauge [8],

$$\begin{aligned} \frac{1}{2} h'_{\mu\nu} + \frac{r_c}{2} \square h_{\mu\nu} - H(1 + Hr_c) h_{\mu\nu} \\ = (2Hr_c - 1) (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu}) \varphi. \end{aligned} \quad (17)$$

Taking the trace of this equation results in the equation of motion for the field  $\varphi$ ,

$$\square \varphi + 4H^2 \varphi = 0. \quad (18)$$

Let us summarize the results about solutions of Eqs. (16), (17) in the case  $Hr_c \neq 1$  [8]. First, there are solutions leaving the brane at rest,  $\varphi = 0$ . They describe a tower of modes of the form  $h_{\mu\nu}(x, y) = \chi_{\mu\nu}^{(m)}(x) F_m(y)$ . The fields  $\chi_{\mu\nu}^{(m)}$  satisfy the four-dimensional equation  $\square \chi_{\mu\nu}^{(m)} = (m^2 + 2H^2) \chi_{\mu\nu}^{(m)}$  for massive spin-2 fields in dS space-time. The functions  $F_m(y)$  obey the following equations,

$$F_m'' + \frac{m^2 - 2H^2}{N^2} F_m = 0, \quad y > 0, \quad (19)$$

$$F_m' - 2HF_m = -m^2 r_c F_m, \quad y = 0, \quad (20)$$

where prime denotes differentiation with respect to  $y$ . There is a continuum spectrum of modes with masses  $m^2 \geq (9/4)H^2$ , and a normalizable mode

$$F_{m_d}(y) = [N(y)]^{-1 + \frac{1}{Hr_c}} \quad (21)$$

with the mass

$$m_d^2 = \frac{3Hr_c - 1}{r_c^2}. \quad (22)$$

Second, there is a discrete mode with non-zero brane bending. The corresponding perturbation of the metric has the form,

$$h_{\mu\nu}(x, y) = \frac{1 - 2Hr_c}{H(1 - Hr_c)} (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu}) \varphi(x). \quad (23)$$

This is a solution with  $m^2 = 2H^2$ . In [8] the effective quadratic action for the two discrete modes was constructed for the case  $Hr_c \neq 1$ . As the masses of the modes are different in this case,  $m_d^2 \neq 2H^2$ , the Lagrangian decomposes into the Lagrangians for the massive spin-2 field  $\chi_{\mu\nu}$  and the scalar field  $\varphi$ . It was found that at  $Hr_c < 1$  the scalar field is a ghost, while the tensor field is well behaved. [It is worth mentioning that the region  $Hr_c < 1$  corresponds to the unphysical case of negative brane tension. Nevertheless, considering this regime does make sense if the value of  $Hr_c$  is close enough to unity.] On the other hand, at  $Hr_c > 1$  the ghost mode coincides with the helicity-0 component of the field  $\chi_{\mu\nu}$ , while the scalar field has the correct sign in front of its kinetic term. At the point  $Hr_c = 1$  the masses of the two modes coincide, and the solution (23) becomes singular. This signals that at the point  $Hr_c = 1$  the Lagrangian for the discrete modes cannot be diagonalized. One concludes that the situation is analogous to the situation in the Abelian model of the previous section.

Let us concentrate on the sector of the localized modes. Keeping these modes only, the expression for metric perturbations at  $Hr_c \neq 1$  reads

$$h_{\mu\nu}(x, y) = \chi_{\mu\nu}^{(m_d)}(x) F_{m_d}(y) + \frac{1 - 2Hr_c}{H(1 - Hr_c)} (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu}) \varphi(x).$$

To obtain a non-singular Lagrangian for the localized modes in the limit  $Hr_c \rightarrow 1$  corresponding to the self-

accelerating Universe we perform a field redefinition (cf. Eq. (6))

$$\chi_{\mu\nu}^{(m_d)}(x) = A_{\mu\nu}(x) - \frac{1 - 2Hr_c}{H(1 - Hr_c)} (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu}) \varphi(x).$$

