

# Fairness in Nurse Rostering

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**Abstract.** Nurse rostering is a complex real world problem requiring the assignment of various shifts to the hospital personnel subject to a set of constraints. Fairness of work allocation is a major human resources issue which has not been sufficiently explored in this field. The quality of a satisfactory solution to a nurse rostering problem instance is usually evaluated based on quantitative criteria. Nevertheless, the individuals have different implicit preferences and availabilities. Hence, the perceived roster quality may not be fully aligned with these quantitative criteria. Therefore, it would be more appropriate to also consider qualitative measures for fair rosters. This study investigates new qualitative measures for fairness and investigates centralised and cooperative meta-heuristic approaches for optimising these measures on a set of existing benchmark instances.

**Keywords:** Nurse rostering, fairness, cooperative search, meta-heuristics, hyper-heuristics, agent-based systems.

## 1 Introduction

The health care sector is under increasing pressure due to the ageing population and increasing cost of ever improving treatments (Rais and Viana, 2011). Moreover, many health care organisations suffer from a shortage of nursing staff. Nurses are responsible for the majority of health care duties and experience a lot of stress on a daily basis. Job dissatisfaction appears to play a key role in the high resignation rates of nurses. Mueller and McCloskey (1990) identified ‘scheduling’ as the second most important factor contributing to job satisfaction, after ‘extrinsic rewards’ such as salary, vacation and benefits. Improving the schedules of nurses thus seems to be a sensible way to increase their satisfaction and consequently their retention (Larrabee et al., 2007).

Nurse rostering is classified as a timetabling problem in the academic literature. The problem usually considers assigning nurses to a set of shifts in such

a way that required shifts are covered by nurses with the best possible skill match. Besides this requirement, the resulting assignment should optimise constraints on the nurses' individual rosters thereby warranting the quality of their work life balance. Burke et al. (2004) present an overview of constraints and objectives that are common to many nurse rostering problems. This survey, in addition, classifies many mathematical as well as heuristic approaches that have been applied to different variants of the nurse rostering problem.

The present paper reports on a methodology for increasing the nurses' satisfaction with respect to their personal schedule, also referred to as roster in the literature. Automatically generated rosters are commonly evaluated by means of a weighted sum objective function, the result of which is proportional to the number of soft constraint violations (Burke et al., 2001). Composite evaluation functions like this are attractive because they are based on crisp mathematical descriptions of the quality measures. However, such approaches do not necessarily compare well with the human way of assessing the quality of a roster. Two rosters with the same objective function value may differ considerably in terms of a pairwise comparison of individual roster's quality. Beddoe and Petrovic (2006) apply a completely different methodology for quality assessment, namely case based reasoning. The approach requires a training phase in which experts, i.e. nurses, are invited to identify poor elements and to modify the roster such that the problems are sorted out. Objective function based approaches and case based reasoning are hard to compare in a quantitative manner (Petrovic and Vanden Berghe, to appear).

Modelling mathematically the perceived quality of a roster by individual nurses would definitely be much harder than the weighted sum approach. Such a model is likely to be blurred by individual time-varying nonlinear dependencies between constraint violations and by the rosters that have been obtained for the fellow nurses. Therefore, some assumptions and simplifications are inevitable.

Assuming that fairness of work distribution among colleagues contributes to a higher job satisfaction, we will concentrate on including fairness measures into nurse rostering models in what follows.

The main aim of this study is to propose a set of new evaluation models for nurse rostering, which capture the concept of fairness of work better than the existing models and hence optimise the rosters accordingly. Different fairness measures are introduced and their attainability is investigated experimentally. We make use of existing data sets that were obtained from a hospital in Belgium and apply centralised and cooperative meta-heuristic approaches to generate fair nurse rosters. Additionally, along with some traditional approaches, an agent-based framework for cooperative meta-heuristic search emphasizing fairness will be described in this study. Each agent is autonomous and capable of executing different meta-heuristic and local search combinations with different parameter settings. They cooperate asynchronously using pattern matching and reinforcement learning to converge towards good quality solutions. Agent based cooperative meta-heuristic approaches are fairly new in nurse rostering. Apart from a number of conference presentations, very little publications are available.

Wang and Wang (2009) developed an agent based approach to self rostering. Haspeslagh et al. (2009) addressed the problem of exchanging nurses between wards and sorting out personnel shortages. ? present a Pareto optimal negotiation approach for leveraging the workload across different wards. To the best of our knowledge, there is scarce research work on fairness and cooperative search in nurse rostering.

