

# String-inspired cosmology

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**Abstract.** I discuss cosmological models either derived from, or inspired by, string theory or M-theory. In particular I discuss solutions in the low-energy effective theory and the role of the dilaton, moduli and antisymmetric form fields in the dimensionally reduced effective action. The pre big bang model is an attempt to use cosmological solutions to make observational predictions. I then discuss the effective theory of gravity found in recent brane-world models where we live on a 3-brane embedded in a five-dimensional spacetime and how the study of cosmological perturbations may enable us to test these ideas.‡

## 1. Motivation

It is not hard to think of questions in cosmology that go beyond the framework of classical general relativity. In an expanding universe as one traces back our past worldline to ever higher energies and densities one reaches an initial ‘big bang’ singularity where the classical theory breaks down. The hope is that a finite quantum theory of gravity exists which remains well-defined. If string theory is the leading candidate for that fundamental quantum theory of gravity then it should provide some insight on the peculiar initial conditions required by the standard big bang. If inflation is to provide a dynamical explanation of those initial conditions [16], then string theory should provide the answer to the question of what the inflaton field is that drives it. As a higher dimensional theory, string theory should also be able to answer why we only observe three large spatial dimensions, and one of time. Even at low energies there are puzzles such as the unresolved embarrassment of the small, but apparently finite, cosmological constant (or dark energy).

But if cosmology needs string theory, then string theory itself needs cosmology. For a start, it has an embarrassment of riches in that string theory (or M-theory) has a large number of possible low-energy vacuum states. To answer the question of why we live in the particular physical state we observe requires a dynamical model of how we got here. The number of large extra dimensions is just one example of the many moduli that parameterise different vacuum states that have to be fixed. A more prosaic reason why I believe string theory needs cosmology is that string theory is a grand theoretical edifice that needs some experimental support. It may be that cosmology is the only arena in which string theorists can make predictions that are observable and could promote it to the status of an empirical science.

The subject of string cosmology is far from being a mature field of research where one could expect some consensus about what might be considered a standard model.

‡ Article based on a talk presented at “The Early Universe and Cosmological Observations: a Critical Review”, Cape Town, July 2001

There are many different approaches reflecting the many different perspectives on both string theory and cosmology. This article inevitably reflects my own personal interests and prejudices.

## 2. What is string theory?

At its most fundamental level, string theory is a theory for calculating scattering amplitudes between asymptotic states where the fundamental objects are one-dimensional strings. The spectrum of states includes a massless spin-2 graviton and as a result string theory has been said to ‘predict gravity’ - though only after apples had already been observed falling to the ground. Nonetheless gravity is an essential component for a would-be theory of cosmology. But a model of asymptotic states seems rather unpromising for a framework in which to study cosmology where we, as observers, are embedded within the model and asymptotic states may be of limited relevance.

In practice, most calculations are done in the low-energy limit where string theory can be described by an effective field theory. In the mid-1980s it was shown that low-energy effective actions can be derived for fields that are consistent low-energy background solutions [1, 2, 3]. All the low-energy effective actions of string theories include a gravitational part in the action [4]:

$$S = \frac{1}{\ell_s^8} \int d^{10}x \sqrt{-g} e^{-\phi} [R + (\nabla\phi)^2 \dots], \quad (1)$$

where  $\ell_s$  is the fundamental string length scale,  $R$  is the Ricci curvature scalar, and  $\phi$  the dilaton. The low-energy effective theory of gravity is familiar to relativists as a Jordan-Brans-Dicke type theory of gravity where the Brans-Dicke parameter takes the value  $\omega = -1$ . If the dilaton were left free then this would be experimentally unacceptable as a theory of gravity so the dilaton is one of the massless fields in the low-energy effective action that must become fixed by the present.

By the end of the 1980’s string theorists had more string theories than they knew what to do with: five low-energy effective 10-dimensional supergravity theories, to which has now been added 11-dimensional supergravity. However the string theory revolution of the 1990s showed that by including higher-dimensional extended objects (two-dimensional membranes or, more generally, *branes*) these six-theories can be related by duality symmetries and are now seen to be part of some more fundamental *M-theory*. These dualities include T-duality which relates solutions on a large toroidal geometry to those on a small one, and S-duality which relates strong and weak coupling limits. While there may be no understanding of what M-theory is (in the way that string theory was understood as a theory of strings) it too can be studied through its different low-energy effective descriptions. In 11-D supergravity there is no dilaton, but there are of course additional geometrical moduli, and a Kaluza-Klein dimensional reduction will still give rise to a dilaton-type field describing the size of the compact dimension.

Branes are not part of the perturbative formulation of string theory and duality symmetries have played a key role in mapping out non-perturbative structure of M-theory. One would certainly hope that they could provide a valuable insight into non-perturbative cosmology where the low-energy descriptions breakdown.

For a more substantive review of the state of the subject see, for example, the review by Duff [5].

