Brane induced gravity from asymmetric warped compactification

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We show that brane induced gravity can be realized as a low energy effective theory of brane worlds with asymmetric warped compactification. A self-accelerating universe without cosmological constant on the brane can be realized in a model where one side of the bulk has finite volume, but the other side has infinite volume. The spin-2 perturbations for brane induced gravity and asymmetric warped compactification models have the same spectrum at low energies. For a de Sitter brane, the spin-2 graviton has mass in the range $0 < m^2 \le 2H^2$, with $m^2 = 2H^2$ in the self-accelerating universe.

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1. Introduction

The discovery of the acceleration of universe has raised significant challenges for cosmology [1]. The simplest solution is the introduction of a tiny cosmological constant by hand. However, it requires an unacceptable fine-tuning. Recently, much efforts have been devoted to the attempt to modify Einstein theory of gravity. However, it is extremely difficult to modify gravity at large distances and, at the same time, provide the mechanism for cosmic acceleration [2].

So far, one successful realization is known in the context of the brane-world scenario where standard model particles are confined to the brane while gravity can propagate in higher-dimensional spacetime. Dvali, Gabadadze and Porrati (DGP) [3] proposed the brane induced gravity model where the 4D Einstein-Hilbert term is assumed to be induced on the brane. In this model, 4D gravity is recovered on short scales but gravity becomes 5D on large scales. Deffayet showed that the DGP model realized the accelerated expansion of the universe at late times without introducing the cosmological constant [4]. The motivation for the induced Einstein-Hilbert term in the action is that it arises from quantum effects of matter fields confined to the brane [3].

We present a new way to realize the brane induced gravity model as an asymmetric warped compactification of the form discussed by Padilla [5]. Warped compactification is a mechanism to recover 4D gravity on the brane via an extra-dimension that shrinks exponentially away from the brane and so confines gravity [6]. Recent developments in string theory suggest that there may be many regions in the extra-dimension with different warped geometries, and with the observable universe on one of the D-branes between different warped geometries [7].

In this letter, we present a simple argument to show that *the brane induced gravity model can be realized as a low energy effective theory of the asymmetric warped compactification model where one side of the brane confines the gravity but the other side does not.*

2. Brane Induced Gravity

Let us consider the 5D action given by

$$S = \int d^5x \sqrt{-{}^{(5)}g}\left[\frac{1}{2\kappa^2}\left({}^{(5)}R + \frac{12}{\ell^2}\right)\right] + \int d^4x\sqrt{-{}^{(4)}g}\left(-\sigma + \frac{1}{2\kappa_4^2}{}^{(4)}R\right). \quad (1)$$

We assume that the tension is tuned $\sigma = 6/\kappa^2\ell$ so that the cosmological constant vanishes on the brane. The reflection symmetry across the brane is imposed. The original DGP model assumed $\sigma = 0$ and $\ell \to \infty$.

In this model, there is a characteristic scale called a cross over scale defined by

$$r_c = \frac{\kappa^2}{2\kappa_4^2}. \quad (2)$$

r_c controls the strength of the brane induced gravity. In the following we assume $r_c \ll \ell$, which is the regime relevant for self-accelerating models. The behavior of gravity on the brane is summarized in Fig.1 [8].

An interesting property of this model is that there is a solution for an accelerating universe without cosmological constant [4, 10]. The Friedmann equation on the brane is given by

$$\pm H = r_c H^2 - \frac{\kappa^2}{6}\rho. \quad (3)$$

The sign is related to the geometry of the bulk spacetime. We notice that in the + branch, the universe approaches de Sitter spacetime at late times, i.e. $\rho \to 0$,

$$H = \frac{1}{r_c}. \quad (4)$$

In order to derive the above results, the junction condition plays the crucial role. The junction condition at the brane is given by

$$K^\mu_\nu = \frac{\kappa^2}{2}\left(-\frac{\sigma}{3}\delta^\mu_\nu - \tilde{T}^\mu_\nu + \frac{1}{\kappa_4^2}{}^{(4)}\tilde{G}^\mu_\nu\right), \quad (5)$$

