

The Amplitude of Dark Energy Perturbations

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Abstract

We propose a model which produces dark energy perturbations large enough to explain the lack of power seen at the quadrupole scale in the cosmic microwave background. If the dark energy is frozen from horizon exit during inflation until dark energy domination, then it is not possible to have perturbations in the dark energy which are large enough. We propose using a tachyonic amplification mechanism to overcome this. The dark energy is taken to be a complex scalar field, where the radial field has a Mexican hat potential. During inflation, the radial component is trapped near the maximum of its potential. At the end of inflation, it rolls down to the minimum. The dark energy today is taken to be a pseudo-Nambu-Goldstone boson. The perturbations generated during inflation are amplified by the rolling of the radial field. We also examine the use of the variable decay mechanism in order to generate an anti-correlation between the dark energy perturbations and the curvature perturbation. We show that using this mechanism then constrains the properties of the dark energy and its evolution from redshift one until today.

1 Introduction

Recently, it has been shown, that there is some evidence, that the dark energy may not be constant on large spatial scales [1, 2]. This is because a smooth dark energy on large scales predicts a temperature/temperature (TT) quadrupole moment in the CMB which is much larger than the observed WMAP value [3, 4]. The original estimate was that the probability of having such a low or lower quadrupole given a cosmological constant was only 0.7% [4]. But, other studies claim the probability is closer to 4% [5].

In reference [6] it was shown that if the dark energy has perturbations which are initially uncorrelated with the perturbations in the other matter components, then this will make the (TT) quadrupole even higher, and so is not favored by the data. Then, in reference [1] it was shown that adding perturbations which are spatially negatively correlated with the initial perturbations in the other components can lower the estimated TT quadrupole. In reference [2], the relative magnitude of the dark energy perturbations ($\delta\rho_Q/\rho_Q$) relative to the curvature perturbation (ζ), on constant density, or equivalently comoving, hyper-surfaces [7, 8, 9], was parameterized as

$$S \equiv \frac{1}{\zeta} \frac{\delta\rho_Q}{\rho_Q}. \quad (1)$$

where the values of the quantities are taken some time before the onset of dark energy domination, which occurs at $z \approx 1$. This parameterization includes the possibility of no perturbations in the dark energy, which occurs when $S = 0$. It was found that

$$\Pr(S > 0|\text{TT}) = 0.04, \quad \Pr(S > 0|\text{TT, TE}) = 0.06 \quad (2)$$

where $\Pr(x|y)$ stands for the probability of x given y and TT and TE are the WMAP temperature and temperature/polarization power spectra. The mean with 68% confidence interval, using the TT and TE, data was

$$S = -11.8 \pm 7.1. \quad (3)$$

The temperature data is already cosmic variance limited at the quadrupole scale, but the polarization data is not, and future measurements may more than double the amount of information available [11, 2].

As first shown in reference [1], for a specific model, and in reference [2], under more general conditions, such large perturbations in the dark energy density, as in Eq. (3), imply a gravitational wave signal which is much larger than allowed by observations. In this paper we propose a mechanism of tachyonically amplifying the initial dark energy perturbations so as to reduce the gravitational wave signal to negligible levels.

In Sec. 2, we review in more detail why the gravitational waves are a problem. The basic amplification mechanism is outlined in Sec. 3. The role of the inflaton field and its relation to the amplification mechanism is investigated in Sec. 4 and Sec. 5. A variable decay reheating mechanism is discussed in Sec. 6. Then, in Sec. 7 an example is given. The conclusions are summarized in Sec. 8.

2 The problem of gravitational waves

The continuity equation for a matter component with equation of state w_Q , can be written as

$$\frac{\rho'_Q}{\rho_Q} = 3(1 + w_Q) \quad (4)$$

where the prime indicates differentiation with respect to the number of e-folds of expansion which is given by

$$N \equiv -\log(a). \quad (5)$$

If we set $a = 1$ today, then at red shift one, when dark energy began to dominate, we have $N \approx 0.7$. In general we will express time in e-folds as this will allow an easy conversion of background equations into perturbation equations via the separate Universe approach [9]. This method was originally used in estimating the curvature perturbation originating from inflation [10]. However, it can also be used for modeling large scale perturbations at any cosmological epoch [9].