### 3. Dilaton-gravity

The basic building block for many string-inspired cosmologies is the rolling radii solution of Mueller[6] for the 10-dimensional dilaton-gravity action (1) in a spatially-flat and homogeneous spacetime

$$ds^2 = -dt^2 + \sum_{i=1}^{10} a_i^2(t) dx_i^2$$

where

$$a_i \propto t^{p_i} \quad \phi \propto K \ln t \quad (2)$$

and the indices obey the generalised Kasner constraints

$$\sum_i p_i = 1 + K, \quad \sum_i p_i^2 = 1.$$

I refer to these as dilaton-vacuum solutions in the sense that only the gravitational fields (in which I include the dilaton) are excited and no matter fields are included. These solutions are in general singular as, for example, the Ricci curvature diverges for  $K \neq 0$  [4]. Higher-order curvature corrections are expected to become important when the curvature approaches the string scale,  $R \sim \ell^{-2}$ , and loop corrections as the dilaton approaches the strong coupling regime  $g_s^2 = e^\phi \sim 1$ .

The original 10-D dilaton-gravity action (1) can be reduced down to 4-D if the fields (and metric) are assumed to be independent of the six internal dimensions. Compactifying on an internal space of volume  $\propto e^{6\beta}$  would yield the four-dimensional action

$$S_4 = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} e^{-\varphi} [R + (\nabla^2 \varphi) - 6(\nabla\beta)^2 \dots]$$

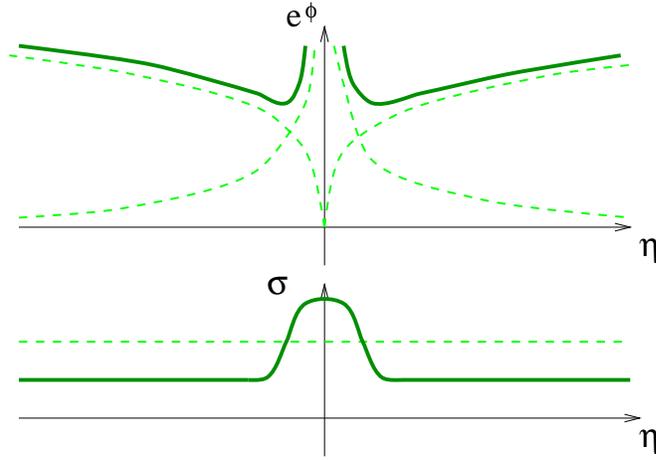
where my ellipsis represent all the moduli associated with the internal geometry other than the token modulus  $\beta$  that I have arbitrarily kept. The symmetries of the internal space determines the physics of what can be re-interpreted as matter fields in a four-dimensional world.

The 4D effective dilaton is given by

$$\varphi = \phi - 6\beta$$

and the effective gravitational coupling  $\kappa^2$  is proportional to  $\ell_s^8$  divided by the volume of the internal space. Thus a hierarchy may be introduced between the higher-dimensional string scale and that apparent in the 4D theory if the internal space is much larger than the fundamental string scale [28].

Neglecting all dependence of the fields and the geometry upon the internal space is of course a huge simplification, but it can be done in a self-consistent manner. Arbitrary perturbations about this background which allow inhomogeneity on the internal space can be decomposed in eigenvalues of the Laplacian on the internal space which then can again be treated as effective fields in the 4D world. If the compactification scale is of order the string scale, then these are very massive excitations that are not part of the low-energy effective theory. In theories where the scale of compactification is much larger than the string scale it may be necessary to include inhomogeneous bulk configurations at intermediate energy scales.



**Figure 1.** Evolution of the dilaton and axion fields as functions of conformal time  $\eta$  in Eq. (3) for a spatially flat Friedmann-Robertson-Walker spacetime.

#### 4. Form fields

Much of my own work on string-inspired cosmology has investigated the dynamical role of antisymmetric form fields that appear alongside the dilaton and moduli in the bosonic sector of the low-energy superstring effective actions. These have often been ignored in previous discussions because their effective density is typically localised in space or time and may be negligible in the asymptotic regime. Nonetheless they can play a crucial role interpolating between dilaton-vacuum solutions and dynamically determining the allowed asymptotic states [7, 8].

As an example consider a spatially flat FRW metric,

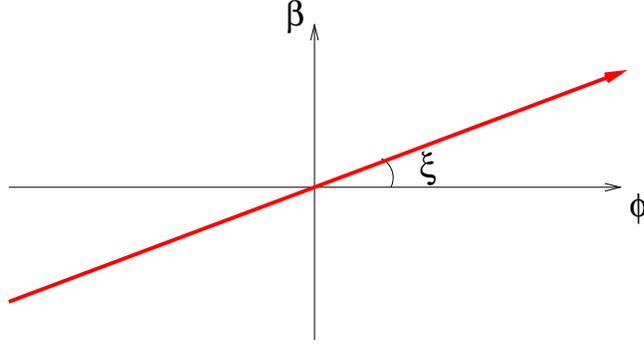
$$ds^2 = a^2(\eta) [-d\eta^2 + dx^2 + dy^2 + dz^2]$$

The model-independent axion in four-dimensions is a pseudo-scalar  $\sigma$  Poincare-dual to the anti-symmetric three-form field strength  $H = *\nabla\sigma$ . The general axion-dilaton solutions (neglecting now for simplicity the modulus field,  $\dot{\beta} = 0$ ) can be given in simple closed form:

