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## Primordial perturbations from inflation

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### Abstract

I review the standard analysis of adiabatic scalar and tensor perturbations produced by slow-roll inflation driven by a single scalar field, before going on to discuss recent work on the role of non-adiabatic modes during and after inflation. Isocurvature perturbations correlated with adiabatic modes may be produced during multi-field inflation and would give valuable information about the physics of the early universe. A significant contribution from correlated isocurvature perturbations is not ruled out by present data, but should be either detected or ruled out by future observations.

### 1. Introduction

In recent years a standard model has emerged for the origin of the large-scale structure of our Universe [1]. Observations of the cosmic microwave background (CMB) reveal primordial anisotropies on the surface of last scattering of the CMB photons. Structure can form from these initial perturbations about a Friedmann-Robertson-Walker (FRW) universe by gravitational instability to form the galaxies, clusters of galaxies and superclusters observed in large-scale surveys. These observations are consistent with an almost scale-invariant initial spectrum of adiabatic density perturbations at last-scattering [2]. Inflation is a dynamical model of the early universe which can explain the origin of perturbations on arbitrarily large scales from small-scale vacuum fluctuations of light fields.

Observational data are increasingly being used to constrain the cosmological parameters of the background FRW model of the universe since last-scattering. But this can only be done in the context of some model for the nature of the primordial perturbations. Constraints on the form of the primordial perturbations yield constraints on the dynamics and high energy physics driving inflation in the very early universe.

## 2. Single-field inflation

The simplest models of inflation are driven by a scalar field,  $\phi$ , slowly rolling down its potential,  $V(\phi)$ , in a spatially flat FRW spacetime with scale factor  $a$ . The classical evolution is determined by the Klein-Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} = -V_\phi, \quad (1)$$

(where  $V_\phi$  denotes  $dV/d\phi$ ) coupled to the Friedmann equation for the Hubble expansion ( $H \equiv \dot{a}/a$ )

$$H^2 = \frac{8\pi}{3M_P^2} \left( V + \frac{1}{2}\dot{\phi}^2 \right). \quad (2)$$

This gives inflation (i.e., accelerated expansion,  $\ddot{a} > 0$ ) for  $V > \dot{\phi}^2$ .

The slow-roll approximation truncates these to a first-order system

$$3H\dot{\phi} \simeq -V_\phi \quad (3)$$

$$H^2 \simeq \frac{8\pi}{3M_P^2} V. \quad (4)$$

This assumes the evolution is potential-dominated ( $V \gg \dot{\phi}^2$ ) and over-damped ( $3H|\dot{\phi}| \gg |\ddot{\phi}|$ ) but can give a useful approximation to the growing mode solution when the dimensionless slow-roll parameters [1] are small

$$\epsilon \equiv \frac{M_P^2}{16\pi} \left( \frac{V_\phi}{V} \right)^2 \ll 1 \quad , \quad |\eta| \equiv \frac{M_P^2}{8\pi} \left| \frac{V_{\phi\phi}}{V} \right| \ll 1. \quad (5)$$

Linear perturbations of a massless field in an FRW background obey the wave equation

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \nabla^2\delta\phi = 0. \quad (6)$$

Arbitrary inhomogeneities can be decomposed into spatial harmonics, such as Fourier modes. Each mode with fixed comoving wavenumber  $k$  has two characteristic timescales

- oscillation period (determined by the physical wavelength)  $a/k$
- damping timescale (determined by the Hubble expansion)  $H^{-1}$

The evolution of each mode is thus naturally split into two regimes

- small scales ( $a/k < H^{-1}$ ) under-damped *oscillations*
- large scales ( $a/k > H^{-1}$ ) over-damped, or *frozen-in*

In a conventional (non-inflationary) matter- or radiation-dominated universe the comoving Hubble length  $H^{-1}/a = \dot{a}^{-1}$  increases with time so that modes are frozen-in ( $k < aH$ ) at early times and only come within the Hubble length at late times ( $k > aH$ ). But in an inflationary era the comoving Hubble length decreases and modes that begin as under-damped oscillators on small scales are stretched by the accelerated expansion beyond the Hubble length. Thus initial zero-point fluctuations of the quantum vacuum on small-scales (where the effects of the cosmological expansion is negligible) leads to a spectrum of overdamped perturbations on large-scales. Linear evolution ensures that the perturbations are described by a Gaussian random field at all times.