Current observational limits of the equation of state of dark energy give [12]

$$w_Q \approx -1 \pm 0.1 \quad (6)$$

at the 68 % confidence interval and so Eq. (4) implies that ρ_Q is close to constant from redshift one till today. Perturbing Eq. (4) gives the large scale

perturbation equation in the flat gauge [9]. In this paper all perturbations of matter variables will be in the flat gauge. If we assume the equation of state of the dark energy is unperturbed then Eq. (4) gives

$$\frac{\delta\rho'_Q}{\delta\rho_Q} = 3(1 + w_Q) \quad (7)$$

which by the same argument as for the background is also approximately constant at least between about redshift one and today. The adiabatic condition is

$$\frac{\delta\rho_Q|_{\text{adiabatic}}}{\rho'_Q} = \frac{\delta\rho_i}{\rho'_i} \quad (8)$$

where i is any of cold dark matter (CDM), baryons, photons or neutrinos. Using Eq. (4) this can be rewritten as

$$\frac{\delta\rho_Q|_{\text{adiabatic}}}{3(1 + w_Q)\rho_Q} = \frac{\delta\rho_i}{3(1 + w_i)\rho_i} \equiv \zeta \quad (9)$$

where w_i is zero for CDM or baryons and a third for photons or neutrinos. The final equivalence follows from the definition of ζ and our flat gauge assumption [9]. Using Eqs. (9), (6), (1) and (3) and that $\rho_Q \sim \rho_{\text{cdm}}$ today, shows that the adiabatic condition is strongly violated. This implies that there must have been more than one light degree of freedom during inflation (see for example reference [13]).

If we assume, as is commonly done, that the dark energy is a light canonical scalar field, then its Lagrangian is given by

$$\mathcal{L}_Q = -\frac{1}{2}\partial_\mu Q\partial^\mu Q - V_Q(Q) \quad (10)$$

where Q is the dark energy scalar field, also known as the ‘quintessence’, and V_Q are the terms in the potential only depending on Q . If during inflation, the quintessence is a light field, then

$$\frac{\partial^2 V_Q}{\partial Q^2} \ll H_{\text{inf}}^2 \quad (11)$$

where H_{inf} is the Hubble parameter during inflation. In which case, Q acquires the usual quantum fluctuations

$$\mathcal{P}_Q(k) = \left(\frac{H_{\text{inf}}}{2\pi}\right)^2 \quad (12)$$

where

$$\mathcal{P}_x(k) \equiv \frac{k^3}{2\pi^2} \langle |\delta x(k)|^2 \rangle \quad (13)$$

is the power spectrum of some quantity x evaluated at wave number k . Current observations give [12]

$$\mathcal{P}_\zeta^{1/2} = 5 \times 10^{-5}. \quad (14)$$

which when combined with Eqs. (1) and (3) give

$$\frac{1}{\rho_Q} \mathcal{P}_{\rho_Q}^{1/2} = 6 \times 10^{-4}. \quad (15)$$

For a canonical scalar field, the dark energy satisfies the Klein-Gordon equation, which when using e-folds as the time parameter is

$$Q'' + \frac{3}{2}(w_T - 1)Q' + \frac{1}{H^2} \frac{\partial V_Q}{\partial Q} = 0 \quad (16)$$

where $w_T = p_T/\rho_T$ is the total equation of state and ρ_T and p_T are the total density and pressure. Eq. (16) has a constant solution provided we can neglect the $(\partial V_Q/\partial Q)/H^2$ term. The other solution is a decaying mode if $w_T < 1$. The first slow parameter is defined as

$$\epsilon_Q \equiv \frac{M_p^2}{2} \left(\frac{1}{V_Q} \frac{\partial V_Q}{\partial Q} \right)^2 \quad (17)$$

where $M_p \equiv 1/8\pi G$ is the reduced Planck mass. The equation of state of the quintessence is given by

$$w_Q \equiv \frac{-\frac{V_Q}{H^2} + \frac{1}{2}Q'^2}{\frac{V_Q}{H^2} + \frac{1}{2}Q'^2} \quad (18)$$