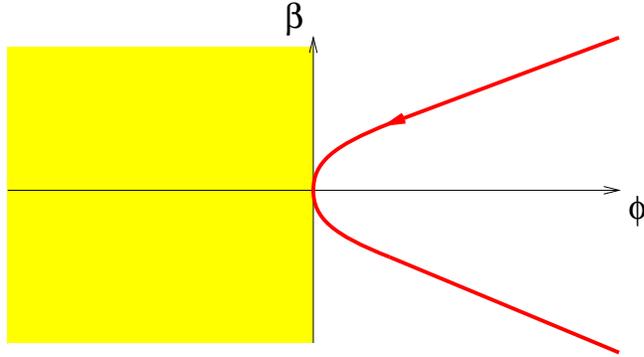
$$\begin{aligned} e^\varphi &= \frac{e^{\varphi_*}}{2} \left[ \left( \frac{\eta}{\eta_*} \right)^{-\sqrt{3}} + \left( \frac{\eta}{\eta_*} \right)^{\sqrt{3}} \right] \\ a &= \frac{a_*}{\sqrt{2}} \left[ \left( \frac{\eta}{\eta_*} \right)^{1-\sqrt{3}} + \left( \frac{\eta}{\eta_*} \right)^{1+\sqrt{3}} \right]^{1/2} \\ \sigma &= \sigma_* \pm e^{-\phi_*} \left[ \frac{(\eta/\eta_*)^{-\sqrt{3}} - (\eta/\eta_*)^{\sqrt{3}}}{(\eta/\eta_*)^{-\sqrt{3}} + (\eta/\eta_*)^{\sqrt{3}}} \right] \end{aligned} \quad (3)$$

These solutions provide a nice example of how the symmetries of the string effective actions (in this case an  $SL(2, \mathbb{R})$  symmetry which is related to the modular invariance of the string compactified on a two-torus [10, 4]) can be used to generate the general axion-dilaton solutions starting from the dilaton-vacuum solutions [9].

The axion remains almost constant except when  $\eta \sim \eta_*$  where its value undergoes a step up or down. The asymptotic solution returns to the dilaton-vacuum solution,



**Figure 2.** Evolution of the dilaton and modulus fields in Mueller's rolling radii solutions.



**Figure 3.** Evolution of the dilaton and modulus fields in axion-dilaton-moduli solutions.

but the presence of the axion nonetheless selects the allowed asymptotic solutions, in particular placing a lower bound on the dilaton field. Note that because the dilaton and axion are massless fields the Einstein frame scale factor is simply  $\tilde{a} = \tilde{a}_* (\tilde{t}/\tilde{t}_*)^{1/3}$ , where  $\tilde{t}$  is the proper cosmic time in the Einstein conformal frame [4].

Reintroducing the modulus  $\beta$ , we see that the dilaton-moduli-vacuum solutions (2) correspond to free particle moving in a straight line in  $(\phi, \beta)$  field space (see figure 2):

$$\begin{aligned}\varphi &= \varphi_* + \sqrt{3}(\cos \xi) \ln(\eta/\eta_*) \\ \beta &= \beta_* + \sqrt{3}(\sin \xi) \ln(\eta/\eta_*).\end{aligned}$$

The axion-dilaton-moduli solutions (figure 3) correspond to a particle moving in  $(\varphi, \beta)$  field space which bounces off a potential  $V(\varphi) \propto e^{-2\varphi}$ :

$$\begin{aligned}\varphi &= \varphi_* - \ln 2 + \ln \left( (\eta/\eta_*)^{\sqrt{3}(\cos \xi)} + (\eta/\eta_*)^{-\sqrt{3}(\cos \xi)} \right) \\ \beta &= \beta_* + \sqrt{3}(\sin \xi) \ln(\eta/\eta_*)\end{aligned}$$

Recently [11] it has been shown that this can lead to a BKL-like chaos approaching the singularity in homogeneous superstring cosmologies. In higher dimensional cosmologies the antisymmetric field strength(s) of the superstring effective actions

have many more degrees of freedom each of which can act as a wall in the dilaton-moduli phase-space. Damour and Henneaux [11] have shown that in a generic solution, i.e., where all the axion-type degrees of freedom are non-vanishing then the solutions are trapped in a compact region of phase-space, bouncing off the walls infinitely many times in the approach to the singularity. Of course, just as in the analogous case of Bianchi IX cosmologies in general relativity, the low-energy effective description itself breaks down before one reaches the singularity, but this behaviour must be an important consideration for any discussion of the nature, or avoidance, of cosmological singularities in string theory.

## 5. The pre big bang

The most concerted campaign to address the practical implications for cosmology of string theory has been the work of Gasperini and Veneziano and their collaborators [12] in developing the *pre big bang* scenario.

There are three basic ingredients/assumptions:

- (i) The universe begins in the asymptotic past in a low-energy, weakly-coupled, vacuum state well-described by a low-energy effective supergravity theory.
- (ii) Small fluctuations about this vacuum state lead to gravitational instability [13] and run-away evolution (collapse in the conformally related Einstein frame [14]) towards a high-curvature, strongly-coupled regime.
- (iii) This run-away instability must be tamed by stringy effects and transformed into a conventional expanding cosmology, possibly by T-duality which could map a high-curvature regime to a low-curvature one.

