

The CDM isocurvature perturbation in the curvaton scenario

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Abstract

We discuss the residual isocurvature perturbations, fully-correlated with the curvature perturbation, that are automatic in the curvaton scenario if curvaton decay is sufficiently late. We contrast these residual isocurvature perturbations with the generally un-correlated ‘intrinsic’ isocurvature perturbation generated by an additional field such as the axion. We present a general formula for the residual isocurvature perturbations, referring only to the generation of the relevant quantity (Cold Dark Matter, baryon number or lepton number) in an *unperturbed* universe. Specific formulas for the residual isocurvature CDM perturbation are given, for most of the commonly-considered CDM candidates.

I. INTRODUCTION

The inhomogeneity of the early Universe first becomes measurable a few Hubble times before cosmological scales enter the horizon. At this stage, well after Big Bang Nucleosynthesis (BBN), we know that the Universe consists primarily of photons, neutrinos, baryonic matter and Cold Dark Matter (CDM) [1]. The perturbations in the densities of these components are conventionally specified by the perturbation in the total energy density, characterised by a spatial curvature perturbation, ζ , and by three isocurvature perturbations

$$\mathcal{S}_i \equiv \frac{\delta(n_i/n_\gamma)}{(n_i/n_\gamma)}, \quad (1)$$

with $i = \text{CDM}, B$ or ν , giving the inhomogeneity in the relevant number density per photon [2]. As we shall see, ζ and the \mathcal{S}_i are constant from BBN to the approach of horizon entry. They set the initial conditions for the subsequent evolution of cosmological perturbations, and we refer to them collectively as the ‘Primordial Density Perturbation’.

Observation is *consistent* with the hypothesis that the isocurvature perturbations are absent, while the curvature perturbation is Gaussian with an almost perfectly flat spectrum [3–5]. On the other hand, spectral tilt, non-Gaussianity and isocurvature perturbations may all be present at some level.

The Primordial Density Perturbation presumably originates during inflation, from the vacuum fluctuation of one or more light scalar fields. According to the usual hypothesis [2], it originates solely from the vacuum fluctuation of the inflaton field, defined as the one whose value determines the end of inflation. This ‘inflaton scenario’ leads to definite expectations about the nature of the primordial density perturbation. There are no isocurvature perturbations, and non-gaussianity will almost certainly exist [6] only at the 10^{-5} level through second-order perturbations.¹ On the other hand, significant spectral tilt is expected, and significant running of the spectral index is quite possible [8,2]. It is also possible that the curvature perturbation is accompanied by gravitational waves of sufficient amplitude to affect the Cosmic Microwave Background (CMB) anisotropy [8,2]. These expectations have achieved the status of a ‘standard model’ of the early Universe, which is now used by almost all groups who analyse CMB and galaxy distribution measurements.

It has been pointed out recently [9] (see also [10,11]) that the inflaton scenario is not the only possible mechanism to generate large scale structure from inflation. The primordial density perturbation may instead originate from the vacuum fluctuation of some ‘curvaton’ field, different from the inflaton field. The curvaton energy density has an isocurvature perturbation, and this generates the curvature perturbation when the curvaton density comes to be a significant fraction of the total before the curvaton finally decays.² This ‘curvaton

¹Bigger non-gaussianity of the χ^2 type can be generated in a multi-field model, but only at the expense of fine-tuning [7].

²Recently, a completely different scheme has been proposed [12–14], in which the responsible field acts by perturbing the inflaton decay rate without ever contributing significantly to the energy density.

scenario' leads to completely different expectations about the nature of the primordial density perturbation. Gravitational waves [15] (see also [16]), and probably also the spectral tilt will be negligible, but non-Gaussianity and isocurvature perturbations are perfectly possible [17].

The curvaton scenario has received a lot of attention [15,17–43] because it opens up new possibilities both for model-building and for observation. The present paper focusses on the possible isocurvature perturbations, following [17]. Because they have the same source as the curvature perturbation, they are fully correlated

$$\mathcal{S}_i(\mathbf{x}) = s_i \zeta(\mathbf{x}), \quad (2)$$

with s_i numbers that depend on the physical model. Following [17], we call this kind of isocurvature perturbation *residual*. A given s_i will be zero if the relevant quantity (CDM, baryon number or lepton number) is created after the curvaton decays. Otherwise it can be calculated within a given early-Universe scenario, and it is generally expected to be of order unity. Our main purpose is to derive specific formulas for the CDM case, for the various possibilities regarding the nature of the CDM that are commonly envisaged.

In either the inflaton or the curvaton scenario, one can suppose that the vacuum fluctuation of one or more additional fields is also relevant. Such additional fields may generate additional isocurvature perturbations, which will in general be *uncorrelated* with the curvature perturbation and at best only partially correlated. We shall call these *intrinsic* isocurvature perturbations. In general, intrinsic isocurvature perturbations and the curvature perturbation are determined by different sectors of the underlying particle theory. This means that in general, there is no reason to expect the magnitude of an intrinsic isocurvature perturbation to be comparable with that of the curvature perturbation. In a particular model though, it may turn out that a single sector of the theory accounts for both the curvature perturbation and an intrinsic isocurvature perturbation, so that their magnitudes are comparable. (See [36] for the only example known so far, arising within the curvaton scenario.)

The plan of the paper is as follows. In Section II we recall the general description of the Primordial Density Perturbation, in terms of quantities which are conserved on super-horizon scales. In Section III we discuss how isocurvature perturbations can originate, first in the inflaton scenario and then in the curvaton scenario, giving a general formula for the residual isocurvature perturbations. In Section IV we apply our formula, to evaluate the residual CDM isocurvature perturbation for each of the commonly-envisaged CDM possibilities. We conclude in Section V.