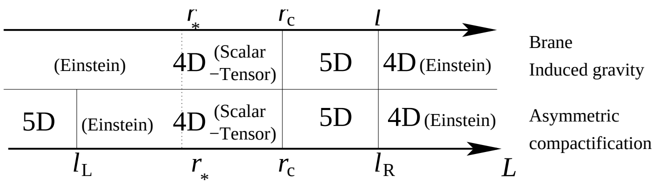

FIG. 1: Behavior of gravity outside a matter distribution with total mass m. If we consider the scale $L < \ell$, gravity is not localized. In the intermediate region $r_c < L < \ell$, gravity looks 5D. 4D Newton gravity is reproduced for $L < r_c$ due to the brane induced gravity, but linearized gravity is not described by Einstein theory because 4D gravity is recovered by massive modes which contain an extra scalar polarization. However, there is a length scale given by $r_* = (m\kappa_4^2 r_c^2/4\pi)^{\frac{1}{3}}$ [9]. Below r_* the non-linear interaction of the scalar mode effectively shields the extra scalar polarization. Then we recover 4D Einstein gravity for $L < r_*$. In the asymmetric warped compactirication model, there is a 5D gravity regime for $L < \ell_L$.

where K^μ_ν is the extrinsic curvature of the brane, ${}^{(4)}G^\mu_\nu$ is Einstein tensor on the brane and $\tilde{T}^\mu_\nu = T^\mu_\nu - (1/3)T\delta^\mu_\nu$, ${}^{(4)}\tilde{G}^\mu_\nu = {}^{(4)}G^\mu_\nu - (1/3)G\delta^\mu_\nu$. The 4D Einstein tensor comes from induced Einstein-Hilbert term.

3. Asymmetric Warped Compactification

Next, let us consider the asymmetric warped compactification model described by the action [5]

$$
\begin{aligned}
S \;=\; & \int_{M_R} d^5x\sqrt{-{}^{(5)}g}\left[\frac{1}{2\kappa_R^2}\left({}^{(5)}R + \frac{12}{\ell_R^2}\right)\right] \\
& + \int_{M_L} d^5x\sqrt{-{}^{(5)}g}\left[\frac{1}{2\kappa_L^2}\left({}^{(5)}R + \frac{12}{\ell_L^2}\right)\right] \\
& - \int d^4x\sqrt{-{}^{(4)}g}\;\sigma, \qquad (6)
\end{aligned}
$$

where M_R, M_L are two AdS spacetimes with different AdS curvature lengths ℓ_R, ℓ_L (See Fig.2). The indices R and L are used to denote the variables in the right and left side of AdS spacetime respectively. We only consider the brane tension σ and do not introduce an induced gravity term ${}^{(4)}R$ in the brane action. Instead of it, we have an asymmetry between the right and left side of the brane.

We can explicitly show the equivalence of this model at low energies with the brane induced gravity model as follows. We first take $\ell_R \gg \ell_L$ and consider the physics on the scale L on the brane with $L \gg \ell_L$. We solve the 5D Einstein equation for the bulk gravitational field in left side of the brane perturbatively in terms of the small parameter $(\ell_L/L)^2$. The solution for $K^\mu_{\nu,L}$ on the brane up to the first order is given by [11]

$$
K^\mu_{\nu,L} = \frac{1}{\ell_L}\delta^\mu_\nu + \frac{\ell_L}{2}{}^{(4)}\tilde{G}^\mu_\nu + \chi^\mu_\nu, \qquad (7)
$$

where χ^μ_ν is an integration constant which should be determined by the conditions at Cauchy horion of the AdS spacetime. In the following we take $\chi^\mu_\nu = 0$. This assumption is appropriate for the background dynamics, and we will consider perturbations about the background in the following section. The appearance of Einstein tensor manifests the localization of gravity in this model. This Einstein tensor is responsible for the recovery of 4D Einstein gravity in the model proposed by Randall and Sundrum [6] where the reflection symmetry across the brane is assumed.