The pre big bang is an honest attempt to use the tools available to us from string theory to make cosmological predictions. It has incurred the wrath of those who seek a dynamical explanation of the peculiar spatial homogeneity and flatness of the standard hot big bang model (such as that offered by inflation [16]) because the initial state must be at least close to the homogeneity and flatness sought [15]. But if a model such as the pre big bang could be shown to be in better agreement with observational data than, say, inflation, then we would have to learn to live with the initial conditions that required. A more pragmatic objection is that the well-understood low-energy effective solutions still contain a singularity at the end of the pre big bang phase and this can only be avoided by introducing much less-well understood loop- or string-corrections as the strong-coupling or high-curvature regime is reached [17].

The pre big bang does offer possible observational tests. The pre big bang phase has a causal structure similar to that of conventional inflation and can lead to the generation of effectively classical fluctuations on super-Hubble scales from an initial vacuum state. In the pre big bang the spectrum of metric perturbations is sharply peaked towards the smallest scales (those leaving the horizon close to the singularity). This could give a stochastic background of gravitational waves with a steep blue spectrum [18] (quite different to the almost scale-invariant spectra from inflation). But it would leave effectively no density perturbations on large scales at the start of the hot big bang [19].

Axion-type fields would have to play a crucial role if the pre big bang (rather than inflation) is to provide a mechanism to seed the large-scale structure in the universe. This is because the kinetic coupling of the axion field can produce an almost scale-invariant spectrum of fluctuations [9]. However because the background

axion density vanishes asymptotically, these are *isocurvature* fluctuations. Durrer and collaborators have estimated the power spectrum of the cosmic microwave background anisotropies that might result from this novel isocurvature source if the axion remained decoupled [20]. While another distinctive prediction, the latest observations of the cosmic microwave background anisotropies may be hard to reconcile with this model [21].

Instead one may be forced to consider a pre big bang model that mimics the *adiabatic* primordial perturbation spectra produced by inflation [16]. It is not widely recognised that in the presence of non-adiabatic perturbations, the large-scale curvature perturbation produced, say, during a pre big bang phase provides only a lower limit on the large-scale curvature perturbation at later times. Isocurvature modes provide an additional source for curvature at later times on arbitrarily large scales if they are coupled to the dominant energy density, as has recently been explicitly demonstrated in the case of two-field inflation [22]. It is quite possible that isocurvature axion fluctuations could be converted into adiabatic curvature perturbations either at, or after, the transition to the post-big bang evolution.§

## 6. Brane-worlds

All the string-inspired cosmological solutions I have discussed thus far are based upon a traditional Kaluza-Klein treatment of the extra-dimensions where the role of the higher-dimensional geometry can be reduced to a collection of effective fields in a more or less conventional four-dimensional theory. In the last couple of years a new approach to dimensional reduction has attracted much attention where matter fields may be constrained to a lower-dimensional surface. This may be valid as a low-energy description of some string theory solutions including branes, where the end-points of strings describe gauge fields on the lower-dimensional surface [25], or the Horava-Witten model where gauge fields live on orbifold fixed planes [26, 27].

If only gravitational fields are able to explore the higher-dimensional model then this raises the possibility that the extra dimensions may be much larger than previously assumed, as the four-dimensional nature of gravity is only tested on scales above about a millimetre [28]. As discussed earlier, the higher dimensional (fundamental) Planck scale may be much smaller than that which is apparent in four-dimensions if the volume of the extra dimensions is large compared with the fundamental Planck scale. This offers a novel geometrical perspective on the hierarchy between different energy scales (such as the disparity between the Planck and supersymmetry scales) found in particle physics, and intriguingly suggests that the regime of quantum gravity may be closer than we thought.

There have been many different approaches to studying the possible cosmological consequences of the brane-world proposal. I will follow my personal bias and restrict my discussion to just one arising from a paper by Randall and Sundrum in 1999 which allows us to study the role of the higher dimensional geometry at the expense of the possible effects of additional bulk fields such as the dilaton and form fields.

§ Since this talk was given two papers [23, 24] have given explicit demonstration of this effect due to late-decay of a massive ‘curvaton’ field. A distinctive observational signature in such a model would be a direct correlation between the curvature perturbation and any residual isocurvature perturbation as they would both be derived from the same curvaton field fluctuations.

### 6.1. Brane-world cosmology

It was noted by Binetruy, Deffayet and Langlois [29] that cosmological solutions based on a naive application of the conventional 4D form of Einstein's equations could be incorrect even at low energies in the brane-world scenario in 5D Einstein gravity. They showed that an observer on a 3-brane with surface density  $\sigma$  embedded in five-dimensional Minkowski spacetime sees a Hubble expansion rate  $H$  given by the very unconventional Friedmann expansion [29]

$$3H^2 = (\kappa_5^2 \sigma)^2 \quad (4)$$

At around the same time Randall and Sundrum [30] proposed a brane-world based upon a flat (Minkowski) 3-brane embedded in curved five-dimensional anti-de Sitter spacetime. This is a consistent solution in five-dimensional general relativity if the brane surface density  $\sigma$  has a critical value,  $\lambda_{RS}$  (see below), which exactly cancels the gravitational effect on the brane of the (negative) cosmological constant in the bulk. More remarkably Randall and Sundrum showed, by considering bulk metric perturbations, the existence of a zero-mode solution in the bulk which yields a massless 4D-graviton on the brane. The correct Newtonian limit can then be recovered on length-scales greater than either the size of the extra dimension, or the anti-de Sitter curvature scale, whichever was the smaller. Thus the four-dimensional Newtonian limit is recovered even when the extra dimension is non-compact. The magnitude of the apparent 4D Planck scale is again a derived quantity, in this case proportional to the brane tension. Thus in this model the brane tension must be positive (in contrast to another proposal of Randall and Sundrum's where the visible brane tension was taken to be negative [31].)