Section 2 reviews approaches and considerations on fairness from the literature. A set of new alternative objective functions for nurse rostering, which we believe are more fair than traditional ones, are introduced in Section 3. In Section 4, we present an agent-based framework for cooperative meta-heuristic search for nurse rostering. The results are discussed in Section 5. Section 6 concludes the paper with a discussion on the contribution and some directions for future research.

## 2 Related work on fairness in nurse rostering

A recent survey on operational research in health care points at the importance of optimising resource planning and scheduling (Rais and Viana, 2011). It is noteworthy that among the extensive number of papers discussed in that review, only a few pay attention to fairness of work distribution. Felici and Gentile (2004), for example, assume that the extent to which the contractual constraints are met is correlated with the nurse’s satisfaction. However, the objective function sums ‘satisfaction’ over all the roster elements, thereby making no distinction between individual nurses’ overall satisfaction. Some work schedules, especially those involving shift work, induce high levels of fatigue. Yuan et al. (2011) present recommendations on shift sequences, days off and overtime so as to reduce the risk of fatigue as much as possible.

One of the most seminal overview papers on nurse rostering (Warner, 1976) puts forward fairness as a quality measure. Warner indicates even distribution of work has a considerable advantage of cyclic schedules, despite their limited flexibility. Burke et al. (2004) noticed also that the majority of nurse rostering papers do not explicitly address fairness. Approaches paying attention to this concern tend to model fairness as a balance constraint on working time accounts, while solving the problem with an optimisation algorithm (Burke et al., 2006; Meyer auf’m Hofe, 2001). It is noteworthy that many recent nurse rostering papers take some work balancing measures into consideration (De Causmaecker and Vanden Berghe, 2011).

Kellogg and Walczak (2007) investigated why only a small number of automated nurse rostering approaches are being used as decision support systems in hospitals. They pointed out, among some other causes, that academic models fail to meet the complex needs that health care organisations face. Interactive rostering, also referred to as self scheduling, is a mostly manual mode of operation that has gained attention particularly because it potentially increases nurses’ satisfaction (Robb et al., 2003). The nurses put together a roster by expressing their preferences and negotiating until the hospital’s requirements are

met. One major drawback of manual self scheduling is that the quality of the result depends on the nurses' ability to cooperate and negotiate, and therefore Rönnerberg and Larsson (2010) advocate an automated approach. They enforce some degree of fairness by means of an auxiliary variable representing the requests by the least favoured nurse, which was included in the objective to be maximised. Along the same lines, we have modelled a set of fairness measures for nurse rostering (Section 3).

Grano et al. (2009) combine aspects from self rostering and from common optimisation methods into a two-stage approach. First, an auction is set up in which nurses can spend a number of points to bid for preferred shifts or days off. After the best bid assignments have been made, the remaining part of the problem is solved with a mathematical solver so as to meet the ward's staffing demands. This hybrid approach was tested on data sets from a real hospital ward but without active involvement of the nurses. The bids were derived from the nurses' preferences instead. Grano et al. (2009) indicated issues that require further research. Knowledge on popular shifts, for example, may influence a nurse to set a bid. In addition, fairness requires that points of ungranted bids should be transferrable to the next scheduling period. The models that were derived in this study (Section 3) attempt to sort out such unfairness issues.

### 3 Modelling of the nurse rostering problem

Most real-world nurse rostering problems can be represented as constraint optimisation problems using 4-tuples  $\langle N, D, S, K, C \rangle$ :

- $N$ : Set of nurses.
- $D$ : Set of days in the current schedule period and in the related parts of the previous and upcoming schedule period.
- $S$ : Set of shift types.
- $K$ : Set of skill types.
- $C$ : Set of constraints.

$x_{n,d,s,k}$  denote the decision variables and  $\forall n \in N, \forall d \in D, \forall s \in S, \forall k \in K$ :

$$x_{n,d,s,k} = \begin{cases} 1 & \text{if employee } n \text{ is assigned to shift } s \text{ and skill } k \text{ at day } d \\ 0 & \text{otherwise} \end{cases}$$

The assignment of values to the decision variables is strongly restricted by constraints. The literature presents various ways to dealing with the constraints by, for example, considering some as hard and some as soft constraints. The former need to be satisfied in order to produce a feasible solution, while the latter need to be satisfied as much as possible in order to find high quality solutions. Most problems are very complex and it is usually impossible to generate solutions satisfying all soft constraints.