The junction condition for the asymmetric warped compactification is given by

$$
\kappa_R^{-2}K^\mu_{\nu,R} - \kappa_L^{-2}K^\mu_{\nu,L} = -\frac{\sigma}{3}\delta^\mu_\nu - \tilde{T}^\mu_\nu. \qquad (8)
$$

Using the solution for $K^\mu_{\nu,L}$ we derive the junction condition for $K^\mu_{\nu,R}$ as

$$
K^\mu_{\nu,R} = \kappa_R^2\left(-\frac{\sigma_{\rm eff}}{3} - \tilde{T}^\mu_\nu + \frac{\ell_L}{2\kappa_L^2}{}^{(4)}\tilde{G}^\mu_\nu\right). \qquad (9)
$$

where $\sigma_{\rm eff} = \sigma - 3/\kappa_L^2\ell_L$. This is completely the same as the junction condition in the brane induced gravity model except for factor 2 that comes from the reflection symmetry in the brane induced gravity model. We find that *the localization of the gravity in the left side plays the same role as brane induced gravity.* We can identify the coupling constant κ_4^2 and the cross-over scale r_c as

$$
\kappa_4^2 = 2\kappa_L^2\ell_L^{-1}\;, \qquad r_c = \frac{\ell_L\kappa_R^2}{2\kappa_L^2}. \qquad (10)
$$

In order to ensure $\ell_L \ll r_c \ll \ell_R$, we must choose the parameters so that $\ell_L\kappa_R^2 \ll \ell_R\kappa_L^2$ and $\kappa_L^2 \ll \kappa_R^2$.

Padilla studied the behavior of gravity in asymmetric brane [5] and noticed that the behavior of gravity is pretty much the same as the brane induced gravity model studied in Ref. [8] (See Fig.1). Indeed, we can explicitly check that the results of [5] can be derived from the results of [8] in the brane induced gravity model using the identification (10).

The cosmological solutions can be easily obtained. We tune the tension $\sigma = 3/\kappa_L^2\ell_L + 3/\kappa_R^2\ell_R$ so that the cosmological constant on the brane vanishes. Then we have the same Friedmann equation (3). Actually we have the late time accelerating universe [5]

$$
H = \frac{1}{r_c} = \frac{2\kappa_L^2}{\ell_L\kappa_R^2}. \qquad (11)
$$

4. Linear Perturbations and Mode Analysis

We now show that the relation between brane induced gravity and asymmetric warped compactification in the background dynamics extends also to the linear perturbations about Minkowski brane and de Sitter brane. We study the structure of the mass spectrum of the discrete

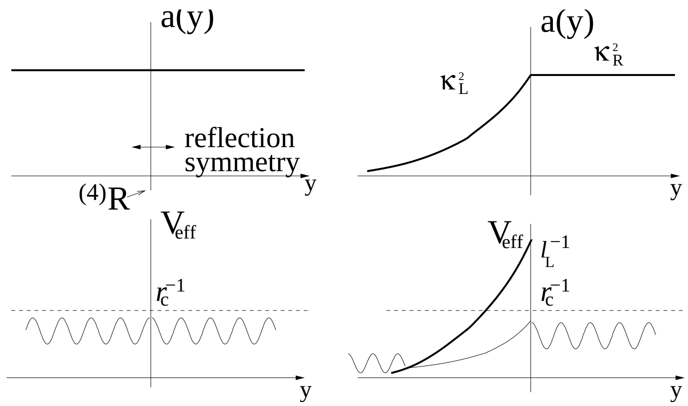

FIG. 2: Warp factor and effective potential for Minkowski brane. In the brane induced gravity model, $\sigma = 0$, the warp factor and the effective potential are given by $a = 1$ and $V_{\rm eff} = 0$, respectively, for $-\infty < y < \infty$ if we take $\ell \to \infty$. In the asymmetric warped compactification model, $\sigma > 0$ and $a_L = e^{y/\ell_L}$, $V_{\rm eff\,L} \propto {\rm e}^{2y/\ell_L}/\ell_L^2$ for $y < 0$, $a_R = 1$, $V_{\rm eff\,R} = 0$ for $y > 0$.

and continuous modes in both the brane induced gravity model and the asymmetric warped compactification model. One can see the brane induced gravity is actually recovered as a low energy limit of the asymmetric warped compactification.

We write the background metric in the bulk as

$$ds^2 = dy^2 + a(y)^2 \gamma_{\mu\nu} dx^\mu dx^\nu, \tag{12}$$

where $\gamma_{\mu\nu}$ is the metric of Minkowski spacetime or de Sitter spacetime. We assume the brane is located at $y = 0$. Let us consider the perturbations $\gamma_{\mu\nu} + h_{\mu\nu}$. Imposing the transverse-traceless condition $\nabla^\mu h_{\mu\nu} = h^\mu_\mu = 0$ and using the separation of variables, i.e. the mode expansion $h_{\mu\nu} = \int dm\ e_{\mu\nu}(x) F_m(y)$, one can rewirte the wave equation in the bulk as the Shrödinger equaiton

$$-\frac{1}{2}\frac{d^2}{dz^2}\Psi_m + V_{\rm eff}\Psi_m = \frac{1}{2}m^2\Psi_m, \tag{13}$$

where $\Psi_m = a^{3/2} F_m$ and z is a conformal coordinate $a(y)dz = dy$. In order to have the normalizable mode we must impose the condition $\int_0^\infty dy a^2 F_m^2 < \infty$ for discrete modes and $\int_0^\infty dy a^2 F_m F_{m'} = \delta(m - m')$ for continuous massive modes.