which, with the observational constraints in Eq. (6), implies

$$\frac{1}{2}H^2 Q'^2 \ll V_Q \quad (19)$$

between redshift one and today. So when ρ_Q dominates the energy density

$$H^2 \approx \frac{1}{3M_p^2} V_Q \quad (20)$$

which when combined with Eqs. (16) and (17) implies that $\epsilon_Q < 1$ is needed for an approximately constant Q solution. As H was larger in the past, the conditions for constant Q may be even better satisfied at previous times. If we assume this to be the case, then we can neglect the third term on the left hand side of Eq. (16). Then, before dark energy domination, the total equation of state can be taken to be unperturbed and using the separate Universe approach [9] we can write an approximate equation for the perturbations in Q

$$\delta Q'' + \frac{3}{2}(w_T - 1)\delta Q' \approx 0. \quad (21)$$

Then, if $w_T < 1$, we only have a constant and a decaying solution. In theory, it is still possible to have a transient period of growth in δQ . This can happen if the initial conditions are such that the decaying and constant mode have a different sign and the decaying mode is initially large. However, those initial conditions are not consistent with Q being a light field whose perturbations are generated during inflation.

If, after inflation, there is a period in which the kinetic energy of a canonical scalar field becomes the dominant energy source then during this time $w_T \approx 1$. Then, during the period of kinetic domination, Eq. (21) has a constant and linear solution for δQ . However, between inflation and the onset of kinetic domination, δQ would have been constant and so this will match onto the constant mode of the solution of Eq. (21).

It is possible that in the past the quintessence had a much steeper potential and so was not frozen. Then its motion would have needed to carry it into a domain where the potential slope was shallow. The ‘tracking’ potentials have this property. They require [14],

$$\frac{1}{V_Q} \frac{\partial^2 V_Q}{\partial Q^2} \left(\frac{1}{V_Q} \frac{\partial V_Q}{\partial Q} \right)^{-2} \geq 1. \quad (22)$$

They are insensitive to perturbations in the initial conditions of the field. This implies that any isocurvature perturbations decay [15, 2, 16]. This makes these potentials unsuitable for providing the large isocurvature perturbations needed in Eq. (15).

Taking the kinetic term in the quintessence to be negligible, we can use $\delta \rho_Q \approx \delta Q \partial V_Q / \partial Q$, $\rho_Q \approx V_Q$ and Eq. (17) in Eq. (15) to get

$$\mathcal{P}_Q^{1/2} = 4 \times 10^{-4} M_p \epsilon_Q^{-1/2}. \quad (23)$$

As the current acceleration requires $\epsilon_Q < 1$, Eq. (23) implies

$$\mathcal{P}_Q^{1/2} > 4 \times 10^{-4} M_p \quad (24)$$

This applies to the value of δQ during dark energy domination. However, as we have discussed, a constant δQ is a likely solution, in which case this bound also applies to δQ during inflation. It then follows from Eqs. (24) and (12) that

$$V_{\text{inf}}^{1/4} > 7 \times 10^{-2} M_p \quad (25)$$

which is inconsistent with the current observational limits on gravitational waves [12]

$$V_{\text{inf}}^{1/4} < 10^{-2} M_p \quad (26)$$

at the 95% confidence level. As proposed in [1], a way round these contradictory bounds is to have δQ grow between inflation and dark energy domination. From Eqs. (26), (25) and (12), we see that we require

$$\frac{\delta Q_f}{\delta Q_i} > 45 \quad (27)$$

where subscript i denotes the initial value at horizon exit* during inflation and subscript f denotes the final value at dark energy domination.

3 Tachyonic Amplification

In reference [1] an amplification mechanism that relied on varying the coefficient of the kinetic term in the Lagrangian was proposed. However, no specific mechanism was given for why the coefficient varied. Here, we propose a mechanism that identifies the coefficient with a scalar field which has a tachyonic potential. Say, we write the quintessence as

$$Q \equiv \phi\theta \quad (28)$$

and we assume initially ϕ is fixed at a constant value ϕ_i . Then the kinetic part of the Lagrangian can be written as

$$\mathcal{L}_{\text{kinetic}} \equiv -\partial_\mu Q \partial^\mu Q = -\phi_i^2 \partial_\mu \theta \partial^\mu \theta \quad (29)$$

*By ‘‘horizon exit’’ we mean $\frac{a}{k}$ becomes larger than $\frac{1}{H}$ due to the growth of a .