Common models distinguish between coverage constraints and time related constraints. The first category includes constraints limiting the deviation between required nurses on a shift, skill, day and nurses actually assigned to

this shift, skill, day. In contrast, time related constraints restrict the assignments within nurses' timetables. The class of time related constraints includes on one hand contractual constraints covering all the permanent rules such as minimum/maximum working time and minimum/maximum consecutive assignments to shifts, days, etc. Contractual constraints are typically grouped into full time nurses, weekend workers and many different part time contracts (30%, 75%, 90%, or never on Wednesday afternoon). It should be clear that the possible contract definitions are endless. On the other hand, personal requests for a day on/off or for another short leave also belong to this class. Some particular time related constraints are noteworthy in a fairness context. Balancing working hours or weekend work among full time nurses are examples of fairness related constraints.

In the present model, a solution is feasible if only one assignment can start for each employee on each day. Also, respecting skill types is considered as a hard constraint, i.e. no assignments are allowed for which a nurse is not qualified. Furthermore, overlapping assignments on two consecutive days are not allowed. Finally, assignments are only considered feasible if they are defined in the coverage constraints. This is motivated by the fact that in practice there are assignments which will never occur, such as a head nurse will never be assigned to a night shift. No such coverage constraints will be defined, and by imposing this constraint as a hard constraint, such assignments will never be made in a solution.

Coverage constraints are considered soft constraints. This allows for assigning nurses to a particular shift, skill, day, even when the maximum has been reached. In practice, this is sometimes necessary in order to meet the nurses' required working time. Another soft constraint is the minimum required rest time between two consecutive shifts, which is typically set to 11 hours.

Multi-skilled nurses can be defined in the present model. Typically, this is used to model primary and secondary skills by assigning weights to the different skill types. Assignments in which a nurse uses a secondary skill are feasible, but will incur a penalty for doing so. The aforementioned time related constraints can be further categorised into three types of soft constraints: counters, series and successive series. Counters are used to limit the occurrence of a specific subject in a particular period. Examples of counter constraints are no night shifts in a weekend or maximum 36 hours worked each week. Series restrict the successive occurrence of a specific subject in the scheduling period, e.g. minimum 3 and maximum 5 consecutive night shifts or no isolated idle days. Finally, successive series are used to restrict the occurrence of two consecutive series. Successive series are used to model constraints such as no early shift after a late shift or a series of night shifts have to be followed by at least 2 free days.

### 3.1 New models considering fairness

Previously proposed approaches often define a weighted sum objective function  $WO$  to evaluate the quality of a roster. It is defined as follows:

$$WO = \sum_{\forall n \in N} \sum_{\forall c \in C} \#violations_{n,c} \times weight_c \quad (1)$$

Despite its simplicity,  $WO$  includes some weaknesses when considering fairness. The function does not allow for distinguishing solutions with the same objective but composed of unbalanced violations with respect to the individuals. Therefore, we suggest in this paper to consider a different objective function  $FO$ , which is equal to the maximum weighted sum of violations in an individual's roster. Again,  $FO$  should be minimised. Good values for  $FO$  are expected to correspond to rosters that are more fair than those obtained when optimising  $WO$ .

$$FO = max_{\forall n \in N} \sum_{\forall c \in C} \#violations_{n,c} \times weight_c \quad (2)$$

Alternatively, a model could also deal with  $N$  individual objectives  $IO$ . This would be the most natural way of modelling objectives for a decentralised approach in which each nurse tries to optimise his/her own schedule.

$$IO = \sum_{\forall c \in C} \#violations_{n,c} \times weight_c \quad (3)$$

Both  $WO$ ,  $FO$  and  $IO$  can be extended with individual nurses' appreciation for constraints to be satisfied.  $WIO$  and  $FIO$  express this by multiplying each violation with the particular nurse's personal weight settings. Obviously, these functions can only be considered fair in case of constrained weights. Without elaborating on the regulations required for individual weights, we present the two functions below.

$$WIO = \sum_{\forall n \in N} \sum_{\forall c \in C} \#violations_{n,c} \times weight_{n,c} \quad (4)$$

$$FIO = max_{\forall n \in N} \sum_{\forall c \in C} \#violations_{n,c} \times weight_{n,c} \quad (5)$$

$$IIO = \sum_{\forall c \in C} \#violations_{n,c} \times weight_{n,c} \quad (6)$$

## 4 Agent-based cooperative meta-heuristic search

Several meta-heuristics have been successfully used to solve the nurse rostering problem (Edmund et al., 2004). However, their performance varies greatly from one problem instance to another and a great deal of computational experiments is required for parameter tuning. Moreover, it is hard to know in advance which meta-heuristic would solve best any nurse rostering problem.