4.1 Mincowski Brane

First we investigate the Mikonwski brane case, $H = 0$ (See Fig.2). In the brane induced gravity model, the junction condition on the brane is given by

$$F_m'(0) = -m^2 r_c F_m(0). \tag{14}$$

Note that the 0-mode solution that satisfies this boundary condition is not normalizable and so it is excluded from the spectrum. The solution for F on the brane is given by [12]

$$|F_m(0)|^2 = |N|^2 \frac{4}{1 + m^2 r_c^2}, \tag{15}$$

where N is the normalization factor. For $m > 1/r_c$, the contribution of massive modes are suppressed, so 4D gravitational interactions are reproduced on the scale $L < r_c$.

In the asymmetric warped compactification model (See Fig.2), from Eq.(13) the mode functions $F_{Lm}(y)$ in the left side AdS bulk are given by [6]

$$F_{Lm}(y) = \frac{e^{2y/\ell_L} H_2^{(1)}(m\ell_L e^{2y/\ell_L})}{H_2^{(1)}(m\ell_L)}, \tag{16}$$

where $H_2^{(1)}$ is a Hankel function of the first kind and we imposed a no-incoming radiation condition at the Cauchy horizon of AdS spacetime. The derivative of F_{Lm} can be written as

$$F_{Lm}'(0) = -\left(\frac{m^2\ell_L}{2} + \frac{m^2\ell_L}{2}\frac{H_0^{(1)}(m\ell_L)}{H_2^{(1)}(m\ell_L)}\right). \tag{17}$$

At low energies $m \ll 1/\ell_L$, we can neglect the second term in Eq.(17). From the junction condition $\kappa_R^{-2} F_{Rm}' = \kappa_L^{-2} F_{Lm}'$, we find

$$F_{Rm}'(0) = -\frac{\ell_L \kappa_R^2}{2\kappa_L^2} m^2 F_{Rm}(0) = -m^2 r_c F_{Rm}(0). \tag{18}$$

Then, we have the same modes in the right side bulk as the brane induced gravity model. The induced gravity effect here is originated from the confinement of nearly massless modes around the brane in the left side AdS bulk.

On the other hand, the second term in the right hand side of Eq.(17) becomes important at high energies $m > 1/\ell_L$. We do not have an induced gravity effect and the 4D theory is modified. It is interesting to note that, in AdS bulk spacetime, the modes with a finite Kaluza-Klein mass are quasi-bound states because it can decay into infinity $y = -\infty$ due to tunnelling (See Fig.2). These phenomena give clear differences between the brane induced model and asymmetric warped compactification model at high energies.

4.2 de Sitter Brane

Next we investigate the de Sitter solution that includes the self-accelerating solution (See Fig.3). We start with the brane induced gravity model, with $V_{\rm eff} = 9H^2/4$ and $a_+ = 1 + H|y|$. In de Sitter spacetime, continuous massive modes start from $m^2 = 9H^2/4$. Again, the 0-mode solution with $m = 0$ that satisfies the boundary condition is not normalizable. Continuous massive modes are too heavy to reproduce the 4D gravity on super-horizon