In terms of the fields  $A_{\mu\nu}$ ,  $\varphi$  the KK decomposition of the metric perturbation reads

$$h_{\mu\nu}(x, y) = A_{\mu\nu}(x) F_{m_d}(y) + \frac{1 - 2Hr_c}{H(1 - Hr_c)} (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu}) \varphi(x) (1 - F_{m_d}(y)).$$

Now, we can take the limit  $Hr_c \rightarrow 1$ , keeping both fields  $A_{\mu\nu}(x)$  and  $\varphi(x)$  finite. Using Eq. (21) we obtain,

$$h_{\mu\nu} = A_{\mu\nu}(x) + \frac{1}{H} (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu}) \varphi(x) \log(1 + Hy). \quad (24)$$

We note in passing that this representation of  $h_{\mu\nu}$  in terms of localized modes can be obtained directly in the case  $Hr_c = 1$  starting from the Einstein equations in the bulk and junction conditions on the brane.

From now on we set  $Hr_c = 1$ . The field  $\varphi$  still obeys Eq. (18). To obtain the equations of motion for the field  $A_{\mu\nu}$  we plug the expression (24) into Eqs. (16), (17). This yields

$$\square A_{\mu\nu} - 4H^2 A_{\mu\nu} = H (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu}) \varphi. \quad (25)$$

Note, that the TT condition for the metric perturbation  $h_{\mu\nu}$  implies that  $A_{\mu\nu}$  must be also TT.

We proceed to construct the effective quadratic action for the fields  $A_{\mu\nu}$  and  $\varphi$ . The full quadratic action has the form:

$$S^{(2)} = \frac{1}{4\kappa^2} \int d^5x \sqrt{-g} \delta g^{MN} \delta G_{MN}^{(5)} + \frac{1}{4\kappa_4^2} \int d^4x \sqrt{-\gamma} \delta g^{\mu\nu} \delta G_{\mu\nu}^{(4)}, \quad (26)$$

where  $\delta G_{MN}^{(5)}$ ,  $\delta G_{\mu\nu}^{(4)}$  are the variations of the five-dimensional and four-dimensional Einstein's tensors, respectively. We impose the gauge (15) and write the action (26) in the Gaussian normal (GN) coordinate frame where the brane is at rest,

$$S^{(2)} = -\frac{1}{2\kappa^2} \left[ \int_{+0}^{+\infty} dy d^4x \sqrt{-\gamma} \left( -\frac{1}{2} \bar{h}^{\mu\nu} \bar{h}''_{\mu\nu} + \frac{1}{2} \bar{h} \bar{h}'' + \frac{1}{N^2} \bar{h}^{\mu\nu} X_{\mu\nu}(\bar{h}) + \frac{H^2}{N^2} \bar{h}^{\mu\nu} \bar{h}_{\mu\nu} - \frac{H^2}{N^2} \bar{h}^2 \right) + \int d^4x \sqrt{-\gamma} \left( \frac{1}{H} \bar{h}^{\mu\nu} X_{\mu\nu}(\bar{h}) + H \bar{h}^{\mu\nu} \bar{h}_{\mu\nu} - H \bar{h}^2 - \frac{1}{2} \bar{h}^{\mu\nu} \bar{h}'_{\mu\nu} + \frac{1}{2} \bar{h} \bar{h}' \right) \right]. \quad (27)$$

Here  $\bar{h}_{\mu\nu}$  is the perturbation of the metric in the GN frame,  $\bar{h} \equiv \bar{h}_{\mu}^{\mu}$  and

$$\begin{aligned} X_{\mu\nu}(\bar{h}) &\equiv \delta G_{\mu\nu}^{(4)} + 3H^2 \bar{h}_{\mu\nu} \\ &= -\frac{1}{2} (\square \bar{h}_{\mu\nu} - \nabla_{\mu} \nabla_{\alpha} \bar{h}_{\nu}^{\alpha} - \nabla_{\nu} \nabla_{\alpha} \bar{h}_{\mu}^{\alpha} + \nabla_{\mu} \nabla_{\nu} \bar{h}) \\ &\quad - \frac{1}{2} \gamma_{\mu\nu} (\nabla_{\alpha} \nabla_{\beta} \bar{h}^{\alpha\beta} - \square \bar{h}) + H^2 \left( \bar{h}_{\mu\nu} + \frac{1}{2} \gamma_{\mu\nu} \bar{h} \right). \end{aligned}$$

In deriving the expression (27) we included the contributions from the first integral in (26) which are proportional to  $\delta(y)$  into the integral over the brane. Let us note that the action, restricted to the gauge (15) does not give (55) and (5 $\mu$ ) Einstein equations. So, a priori, the latter equations must be imposed as constraints. However, we will

see that the equations of motion coming from the effective action imply that the metric perturbations are TT. This, in turn, implies that the (55) and (5 $\mu$ ) Einstein equations are automatically satisfied.