Recently, the interest in cooperative search to solve combinatorial optimisation problems has risen considerably due to its success to provide novel ways to

combine the strength of different meta-heuristics. Current research has shown that the parallel and cooperation of several meta-heuristics could improve the quality of the solutions that each of them would be able to find by itself working on a stand alone basis (Ouelhadj and Petrovic, 2010; Cancino et al., 2010). Moreover, parallel and cooperative approaches can provide more powerful and robust problem solving environments in a variety of problem domains. The main motivation of cooperative search is to enhance the robustness of the search by the use of different combinations of meta-heuristics and parameter settings, leading to high quality solutions for different problem instances.

Blum and Roli (2003); Clearwater et al. (1992); Hogg and Williams (1993); Toulouse et al. (1999); Crainic and Toulouse (2008) described cooperative search as a search performed by agents that exchange information about states, models, entire sub-problems, solutions or other search space characteristics. Cooperative search has been successfully used to solve a number of difficult combinatorial optimisation problems, such as multi-commodity location with balancing requirements (Crainic et al., 1995, 1997), capacitated network design (Crainic and Gendreau, 2002), vehicle routing problem (Bouthillier and Crainic, 2005), quadratic assignment (James et al., 2009), labour constraint scheduling (Cavalcante et al., 2001), permutation flow shop scheduling (Ouelhadj and Petrovic, 2010; Vallada and Ruiz, 2009). These studies have shown that the combination of several different meta-heuristics with different parameter settings increases the robustness of the global search relative to variations in problem instance characteristics.

To the best of our knowledge, cooperative meta-heuristic search has never been used to solve the nurse rostering problem. The following section describes an agent-based framework for cooperative meta-heuristic search that has been used to solve fairness in nurse rostering.

#### 4.1 The agent-based framework for cooperative meta-heuristic search

We propose an agent-based system for cooperative meta-heuristic search composed of a population of autonomous meta-heuristic agents. The meta-heuristic agents run in parallel and can execute different meta-heuristic and heuristic combinations with different parameter settings. The meta-heuristic agents cooperate asynchronously to exchange information on the search space using pattern matching and reinforcement learning (Figure 1). The agent based system is composed of a launcher agent and meta-heuristic agents (Martin et al., 2012).

- **Launcher agent:** It reads problem instances from a configuration file, configures the meta-heuristic agents, and gathers the solutions from the meta-heuristic agents for a given problem instance.
- **Meta-heuristic agent:** A meta-heuristic agent implements a given meta-heuristic or combinations of local search heuristics (Figure 1). The meta-heuristic agents perform a local search on complete solutions to improve their local solutions using different meta-heuristic and heuristic combinations

with different parameter settings. The meta-heuristic agents cooperate and communicate asynchronously in order to combine the strength of several independent meta-heuristics, and to improve the quality of the solutions that each of them would be able to find by itself working on a stand alone basis.

## 4.2 Ontologies

Since the inert-agent cooperation involves the exchange of conversations, it is important to define ontologies to provide a commonly agreed vocabulary which will be shared across the agents. Ontologies are defined as a set of general representational primitives to model some domain Gruber (1993) and as such are semantic.

The ontology currently used by the framework generalises the notions of: *schedule*, *pattern*, and *individual elements*. In the ontology these are called *SolutionData*, *HeuristicData* and *NodeData* objects respectively.

- **NodeData:** It represents the individual elements of a schedule. For example, a NodeData object defines a quadruplet of (nurse, day, shift type, skill type).
- **HeuristicData:** A HeuristicData object contains two NodeData objects. It stores information about the pair. These include the distance between the two nodes and also the frequency score.
- **SolutionData:** A SolutionData represents the current schedule and its objective function value. SolutionData objects also contain lists of HeuristicData and NodeData objects.