and perturbations in Q can be rewritten using Eq. (28) as

$$\delta Q_i = \phi_i \delta \theta. \quad (30)$$

If after inflation, and before dark energy domination, ϕ_i changes to ϕ_f while $\delta \theta$ remains constant, then from Eqs. (28) and (30), the final perturbation in the dark energy perturbation will be

$$\delta Q_f = \delta Q_i \frac{\phi_f}{\phi_i} \quad (31)$$

Substituting this equation into Eq. (27) gives the bound needed to solve the gravitational wave problem as

$$\frac{\phi_f}{\phi_i} > 45. \quad (32)$$

A natural way of implementing this amplification mechanism is to take the quintessence to be a complex scalar field with a Lagrangian given by

$$\mathcal{L}_Q = -\partial_\mu \Phi (\partial^\mu \Phi)^* - V(\Phi) \quad (33)$$

where $*$ denotes complex conjugation and V is the potential. Defining

$$\Phi = \frac{1}{\sqrt{2}} \phi e^{i\theta} \quad (34)$$

then the Lagrangian is of the form

$$\mathcal{L} = -\frac{1}{2} (\phi^2 \partial_\mu \theta \partial^\mu \theta + \partial_\mu \phi \partial^\mu \phi) - V(\Phi). \quad (35)$$

So that if θ is constant, ϕ is a canonical scalar field, and if ϕ is constant then

$$Q \equiv \phi \theta \quad (36)$$

is a canonical scalar field.

One proposed model for the quintessence is a pseudo-Nambu-Goldstone boson [17]. This has the advantage of suppressing unwanted couplings with Standard Model fields by imposing a discrete symmetry in the Lagrangian. In this case the radial field will have a Mexican hat type potential

$$V_\phi = V_0 \left(\frac{\eta_0 \phi^2}{4M_p^2} - 1 \right)^2 \quad (37)$$

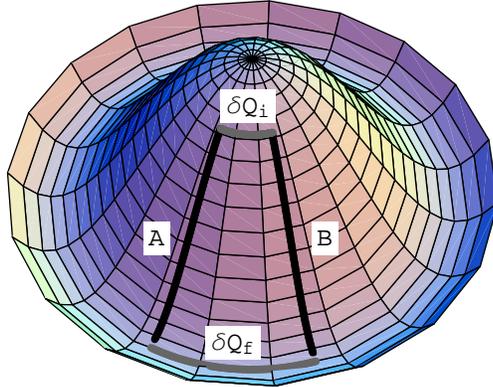


Figure 1: This figure shows how the amplification of δQ takes place. The Mexican hat potential, whose radial form is given in Eq. (37), is displayed. The large scale perturbation, δQ , can be thought of as the difference, in the circumference direction, between two separate homogeneous Universes [9], whose trajectories in field space (thick black lines) are labeled by ‘A’ and ‘B’. The initial (δQ_i) and final (δQ_f) perturbations are drawn with thick gray lines.

where V_0 is the potential at the maximum and η_0 is the absolute value of the second slow roll parameter at the maximum. The final value of ϕ is at the minimum of its potential

$$\phi_f = \frac{2M_p}{\sqrt{\eta_0}}. \quad (38)$$

Substituting Eq. (38) into Eq. (32) gives the bound

$$\phi_i < \frac{M_p}{22\sqrt{\eta_0}} \quad (39)$$

needed to solve the gravitational wave problem.

This tachyonic amplification mechanism has also been used in constructing models of moduli inflation [18] and in the curvaton scenario [19] in order to get inflation at a low energy scale. A graphical representation of the amplification is given in Fig. 1.

4 Radial field as the inflaton

In this section we investigate whether the radial field (ϕ), in Eq. (34), can be used as the inflaton. The Klein-Gordon equation for ϕ is

$$\phi'' + \frac{3}{2}(w_T - 1)\phi' + \frac{1}{H^2} \frac{\partial V_\phi}{\partial \phi} = 0. \quad (40)$$

Assuming that the potential of ϕ dominates the energy density, Eq. (40) can be solved by making the approximations that $w_T \approx -1$,

$$H \approx \frac{1}{M_p} \sqrt{\frac{V_0}{3}} \quad (41)$$

and that ϕ is small enough that we can neglect the quartic part of the potential given by Eq. (37). We can then obtain the growing mode solution

$$\phi = \frac{2M_p}{\sqrt{\eta_0}} e^{\frac{3}{2}N(1 - \sqrt{1 + 4\eta_0/3})}. \quad (42)$$