The metrics in GN and bulk frames are related by the gauge transformation

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{2N}{H} \nabla_{\mu} \nabla_{\nu} \varphi - 2HN \gamma_{\mu\nu} \varphi. \quad (28)$$

To obtain the effective action for  $A_{\mu\nu}$ ,  $\varphi$  one takes  $h_{\mu\nu}$  in the form (24) and plugs the expression (28) in (27). Note that in spite of the logarithmic growth of the metric perturbation (24) into the fifth dimension, the integrals over  $y$  in (27) are finite due to the presence of  $1/N^2(y)$  warp factors. After straightforward calculation one obtains

$$\begin{aligned} S_{eff} &= \frac{1}{\mathcal{V}^2 H} \int d^4 x \sqrt{-\gamma} \left\{ -A^{\mu\nu} X_{\mu\nu}(A) - H^2 A^{\mu\nu} A_{\mu\nu} + H^2 A^2 \right. \\ &\quad \left. - H(A^{\mu\nu} \nabla_{\mu} \nabla_{\nu} \varphi - A \square \varphi - 3H^2 A \varphi) - \frac{9H^2}{4} \varphi (\square + 4H^2) \varphi \right\}, \end{aligned} \quad (29)$$

where we introduced the notation  $A \equiv A_{\mu}^{\mu}$ . Let us stress that it would be incorrect to impose the TT condition on the field  $A_{\mu\nu}$  in the action (29). In particular, assuming  $A_{\mu\nu}$  to be TT would lead to the absence of the term describing mixing between the fields  $A_{\mu\nu}$  and  $\varphi$  in (29), and hence, to incorrect field equations. On the other hand, let us show that the equations of motion following from the action (29) entail both the TT conditions and the field equations (18), (25). By varying the action (29) we obtain

$$\begin{aligned} -2X_{\mu\nu}(A) - 2H^2 A_{\mu\nu} + 2H^2 \gamma_{\mu\nu} A \\ - H \nabla_{\mu} \nabla_{\nu} \varphi + H \gamma_{\mu\nu} \square \varphi + 3H^3 \gamma_{\mu\nu} \varphi = 0, \quad (30) \\ -\nabla_{\mu} \nabla_{\nu} A^{\mu\nu} + \square A + 3H^2 A - \frac{9H}{2} (\square + 4H^2) \varphi = 0. \end{aligned} \quad (31)$$

Taking the covariant divergence and the trace of the first equation one obtains

$$\nabla_{\mu} A_{\nu}^{\mu} - \nabla_{\nu} A = 0, \quad (32)$$

$$2\nabla_{\mu} \nabla_{\nu} A^{\mu\nu} - 2\square A + 3H(\square + 4H^2)\varphi = 0, \quad (33)$$

where in deriving (32) we made use of the identity  $\nabla_{\mu} X_{\nu}^{\mu}(A) \equiv 0$ . Equations (32), (33) imply Eq. (18). Now, Eqs. (31), (32), (18) yield the TT conditions,

$$\nabla_{\mu} A_{\nu}^{\mu} = 0, \quad A = 0. \quad (34)$$

With the use of Eq. (18) and the TT conditions equation (30) is reduced to Eq. (25).