Perturbations of a massive or self-interacting field have an additional oscillation timescale set by the effective mass,  $m^2 \equiv V_{\phi\phi}$ . Massive fields ( $m^2 \geq 9H^2/4$ ) remain underdamped even on large-scales and effectively no perturbations are generated. But any light field ( $m^2 < 9H^2/4$ ) acquires a spectrum perturbations  $\langle \delta\phi^2 \rangle \simeq (H/2\pi)^2$  at Hubble-crossing. In particular, the inflaton must be light ( $|\eta| \ll 1$ ) during slow-roll inflation.

### 3. Cosmological perturbations

Arbitrary perturbations of an FRW cosmology can be decomposed into two types

- *adiabatic perturbations* perturb the solution along the same trajectory in phase-space as the background solution. Thus perturbations in any scalar  $x$  can be described by a unique perturbation in expansion with respect to the background

$$H\delta t = H \frac{\delta x}{\dot{x}} \quad \forall \quad x, \quad (7)$$

e.g., the adiabatic density perturbation  $\delta\rho/\rho \propto H\delta\rho/\dot{\rho}$ .

- *entropy perturbations* perturb the solution off the background solution

$$\frac{\delta x}{\dot{x}} \neq \frac{\delta y}{\dot{y}} \quad \text{for some } x \text{ and } y. \quad (8)$$

One example is an *isocurvature* perturbation of the baryon-photon ratio  $S = \delta(n_B/n_\gamma) = (\delta n_B/n_B) - (\delta n_\gamma/n_\gamma)$ .

Although the amplitude of adiabatic perturbations (such as the density perturbation) is notoriously gauge-dependent, the adiabaticity condition (7) is not. Moreover, from this definition it is clear that purely adiabatic perturbations (along the background trajectory) must remain adiabatic on large scales and cannot generate entropy perturbations (off that trajectory) [5, 3].

If the perturbed expansion,  $H\delta t$ , is evaluated in the spatially-flat gauge [4] then it coincides with the gauge-invariant scalar curvature perturbation,  $\zeta$ , on uniform-density hypersurfaces [5], first introduced by Bardeen, Steinhardt and Turner [6]. This is a particularly useful quantity as it remains constant for adiabatic perturbations in the large-scale limit.

In single-field inflation the over-damped perturbations of the inflaton field on large scales are adiabatic perturbations of the growing mode solution and entropy perturbations vanish in the large-scale limit [3]. Thus single-field inflation models predict a primordial adiabatic perturbation on large-scales whose amplitude can be calculated in terms of the scalar field perturbations at Hubble-crossing ( $k = aH$ ) during inflation. This in turn can be related to the inflaton potential  $V(\phi)$  in the slow-roll approximation [7]

$$A_S^2 = \left\langle \left( \frac{H\delta\phi}{\dot{\phi}} \right)^2 \right\rangle_{k=aH} \simeq \frac{32}{75} \frac{V}{\epsilon M_P^4} \quad (9)$$

The time-dependence of the inflaton field during inflation leads to a scale dependence of the resulting spectrum, which in the slow-roll approximation is determined by the slow-roll parameters

$$n_S - 1 \equiv \frac{d \ln A_S^2}{d \ln k} \simeq -6\epsilon + 2\eta. \quad (10)$$

In the extreme slow-roll limit this yields the scale-invariant Harrison-Zel'dovich spectrum,  $n_S = 1$ .