The contribution to ζ from ϕ is [9, 10]

$$\zeta_\phi = \frac{\partial N}{\partial \phi_i} \delta \phi_i \quad (43)$$

where subscript i denotes the initial value of a quantity. In order for ζ to be completely correlated with ρ_Q we require $\mathcal{P}_{\zeta_\phi} \ll \mathcal{P}_\zeta$. Substituting Eq. (42) into Eq. (43) gives

$$\zeta_\phi = \frac{2}{3(1 - \sqrt{1 + 4\eta_0/3})} \frac{\delta \phi_i}{\phi_i} \quad (44)$$

The power spectrum for δQ can be obtained by solving Eq. (44) for ϕ_i and substituting with Eq. (38) into Eq. (31). Then, as δQ and $\delta \phi$ are perpendicular in field space we can use $\mathcal{P}_{Q_i} = \mathcal{P}_{\phi_i}$ in this result to get

$$\mathcal{P}_{Q_f}^{1/2} = M_p \mathcal{P}_{\zeta_\phi}^{1/2} \frac{3(-1 + \sqrt{1 + 4\eta_0/3})}{\sqrt{\eta_0}}. \quad (45)$$

Then the bound in Eq. (24) implies

$$\mathcal{P}_{\zeta_\phi}^{1/2} > \frac{4 \times 10^{-4} \sqrt{\eta_0}}{3(-1 + \sqrt{1 + 4\eta_0/3})}. \quad (46)$$

As $\eta_0 > 0$, Eq. (46) implies

$$\mathcal{P}_{\zeta\phi}^{1/2} > 1 \times 10^{-4} \quad (47)$$

which, as can be seen from Eq. (14), is larger than the measured total ζ . It follows that the radial field is not suited to be the inflaton.

5 Separate inflaton

As seen from Eq. (43), the contribution of ϕ to the total ζ can be suppressed by suppressing $\delta\phi$. This can be achieved if ϕ is fixed at a constant value while the wavelengths responsible for structure formation are exiting the Hubble horizon during inflation. This may come about if this era of inflation is driven by some other potential dominated scalar field which also traps ϕ in an induced local high curvature minimum in the potential close to the origin of ϕ . At the end of this initial era of inflation, this trapping mechanism would need to disappear as ϕ still needs, at some point before today, to roll down to its origin so as to induce the needed amplification in the perturbations of Q .

Depending on the initial value of ϕ , it may drive a second phase of inflation. We have to make sure that this second phase isn't too long or else the structure formation wave numbers will leave the horizon during this phase and we have to face the problem mentioned in Sec. 4. The modes relevant to large scale structure have wavelengths between one and a thousandth of the Hubble length today. To estimate the number of efolds of inflation after the first of the large scale structure modes exits the horizon we can, for most models, neglect the change of energy scales from then until the end of inflation. Also, assuming there are no phases in the Universe when $w_T > 1/3$, the number of efolds is bounded by [20]

$$N_* \lesssim 67 - 2 \log_{10} \left(\frac{M_p}{V_{\text{end}}^{1/4}} \right) - 0.8 \log_{10} \left(\left(\frac{V_{\text{end}}}{\rho_{\text{reheat}}} \right)^{1/4} \right). \quad (48)$$

Substituting the gravitational wave bound (Eq. (26)) into Eq. (48) gives

$$N_* \lesssim 63 - 0.8 \log_{10} \left(\left(\frac{V_{\text{end}}}{\rho_{\text{reheat}}} \right)^{1/4} \right). \quad (49)$$