It should be emphasised that the Randall-Sundrum model falls into the class of string-inspired rather than string-derived solutions. It draws on the notion of localised matter fields on a brane, but then invokes Einstein gravity in a 5D vacuum. This might be contrasted, for instance, with the earlier work of Lukas, Ovrut and Waldram [27] who went to considerable lengths to derive a 5D model from Horava-Witten theory which inevitably includes scalar fields and form fields in the bulk. They found an analogous solution for Minkowski 3-branes embedded in a curved (but conformally flat) 5D spacetime. However the simplicity of Randall and Sundrum's model has attracted considerable attention.

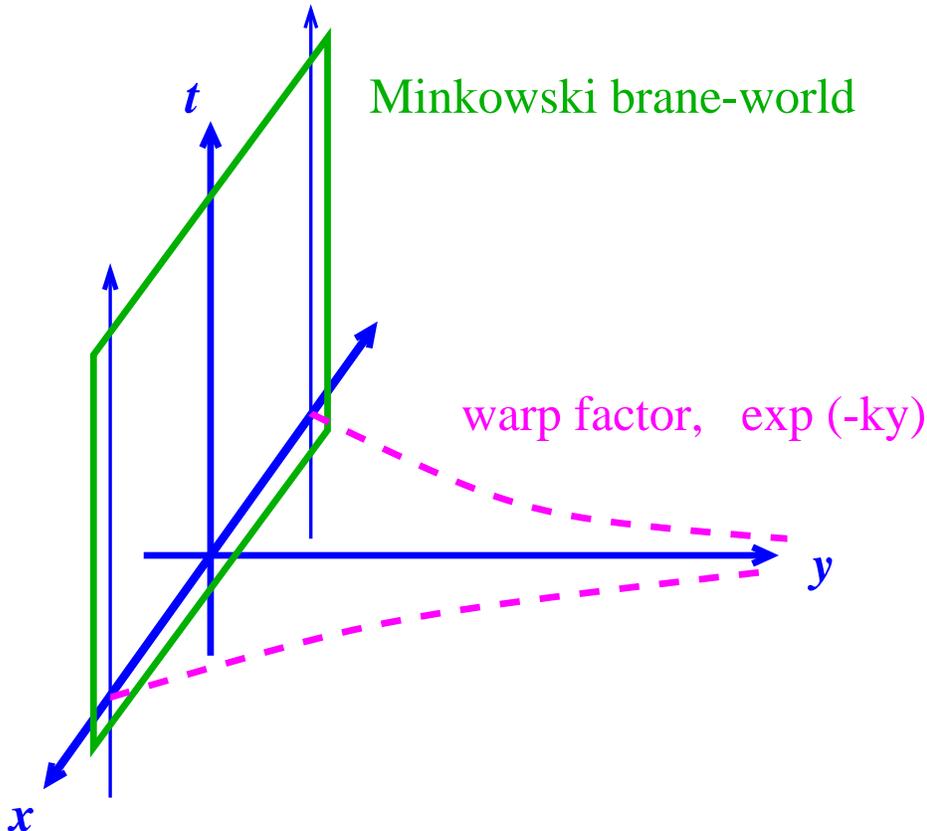
In many ways studying the Randall-Sundrum model of brane-world gravity is analogous to using Brans-Dicke gravity to model conventional string theory solutions. The full theory is expected to have many more light degrees of freedom in the perturbative limit whose couplings and interactions could play an important role, but it is useful to have a simplified but self-consistent model in which one can study the non-linear gravitational dynamics. It is in this spirit that I will discuss brane-world cosmologies based on the Randall-Sundrum model.

A negative bulk cosmological constant  $\Lambda_5 < 0$  plus a positive brane tension  $\lambda > 0$  is just what is required to recover a viable Friedmann equation at low energies ( $\rho \ll \lambda$ ) on the brane with total surface density  $\sigma = \lambda + \rho$  [32]. The modified Friedmann equation (4) becomes

$$3H^2 = \frac{1}{2}\Lambda_5 + \kappa_5^4 (\lambda + \rho)^2$$

For a suitably chosen value of brane tension  $\lambda_{RS}^2 = -\Lambda_5/2\kappa_5^4$  we have