## 4.3 Asynchronous cooperation by pattern matching and reinforcement learning for nurse rostering

The meta-heuristic agents perform local search on complete solutions using the assigned meta-heuristics. Meta-heuristic search is undertaken using a novel cooperation mechanism where agents exchange good patterns of good solutions. These are shared amongst the meta-heuristic agents which then build new solutions based on these good patterns. The cooperation is achieved through the exchange of a conversations between the meta-heuristic agents.

A nurse rostering solution is composed of a set of assignments that are defined as quadruples of (nurse, day, shift type, skill type).

The cooperation is achieved through the exchange of conversations between the meta-heuristic agents. A conversation involves an initiator meta-heuristic agent and responders meta-heuristic agents. At the start of the search any meta-heuristic agent can be the initiator. However, the selection of the current initiator is determined in the previous conversation, which will be described below.

The asynchronous cooperative search is undertaken as follows:

- A meta-heuristic agent taking on the role of initiator starts a conversation. It takes a new schedule either generated from a previous conversation or supplied by the launcher agent. The new schedule is then improved by the initiator meta-heuristic agent. When an improved solution is generated, it is sent to the other meta-heuristic agents.
- The meta-heuristic agents have also generated their best-so-far schedules using their meta-heuristics. They break up the solutions sent from the initiator and their own into pairs. The pairs are then compared and only those that are the common to both schedules are kept. HeuristicData objects are created from these pairs storing the first and second elements of the pair. These are then sent by the meta-heuristic agents to the initiator. The meta-heuristic agents also send the value of their best solution found so far. These solutions will be used by the initiator to determine which meta-heuristic agent will be the new initiator in the next conversation.
- Upon receiving the HeuristicData objects from the meta-heuristic agents, the initiator pools them. Each HeuristicData object is scored by counting how frequently it occurs in the pool. The initiator then tries to build a linked list from these high scoring HeuristicData objects.

For example, if the pool contains the following HeuristicData objects with first and second elements expressed here as pairs (4,7) (6,1) (7,2) (2,6) (5,9) (3,8), the linked list generated from the HeuristicData objects will have the following order (4,7) (7,2) (2,6) (6,1). Any HeuristicData objects not linked in this way are stored in an unlinked list (5,9) (3,8).

- The initiator then determines which meta-heuristic agent is going to be the initiator in the next conversation. This is done by pooling all the values the best solutions found so far of the meta-heuristic agents and then identifying which meta-heuristic agent has the best objective function value. The meta-heuristic agent with the best objective value will be the new initiator in the next conversation. The initiator then sends these lists of linked and unlinked of HeuristicData objects to the meta-heuristic agents. In the same message it also indicates which meta-heuristic agent will be the new initiator in the next conversation.
- The meta-heuristic agents receive the list of HeuristicData objects. Both initiator and meta-heuristic agents then create a new solution using both the linked and the unlinked lists, as well as their current best solution. The new solution is created by trying to build first a list of numbers from the linked HeuristicData objects. The unlinked HeuristicData objects are used next to supply more numbers. Finally the meta-heuristic agent's best-so-far solution provides any missing numbers. In this way a new unique schedule is generated and the objective function value is calculated.
- The conversations are repeatedly exchanged between the meta-heuristic agents for a maximum number of conversations set in the configuration file of the launcher agent.

## 5 Experiments

### 5.1 Experimental setup

Experiments have been conducted to compare the fairness of the different objective functions using both centralised and agent-based cooperative meta-heuristic approaches. The experiments to conduct are presented in Table 1.

The experiments have been carried out using four different scenarios. These scenarios are based on existing wards in a Belgian hospital: emergency, geriatrics, psychiatry and reception (Bilgin, 2008). Table 2 gives an overview of the instance characteristics. For each ward, two cases are considered: one where all the nurses have the same contract, and one where each nurse has an individual contract with both common and personalised constraints. The amount of constraints defined in each contract greatly differs between the instances. For example, nurses in the geriatrics ward are only subject to two constraints (limiting working hours and the number of consecutive days worked). In the psychiatry ward on the other hand, a large number of constraints are specified in each contract, restricting working time as well as specific patterns. In the cases where for each nurse an individual contract is defined, again the number of specified constraints differs greatly. Some of these constraints will be personalised, but there still exist a number of common constraints which apply to all nurses in the ward. For both cases, nurses follow one contract during the scheduling period, i.e. they do not change contract types during this period.