Gravitational waves, corresponding to tensor metric perturbations independent of the scalar perturbations to linear order [8], are another massless degree of freedom excited during inflation and frozen-in on large scales to give [7]

$$A_T^2 = \left\langle \left( \frac{H}{M_P} \right)^2 \right\rangle_{k=aH} \simeq \frac{32}{75} \frac{V}{M_P^4}, \quad (11)$$

with spectral tilt

$$n_T \equiv \frac{d \ln A_T^2}{d \ln k} \simeq -2\epsilon. \quad (12)$$

$A_S^2$  and  $A_T^2$  describe the contribution of scalar and tensor perturbations to the the CMB anisotropies on large angular scales. A prediction of single-field slow-roll models of inflation is that there should be a consistency condition relating the scalar-tensor ratio to the tensor tilt [7]:

$$\frac{A_T^2}{A_S^2} \simeq -\frac{1}{2} n_T. \quad (13)$$

Unfortunately there is no guarantee that the tensor contribution is large enough to ever be detectable (let alone its tilt measurable). Currently favoured hybrid-type inflation models generally occur at low energies with  $\epsilon \ll 1$  and  $A_T^2 \ll A_S^2$  [30].

## 4. Non-adiabatic effects

### 4.1. Single field

In single-field models the Hubble damping generally causes the decaying mode solution to the Klein-Gordon equation to rapidly decay, suppressing any non-adiabatic perturbations on large scales. Nonetheless, even in single-field inflation it is possible to significantly alter the curvature perturbation on finite, but super-Hubble scales. Non-adiabatic (decaying mode) and/or gradient terms may have an effect if  $z \equiv a\dot{\phi}/H$  (which is a monotonic increasing function of time in the slow-roll approximation) decreases back below its value at Hubble-crossing [9]. This is possible in some models of inflation where the slope of the inflaton potential decreases abruptly [10] and the inflaton field enters a transient friction-dominated ‘fast-roll’ regime [11] described by  $\ddot{\phi} \simeq -3H\dot{\phi}$ , leading to  $z \propto a^{-2}$  for a finite period.

Another case in which the instantaneous value of the scalar curvature perturbation calculated at horizon-crossing may not be a good estimate of the final value on large scales is when the inflaton stops [12],  $\dot{\phi} = 0$ . In this case the apparent divergence of  $\zeta = H\delta\phi/\dot{\phi}$  is transient if at the same time  $V_\phi \neq 0$  and, ironically, the slow-roll approximation  $\zeta \simeq V^{3/2}/V'$  can give a better estimate of the final value, giving as it does an estimate of the perturbation in the growing-mode solution [9].

### 4.2. Two fields

If more than one light scalar field exists during inflation then there is more than one allowed phase-space trajectory for FRW cosmologies. A spectrum of entropy perturbations (off the background trajectory) will be generated on large scales from initial vacuum fluctuations on small scales.

In the case of two canonical light fields  $\varphi_1$  and  $\varphi_2$  the background trajectory is described by  $\dot{\varphi}_1$  and  $\dot{\varphi}_2$  and arbitrary field perturbations can be decomposed along and orthogonal to the background trajectory [3]:

$$\delta\sigma \equiv \cos\theta\delta\varphi_1 + \sin\theta\delta\varphi_2, \quad \delta s \equiv -\sin\theta\delta\varphi_1 + \cos\theta\delta\varphi_2 \quad (14)$$

where the angle of the trajectory in field-space is given by  $\tan\theta = \dot{\varphi}_2/\dot{\varphi}_1$ . At any instant, the adiabatic perturbation  $\delta\sigma$  determines the scalar curvature perturbation  $R = H\delta\sigma/\dot{\sigma}$ , while the entropy perturbation determines the isocurvature perturbation  $S \propto \delta s$  at that time.

The coupled evolution equations for  $\delta\sigma$  (in arbitrary gauge) and  $\delta s$  (which is automatically gauge invariant) were derived in Ref. [3] (for a different treatment

of the same equations see [13]). In the slow-roll limit, and on large scales, the equations can be reduced to [3, 14]

$$\begin{aligned} 3H\dot{\delta\sigma} + (V_{\sigma\sigma} - \dot{\theta}^2)\delta\sigma &= 2(\dot{\theta}\delta s) - 2\left(\frac{V_\sigma}{\dot{\sigma}} + \frac{\dot{H}}{H}\right)\dot{\theta}\delta s, \\ 3H\dot{\delta s} + (V_{ss} + 3\dot{\theta}^2)\delta s &= 0 \end{aligned} \quad (15)$$