Before proceeding further, let us make the following comment. The first line in (29) coincides with the quadratic Lagrangian for the Fierz–Pauli theory of massive graviton with the mass  $m^2 = 2H^2$  in dS background. Thus, it is invariant [10] under gauge transformations

$$A_{\mu\nu} \mapsto A_{\mu\nu} + (\nabla_{\mu} \nabla_{\nu} + H^2 \gamma_{\mu\nu}) \omega(x). \quad (35)$$

The mixing of the graviton field  $A_{\mu\nu}$  with the scalar  $\varphi$  in (29) breaks this symmetry explicitly. Let us restore the symmetry by introducing the Stückelberg field  $\psi$ . Namely, let us consider the following action,

$$\begin{aligned}
\tilde{S}_{eff} = \frac{1}{\varkappa^2 H} \int d^4x \sqrt{-\gamma} \left\{ -A^{\mu\nu} X_{\mu\nu}(A) - H^2 A^{\mu\nu} A_{\mu\nu} + H^2 A^2 \right. \\
+ \frac{3}{4} \left( \left[ (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu}) \varphi - \frac{2H}{3} A_{\mu\nu} \right]^2 - \left[ (\square + 4H^2) \varphi - \frac{2H}{3} A \right]^2 \right) \\
\left. - \frac{3}{4} \left( \left[ (\nabla_\mu \nabla_\nu + H^2 \gamma_{\mu\nu}) \psi - \frac{2H}{3} A_{\mu\nu} \right]^2 - \left[ (\square + 4H^2) \psi - \frac{2H}{3} A \right]^2 \right) \right\}. \quad (36)
\end{aligned}$$

Note that the terms with four derivatives in (36) cancel out after integration by parts. The action (36) is invariant under the gauge transformation (35) supplemented by

$$\varphi \mapsto \varphi + \frac{2H}{3} \omega, \quad \psi \mapsto \psi + \frac{2H}{3} \omega. \quad (37)$$

It is straightforward to check that in the gauge  $\psi = 0$  the action (36) reduces to (29). The form of the Lagrangian (36) is similar to the Lagrangian (4) of the Abelian model of Sec. II; it corresponds to equal coefficients in front of the Lagrangians for the Stückelberg fields,  $f_1^2 = f_2^2$ . In fact, one can check, that the quadratic action for the perturbations about the self-accelerating Universe can be written as an action for a massive graviton with the mass  $m^2 = 2H^2$  and two Stückelberg fields in dS space-time at any value of the parameter  $Hr_c$ . We will not pursue this issue further in this paper.

#### IV. HAMILTONIAN FOR HELICITY-0 EXCITATIONS

Let us come back to the four-dimensional theory with quadratic action (29). Our aim now is to show that this theory has ghost in dS space-time. We adopt the approach of Refs. [9, 10] and construct the Hamiltonian for the helicity-0 excitations.

To simplify the formulas we set in this section the four-dimensional Planck mass equal to one,  $\varkappa^2 H = 2\varkappa_4^2 = 1$ . We use the following explicit form of dS metric:

$$ds^2 = -dt^2 + f^2(t) \delta_{ij} dx^i dx^j, \quad (38)$$

where

$$f(t) = e^{Ht}, \quad (39)$$

and  $i$  runs from 1 to 3. One decomposes various tensors into spatial and time components, and introduces

$$A_{ij}, \quad N_i = A_{0i}, \quad n = -A_{00}/2. \quad (40)$$

These notations are reminiscent of the standard notations in the Hamiltonian approach to gravity. Indeed, the first term in (29) represents the quadratic action for gravitational perturbations in dS background with  $\delta g_{\mu\nu} = A_{\mu\nu}$ .

The quantities in Eqs. (40) correspond to linear perturbations of the spatial metric,  $g_{ij} = f^2 \delta_{ij} + A_{ij}$ , the shift functions,  $N_i = g_{0i} = A_{0i}$ , and the lapse function,  $N = (-g^{00})^{-1/2} = 1 + n$ . The next step is to perform the spatial Fourier decomposition. We are interested in the helicity-0 sector, as we expect the ghost mode to be present there. (As to the helicity  $\pm 2$  and  $\pm 1$  sectors, they are not affected by the presence of the scalar  $\varphi$  and do not contain instabilities [10].) So, we write

$$f^{-1/2} A_{ij} = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k}\mathbf{x}} \left\{ \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) \alpha_1(\mathbf{k}, t) + \frac{k_i k_j}{k^2} \alpha_2(\mathbf{k}, t) \right\}, \quad (41a)$$

$$f^{1/2} N_i = -i \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k}\mathbf{x}} \frac{k_i}{|k|} \nu(\mathbf{k}, t), \quad (41b)$$

$$f^{3/2} n = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k}\mathbf{x}} n(\mathbf{k}, t), \quad (41c)$$

$$H f^{3/2} \varphi = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k}\mathbf{x}} \phi(\mathbf{k}, t). \quad (41d)$$