II. CONSERVED PERTURBATIONS

The discussion of the evolution of perturbations while they are outside the horizon is facilitated by the existence of perturbations that are, under suitable circumstances, conserved [44,42]. The conserved perturbations are conveniently defined with reference to the slices of spacetime which have zero intrinsic curvature perturbation (spatially flat slices), because on super-horizon scales the local expansion of the Universe between such slices is uniform [44,42]. By virtue of this uniformity, a perturbation will be conserved on super-horizon scales if it is constructed according to the following recipe [42]. Consider a quantity $f(\mathbf{x}, t)$, with t

the coordinate time labelling spatially flat slices. If f decreases (or increases) monotonically with the comoving volume \mathcal{V} according to an equation of the form

$$\mathcal{V} \frac{\partial f}{\partial \mathcal{V}} = y(f), \quad (3)$$

then the perturbation

$$X_f \equiv -H \frac{\delta f}{\dot{f}} \quad (4)$$

is conserved [42], where δf is to be evaluated on spatially flat slices. In practice we find conserved quantities of three types.

a. The total energy density perturbation. The quantity

$$\zeta \equiv -H \frac{\delta \rho}{\dot{\rho}} = \frac{\delta \rho}{\rho + P} \quad (5)$$

is conserved provided that the pressure P of the Universe is a unique function of the energy density ρ , or equivalently if the perturbations satisfy the adiabatic condition

$$\delta P / \delta \rho = \dot{P} / \dot{\rho}. \quad (6)$$

More general, if the pressure is non-adiabatic,

$$\dot{\zeta} = -\frac{H}{\rho + P} \delta P_{\text{nad}}, \quad (7)$$

where

$$\delta P_{\text{nad}} \equiv \delta P - \frac{\dot{P}}{\dot{\rho}} \delta \rho. \quad (8)$$

b. Separate energy density perturbations. Let ρ_i and P_i refer to some component which by itself satisfies the adiabatic condition Eq. (6), and which does not exchange energy with any other component. Then the quantity

$$\zeta_i \equiv -H \frac{\delta \rho_i}{\dot{\rho}_i} = \frac{\delta \rho_i}{\rho_i + P_i} \quad (9)$$

is conserved. If the component is radiation ($P_i = \rho_i/3$),

$$\zeta_i \equiv \frac{1}{4} \frac{\delta \rho_i}{\rho_i} \quad (\text{radiation}), \quad (10)$$

and if it is matter ($P_i = 0$)

$$\zeta_i \equiv \frac{1}{3} \frac{\delta \rho_i}{\rho_i} \quad (\text{matter}). \quad (11)$$

The conservation of ζ_i in these cases is an immediate consequence of the dependence of the local energy densities on comoving volume \mathcal{V} (namely $\rho_i \propto \mathcal{V}^{-1}$ for matter and $\zeta_i \propto \mathcal{V}^{-4/3}$ for radiation).

c. Number density perturbation. Let n_i be any conserved number density. Then, since $n_i \propto \mathcal{V}^{-1}$ the quantity

$$\tilde{\zeta}_i \equiv \frac{1}{3} \frac{\delta n_i}{n_i}, \quad (12)$$

is conserved.

A. The primordial density perturbation

The epoch $T \sim 10$ MeV, marking the beginning of the Big-Bang Nucleosynthesis process, is the earliest one of which we have definite knowledge. After T falls below 1 MeV, the energy density is dominated by radiation in the form of photons and neutrinos, and there is also baryonic matter and presumably Cold Dark Matter (CDM);³ The number of particles is conserved for each component, and there is no exchange of energy between components. As a result the following quantities are conserved until the approach of horizon entry:

$$\zeta_{\text{cdm}} \equiv \frac{1}{3} \frac{\delta \rho_{\text{cdm}}}{\rho_{\text{cdm}}} \quad (13)$$

$$\zeta_B \equiv \frac{1}{3} \frac{\delta \rho_B}{\rho_B} \quad (14)$$

$$\zeta_\nu \equiv \frac{1}{4} \frac{\delta \rho_\nu}{\rho_\nu} \quad (15)$$

$$\zeta_\gamma \equiv \frac{1}{4} \frac{\delta \rho_\gamma}{\rho_\gamma} \quad (16)$$

The three isocurvature perturbations Eq. (1) are given by

$$\mathcal{S}_i \equiv 3(\zeta_i - \zeta_\gamma), \quad (17)$$

and they too are conserved until the approach of horizon entry. The total energy density/curvature perturbation, Eq. (5), is given by

$$\zeta = -H \frac{\sum \rho_i \zeta_i}{\dot{\rho}} \quad (18)$$

$$\simeq (1 - f_\nu) \zeta_\gamma + f_\nu \zeta_\nu, \quad (19)$$

during the primordial era when the matter density is negligible and $f_\nu = \rho_\nu/\rho_\gamma$ is a constant. The curvature perturbation ζ is thus constant during the radiation dominated era to high accuracy until the approach of horizon entry.

The smallest cosmological scale, enclosing say $M \sim 10^6 M_\odot$, enters the horizon at $T \sim 1$ keV. The curvature perturbation ζ and the isocurvature perturbations \mathcal{S}_i therefore have

³If non-relativistic particles decaying well after nucleosynthesis have a significant effect, we are assuming here that the perturbation in their energy density is adiabatic and hence need not be considered separately.

constant values on all cosmological scales in the regime $1 \text{ keV} \lesssim T \lesssim 1 \text{ MeV}$. We are calling them collectively the Primordial Density Perturbation, and through the Einstein field equation and the Boltzmann hierarchy, they specify the subsequent linear evolution of the entire set of cosmological perturbations [2,45]. They are therefore determined by observation.

It is found from observation [3–5] that the primordial curvature perturbation is almost Gaussian, with an almost scale-independent spectrum given by $\mathcal{P}_\zeta^{\frac{1}{2}} \simeq 5 \times 10^{-5}$. Current observations provide only upper bounds on the isocurvature perturbations [46,47,31,4,48,49], which depend on the degree of correlation between them and the curvature perturbation. In this paper we are concerned with the fully-correlated perturbations that can be produced only in the curvaton scenario. For them, the ratios $\mathcal{S}_i(\mathbf{x})/\zeta(\mathbf{x})$ are constants determined by the particle physics model.

The observational bound in the case of $s_B = \mathcal{S}_B(\mathbf{x})/\zeta(\mathbf{x})$ is [31] at 95% confidence level

$$-0.53 < s_B < 0.43. \quad (20)$$

The bound on $s_{\text{cdm}} = \mathcal{S}_{\text{cdm}}(\mathbf{x})/\zeta(\mathbf{x})$ is tighter by precisely the factor $\Omega_B/\Omega_{\text{cdm}}$ [31], and taking that ratio as 1/6 we get

$$-0.09 < s_{\text{cdm}} < 0.07. \quad (21)$$

The observational bound on s_ν is [49]

$$-0.14 < s_\nu < 0.47. \quad (22)$$

This bound has been calculated for negligible lepton asymmetry, whereas according to Eq. (29) below a non-zero lepton asymmetry is required [17] to produce an neutrino isocurvature density perturbation. Nevertheless, because the lepton asymmetry is small the bound may be expected to provide a good approximation.