The agent-based framework for cooperative search is implemented on the open source FIPA compliant development platform JADE. The meta-heuristic agents implemented in the framework are the following:

- **Tabu search agents:** The tabu search agents implement the basic tabu search. In the basic tabu search, the search starts from a feasible solution and iteratively moves from the current solution to its best neighbouring solution using moves even if that move worsens the objective function value. To avoid cycling, moves which would give the same solution as the recently examined one are forbidden or declared tabu for a certain number of iterations, set to seven in the tabu list. In addition, an aspiration criterion is defined to accept tabu moves that with an objective function value better than the best available so far.
- **Simulated annealing agents:** The simulated annealing agents implement the basic simulated annealing with geometric and logarithmic cooling schedule. The search starts from a feasible solution and iteratively moves from the current solution to its best neighbouring solution. Improving solutions are accepted, while non improving solutions are accepted with a probability  $\exp \frac{\Delta}{t}$  where  $\Delta$  is the change in the objective function value, and  $t$  is the temperature which controls the acceptance probability. The temperature gradually decreases using a geometric and logarithmic cooling schedule.
- **Greet heuristic**

The meta-heuristic agents use the following neighbourhoods:

- Insert shift
- Delete shift
- Swap shifts...

Objective function	Centralised optimisation	Decentralised optimisation
<i>WO</i>	x	x
<i>FO</i>	x	x
<i>IO</i>	-	x
<i>WIO</i>	x	x
<i>FIO</i>	x	x
<i>IIO</i>	-	x

Table 1: Experimental setup

For each nurse  $n \in N$ , the quality of his/her individual roster is given by  $q_n$ . Several metrics based on  $q_n$  are used to compare the different objective functions in terms of fairness. First, the average roster quality  $\mu$  of all nurses is calculated as well as the standard deviation  $\sigma$ . Based on these two statistical properties, the relative standard deviation *RSD* is calculated (Equation 7). For an indication about the difference between the best and worst individual roster, *diff* is used (Equation 8). This metric shows the quality of the worst roster compared to the best roster. Finally, the overall quality of the solution is measured by two metrics:  $q_{solution}^{WO}$  and  $q_{solution}^{FO}$ . The former corresponds to the sum of all coverage constraint violations and the quality of the nurses' roster measured by *WO*. Similarly,  $q_{solution}^{FO}$  is calculated as the sum of all coverage violations and the value of *FO*.

$$RSD = 100 \times \frac{\sigma}{\mu} \tag{7}$$

$$diff = 100 \times \frac{\max_{\forall n \in N}(q_n) - \min_{\forall n \in N}(q_n)}{\max_{\forall n \in N}(q_n)} \tag{8}$$

Instance	Nr of nurses	Nr of shifts	Nr of skills	Planning period
Emergency	27	27	4	28 days
Geriatrics	21	9	2	28 days
Psychiatry	19	14	3	31 days
Reception	19	19	4	42 days

Table 2: Instance characteristics

The experiments have been carried out on an Intel Core 2 Duo at 3.16GHz with 4GB RAM operating on Windows 7. Each run was repeated 10 times, with computation time limited to 10 minutes.

## 5.2 Experimental results of the centralised approach

Table 3 shows the results of the centralised approach for the different instances. A low *RSD* indicates individual rosters being closer to each other in terms of quality, resulting in an overall fairer solution. As can be seen in Table 3, using objective *FO* results in lower *RSD* values for all instances and all cases. This means that by optimising *FO*, fairness will be better guaranteed than with *WO*. The same conclusion is supported by the values of *diff*. As with *RSD*, a lower value of *diff* indicates a better final solution in terms of fairness. More specifically, a low *diff* means that the difference between the worst roster and the best roster is small. Optimising *FO* results in lower *diff* values for all the instances and all the cases, thus again resulting in a fairer distribution of duties among the nurses.

Instance	<i>RSD</i>		<i>diff</i>	
	WO	FO	WO	FO
Emergency-i	19,24%	7,40%	54,00%	24,11%
Emergency-d	19,10%	8,35%	62,02%	30,91%
Geriatrics-i	32,23%	16,07%	65,08%	45,93%
Geriatrics-d	52,53%	27,67%	79,37%	66,69%
Psychiatry-i	15,97%	10,75%	47,18%	42,56%
Psychiatry-d	35,44%	29,15%	66,91%	65,20%
Reception-i	55,20%	35,94%	79,18%	59,42%
Reception-d	60,40%	38,70%	95,41%	89,24%

Table 3: Results for the centralised approach. Instance-i refers to cases with identical contracts, instance-d refers to cases with different contracts.