For  $\dot{\theta} = 0$  the equations are decoupled and reduce to the standard slow-roll equations for canonical field perturbations. But for a curved trajectory the entropy perturbation  $\delta s$  appears as an extra driving term for  $\delta\sigma$ . As a result the curvature perturbation is no longer constant on large scales. This additional contribution to the spectrum of scalar curvature perturbations at the end of inflation weakens the consistency condition (13) of single-field inflation to an inequality [15, 16, 17]

$$\frac{A_T^2}{A_S^2} \leq -22n_T. \quad (16)$$

Langlois [18] was the first to point out that another consequence of this coupling is that any residual isocurvature perturbation after multi-field inflation will in general be correlated with the curvature perturbation. The adiabatic and entropy field perturbations at horizon crossing are, by their construction (14), independent random fields

$$\langle\delta\sigma^2\rangle_{k=aH} = \langle\delta s^2\rangle_{k=aH} \simeq (H/2\pi)^2, \quad \langle\delta\sigma\delta s\rangle_{k=aH} = 0. \quad (17)$$

The subsequent evolution on large scales can be parameterised by a transfer matrix, which has the general form [19]

$$\begin{pmatrix} R \\ S \end{pmatrix} = \begin{pmatrix} 1 & T_{RS} \\ 0 & T_{SS} \end{pmatrix} \begin{pmatrix} R \\ S \end{pmatrix}_{k=aH} \quad (18)$$

The form of this matrix is determined by the general definition of adiabatic and entropy perturbations [5] introduced in section 3. and the evolution of two field perturbations during inflation in Eq. (15) provides just one example.

The two coefficients  $T_{RS}$  and  $T_{SS}$  are model-dependent transfer functions which determine the final power spectra

$$\langle R^2 \rangle \propto 1 + T_{RS}^2, \quad \langle S^2 \rangle \propto T_{SS}^2, \quad \langle RS \rangle \propto T_{RS}. \quad (19)$$

For example, if all the particle species present after inflation are in thermal equilibrium determined by a single temperature then there is a unique phase-space trajectory for the FRW cosmologies and only adiabatic perturbations are possible,  $T_{SS} = 0$ . The other extreme is that one species remains completely decoupled after Hubble-crossing corresponding to  $T_{RS} = 0$ , leaving uncorrelated isocurvature perturbations. But the general case is that non-zero isocurvature perturbations survive and are correlated with the curvature perturbations.

## 5. Curvaton model for the origin of structure

Taking into account the effect upon the large-scale curvature perturbations of entropy perturbations possible in multi-field or multi-fluid cosmological models leads to the realisation that the curvature perturbation calculated at Hubble-crossing during inflation only gives a *lower bound* for the curvature perturbation at the start of the epoch of structure formation. In a recent paper with David Lyth [20], I addressed the question of whether it is possible to have a viable model of structure formation if there is effectively no curvature perturbation produced on large scales during inflation. The answer is yes!

In our model the *curvaton* is supposed to be a light field during inflation ( $m_s^2 \equiv V_{ss} \ll H^2$ ) which does not affect the dynamics during inflation, i.e., is not the inflaton, and thus its perturbations correspond to isocurvature perturbations. If this field is decoupled from other inflation or matter fields then the long-wavelength perturbations are effectively frozen-in until the Hubble rate  $H$  in the expanding universe drops to below the mass  $m_s$ . At this point the curvaton field begins to oscillate and its energy density redshifts as  $\rho_s \propto a^{-3}$ . This grows relative to the radiation density  $\rho_\gamma \propto a^{-4}$  and will come to dominate the energy density of the universe unless the field decays. This is the well-known Polonyi (or moduli) problem associated with massive weakly-coupled fields. But a late-decaying field, so long as it decays before nucleosynthesis, may not be a bad thing. Late entropy release can dilute the abundance of other dangerous relics and can be used as a model for baryogenesis or leptogenesis. Crucially, its perturbations can also produce a large-scale curvature perturbation [23, 20, 21].