B. Conserved number densities

To describe the origin of the isocurvature perturbations in the early Universe, we need to consider [17,42] the CDM number density n_{cdm} , the density of baryon number n_B and the density of lepton number n_L . Their perturbations may be characterised by [17]

$$\tilde{\zeta}_i \equiv -H \frac{\delta n_i}{\dot{n}_i}, \quad (23)$$

with $i = \text{cdm}, B$ or L . We can take the epoch of creation of CDM, baryon number or lepton number to be the epoch after which the relevant quantity (CDM particle number, baryon number or lepton number) is conserved. Each quantity $\tilde{\zeta}_i = \delta n_i/3n_i$ is then conserved after the relevant epoch of creation [42].

In the case of CDM, ζ_{cdm} in Eq. (13) and $\tilde{\zeta}_{\text{cdm}}$ Eq. (23) will usually be equal from the moment of CDM creation, and in any case are equal by the primordial epoch. The exception is the case of axionic CDM though, where the axion mass is initially temperature-dependent

so that only $\tilde{\zeta}_{\text{cdm}}$ is conserved after creation. In that case, ζ_{cdm} , will be equal to $\tilde{\zeta}_{\text{cdm}}$ after the mass becomes constant.

It follows from Eqs. (17) and (19) that

$$\mathcal{S}_{\text{cdm}} = 3(\tilde{\zeta}_{\text{cdm}} - \zeta_\gamma) \quad (24)$$

$$= 3(\tilde{\zeta}_{\text{cdm}} - \zeta). \quad (25)$$

The final equality assumes that $\mathcal{S}_\nu = 0$, but the modification in the case of nonzero \mathcal{S}_ν is quite straightforward [17] and there is no need to consider it here.

In the case of baryonic matter, there is no useful definition of ζ_B before the quark-hadron transition, since baryon number is not carried by a particular energy density. Afterwards, $\zeta_B = \tilde{\zeta}_B$ leading to

$$\mathcal{S}_B = 3(\tilde{\zeta}_B - \zeta_\gamma) \quad (26)$$

$$= 3(\tilde{\zeta}_B - \zeta). \quad (27)$$

In the case of neutrinos, ζ_ν is of course always defined provided that neutrinos exist but it is a useful quantity only after neutrinos freeze out of thermal equilibrium. After that epoch [17],

$$\mathcal{S}_\nu = \frac{135}{7} \left(\frac{\xi}{\pi} \right)^2 (\tilde{\zeta}_L - \zeta_\gamma) \quad (28)$$

$$\simeq \frac{135}{7} \left(\frac{\xi}{\pi} \right)^2 (\tilde{\zeta}_L - \zeta). \quad (29)$$

In these expressions, $\tilde{\zeta}_L$ is the perturbed net lepton number, $\delta n_L/3n_L$, and ξ is the neutrino asymmetry parameter, constrained by nucleosynthesis to $|\xi| < 0.07$. The second equality is a good approximation [17] by virtue of the bound on $|\xi|$.

In the curvaton scenario, $\tilde{\zeta}_L$ is negligible compared with ζ if the neutrino asymmetry is created before the curvaton contributes significantly to the energy density [17]. Then, the present observational bound Eq. (22) leads to $-\xi < 0.27$, which is competitive with the nucleosynthesis bound.

III. INTRINSIC AND RESIDUAL ISOCURVATURE PERTURBATIONS

A. The inflationary initial condition

Inflation is supposed to set the initial condition for the subsequent evolution of the Universe, through the values of the light scalar fields which exist at the end of inflation. We are here defining a field as light if the second derivative of the potential in that direction is much less than H^2 . This means that a light field has a flat spectrum of perturbations with spectrum $(H_*/2\pi)^2$, with the star denoting the epoch of horizon exit.

To handle the perturbed Universe, we adopt what has been called the separate universe picture [44,2], which means that the local evolution of regions within our presently observable Universe is the same as that of an unperturbed FRW universe. One assumes that the

Universe is smoothed on the shortest relevant scale. At least for cosmological scales the separate universe picture seems very likely to be valid, since such scales are far larger than the Hubble scale in the very early Universe.⁴ As a result, physical processes in the very early Universe are unlikely to distinguish between the shortest cosmological scale (say, 1 Mpc) and the longest one corresponding to the present Hubble scale ($\sim 10^4$ Mpc). And the separate universe assumption is certainly valid on the latter scale, or else the whole concept of an using an unperturbed FRW background to describe our observable Universe makes no sense.

For an unperturbed FRW universe, a generic quantity g (such as energy density, pressure, number density or a scalar field) has an evolution of the form

$$g(\phi_1, \phi_2, \dots, N) , \quad (30)$$

where ϕ_i are the values of the light fields specified at some initial epoch. We are specifying the epoch at which g is to be evaluated by the amount of expansion

$$N \equiv \frac{1}{3} \ln(\mathcal{V}/\mathcal{V}_{\text{initial}}) \equiv \ln(a/a_{\text{initial}}) \equiv \int_{t_{\text{initial}}}^t H dt . \quad (31)$$

In the separate universe picture, the locally-defined generic quantity g has the same functional form as in the unperturbed Universe, Eq. (30), but with position-dependent arguments;

$$g(\phi_1(\mathbf{x}), \phi_2(\mathbf{x}), \dots, N(\mathbf{x}, t)) , \quad (32)$$

where (\mathbf{x}, t) are the coordinates (gauge) used to describe the perturbations, and N is the locally-defined defined quantity.⁵

The first-order perturbation can be written as

$$\delta g = \sum_i \frac{\partial g}{\partial \phi_i} \delta \phi_i(\mathbf{x}) + \frac{\partial g}{\partial N} \delta N(\mathbf{x}, t) . \quad (33)$$

For a given wavenumber k/a one can choose the initial time to be a few Hubble times after horizon exit during inflation. Then the $\delta \phi_i(k)$ are uncorrelated gaussian quantities, with an almost flat spectrum

$$\mathcal{P}(k) = (H_*/2\pi)^2 , \quad (34)$$

where the star denotes the epoch of horizon exit $aH = k$.

Any conserved quantity X_f , defined in Eq. (4), can be calculated by evaluating δf on a spatially flat time-slice as a function of the initial field perturbations . This is separated

⁴In [44,2] the separate universe picture at a given epoch is taken to be valid for any smoothing scale bigger than the Hubble distance. That is a stronger assumption, which might not be valid in the presence of short-distance phenomena like preheating [51].