This upper limit is reduced by any subsequent periods of inflation or additional periods of matter domination. A lower bound on N_* is placed by requiring that all the structure formation modes exit during the first phase of inflation. Combined with Eq. (49), this gives

$$10 < N_* < 63. \quad (50)$$

Substituting Eq. (42) into Eq. (32) and solving for N we get

$$N_\phi > \frac{1}{\eta_0}(2 + \sqrt{3 + 4\eta_0}) \quad (51)$$

where we now distinguish the e-folds produced by ϕ with a subscript. Any e-folds of inflation due to ϕ will be subtracted from the amount of e-folds before the end of the first phase of inflation in which structure formation modes left the horizon [20]. This means that in order for the structure formation modes not to exit the horizon during any inflation provided by ϕ , we need

$$N_\phi \ll N_* \quad (52)$$

Using Eqs. (49), (51) and (52) we get the limit

$$\eta_0 > 0.06 \quad (53)$$

which when combined with Eq. (38) implies the vev of ϕ satisfies the limit

$$\phi_f < 8M_p. \quad (54)$$

The potential for Q needs to be periodic and can have the form [17]

$$V_Q = V_{Q,0}(1 + \cos(Q/\phi)) \quad (55)$$

where $V_{Q,0}$ is a constant. Then if the current phase of acceleration is due to Q slow rolling towards a minimum and we take $\phi = \phi_f$, we need

$$Q \gtrsim M_p \quad (56)$$

which combined with Eq. (28) implies

$$\phi_f \gtrsim M_p. \quad (57)$$

So in order to satisfy both constraints, Eqs. (54) and (57) we set $\phi_f \sim M_p$ or $\eta_0 \sim 4$. The small number of efolds of inflation produced by ϕ and the matter dominated phase resulting from the oscillations of ϕ around M_p should be sufficient to dilute the relativistic decay products of the previous era of inflation to negligible levels. Then either ϕ decays to reheat the Universe or there may be further phases of inflation at lower energy scales such as those needed to cure the moduli problem. Any additional inflation will subtract from the upper bound in Eq. (50) which means that the additional efolds should be less than that upper bound. Curing the moduli problem should not pose a problem in this regard as that can be solved by about ten efolds of thermal inflation [21].

6 Variable Decay

In the separate Universe approach, ζ can be seen to be the perturbation in the number of efolds of expansion needed to reach a fixed energy density [9, 10]. It follows that in order for $\delta\rho_Q$ and ζ to be completely anti-correlated we require that the amount of expansion undergone, to reach a fixed density, depend on the initial value of Q . In reference [1] a curvaton based mechanism [22] was proposed. This entailed adding an extra field, the curvaton, which was taken to be strongly coupled with the quintessence during inflation and then decoupled after inflation.

In reference [2] a variable decay mechanism [23, 24] was used. This mechanism can be implemented by a non-renormalizable coupling between the inflaton (ψ), a fermion (q) and anti-fermion (\bar{q}) and the quintessence (Q):

$$\mathcal{L}_{\text{reheat}} = - \left(\frac{Q}{M_p} \right) \bar{q}q\psi. \quad (58)$$

Then, the decay rate of the inflaton into the fermions is

$$\Gamma = \left(\frac{Q}{M_p} \right)^2 \frac{m_\psi}{8\pi}. \quad (59)$$

As Q is inhomogeneous, the decay rate will be inhomogeneous and at reheating this leads to [23]

$$\zeta = -\frac{1}{3} \frac{\delta Q}{Q}. \quad (60)$$

As $\delta Q/Q$ is constant, it does not matter at which point we take this value. Substituting Eqs. (23) and (14) into Eq. (60) and rearranging terms gives

$$\epsilon_Q \approx \frac{8M_p^2}{Q^2}. \quad (61)$$

Substituting Eqs. (61), (55) and (38) into Eq. (17), using $\eta_0 = 4$ and solving for Q gives

$$Q \approx 5M_p \quad (62)$$

and also implies

$$\epsilon_Q \approx 0.3. \quad (63)$$

The effect on the equation of state can be seen by numerically evolving the Klein-Gordon and Friedman equations with the potential given in Eq. (55) and the initial condition given in Eq. (62). The result is plotted in Fig. 2. By comparing with the probability contours in Fig. 10 of [12], it can be seen that these values are consistent with current observational bounds.