As an example consider a complex scalar field  $\phi = |\Sigma|e^{is/v}$  whose modulus is fixed,  $|\Sigma| \sim v$ , by a mexican-hat potential with large effective mass  $m_{|\Sigma|} \sim v$ , but whose U(1) symmetry is only broken by non-renormalisable terms so that  $s$  has a small mass  $m_s \sim v^2/M_P$ . The radial vev is stabilised, but the pseudo-Goldstone boson  $s$  will acquire an almost scale-invariant spectrum of isocurvature fluctuations during a period of inflation, with Hubble rate  $H$ , if we have

$$\frac{v^2}{M_P} \ll H \ll v. \quad (20)$$

Assuming that after inflation  $s$  decays only with gravitational strength,  $\Gamma \sim m_s^3/M_P^2$  then we naturally have  $\rho_s \sim \rho_\gamma$  at the decay time and hence [20]

$$\langle \zeta^2 \rangle \sim \langle (\delta s/s)^2 \rangle \sim (H/v)^2. \quad (21)$$

If the decay products thermalise completely then the isocurvature perturbation during inflation,  $\delta s$ , is converted into an adiabatic curvature perturbation,

$\zeta$ . It is then indistinguishable from conventional inflaton models for structure formation, other than violating the consistency condition (13), but respecting the inequality (16).

An interesting alternative possibility is that, because the curvaton decay can occur relatively late, some particle species have already dropped out of equilibrium when the curvaton decays. These species would then have an isocurvature perturbation relative to the radiation produced by the curvaton decay, but one that would be 100% correlated with the curvature perturbation. A recent example of a late-decaying field that could produce correlated curvature and isocurvature perturbations is the sneutrino field in leptogenesis models [22].

A similar model for the origin of large-scale structure from initially isocurvature axion perturbations has been recently proposed by Enqvist and Sloth [23] in the context of the pre big bang scenario [24]. In this case the rapid increase in the Hubble rate during the pre big bang phase must be compensated by the rapidly growing dilaton coupling to yield a scale-invariant spectrum of axion perturbations [25]. This appears to be the only possible origin of cosmological structure in pre big bang type models where essentially no curvature perturbation is produced on large scales during the ‘inflationary’ phase [24].

## 6. Observational data

The form of the primordial perturbation spectra is increasingly being constrained by astronomical observations, especially cosmic microwave background experiments. The overall amplitude of the anisotropies on large scales is still set by the COBE data [26] which gives an amplitude  $A^2 = 1.9 \times 10^{-5} \pm 10\%$ . A recent compilation comparing data against FRW models with adiabatic scalar and tensor perturbations by Wang, Tegmark and Zaldarriaga [2] gives a constraint on the spectral index  $0.80 < n_S < 1.03$ , and an upper limit on the contribution from gravitational waves,  $A_T^2/A_S^2 < 0.008$ , with the spectral index  $n_T$  being unbounded.

Most studies of non-adiabatic perturbation spectra to date have considered only uncorrelated isocurvature perturbations which tend to only give an additional source of anisotropies on large angular scales and hence their contribution to the CMB anisotropies is severely constrained [27]. However I have emphasized that it is natural in inflation models to consider isocurvature perturbations *correlated* with the standard adiabatic mode [28, 29, 19]. In particular the cross-correlation can decrease the CMB anisotropies on large angular scales and a significant contribution from isocurvature modes cannot be ruled out from current data. Indeed considering correlated CDM-isocurvature modes (with scale-invariant correlation angle  $\Delta$ ) the best-fit to the current CMB data [19] has  $n_S = 0.8$ ,  $\cos \Delta = 1$  and



a similar contribution to large-angle anisotropies from adiabatic and isocurvature modes. Future CMB data from the MAP satellite would certainly be able to distinguish between such a model and a purely adiabatic spectrum.

## 7. Conclusions

It is quite possible that the primordial perturbations observed by future CMB experiments will remain consistent with the scale-invariant Gaussian spectrum of adiabatic density perturbations proposed more than thirty years ago by Harrison and Zel'dovich. In this case we would be able to extract little about the physics of inflation and would only be able to place bounds on allowed deviations from the extreme slow-roll limit.

If we are to learn more about the dynamical history of the early universe we need to find deviations from scale-invariance or traces of either tensor perturbations or non-adiabatic effects which, though not evident in current observations, could be detected by future experiments.

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