⁵The final expression of Eq. (31) is now $\int H d\tau$ where H is the locally-defined quantity Hubble expansion and $\tau(t)$ is the local proper time.

from an initial spatially flat time-slice during inflation by a uniform expansion, i.e., $\delta N = 0$, which yields from Eq. (33)

$$X_f = -\frac{H}{\dot{f}}\delta f = -\frac{H}{\dot{f}}\left(\frac{\partial f}{\partial\phi_i}\right)_N\delta\phi_i(\mathbf{x}), \quad (35)$$

In particular we can write the total curvature perturbation as

$$\zeta = -\frac{H}{\dot{\rho}}\delta\rho = -\frac{H}{\dot{\rho}}\sum_i\left(\frac{\partial\rho}{\partial\phi_i}\right)_N\delta\phi_i(\mathbf{x}), \quad (36)$$

This is a generalisation of the approach advocated by Sasaki and Stewart [52] as the simplest way to calculate the curvature perturbation in multi-field models. They calculated $\zeta = \delta N$ in the particular case where N is the local expansion between an initial spatially flat time-slice and a final uniform-density slice [44]. Taking $g = \rho$ and requiring $\delta\rho = 0$ in Eq. (33) reproduces this expression:

$$\zeta = \delta N = -\sum_i\left(\frac{\partial N}{\partial\phi_i}\right)_\rho\delta\phi_i(\mathbf{x}). \quad (37)$$

where we have used $dN/d\rho = H/\dot{\rho}$ along the background solution.

The general expression Eq. (33) has not been written down before. But it is all that is required in the separate universe picture to calculate the conserved quantities ζ_i Eq. (23) produced from field fluctuations during inflation, and hence describe the Primordial Density Perturbation. In what follows we will be interested in the case of isocurvature perturbations corresponding to conserved number densities n_i , and our main result will be Eq. (46), which along with Eq. (66) is a further example of Eq. (33).

B. The inflaton scenario

During single-field slow-roll inflation,⁶ the inflaton field satisfies $3H\dot{\phi} = -V'(\phi)$, which can be integrated to give ϕ as a unique function $\phi(N)$ of the integrated expansion N , up to a constant of integration.

In the the inflaton scenario, ϕ sets the initial condition for both the energy density and the pressure. As a result, the pressure perturbation is adiabatic, making ζ constant. Using Eq. (5) with $\delta\rho \simeq V'\delta\phi$ one finds that a few Hubble-times after horizon exit the well-known formula

$$\zeta \simeq -\frac{H}{\dot{\phi}}\delta\phi, \quad (38)$$

where the right hand side can be evaluated at horizon exit. In the inflaton scenario, this value is maintained until the primordial epoch.

⁶We adopt the now-standard terminology, where ‘single-field’ means that there is an essentially unique slow-roll trajectory while ‘multi-field’ means that there is a family of slow-roll trajectories.

A primordial isocurvature perturbation \mathcal{S}_i can be generated in the inflaton scenario if the relevant number density n_i at the time of its creation depends on some scalar field χ as well as the inflaton field;

$$n_i(\phi(\mathbf{x}), \chi(\mathbf{x}), N(\mathbf{x})). \quad (39)$$

We shall call this kind of isocurvature perturbation an *intrinsic* one, as opposed to the ‘residual’ one that can appear only in the curvaton scenario. The known cases where it is possible are when the CDM comes initially from an oscillating field (axion or Wimpzilla), or when there is Affleck-Dine baryo- or lepto-genesis. The qualification ‘possible’ is here crucial, because the axion or Affleck-Dine field has to be light during inflation so as to acquire a perturbation, and also because that perturbation has to survive until n_i is created. This does not happen if the CDM axions come mostly from the oscillation of Peccei-Quinn cosmic strings, or if the Affleck-Dine field has the generic effective mass-squared of order $\pm H^2$.

The possibilities for obtaining an intrinsic isocurvature perturbation are very limited, so that, for example, there is no known way of obtaining an intrinsic CDM isocurvature perturbation if the CDM consists of the relic abundance of the Lightest Supersymmetric Particle obtained when it decouples from thermal equilibrium. Even when there is an intrinsic isocurvature perturbation, there is in general no reason to expect its magnitude is comparable with that of the adiabatic perturbation. This is because the magnitudes of the two perturbations depend on different and in general unrelated parameters, as a result of the fact that the perturbations are caused by different fields.

A final and very important property of an intrinsic isocurvature perturbation is that it is uncorrelated with the adiabatic perturbation. Again, this comes from the fact that two different fields are involved.

With one possible exception, the above discussion applies also to multi-field inflation, if we define the inflaton as the specific linear combination of fields corresponding to the inflaton trajectory at the *end* of inflation. The possible exception concerns the correlation; there is correlation in the exceptional case that the combination of fields orthogonal to the inflaton was responsible for the isocurvature perturbation [50]. However, no example of this has been proposed in the context of particle physics; in particular, no model has been exhibited where this combination is the axion or an Affleck-Dine field.

C. The curvaton scenario

In the curvaton scenario, the value Eq. (38) of ζ generated by the inflaton is supposed to be negligible compared with the observed value of order 10^{-5} , and to remain negligible until the curvaton field σ starts to oscillate. The density ρ_σ is supposed to be negligible when the oscillation starts, but during radiation domination it grows like $a(t)$ relative to the radiation density. Until the curvaton starts to decay, the energy densities of the radiation and the curvaton are decoupled, so that ζ_r is constant and negligible, while ζ_σ is constant but not negligible. From Eq. (36) we then have

$$\zeta(t, \mathbf{x}) = -\frac{H}{\dot{\rho}} \left(\frac{\partial \rho}{\partial \sigma} \right) \delta\sigma(\mathbf{x}) = f(t)\zeta_\sigma(\mathbf{x}), \quad (40)$$

where $\zeta_\sigma = -H\delta\sigma/\dot{\sigma}$ and

$$f \equiv \frac{\dot{\rho}_\sigma}{\dot{\rho}} = \frac{3\rho_\sigma}{4\rho_r + 3\rho_\sigma}, \quad (41)$$

is a growing function of time.

When curvaton decay is complete, the Universe is supposed to be thermalised, so that the temperature determines the entire future evolution of the energy density and pressure. (Non-thermalised components such as CDM are supposed to give a negligible contribution to these quantities until at least the primordial epoch.) As a result, the pressure in each separate universe is once again a practically unique function of energy density, leading to a practically constant value of ζ given by Eq. (43). Following [17,28] we define

$$r \equiv \zeta/\zeta_\sigma, \quad (42)$$

where ζ on the left-hand-side is evaluated after curvaton decay is complete, and ζ_σ on the right-hand-side is evaluated before the decay starts.