Note that we use the same notation  $n$  for the Fourier transform of the lapse function; this will not lead to confusion. The  $f$  factors on the l.h.s of Eqs. (41) are chosen in such a way that the time-dependence of the resulting Hamiltonian is simplified, see below.

All but two degrees of freedom introduced in (41) are non-dynamical and are eliminated by making use of the constraints. The details of this procedure are presented in Appendix. Once this is done the Hamiltonian is expressed in terms of two dynamical variables, the brane bending mode  $\phi$  and the helicity-0 excitation of the graviton  $v \equiv \alpha_1 - \alpha_2$ . The Hamiltonian has the form,

$$\begin{aligned}
\mathcal{H}_0 = \int d^3k \left\{ \left( \frac{\hat{k}^4}{3H^4} + \frac{\hat{k}^2}{H^2} + \frac{3}{2} \right) |\pi_v|^2 + \frac{2\hat{k}^2}{3H^2} \pi_v \pi_\phi^* \right. \\
- \left( \frac{\hat{k}^2}{3H} - \frac{H}{2} \right) \pi_v v^* - \frac{\hat{k}^2}{2H} \pi_v \phi^* - \left( \frac{\hat{k}^2}{3H} - \frac{5H}{2} \right) \pi_\phi \phi^* \\
\left. + \frac{\hat{k}^2}{6} v \phi^* - \frac{\hat{k}^4}{12H^2} |\phi|^2 \right\}, \quad (42)
\end{aligned}$$

where the canonical momenta  $\pi_v, \pi_\phi$  are conjugate to the

variables  $v$ ,  $\phi$ , respectively, and

$$\hat{k}^2(t) = \frac{k^2}{f^2(t)}. \quad (43)$$

For all fields in the expression (42) one has  $v(-\mathbf{k}, t) = v^*(\mathbf{k}, t)$ , etc. Note that the Hamiltonian (42) explicitly depends on time via  $\hat{k}$ . To simplify the Hamiltonian we perform a canonical transformation to variables  $\tilde{v}$ ,  $\tilde{\pi}_v$ , etc. The transformation is conveniently parametrized by the generating function  $\Phi(v, \phi, \tilde{\pi}_v, \tilde{\pi}_\phi; t)$  such that (see, e.g. [12]),

$$\tilde{v} = \frac{\partial \Phi}{\partial \tilde{\pi}_v^*}, \quad \pi_v = \frac{\partial \Phi}{\partial v^*}, \quad \text{etc.} \quad (44)$$

The Hamiltonian transforms in the following way,

$$\mathcal{H}_0 \mapsto \mathcal{H}_0 + \frac{\partial \Phi}{\partial t}, \quad (45)$$

where the partial time derivative acts on the explicit time dependence of the generating function. With the choice

$$\begin{aligned} \Phi = H \int d^3k \left\{ -\frac{\hat{k}^2}{6H^2} |\tilde{\pi}_v|^2 + \left( \frac{\hat{k}^2}{6H^2} - \frac{1}{2} \right) |\tilde{\pi}_\phi|^2 \right. \\ \left. + \frac{\tilde{\pi}_v \tilde{\pi}_\phi^*}{2} + \frac{\tilde{\pi}_v v^*}{\sqrt{6}} - \frac{\tilde{\pi}_\phi v^*}{\sqrt{6}} + \frac{\hat{k}^2}{\sqrt{6}H^2} \tilde{\pi}_\phi \phi^* \right\}, \end{aligned} \quad (46)$$

one obtains

$$\begin{aligned} \mathcal{H}_0 = \int d^3k \left\{ \left( \hat{k}^2 + \frac{3H^2}{4} \right) \frac{|\tilde{\pi}_v|^2}{2} - H^2 \tilde{\pi}_v \tilde{\pi}_\phi^* \right. \\ \left. + \left( -\hat{k}^2 + \frac{5H^2}{4} \right) \frac{|\tilde{\pi}_\phi|^2}{2} + \frac{|\tilde{v}|^2}{2} - \frac{|\tilde{\phi}|^2}{2} \right\}. \end{aligned} \quad (47)$$

From this expression, it immediately follows that the mode  $\tilde{\phi}$  is ghost.