Figures 2 and 3 show some statistical properties of solutions obtained with the different objective functions. In these figures, box plots are shown representing  $q_n, \forall n \in N$ . Fair solutions are characterised by small boxes and whiskers close to the hinges, as this indicates that all individual roster qualities lie close to each other. In general, the plots support the conclusion that objective *FO* generates fairer solutions. Furthermore, the plots in Figures 2 and 3 show that, in general, the average individual roster quality does not differ when using *WO* or *FO*. The average individual roster quality will thus not be worse even when a fairer distribution of individual high quality rosters is achieved and that is a very important finding.

An alternative representation of the difference between *WO* and *FO* is provided in Figure 4. These line charts show, for each nurse, the average individual roster quality and the standard deviation over ten repeated runs for the emergency ward with different contracts. These charts support the aforementioned conclusions that *FO* achieves a fairer distribution of rosters than *WO*. This is visualised in Figure 4a by the flattened out curve shaped by rosters obtained with *FO*, compared to the inconsistent, spiked curve due to *WO*. In Figure 4b this is shown by the *FO* curve being below the *WO* curve, meaning that the standard deviation is smaller when using *FO*.

When evaluating the overall solution quality, two metrics are examined. First,  $q_{solution}^{WO}$  obtained with both *WO* and *FO* are compared. These values represent the actual roster quality of the solutions obtained. Second,  $q_{solution}^{FO}$  obtained with both *WO* and *FO* are compared. This way, the effect of the worst quality individual roster becomes much more important. As can be seen in the results given in Table 4, there exists no consistent difference between  $q_{solution}^{WO}$  obtained with *WO* or *FO*. However, for all instances, the value of  $q_{solution}^{FO}$ , obtained with *FO* is smaller than the value obtained with *WO*. This again indicates that by using *WO*, a much less fair distribution of rosters is present in final solutions than with *FO*.

These findings are favorable for the use of *FO*, since this means that not only is the average individual roster quality maintained (as was concluded earlier in the text), but that also the overall solution quality is not affected in a consistent way when using *FO* in stead of the typically used *WO*. This again is an important finding and advocates the use of *FO* when a fairer distribution of individual roster quality is wanted.

Instance	$q_{solution}^{WO}$			$q_{solution}^{FO}$		
	WO	FO	gap	WO	FO	gap
Emergency-i	192568,5	169096,43	-12,19%	312266,5	194232,9	-37,80%
Emergency-d	215296,7	167200	-22,34%	368762,5	192361,7	-47,84%
Geriatrics-i	35930	46985,714	30,77%	84428,57	64662,86	-23,41%
Geriatrics-d	53717,86	69732,5	29,81%	114888,6	98065	-14,64%
Psychiatry-i	147960	145491,43	-1,67%	210237,5	165270	-21,39%
Psychiatry-d	129340	125936,25	-2,63%	217982,5	181532,5	-16,72%
Reception-i	80909,38	103832,14	28,33%	227838,8	170684,3	-25,09%
Reception-d	57749,17	61478	6,46%	137450,8	101305	-26,30%

Table 4: Results for the centralised approach. Instance-i refers to cases with identical contracts, instance-d refers to cases with different contracts.

## 6 Conclusion

The present paper introduced a set of new fairness measures and included them in objective functions of the nurse rostering problem. Experiments with an existing approach and existing datasets revealed surprisingly interesting facts. By explicitly modelling fairness into the objective function, the resulting rosters prove to be more fair without deteriorating the quality with respect to other constraints. This conclusion should be adopted in future nurse rostering work because the modelling effort is limited and the complexity of the problem is not affected. The authors believe that this finding can help closing the gap between theory and practice in nurse rostering.