In the sudden decay approximation we have

$$\zeta \simeq f(t_{\text{dec}})\zeta_\sigma. \quad (43)$$

Hence we have $r \sim (\rho_\sigma/\rho)_{\text{dec}}$. Equation (43) gives a good approximation (to within 10%) to the precise numerical solution [28]. Note that r is found to be always less than (or equal to) one and, in particular, $r \rightarrow 1$ in the limit $\rho_\sigma/\rho \rightarrow 1$.

In the curvaton scenario there might be an intrinsic primordial isocurvature perturbation due to some other field χ , different from both the inflaton ϕ and the curvaton σ . If present, it will have the features discussed already in the case of isocurvature perturbations for the inflaton scenario. It will generally be uncorrelated with the residual isocurvature perturbation and accordingly we ignore it from now on.

We are interested in a different type of primordial isocurvature perturbation, which we term residual [17]. A residual isocurvature perturbation can arise because in the curvaton case, the separate universes are not identical while the curvaton is oscillating. This is because they are characterised by the two independent local energy densities ρ_ϕ and ρ_σ . Thus, a residual isocurvature perturbation \mathcal{S}_i will inevitably be present at some level unless the relevant number density n_i is created after the curvaton has decayed.

In the sudden-decay approximation, it is easy to arrive at a general formula for the residual isocurvature perturbations. Note first that before curvaton decay, ρ_r is supposed to be uniform on the flat slicing. The perturbation of the number density $n_i(\rho_r, \rho_\sigma)$ on this slicing is therefore just given by

$$\delta n_i = \left(\frac{\partial n_i}{\partial \rho_\sigma} \right) \delta \rho_\sigma. \quad (44)$$

As a result the perturbation, Eq. (12), can be written as

$$\tilde{\zeta}_i = \frac{1}{3} \left(\frac{\partial n_i}{\partial \rho_\sigma} \right) \frac{\delta \rho_\sigma}{n_i}. \quad (45)$$

Using Eq. (42) for the total curvature perturbation after curvaton decay, we have

$$\frac{\tilde{\zeta}_i}{\zeta} = \frac{1}{r} \frac{\rho_\sigma}{n} \left(\frac{\partial n_i}{\partial \rho_\sigma} \right). \quad (46)$$

This expression can be evaluated at any epoch after the conservation of n_i but before curvaton decay, since both n_i and ρ_σ are proportional to a^{-3} . Courtesy of the separate universe assumption, its evaluation requires only a knowledge of the function $n_i(\rho_r, \rho_\sigma)$ in an unperturbed universe. This is similar in spirit to the calculation in [52] of the curvature perturbation produced by multi-field inflation. The separate universe assumption can be verified by explicit calculation in all three of the CDM cases that we shall deal with in the next section, since the situation there is sufficiently simple, just as it can be verified in the case of multi-field inflation.

In the case where the CDM or baryon number is created before the curvaton decays, the residual CDM or baryonic matter isocurvature perturbation is thus given by Eq. (2) with

$$s_{\text{cdm}} = 3 \left[\frac{1}{r} \frac{\rho_\sigma}{n_{\text{cdm}}} \left(\frac{\partial n_{\text{cdm}}}{\partial \rho_\sigma} \right) - 1 \right] \quad (47)$$

$$s_B = 3 \left[\frac{1}{r} \frac{\rho_\sigma}{n_B} \left(\frac{\partial n_B}{\partial \rho_\sigma} \right) - 1 \right]. \quad (48)$$

In the opposite case where the curvaton decays and its decay products are thermalised before the CDM or baryon number is created, the CDM or baryon number at creation is simply a function of the local energy density, as in the inflaton scenario, and we have $s_i = 0$.

IV. THE RESIDUAL CDM ISOCURVATURE PERTURBATION

For the rest of this paper, we focus on the residual CDM isocurvature perturbation, which is present in the curvaton scenario if the CDM is created before the curvaton has decayed. We shall work out Eq. (47) in the cases that the CDM (i) is a WIMP (ii) is an axion and (iii) comes from an oscillating field with constant mass, such as a Wimpzilla. We shall also compare our results with present observational bounds.

Before looking at the individual cases though, we recall two results already considered in [17]. The first result holds in the case that the CDM is created at an epoch when (ρ_σ/ρ) is negligible compared with the value when the curvaton decays, which means in particular that the CDM must be created well before the curvaton decays. If the curvaton density has a negligible effect on n_{cdm} , Eq. (47) becomes

$$s_{\text{cdm}} = -3. \quad (49)$$

This is too big to be compatible with the current observational bound given in Eq. (21), and is independent of the nature of the CDM. It follows that in the curvaton scenario, CDM cannot be created significantly before the curvature perturbation achieves its final value. In particular, CDM cannot be created just after inflation, as would be the case if the CDM consisted of heavy weakly interacting particles created from the vacuum. (This kind of CDM was originally envisaged in [53,54], and later [55] called Wimpzilla CDM.)

The other result holds when the CDM is created by the decay of the curvaton itself. The epoch of CDM creation then corresponds to the epoch when the curvaton decay is complete. The resulting local CDM density is a fixed multiple of the curvaton number density well before decay. The fractional perturbations are thus equal and hence

$$\tilde{\zeta}_{\text{cdm}} = \zeta_{\sigma}. \quad (50)$$

In terms of r , defined in Eq. (42), the prediction Eq. (50) leads to

$$s_{\text{cdm}} = 3 \left(\frac{1-r}{r} \right). \quad (51)$$

This is compatible with Eq. (21) only if $1-r < 0.03$, which means that the curvaton has to dominate before it decays. Note that $r \leq 1$ [28] and hence in this case there is a positive correlation between the primordial curvature perturbation ζ and isocurvature perturbation \mathcal{S} , i.e., $s > 0$.

A. Weakly interacting massive particles

If the CDM consists of weakly interacting massive particles (WIMPs), it is initially in thermal equilibrium. The number density of the WIMPs is conserved only after the epoch when the WIMPs fall out of thermal equilibrium (the epoch of freezeout). In the present context that epoch should be taken as the one when the CDM is created. In contrast with the axion case that we shall discuss next, no scalar field can be involved in the creation of WIMP CDM and therefore there can be no intrinsic isocurvature perturbation.