7 Example Inflaton

In addition to generating perturbations for large scale structure, inflation should also solve the flatness and related problems [25]. The simple quadratic potential in the chaotic inflation context [25] may be used for this

$$V_\psi = \frac{1}{2}m_\psi^2\psi^2. \quad (64)$$

The Klein-Gordon equation for ψ can be obtained by substituting Q for ψ in Eq. (16). The slow roll approximation can be used in this case. This entails setting $w_T = -1$, neglecting the kinetic term contribution to H and the ψ'' term, giving

$$-\psi' + \frac{M_p^2}{V_\psi} \frac{\partial}{\partial \psi} V_\psi = 0 \quad (65)$$

which has the solution

$$\psi = M_p \sqrt{2 + 4N} \quad (66)$$

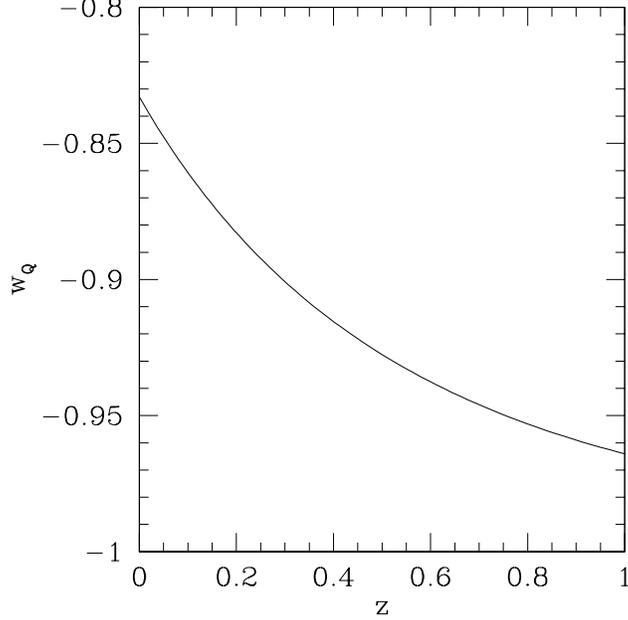


Figure 2: The evolution of the equation of state of the quintessence (w_Q), with redshift z , when the potential is of the form Eq. (55) and the initial condition is given in Eq. (62). The quintessence is taken to make up 73 % of the energy density today.

where the e-folds $N = -\log(a)$ have been set to zero when the slow roll conditions are first violated. The curvature perturbation from ψ can be obtained from Eqs. (66) and (64)

$$\mathcal{P}_{\zeta_\psi}^{1/2} = \frac{\partial N}{\partial \psi} \frac{H}{2\pi} = \frac{m_\psi}{2\pi\sqrt{6}M_p}(1 + 2N). \quad (67)$$

In order to have the perturbations from ψ sub-dominant we require $\mathcal{P}_{\zeta_\psi} \ll \mathcal{P}_\zeta$ which using Eqs. (14) and (50) implies

$$m_\psi \lesssim 10^{-5} M_p. \quad (68)$$

During this time, the ϕ field needs to be trapped near its origin. This can be achieved by adding a coupling term to the potential in Eq. (64) [26]

$$V = \frac{1}{2} (m_\psi^2 + g^2(\phi - \phi_i)^2) \psi^2 \quad (69)$$

where ϕ_i is the trapping point and g is a dimensionless coupling constant. The trapping also requires that the ϕ be heavy:

$$\frac{\partial^2 V}{\partial \phi^2} \gtrsim H^2 \quad (70)$$

which, when used with Eq. (69) and $\phi = \phi_i$, implies

$$m_\psi \lesssim \sqrt{6}gM_p. \quad (71)$$

In order for the coupling not to change the inflaton potential through radiative corrections, $g < 10^{-3}$ is required [25, 27]. It follows that Eq. (71) is easily satisfied due to Eq. (68).

During this first phase of inflation, the quintessence is virtually a free field and it acquires the usual quantum fluctuations, which using Eqs. (66) and (64) are given by

$$\mathcal{P}_{Q_i}^{1/2} = \frac{H_*}{2\pi} = \frac{m_\psi \sqrt{1 + 2N_*}}{2\pi\sqrt{3}} \quad (72)$$

where subscript $*$ denotes the value of a quantity at horizon crossing.