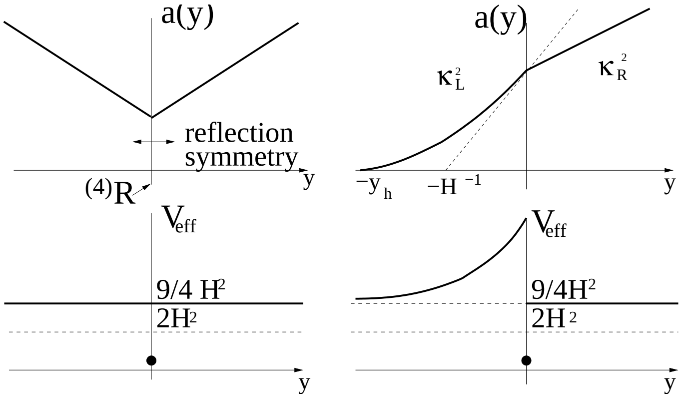

FIG. 3: Warp factor and effective potential for de Sitter brane. The warp factor in the brane induced gravity models is given by $a_+(y) = 1 + H|y|$ in + branch. For asymmetric compactification $a_R(y) = 1 + Hy$ for $y > 0$, $a_L(y) = H\ell_L \sinh\left(\frac{y_h+y}{\ell_L}\right)$ for $y < 0$, where y_h is determined by $\sinh(y_h/\ell_L) = 1/H\ell_L$.

scales. However unlike the Minkowski brane case, Eq. (13) shows that there exists a normalizable discrete mode in de Sitter case that is given by

$$F_m(y) = a(y)^{-\frac{3}{2}-\sqrt{\frac{9}{4}-\frac{m^2}{H^2}}}. \tag{19}$$

Imposing the junction condition, we find one discrete mode with mass

$$\frac{m_d^2}{H^2} = \frac{1}{(Hr_c)^2}\left(3(Hr_c) - 1\right). \tag{20}$$

We should remember that the Hubble parameter is $H \geq 1/r_c$. Then we find $0 < m_d^2 \leq 2H^2$. For large Hr_c where induced gravity effect is large, we have a nearly massless mode $m_d^2 \to 0$. This mode is responsible for the recovery of 4D gravity for $1 \ll Hr_c$ on super-horizon scales. The self-accelerating universe is a de Sitter solution with $Hr_c = 1$, then Eq. (20) shows that $m_d^2 = 2H^2$.

Let us now consider the asymmetric compactification model. We assume $\ell_R \to \infty$ for simplicity. The normalizable solutions for $0 < m^2 < 9/4H^2$ in the left side of the bulk can be solved as

$$F_L(y) = \frac{\sinh^{-2}\left(\frac{y_h+y}{\ell_L}\right) Q^2_{-\frac{1}{2}-\nu}\left(\coth\left(\frac{y_h+y}{\ell_L}\right)\right)}{\sinh^{-2}\left(\frac{y_h}{\ell_L}\right) Q^2_{-\frac{1}{2}-\nu}\left(\coth\left(\frac{y_h}{\ell_L}\right)\right)}, \tag{21}$$

where $\nu^2 = 9/4 - m^2/H^2$ and Q^α_β are associated Legendre functions of the second kind. At low energies $H\ell_L \ll 1$, we have $F'_L \sim -m^2\ell_L/2$, thus we have the same boundary condition for F_R and the same mass for the discrete mode in the right side of the brane as the brane induced gravity model.

Note that the mass of the discrete mode is nearly 0 even at high energies $H > 1/\ell_L$. Thus on super-horizon scales, 4D physics is recovered even at high energies. This is because at high energies the warp factor approaches $a_L(y) \to a_-(y) = 1 + Hy$ for $y < 0$. The behavior of the left side bulk solution is the same as the − branch solution in the DGP model while the right side one behaves like the + branch solution in the DGP model. Although the right side of the bulk volume is infinite, the left side of the bulk has finite volume, due to the horizon in the bulk $y = -y_h \to -H^{-1}$. Then a normalizable 0-mode appears in the left side. This supports a nearly massless mode and one can recover 4D gravity on super-horizon scales.

We should emphasize that the mass of the discrete mode lies in the range $0 < m_d^2 \leq 2H^2$ for the spin-2 graviton. Spin-2 massive graviton contains an extra scalar polarization. In 4D massive gravity theory where a Pauli-Fierz mass terms is introduced by hand [13], this scalar polarization becomes a ghost if the mass lies in the range $0 < m^2 < 2H^2$ [14]. However, this conclusion depends on the introduction of an explicit mass term in the lagrangian and we cannot extrapolate this result to brane world models. In addition to spin-2 graviton, a spin-0 mode called the radion could be physical in the brane world and this raises another danger of a possible ghost [9]. A detailed analysis of gravity in de Sitter solution is needed to conclude that we have a consistent realization of massive gravity in the de Sitter brane. This is a crucial question to be addressed to realize the self-accelerating universe consistently [15].

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