$$3H^2 = \kappa_4^2 \rho + \kappa_5^4 \rho^2 \quad (5)$$



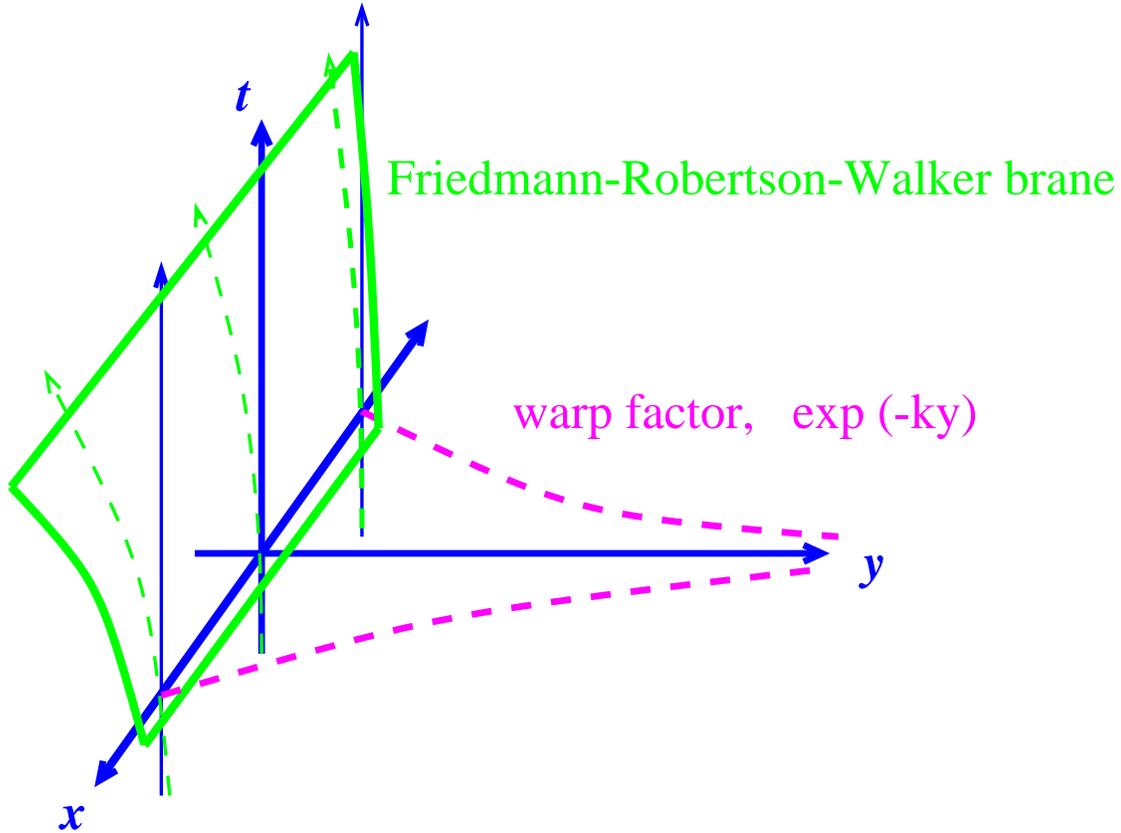
**Figure 4.** Embedding of Randall-Sundrum (Minkowski) brane in five-dimensional anti-de Sitter spacetime in Poincaré coordinates:  $ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$ .

where the four-dimensional Newton's constant is  $\kappa_4^2/8\pi = \kappa_5^4\lambda_{RS}/4\pi$  and we recover the standard Hubble expansion rate for  $\rho \ll \lambda_{RS}$ . Note that once again there may be a hierarchy between the four dimensional Planck scale and the fundamental higher-dimensional Planck scale if the brane tension  $\sigma$  is significantly below the Planck scale.

The brane-world thus offers a novel perspective on the cosmological constant problem [33, 34]. Instead of asking why the vacuum energy density of our world is so small, the question becomes one of why the brane surface density is so close to the critical Randall-Sundrum value.

The modified Friedmann equation admits a non-standard high-energy regime for  $\lambda_{RS} < \rho \ll M_5$  where one expects the equation (5) still to apply while matter fields remain confined to the brane. In particular it would lead to a modified evolution during inflation. The additional quadratic term in the Friedmann equation leads to additional Hubble damping which assists conventional slow-roll inflation driven by an inflaton which is one of the matter fields on the brane [35] and can even yield inflation for potentials that would otherwise be too steep [36, 37].

Randall-Sundrum cosmology offers not just a modified system of effective 4D dynamics but also a genuinely five-dimensional perspective on our cosmological



**Figure 5.** Embedding of FRW brane in five-dimensional anti-de Sitter spacetime using same Poincaré coordinates in the bulk as Fig. 4.

models. The maximal symmetry of the 3D spatial slices of an FRW brane imposes a strong constraint on the allowed 5D geometry. This enables one to find the explicit form of the bulk solution for any non-static<sup>||</sup> FRW brane-world [38] and this bulk solution turns out to be Schwarzschild-anti-de Sitter spacetime [39, 40]. After years of explaining to your mother (or your undergraduate students) that it is not necessary to imagine that our universe is expanding ‘into anything’, it is now perfectly acceptable for you to tell them that our universe could be expanding ‘into’ a five-dimensional Schwarzschild-anti-de Sitter bulk.

The Schwarzschild mass of the bulk spacetime turns up as an additional constant of integration in the most general 4D modified Friedmann equation, controlling the energy density of a dark radiation term [38]. <sup>¶</sup> This is an example of a five-dimensional effect that (unlike the quadratic density term) is not automatically suppressed at low

<sup>||</sup> The only exception I know of to this generalised Birkhoff theorem is the *static* FRW solution of Gergely and Maartens [41].

<sup>¶</sup> One of the many intriguing aspects of the brane-world scenario is the holographic principle which asserts that a bulk gravitational field can be described by a non-gravitational field theory on the boundary of spacetime. The most famous example is the entropy of a black hole which is proportional to the area of its event horizon rather than its volume. Similarly the gravitational field of the bulk black hole turns up as an effective radiation density on the brane [42].

energies. However if this dark radiation is not to disrupt the standard successful model of primordial nucleosynthesis its dynamical effect is constrained to always be small.