The calculation of the residual isocurvature perturbation after freeze-out in terms of gauge-invariant linear perturbations is given in an Appendix A. However we shall show here how the same result can be obtained working only in terms of the unperturbed background equations, via Eq. (47).

In either case we work in the approximation of sudden freeze-out, denoting the moment of freeze-out by a star. This moment is determined by an equation of the form

$$\Gamma(T_*)/H_* = K, \quad (52)$$

where K is of order 1 and Γ is the interaction rate per CDM particle. The temperature dependence of the interaction rate given in terms of a dimensionless parameter

$$\alpha \equiv \frac{d \ln \Gamma}{d \ln T}, \quad (53)$$

with $\alpha > 2$ if there is initially thermal equilibrium at high temperatures.

The number density of non-relativistic particles ($m > T_*$) at freeze-out is

$$n_* = g_* \left(\frac{m T_*}{2\pi} \right)_*^{\frac{3}{2}} e^{-m/T_*}, \quad (54)$$

where m is the mass and g_* the internal degrees of freedom of the WIMP. The annihilation cross-section for non-relativistic WIMPs is weakly dependent on temperature, with $\langle \sigma_{AV} \rangle \propto$

$T^{p/2}$ where $p = 0$ for s -wave annihilation and $p = 2$ for p -wave annihilation [1]. The rate of change of the interaction rate Γ is thus mainly determined by the rapidly decreasing number density if $m > T$, yielding

$$\Gamma = n\langle\sigma_{Av}\rangle \propto T^{3/2+(p/2)}e^{-m/T}, \quad (55)$$

and hence $\alpha = (m/T) + (3/2) + (p/2)$.

After freeze-out interactions are negligible and the number density is diluted as the universe expands, so we have $n = n_*(a_*/a)^3$. During any era when the number of effective species is constant, we have $a \propto 1/T$ and hence

$$n = n_*(T/T_*)^3 = g_*(m/2\pi T_*)^{\frac{3}{2}}e^{-m/T_*}T^3, \quad (56)$$

This is the expression that we want to insert into Eq. (47). The partial derivative is at fixed ρ_r and therefore at fixed temperature, $T < T_*$, so that Eq. (47) can be written

$$s_{\text{cdm}} + 3 = \frac{3}{r} \left(\frac{\rho_\sigma}{n} \frac{\partial n}{\partial \rho_\sigma} \right)_T. \quad (57)$$

The local CDM number density after decoupling varies with the local curvaton density because the curvaton density at freeze-out, $\rho_{\sigma*}$, affects the expansion rate, H_* in Eq. (52), and hence the local freeze-out temperature, T_* . Hence we can write

$$\left(\frac{\rho_\sigma}{n} \frac{\partial n}{\partial \rho_\sigma} \right)_T = \left(\frac{m}{T_*} - \frac{3}{2} \right) \left(\frac{\rho_\sigma}{T_*} \frac{\partial T_*}{\partial \rho_\sigma} \right)_T \quad (58)$$

where we have used Eq. (56) to obtain $(\partial n / \partial T_*)_T = (m/T_*^2 - 3/2T_*)n$.

Assuming that the curvaton is oscillating at freeze-out (i.e., the curvaton mass is greater than the Hubble rate, $m_\sigma > H_*$) we have

$$\rho_\sigma = \rho_{\sigma*} \left(\frac{a_*}{a} \right)^3 = \rho_{\sigma*} \left(\frac{T}{T_*} \right)^3, \quad (59)$$

and hence

$$\left(\frac{\rho_\sigma}{T_*} \frac{\partial T_*}{\partial \rho_\sigma} \right)_T = \left(\frac{T_*}{\rho_\sigma} \frac{d\rho_{\sigma*}}{dT_*} - 3 \right)^{-1}. \quad (60)$$

Finally then we must determine $d\rho_{\sigma*}/dT_*$ from Eq. (52), which gives

$$\begin{aligned} dK &= \left[\alpha_* \frac{dT_*}{T_*} - \frac{1}{2} \frac{d\rho_*}{\rho_*} \right] \frac{\Gamma(T_*)}{H_*}, \\ &= \left[(\alpha_* - 2(1 - \Omega_{\sigma*})) \frac{dT_*}{T_*} - \frac{1}{2} \Omega_{\sigma*} \frac{d\rho_{\sigma*}}{\rho_{\sigma*}} \right] \frac{\Gamma(T_*)}{H_*}, \\ &= 0. \end{aligned} \quad (61)$$

and hence

$$\frac{T_* d\rho_{\sigma^*}}{\rho_{\sigma^*} dT_*} = \frac{2(\alpha_* - 2) + 4\Omega_{\sigma^*}}{\Omega_{\sigma^*}}. \quad (62)$$

Combining Eqs. (57), (58), (60) and (62) we then obtain the residual isocurvature perturbation,

$$s_{\text{cdm}} = 3 \left[\frac{\Omega_{\sigma^*}}{r} \left(\frac{m}{T_*} - \frac{3}{2} \right) \frac{1}{2(\alpha_* - 2) + \Omega_{\sigma^*}} - 1 \right]. \quad (63)$$

In the usual case that freeze-out occurs during radiation domination (requiring $\Omega_{\sigma^*} \ll 1$) with the WIMP a neutralino, then m/T_* is of order 20. In this case the annihilation cross-section is approximately constant while the number density drops rapidly and hence

$$\alpha_* \approx \left(\frac{d \ln n}{d \ln T} \right)_* \approx \frac{m}{T_*}, \quad (64)$$

and Eq. (63) reduces to

$$s_{\text{cdm}} \approx 3 \left[\frac{\Omega_{\sigma^*}}{2r} - 1 \right]. \quad (65)$$

The first term in the bracket is thus roughly of order $\Omega_{\sigma^*}/\Omega_{\sigma \text{ dec}}$ (using the sudden-decay approximation Eq. (43) which gives $r \sim \Omega_{\sigma \text{ dec}}$). If freezeout occurs well before the curvaton decays and while the curvaton accounts for only a small fraction of the energy density (i.e., for $\Omega_{\sigma^*} \ll \Omega_{\sigma \text{ dec}}$), this makes $s_{\text{cdm}} \simeq -3$ in accordance with the general expectation [17] for this regime, which violates the observational bound.