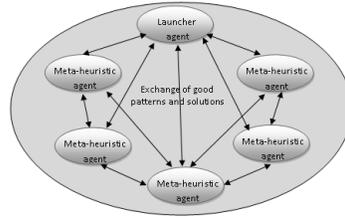


Fig. 1: Agent-based framework for cooperative meta-heuristic search

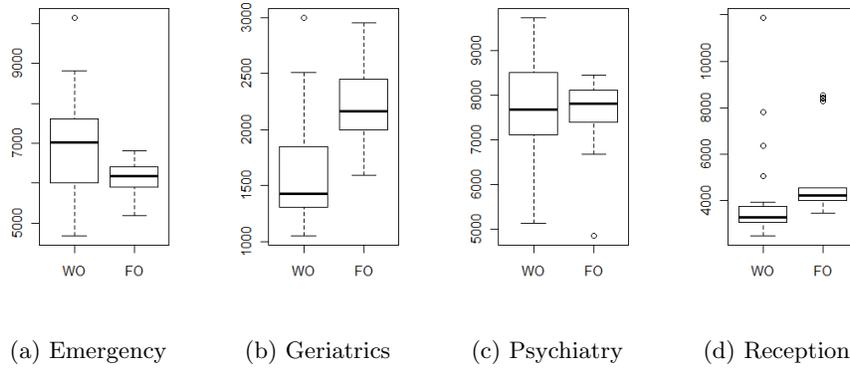


Fig. 2: Distribution of fairness in case of identical contracts

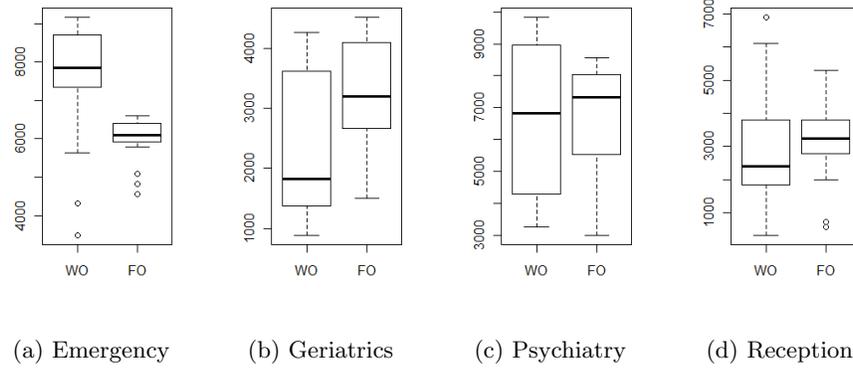
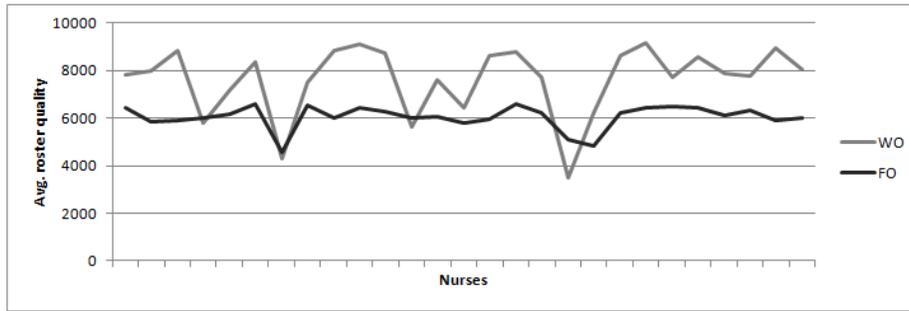
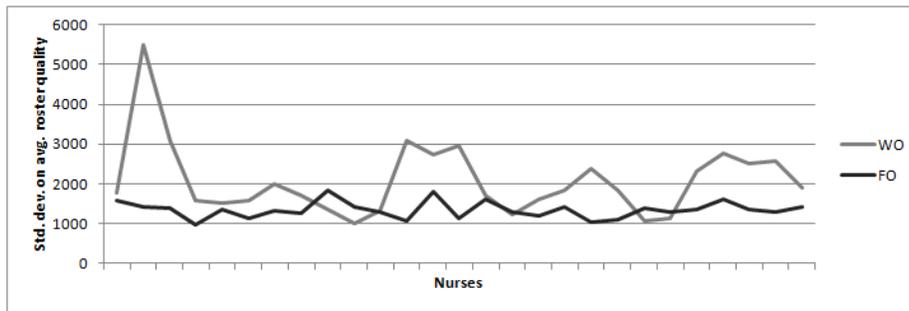


Fig. 3: Distribution of fairness in case of different contracts



(a) Average roster quality for each nurse.



(b) Standard deviation on roster quality for each nurse

Fig. 4: Comparison of roster quality for each nurse between *WO* and *FO* for the emergency ward where all nurses have different contracts.

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