### 6.2. Covariant brane-world gravity

The modified Friedmann equation is an example of non-linear gravitational solutions that are possible in the Randall-Sundrum model. An elegant way to describe the effective gravity seen by an observer on the brane was presented by Shiromizu, Maeda and Sasaki [43] who derived the effective Einstein equations for an observer on a 3-brane embedded at a  $Z_2$ -symmetric fixed point in a five-dimensional Einstein gravity.

Gauss's equation relates the intrinsic curvature of the five-dimensional spacetime to the intrinsic curvature of the brane and the extrinsic curvature of its embedding,  $K_{\mu\nu}$ . For a brane located at a fixed point in a  $Z_2$ -symmetric bulk the extrinsic curvature is completely determined by the tension  $\lambda$  and energy-momentum tensor,  $T_{\mu\nu}$ , on the brane

$$K_{\mu\nu} = \kappa_5^2 \left( -T_{\mu\nu} + \frac{1}{3}(T - \lambda)g_{\mu\nu} \right) \quad (6)$$

The case of an asymmetric bulk is considered in Ref.[46]. The observer sees only the matter fields, with energy-momentum tensor  $T_{\mu\nu}$ , the induced metric on the brane,  $g_{\mu\nu}$ , and the projected five-dimensional Weyl tensor  $E_{\mu\nu}$  which appear in the equations for the 4D Einstein tensor on the brane [43]

$$G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = \kappa_4^2 T_{\mu\nu} + \kappa_5^4 \Pi_{\mu\nu} - E_{\mu\nu}, \quad (7)$$

where the effective cosmological constant on the brane is

$$\Lambda_4 = \frac{1}{2}\Lambda_5 + \frac{\kappa_5^4}{12}\lambda^2,$$

and the quadratic energy-momentum tensor

$$\Pi_{\mu\nu} \equiv -\frac{1}{4}T_{\mu\lambda}T_{\nu}^{\lambda} + \frac{1}{12}TT_{\mu\nu} + \frac{1}{8}g_{\mu\nu} \left( T_{\lambda\kappa}T^{\lambda\kappa} - \frac{1}{3}T^2 \right).$$

represents the effect of high-energy corrections to the local gravitational field, such as the  $\rho^2$  term in the modified Friedmann equation.

The effect of the non-local 5D gravitational field on the brane is described by the projected Weyl tensor which appears as an effective energy-momentum tensor

$$\tilde{T}_{\mu\nu} \equiv -\kappa_4^{-2}E_{\mu\nu} = \left( \frac{4}{3}\tilde{u}_{\mu}\tilde{u}_{\nu} + \frac{1}{3}g_{\mu\nu} \right) \tilde{\rho} + \tilde{\pi}_{\mu\nu},$$

with energy density  $\tilde{\rho}$ , momentum  $\tilde{\rho}\tilde{u}^{\mu}$  and anisotropic pressure  $\tilde{\pi}_{\mu\nu}$ . The evolution of the Weyl energy and momentum on the brane follows from energy-momentum conservation equations, driven by the Weyl pressure and coupled to the quadratic tensor  $\Pi_{\mu\nu}$  [48].

The isotropic Weyl-pressure is  $\tilde{\rho}/3$  due to the tracefree property of the Weyl tensor, but the anisotropic pressure is in general undetermined by the 4D equations and hence this system of 4D equations is not closed. Hence it may be of limited use in actually deriving brane-world solutions unless the anisotropic stress from the bulk is known or can be prescribed for some reason, as in the case of FRW cosmologies where all anisotropies are required to vanish simply by the symmetry of the spacetime. When other fields are included in the bulk (as for instance in the heterotic M-theory solutions of Lukas, Ovrut and Waldram [27]) there are further undetermined terms in

the projected 4D equations on the brane representing energy transfer to or from the bulk fields and this need not be vanishing even for FRW cosmologies on the brane [44]. Nonetheless the 4D effective equations are required for the correct 4D interpretation of the 5D solutions.

For example, it is often supposed that moving matter on the brane must ‘lose energy’ to the gravitational field in the bulk [45], whereas in five-dimensional vacuum Einstein gravity, the local energy-momentum conservation,  $\nabla^\mu T_{\mu\nu} = 0$ , is guaranteed by the Gauss-Codazzi equation [43]. The answer is that moving matter on the brane *can* generate bulk gravitational waves but that this is due to energy-momentum transfer between the quadratic tensor,  $\Pi_{\mu\nu}$ , and the effective Weyl-fluid. The covariant derivative of Eq. (7), plus the Bianchi identity  $\nabla^\mu G_{\mu\nu} = 0$ , yields

$$\nabla^\mu E_{\mu\nu} = \kappa_5^4 \nabla^\mu \Pi_{\mu\nu},$$

without affecting the local conservation of the conventional four-dimensional energy-momentum tensor  $T_{\mu\nu}$ .

### 6.3. Cosmological perturbations

Although quadratic terms in the modified Friedmann equation may be significant in the high-energy regime ( $\rho > \lambda_{RS}$ ) the effect at low-energies (say, after nucleosynthesis) becomes negligibly small. Any evidence of an unconventional past could show up in any primordial perturbations generated at early times.