On the other hand, if the curvaton dominates the energy density before freeze-out so that $\Omega_{\sigma^*} \sim \Omega_{\sigma \text{ dec}} \sim 1$ (or if freeze-out occurs close to curvaton decay with $\Omega_{\sigma^*} \sim \Omega_{\sigma \text{ dec}} < 1$), there could be a regime where s_{cdm} is small enough to satisfy observational bounds. Note that if the CDM freezes-out when the curvaton has come to dominate the energy density ($\Omega_{\sigma^*} \simeq 1$) but before it decays, then the CDM abundance relative to the radiation density will be significantly diluted when the curvaton decays. Thus m/T_* could be significantly smaller than usually assumed. In order to determine the allowed regime more precisely would require more detailed numerical modeling going beyond the sudden freeze-out and sudden decay approximations that we have used here.

B. Axion CDM

If the CDM consists of axions, it is created when the rising axion mass $m(T)$ becomes equal to the falling Hubble parameter H because only then does the approximately homogeneous axion field become free to oscillate. If axionic strings are present, the axions radiated from them will probably be the dominant creation mechanism. In that case there can be no intrinsic CDM isocurvature perturbation. If instead such strings are absent, the axion field $\chi(\mathbf{x}, t)$ oscillates independently at each comoving position, with an initial value that is the same as it was during inflation. (This is the separate universe assumption, which in the present case can be verified by explicit calculation [56] from the axion field equation.)

Then, the axion density is proportional to the square of the initial axion field value, and a perturbation in this value will generate an intrinsic isocurvature perturbation

$$\mathcal{S}_{\text{cdm}} = \frac{2\delta\chi}{\chi}. \quad (66)$$

We are here concerned instead with the residual isocurvature perturbation, which is always present if the curvaton decays after the epoch of axion creation. We work in the approximation that the oscillation starts suddenly and is immediately harmonic, which is known to be reasonable in most of parameter space [56]. Then, if axion creation occurs during radiation domination the calculation is very similar to the one that we gave for the WIMP case. The freezeout epoch is determined by

$$m(T_*)/H_* = K, \quad (67)$$

with K conventionally set equal to 1 exactly. The increasing mass is given accurately by

$$m \propto T_*^{-\beta}, \quad (68)$$

with $\beta = -3.7$ [57]. Assuming that there is no intrinsic perturbation, the axion number density n_* at creation is proportional to $m(T_*)$. Repeating the calculation of the WIMP case we find for the axion case

$$s_{\text{cdm}} = 3 \left[\frac{\Omega_{\sigma^*}}{\Omega_{\sigma \text{ dec}}} \frac{3 + \beta}{2\beta + 4 - \Omega_{\sigma^*}} - 1 \right]. \quad (69)$$

In contrast with Eq. (63), this always gives $s_{\text{cdm}} \simeq -3$. We conclude that if the CDM consists of axions, the curvaton cannot decay after the CDM is created at $T \sim \text{GeV}$.

C. CDM from an oscillating field of fixed mass

The previous calculation does not apply if axion creation occurs at low temperatures, and in particular if it occurs while the curvaton density dominates. In such a case, the temperature of the sub-dominant radiation will have a negligible effect on the axion mass, which will therefore have its vacuum value.

More generally, the CDM might consist of particles corresponding to the oscillation of some scalar field other than the axion, in which case the mass will generally be at the vacuum value whether or not creation takes place during matter domination. This possibility is envisaged in the rather attractive proposal of Moroi and Randall [58], whereby a modulus decays before nucleosynthesis into both baryonic matter and CDM. The decay occurs because the modulus mass is 10 to 100 TeV instead of the usual 1 TeV or so. In contrast with earlier proposals though, the high mass is explained naturally, arising from the fact that in this scenario SUSY-breaking is anomaly-mediated as opposed to gravity-mediated. The anomaly-mediation also ensures that matter will not be over-produced by the decay. Of course, this idea has been worked out only under within the inflaton scenario and it remains to be seen whether it can be viable in the curvaton scenario, with the oscillation starting before the curvaton decays as we are envisaging.

Without focusing on any particular scenario, we consider the residual CDM isocurvature perturbation that is produced if the CDM does originate from the oscillation of some field with fixed mass. With constant mass, corresponding to $\beta = 0$, Eq. (69) can be written

$$s_{\text{cdm}} = 3 \left(\frac{f_*}{\Omega_{\sigma, \text{dec}}} - 1 \right), \quad (70)$$

where f is defined in Eq. (41). Using Eq. (40) this equation corresponds actually to

$$\tilde{\zeta}_{\text{cdm}} = \zeta_*. \quad (71)$$

This in turn is simply a consequence of the fact that the oscillation starts on a slice of uniform energy density, corresponding to local Hubble parameter $H = m$.

If the oscillation starts well before the curvaton decays, and while the curvaton accounts for only a small fraction of the energy density, this again gives $s_{\text{cdm}} \simeq -3$ in contradiction with observation. However, in contrast with the cases of WIMP and axionic CDM, s_{cdm} in the present case vanishes in the limit that the curvaton completely dominates the energy density when the CDM is created (ie. when the CDM oscillation starts). Using Eq. (70) one finds in this regime

$$s_{\text{cdm}} \simeq -\frac{4}{3} \left(\frac{\rho_r}{\rho_\sigma} \right)_* + \left(\frac{\rho_r}{\rho_\sigma} \right)_{\text{dec}}. \quad (72)$$

Unfortunately, the sudden-oscillation and sudden-decay approximations both become inadequate in this regime, so that Eq. (72) will not in fact be correct. Departures in the sudden-oscillation can be calculated once the potential of the oscillating is known but no relevant calculation has been published so far. Even in the absence of such a calculation though, it follows just from continuity that there will be a regime in which s_{cdm} is within the present observational bound, while being big enough to be observable in the future.

V. CONCLUSION

The observed primordial curvature perturbation may well be accompanied by isocurvature perturbations in CDM, baryonic matter or neutrino. In the curvaton scenario, such a perturbation is inevitably generated by the curvaton perturbation, unless the curvaton decays before the relevant quantity (CDM, baryon number or lepton number) is created. This ‘residual’ isocurvature perturbation is fully correlated with the curvature perturbation.

We presented a general expression, Eq. (46), which allows the residual isocurvature perturbation to be evaluated once the mechanism for generating the relevant quantity is specified. We then evaluated the residual CDM isocurvature perturbation for all the commonly considered candidates in the sudden decay approximation, and compared it with the observational upper bound [31]. We first recalled previously-known results [17]. The CDM cannot be created right after inflation in the curvaton scenario, as in the case of CDM in the form of super-heavy weakly interacting massive particles (Wimpzillas), created from the vacuum energy [54,55]. But if the curvaton dominates the energy density before it decays, then the CDM can be created from that decay.