Two comments are in order. First, the Hamiltonian (47) is explicitly time-dependent via  $\hat{k}^2(t)$  (see Eq. (43)), and hence is not conserved. Instead, the conserved energy in dS space-time [9, 10] is

$$E = \int d^3x T_\mu^0 \bar{\xi}^\mu = \mathcal{H} + \int d^3x T_i^0 H x_i, \quad (48)$$

where  $T_\mu^\nu$  is the energy-momentum tensor of the system, and  $\bar{\xi}^\mu = (-1, H x_i)$  is the Killing vector of dS space-time. However, when one considers the system at distances much shorter than  $1/H$  (which is the case relevant to the ultraviolet stability), the second term in (48) can be neglected, and the energy coincides with the Hamiltonian. Second, the expression (47) is not diagonal, reflecting the resonance between the two modes. The situation again is similar to the case of the Abelian model considered in Sec.II. However, for the modes with wavelengths much smaller than the horizon size,  $\hat{k}^2 \gg H^2$ , one can neglect the  $O(H^2)$  terms in (47), and the two modes decouple.

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## APPENDIX A: ELIMINATION OF NON-DYNAMICAL VARIABLES

Substitution of the Fourier decomposition (41) into the action (29) yields the Lagrangian for the helicity 0 sector:

$$\begin{aligned} L_0 = \int d^3k \left\{ -\frac{1}{2} |\dot{\alpha}_1 - 2H\alpha_1 - 2Hn|^2 - (\dot{\alpha}_1 - 2H\alpha_1 - 2Hn)(\dot{\alpha}_2 - 2H\alpha_2 - 2|\hat{k}|\nu - 2Hn)^* \right. \\ \left. + (\dot{\phi} - H\phi)(\dot{\alpha}_1 - 2H\alpha_1 - 2Hn)^* + \frac{1}{2} (\dot{\phi} - H\phi)(\dot{\alpha}_2 - 2H\alpha_2 - 2|\hat{k}|\nu - 2Hn)^* - \hat{k}^2 \phi \alpha_1^* + \frac{\hat{k}^2}{2} |\alpha_1|^2 \right. \\ \left. + H^2 |\alpha_1|^2 + 2H^2 \alpha_1 \alpha_2^* + H^2 |\nu|^2 + n^* \left[ (4H^2 + 2\hat{k}^2) \alpha_1 + 2H^2 \alpha_2 - \hat{k}^2 \phi \right] - \frac{9}{8} |\dot{\phi}|^2 + \frac{9\hat{k}^2}{8} |\phi|^2 - \frac{9H^2}{2} |\phi|^2 \right\}, \end{aligned} \quad (A1)$$

where  $\hat{k}$  is defined by Eq. (43). In deriving (A1) we omitted total derivative terms and used the relations  $(\alpha_1(\mathbf{k}))^* = \alpha_1(-\mathbf{k})$ , etc. The variable  $\nu$  can be integrated out explicitly. From the resulting Lagrangian one determines the canonical momenta:

$$\pi_1 \equiv \frac{\partial L}{\partial \dot{\alpha}_1^*} = - \left( 1 + \frac{2\hat{k}^2}{H^2} \right) \left( \dot{\alpha}_1 - \frac{3H}{2}\alpha_1 - 2Hn \right) - \left( \dot{\alpha}_2 - \frac{3H}{2}\alpha_2 - 2Hn \right) + \left( 1 + \frac{\hat{k}^2}{H^2} \right) \left( \dot{\phi} - \frac{5H}{2}\phi \right), \quad (\text{A2a})$$