If we wish to consider arbitrary linear perturbations about an FRW brane in a (Schwarzschild-)anti-de Sitter bulk, then the projected field equations do not form a closed system due to the possibility of anisotropic stresses. The standard approach to studying linear perturbations in a higher dimensional theory is to decompose arbitrary perturbations into harmonics on the spatial sections. If the wave equation is separable then one can follow the time-dependence of each spatial mode separately. This is always possible, for instance, for perturbations about an FRW background where 3D spatial eigenmodes are Fourier modes (in flat space) or (hyper-)spherical harmonics in curved space. However, for an FRW brane it is not in general possible to separate the time-dependence of each 3D mode from the bulk-dependence in a brane-based coordinate system. One is then left with a partial differential equation to solve for which it has proved difficult to come up with analytic results except in special cases.

If the bulk is 5D anti-de Sitter it is always possible to write down the general separable solution in a static coordinate system [55, 54]. This indeed was the approach used to study the Newtonian limit by Randall and Sundrum in their original paper. But although a Minkowski brane remains at fixed location, an FRW brane will be moving with respect to the static bulk coordinates and applying the correct boundary conditions at the brane couples together different bulk modes. So although the *general* solution is known, the *particular* solution that obeys the correct boundary conditions is not.

As a consequence there is no zero-mode solution for the bulk gravitational field in a simple FRW cosmology such as a radiation- or matter-dominated universe. Although this mode mixing will be suppressed on scales where the cosmological expansion/brane motion can be neglected, the lack of quantitative results means that the magnitude of any deviations from the standard evolution of cosmological perturbations remains uncertain. On small (sub-Hubble) scales we expect the Minkowski results to remain valid. And on large scales there is a solution where the metric perturbations remain

constant when all gradient terms can be neglected [53]. But around horizon-crossing, where the cosmological expansion cannot be neglected, there could be some mode-mixing [47].

There have been many attempts to extend to the brane-world either metric-based or covariant approaches to cosmological perturbation theory, reviewed in Ref. [48]. These have, however, largely been restricted to developing the required formalism and have stopped short of providing quantitative results.

The only cases in which the 5D perturbations are properly understood are those special cases in which the bulk equations are separable in a brane-based coordinate system. For 5D vacuum Einstein gravity this restricts the brane to only de Sitter (or anti-de Sitter or Minkowski) cosmologies in 5D anti-de Sitter. Although a mathematically special case, de Sitter provides an idealised model for conventional slow-roll inflation. Thus one can calculate the spectrum of large-scale metric perturbations generated from initial vacuum fluctuations that cross outside the cosmological horizon during the de Sitter phase [35, 50, 49]. What one finds is that large-scale perturbation spectra are *only* generated in the zero-mode, i.e., the massless 4D graviton, and the massive bulk modes are never excited. There is an enhancement to the amplitude of gravitational waves (tensor modes) generated at a given energy scale if that energy is greater than  $\lambda_{RS}$ . But if inflation is driven by a scalar field on the brane then the scalar curvature perturbations are also enhanced and enhanced more than the gravitational waves [35]. However, surprisingly, the consistency relation found between the ratio of tensor to scalar amplitudes and the tilt of the gravitational wave spectrum in conventional single-field inflation still holds [51], and the Randall-Sundrum gravity misses another opportunity to distinguish itself observationally from a purely four-dimensional theory [52].

## 7. Conclusions

String cosmology is far from being a mature subject in which there is any great consensus about how to tackle cosmological questions or even which cosmological questions should be tackled. In this review I have naturally focussed on those questions that interest me and the solutions I have sought. My own hope is that it is through the nature of primordial perturbation spectra being measured with ever-increasing precision by observational cosmologists that we may learn about high-energy phenomena in our Universe.

String theory requires us to contemplate cosmological solutions in a higher-dimensional spacetime. In traditional Kaluza-Klein compactifications this yields a lower-dimensional spacetime described by dilaton-gravity theory with additional low-energy matter fields arising from the ‘internal’ geometry of the extra dimensions. Higher-dimensional effects, corresponding to excitations of the higher harmonics on the internal space, are then suppressed at energy scales below that set by the compactification scale.

String theory is best understood in terms of effective field theories in the low-energy limit. The pre big bang scenario is an honest attempt to use the low energy effective actions to describe a cosmological prehistory of our Universe. To date it has failed to provide observational predictions that are in any better agreement with the cosmological observations than those of conventional inflation models. At the same time questions remain about the nature of the transition from pre- to post-big bang phase where a deeper understanding of string (or M-) theory may be required.

A different model for our place in the higher-dimensional spacetime has been provided by the brane-world scenario in which matter fields are restricted to lower-dimensional surfaces, or branes. In order to study the consequences of this novel geometrical setting I have focussed on the simple model proposed by Randall and Sundrum of a single brane embedded in 5D vacuum spacetime described by Einstein gravity. This is simple enough for the homogeneous and isotropic cosmological solutions to have been well-understood, but the understanding of cosmological perturbations is still largely restricted to de Sitter inflation and the large-scale limit. There is some hope that novel 5D effects may survive even at low energies due to the non-local gravitational field but these remain the hardest to calculate and their effect remains uncertain.

If string theory is to be vindicated experimentally then it seems likely that cosmology will provide the key. But the challenge remains to provide observational predictions that, without violating existing experimental bounds, go beyond those effects already known to the four-dimensional cosmologist.

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