We then went on to consider new cases, using Eq. (47). After a fairly complicated calculation, we derive Eq. (63) for the case of CDM falling out of thermal equilibrium (weakly interacting massive particle or WIMP CDM). It is compatible with observation only if the CDM is created when the curvaton density has a rather specific value. Modifying the previous calculation slightly, we arrive at the case of axion CDM given by Eq. (69), which we find is too big in all cases. These results mean that axions, and probably also WIMPS, have to be created *after* curvaton decay. Such a requirement is not however a strong constraint on the curvaton scenario, since WIMPS and axions are both typically created rather late (at temperatures respectively of order 10 GeV and 1 GeV).

Finally, we consider the case of CDM created from the oscillation of a scalar field with fixed mass. As the created particle would need to be massive and very weakly interacting to avoid subsequent thermalisation, we are again dealing with a Wimpzilla. We derive Eq. (70) for the residual isocurvature perturbation in this case. It again gives $s_{\text{cdm}} = -3$ in the limit where the curvaton density when the oscillation starts is much smaller than its final value. In contrast with the previous cases though, it gives $s_{\text{cdm}} = 0$ in the opposite limit where curvaton domination is complete when the oscillation starts. There must therefore be an intermediate regime where s_{cdm} in this case is of potentially observable magnitude.

We can summarise the situation as follows. The residual CDM isocurvature perturbation is absent if the CDM is created after curvaton decay. Otherwise it is present, and it is typically either too big to be compatible with observation, or completely negligible. There are cases though where it can be of observable magnitude. In particular, this can occur if the CDM comes from the decay of the curvaton, or from the oscillation of a scalar field of fixed mass. In both of these cases, the residual isocurvature perturbation vanishes in the limit that the curvaton completely dominates the energy density at the time of the CDM creation, but isocurvature perturbations can exist at a detectable level if the domination is not complete.

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APPENDIX A: GAUGE-INVARIANT CALCULATION

In this section we reproduce the calculation of the CDM perturbation given in section IV in terms of manifestly gauge invariant perturbations.

To determine the CDM isocurvature perturbation we need to determine the gauge-invariant perturbation in the the CDM number density on spatially flat slices, Eq. (12), (or equivalently the curvature perturbation on uniform number-density slices)

$$\tilde{\zeta}_{\text{cdm}} \equiv \frac{1}{3} \frac{\delta n_{\text{cdm}}}{n_{\text{cdm}}} - \psi. \quad (\text{A1})$$

where δn_{cdm} and ψ are the gauge-dependent perturbed number density and curvature perturbation respectively.

In the *sudden freeze-out* approximation, we assume that the CDM remains in equilibrium abundance, with number density given by Eq. (54), until the local interaction rate per expansion time falls below the critical value

$$\left. \frac{\Gamma}{H} \right|_f = K. \quad (\text{A2})$$

Thereafter the CDM number is conserved and hence the curvature perturbation $\tilde{\zeta}_{\text{cdm}}$ in Eq. (A1) remains constant on large scales after freeze-out. The physical condition (A2) picks out a physical hypersurface Σ_f which marks the transition between equilibrium and freeze-out. Thus we can determine $\tilde{\zeta}_{\text{cdm}}$ in Eq. (A1) after freeze-out from the curvature perturbation, ψ_f , and perturbed number density, δn_f , on this hypersurface.

A uniform temperature slice at this time Σ_T has no perturbation in the equilibrium number density, $\delta n_T = 0$, but may have a curvature perturbation $\psi_T = -\zeta_r$. (In the curvaton scenario we have $\zeta_r \ll \zeta_\sigma$ before the curvaton decays.) We obtain ψ_f and δn_f via a gauge-shift from the hypersurface Σ_T to the hypersurface Σ_f . This corresponds to a coordinate-shift

$$\delta t_{fT} = \frac{\delta(\Gamma/H)_T}{(\dot{\Gamma}/H) - (\dot{H}\Gamma/H^2)}. \quad (\text{A3})$$

We then have

$$\psi_f = \psi_T + H\delta t_{fT}, \quad (\text{A4})$$

$$\delta n_f = \delta n_T - \dot{n}\delta t_{fT}, \quad (\text{A5})$$

and hence

$$\tilde{\zeta}_{\text{cdm}} = \zeta_r + \frac{1}{3} \left(\frac{m}{T_*} - \frac{3}{2} \right) H_* \delta t_{fT}, \quad (\text{A6})$$

where we have assumed $H^{-1}\dot{T}/T = -1$.

Assuming $\Gamma/H \propto T^\alpha/\rho^{1/2}$ (where $\alpha > 2$ for CDM to fall out of equilibrium with decreasing temperature) we have from Eq. (A3)

$$\delta t_{fT} = \left(\frac{4 - \Omega_\sigma}{2(2 - \alpha) - \Omega_\sigma} \right)_* \frac{\delta\rho_T}{\dot{\rho}_*}, \quad (\text{A7})$$

and using

$$\frac{H\delta\rho_T}{\dot{\rho}} = \zeta_r - \zeta = \frac{3\Omega_\sigma}{4 - \Omega_\sigma} (\zeta_r - \zeta_\sigma), \quad (\text{A8})$$

we then have

$$H_* \delta t_{fT} = \left(\frac{3\Omega_{\sigma^*}}{2(\alpha - 2) + \Omega_{\sigma^*}} \right)_* (\zeta_\sigma - \zeta_r). \quad (\text{A9})$$

Finally we then obtain from Eq. (A6) and (A9)

$$\tilde{\zeta}_{\text{cdm}} = \zeta_r + \left(\frac{m}{T_*} - \frac{3}{2} \right) \frac{\Omega_{\sigma^*}}{2(\alpha - 2) + \Omega_{\sigma^*}} (\zeta_\sigma - \zeta_r). \quad (\text{A10})$$

If we have adiabatic perturbations ($\zeta_r = \zeta_\sigma$) before freeze-out, we must have $\tilde{\zeta}_{\text{cdm}} = \zeta_r$ from Eq. (A10), and hence $s_{\text{cdm}} = 0$ after freeze-out.

In the curvaton scenario when the CDM decouples before the curvaton decays, we have $\zeta_\sigma \gg \zeta_r$ and the primordial curvature perturbation is given by $\zeta = r\zeta_\sigma$. The residual CDM isocurvature perturbation is then given by Eq. (63) of section III.