$$\pi_2 \equiv \frac{\partial L}{\partial \dot{\alpha}_2^*} = - \left( \dot{\alpha}_1 - \frac{3H}{2}\alpha_1 - 2Hn \right) + \frac{1}{2} \left( \dot{\phi} - \frac{5H}{2}\phi \right), \quad (\text{A2b})$$

$$\pi_\phi \equiv \frac{\partial L}{\partial \dot{\phi}^*} = \left( 1 + \frac{\hat{k}^2}{H^2} \right) \left( \dot{\alpha}_1 - \frac{3H}{2}\alpha_1 - 2Hn \right) + \frac{1}{2} \left( \dot{\alpha}_2 - \frac{3H}{2}\alpha_2 - 2Hn \right) - \left( \frac{9}{4} + \frac{\hat{k}^2}{2H^2} \right) \left( \dot{\phi} - \frac{5H}{2}\phi \right), \quad (\text{A2c})$$

and the Hamiltonian:

$$\begin{aligned} \mathcal{H}_0 = \int d^3k \left\{ -\frac{1}{12}|\pi_1|^2 + \left( \frac{5}{12} + \frac{\hat{k}^2}{H^2} \right) |\pi_2|^2 - \frac{1}{3}|\pi_\phi|^2 - \frac{7}{6}\pi_1\pi_2^* - \frac{1}{3}\pi_1\pi_\phi^* - \frac{1}{3}\pi_2\pi_\phi^* + \frac{3H}{2}\pi_1\alpha_1^* + \frac{3H}{2}\pi_2\alpha_2^* + \frac{5H}{2}\pi_\phi\phi^* \right. \\ \left. + \hat{k}^2\alpha_1\phi^* - \left( \frac{\hat{k}^2}{2} + H^2 \right) |\alpha_1|^2 - 2H^2\alpha_1\alpha_2^* - \frac{9}{8}\hat{k}^2|\phi|^2 + n^* [2H\pi_1 + 2H\pi_2 - (4H^2 + 2\hat{k}^2)\alpha_1 - 2H^2\alpha_2 + \hat{k}^2\phi] \right\}. \end{aligned}$$

Let us study the set of constraints of the system. From the Hamiltonian one obtains the primary constraint

$$\chi_1 \equiv \pi_1 + \pi_2 - \frac{\hat{k}^2 + 2H^2}{H}\alpha_1 - H\alpha_2 + \frac{\hat{k}^2}{2H}\phi = 0. \quad (\text{A3})$$

Taking the time derivative of Eq. (A3), using equations of motion and the relation

$$\frac{d\hat{k}^2}{dt} = -2H\hat{k}^2, \quad (\text{A4})$$

we find the secondary constraint,

$$\chi_2 \equiv \frac{\pi_1}{H} + \left( \frac{1}{H} - \frac{2\hat{k}^2}{3H^3} \right) \pi_2 + \frac{2}{3H}\pi_\phi - \frac{4}{3}\alpha_1 - \frac{2}{3}\alpha_2 = 0. \quad (\text{A5})$$

Finally, vanishing of the time derivative of Eq. (A5) implies the constraint

$$2n + 2\alpha_1 + \alpha_2 = 0. \quad (\text{A6})$$

Note, that Eq. (A6) is equivalent to the tracelessness condition  $A_\mu^\mu = 0$ . We use Eq. (A6) to eliminate the variable  $n$  from the Hamiltonian. The constraints (A3), (A5) are second class and have canonical Poisson bracket:

$$\{\chi_1, \chi_2\} = 1. \quad (\text{A7})$$

Let us reduce the number of degrees of freedom by performing a canonical transformation to variables  $\tilde{\pi}_1, \tilde{\alpha}_1$ , etc., such that

$$\tilde{\pi}_2 = \chi_1, \quad \tilde{\alpha}_2 = \chi_2. \quad (\text{A8})$$

To find a suitable transformation we introduce the generating function  $\Phi$  depending on the old coordinates  $\alpha_1, \alpha_2, \phi$  and the new momenta  $\tilde{\pi}_1, \tilde{\pi}_2, \tilde{\pi}_\phi$  such that,

$$\tilde{\alpha}_1 = \frac{\partial \Phi}{\partial \tilde{\pi}_1^*}, \quad \pi_1 = \frac{\partial \Phi}{\partial \alpha_1^*}, \quad \text{etc.} \quad (\text{A9})$$