If Q is the variable decay field, it follows from Eq. (58) that in order for quantum corrections not to significantly modify Eq. (64) we need [25]

$$Q_i^2 \lesssim 10^{-5}M_p^2. \quad (73)$$

It is important to check that this is not smaller than the perturbations in Eq. (72), else the perturbation will be suppressed [28]. Using Eqs. (72) and (50) gives

$$\frac{\mathcal{P}_{Q_i}^{1/2}}{Q_i} < \frac{m_\phi}{Q_i} \quad (74)$$

which shows $\delta Q \ll Q$ is compatible with the bounds in Eqs. (73) and (68). Also, if we take $\phi_i \sim Q_i$ then from Eq. (73) and Eq. (42), with $\eta_0 = 4$, the amount of inflation from ϕ is bounded by

$$N_\phi \gtrsim 3. \quad (75)$$

The density at reheating is given by solving the decay rate given in Eq. (59) to be $\Gamma = 3H_{\text{reheat}}$ with the constraints Eqs. (73) and (68) to give

$$\rho_{\text{reheat}}^{1/4} < 10^{-6}M_p. \quad (76)$$

Substituting this into Eq. (48) and using Eqs. (64) and (68) reduces the upper bound of N_* by about one. Combined with the reduction due to the inflation caused by ϕ , Eq. (75), this gives

$$10 < N_* < 58. \quad (77)$$

Finally, we can estimate the scalar spectral index. We can set η_0 to zero as we are assuming the radial field (ϕ) is trapped at ϕ_i while the structure formation modes are exiting the horizon. The spectral index is given by

$$n_s - 1 \equiv \frac{\partial \log(\mathcal{P}_\zeta)}{\partial \log(k)} \quad (78)$$

As we are assuming the perturbations from ψ contribute negligibly to ζ , it follows that ζ is completely spatially anti-correlated with δQ and so will have the same spectral index as δQ . As \mathcal{P}_Q depends on the value of the Hubble parameter, Eq. (12), when $k = e^{-N}H$, and $H' \ll H$, we can rewrite Eq. (78) as

$$n_s - 1 = \frac{2\partial \log(H)}{\partial \log(e^{-N}H)} \approx -2 \frac{\partial \log(H)}{\partial N}. \quad (79)$$

Then, substituting Eqs. (64) and (66) into Eq. (79) gives

$$n_s = 1 - \frac{2}{1 + 2N}. \quad (80)$$

The current observational bound on the spectral index is [12]

$$n_s > 0.94 \quad (97.5\% \text{ C.I.}) \quad (81)$$

which when combined with Eq. (80) implies that

$$N_* > 16 \quad (82)$$

and using the upper bound in Eq. (77) in Eq. (80)

$$n_s < 0.98 \quad (83)$$

to be consistent with the inflaton potential in Eq. (64) in this tachyonic amplification scenario.

8 Conclusions

In this article we have explained a mechanism for amplifying dark energy perturbations. The motivation for this was that sufficiently large dark energy perturbations which are anti-correlated with the other perturbations provide a possible explanation for the low CMB TT quadrupole. However, if the perturbations are generated during inflation and frozen until today then they are not of sufficient size.

We proposed using a tachyonic amplification mechanism to overcome this problem. The dark energy is modeled as a complex scalar field whose radial component has a Mexican hat potential. The part of the dark energy (the ‘quintessence’) that causes the acceleration today is modeled as a pseudo-Nambu-Goldstone boson.

It was shown that there is a difficulty in using the radial component of the dark energy as the inflaton. This is because it then produces a non-negligible curvature perturbation which is uncorrelated with the quintessence perturbation. We show that a working mechanism can be achieved if the inflaton is a separate field and the radial component is trapped near the maximum of its potential during inflation. After inflation, the trapping mechanism needs to be released so that the radial component can roll down to its minimum. This change in the radial component provides the necessary amplification.

We also showed how the anti-correlation can be achieved by using the variable decay mechanism. By coupling the quintessence to the inflaton and a fermion/anti-fermion pair, the perturbations in the quintessence field cause perturbations in the curvature perturbation by varying the time at which the inflaton decays. We showed how this then constrains the quintessence potential and initial value of the quintessence.

Finally, we looked at the quadratic potential as an example inflaton. We showed how it could satisfy all the necessary constraints and we put limits on the spectral slope and the duration of the initial period of inflation.

The current statistical significance of the detection of dark energy perturbations is modest, however, future CMB polarization data may improve this [2]. It may also be possible to use CMB polarization and polarization/temperature cross-correlation data to distinguish between dark energy perturbations and other explanations for the low CMB temperature quadrupole [2]. A detection of a non-homogenous dark energy would provide a valuable new window on both the nature of the dark energy and the process which

generates the primordial perturbations.

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