The requirements (A8) imply the following equations:

$$\begin{aligned} \tilde{\pi}_2 &= \frac{\partial \Phi}{\partial \alpha_1^*} + \frac{\partial \Phi}{\partial \alpha_2^*} - \frac{\hat{k}^2 + 2H^2}{H}\alpha_1 - H\alpha_2 + \frac{\hat{k}^2}{2H}\phi, \\ \frac{\partial \Phi}{\partial \tilde{\pi}_2^*} &= \frac{1}{H} \frac{\partial \Phi}{\partial \alpha_1^*} + \left( \frac{1}{H} - \frac{2\hat{k}^2}{3H^3} \right) \frac{\partial \Phi}{\partial \alpha_2^*} \\ &\quad + \frac{2}{3H} \frac{\partial \Phi}{\partial \phi^*} - \frac{4}{3}\alpha_1 - \frac{2}{3}\alpha_2. \end{aligned}$$

As a solution we choose

$$\begin{aligned} \Phi = \int d^3k \left\{ \frac{2\hat{k}^2}{3H^3}\tilde{\pi}_1\tilde{\pi}_2^* + \left( \frac{1}{2H} - \frac{\hat{k}^2}{9H^3} \right) |\tilde{\pi}_2|^2 + \frac{2}{3H}\tilde{\pi}_2\tilde{\pi}_\phi^* \right. \\ \left. + \tilde{\pi}_1\alpha_1^* - \tilde{\pi}_1\alpha_2^* + \frac{2}{3}\tilde{\pi}_2\alpha_1^* + \frac{1}{3}\tilde{\pi}_2\alpha_2^* + \tilde{\pi}_\phi\phi^* \right. \\ \left. + \frac{\hat{k}^2 + H^2}{2H}|\alpha_1|^2 + H\alpha_1\alpha_2^* - \frac{\hat{k}^2}{2H}\alpha_1\phi^* + \frac{3\hat{k}^2}{8H}|\phi|^2 \right\}. \end{aligned}$$

The transformation generated by this function has the form,

$$\tilde{\alpha}_1 = \frac{2\hat{k}^2}{3H^3}\tilde{\pi}_2 + \alpha_1 - \alpha_2, \quad (\text{A10a})$$

$$\begin{aligned} \tilde{\alpha}_2 &= \frac{2\hat{k}^2}{3H^3}\tilde{\pi}_1 + \left( \frac{1}{H} - \frac{2\hat{k}^2}{9H^3} \right) \tilde{\pi}_2 + \frac{2}{3H}\tilde{\pi}_\phi \\ &\quad + \frac{2}{3}\alpha_1 + \frac{1}{3}\alpha_2, \quad (\text{A10b}) \end{aligned}$$

$$\tilde{\phi} = \frac{2}{3H}\tilde{\pi}_2 + \phi, \quad (\text{A10c})$$

$$\pi_1 = \tilde{\pi}_1 + \frac{2}{3}\tilde{\pi}_2 + \frac{\hat{k}^2 + H^2}{H}\alpha_1 + H\alpha_2 - \frac{\hat{k}^2}{2H}\phi, \quad (\text{A10d})$$

$$\pi_2 = -\tilde{\pi}_1 + \frac{1}{3}\tilde{\pi}_2 + H\alpha_1, \quad (\text{A10e})$$



$$\pi_\phi = \tilde{\pi}_\phi - \frac{\hat{k}^2}{2H}\alpha_1 + \frac{3\hat{k}^2}{4H}\phi. \quad (\text{A10f})$$

Using these expressions one can check explicitly that the conditions (A8) are satisfied. Making use of the constraints we set  $\tilde{\pi}_2 = \tilde{\alpha}_2 = 0$ , and we are left with two dynamical variables  $v \equiv \tilde{\alpha}_1 = \alpha_1 - \alpha_2$ ,  $\tilde{\phi} = \phi$  and

their conjugate momenta. In the derivation of the resulting Hamiltonian one should account for the explicit time dependence of the generating function according to Eq. (45). A straightforward calculation yields the Hamiltonian (42) of the main text.

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