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# The physics of thermal light second-order interference beyond coherence

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**Abstract.** A novel thermal light interferometer was recently introduced in Ref. [1]. Here, two classically correlated beams, obtained by beam splitting a thermal light beam, propagate through two unbalanced Mach-Zehnder interferometers. Remarkably, second-order interference between the long and the short paths in the two interferometers was predicted independently of how far, in principle, the length difference between the long and short paths is beyond the coherence length of the source. This phenomenon seems to contradict our common understanding of second-order coherence. We provide here a simple description of the physics underlying this effect in terms of two-photon interference.

## 1. Introduction and motivation

The discover of the Hanbury Brown and Twiss (HBT) effect [2, 3] in 1956 triggered an intense debate about the physics behind this effect and even its correctness. To quote Hanbury Brown, the reason behind a lot of this criticism was that “many physicists had failed to grasp how profoundly mysterious light really is”. However, as often happens in science for a new phenomenon which is initially not well understood and received, the HBT effect paved the way to the development of an entire new field, the field of quantum optics [4, 5, 6, 7, 8]. One can consider this discovery as a precursor of fundamental studies and experiments based on multi-photon interference and correlations [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 1, 19], with intriguing applications in imaging [20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30], quantum information processing [31, 32, 33, 34, 35, 36], and metrology [37, 38, 39, 40, 41, 42, 43, 44].

In the temporal domain [2], the HBT effect consists of beam splitting a thermal beam and measuring the correlation in time between the photon intensities at the beam splitter output ports (Fig. 1(a)). Two-photon interference is observed for a detection time delay small compared to the coherence time of the source. Indeed, two-fold detection events have twice the chance to occur at equal detection times than with a

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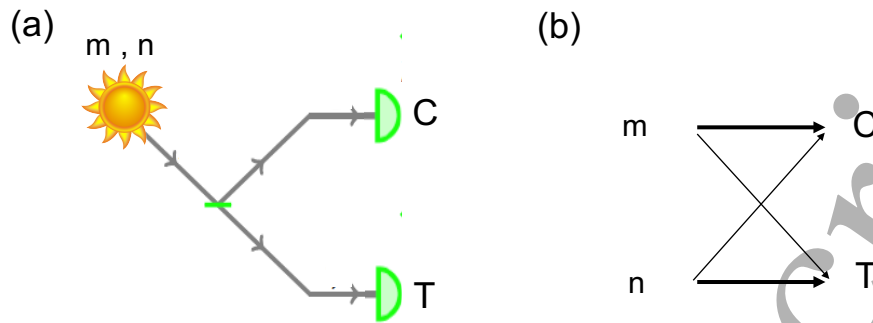


Figure 1: (a) Hanbury Brown and Twiss (HBT) interferometer: The classically correlated beams obtained by beam splitting the light emitted by a thermal source are measured by performing joint detections at the two output ports of the balanced beam splitter. (b) Two-photon interference is observed at detection time delays small compared with the coherence time of the source. This corresponds to the interference of two two-photon detection amplitudes corresponding to the two possible indistinguishable paths (depicted with bold and thin arrows, respectively) two photons  $m$  and  $n$  can undertake to trigger a joint detection.

time delay much larger than the coherence time. This effect arises from the interference between the two possible indistinguishable pairs of paths that two detected photons may have undertaken from the source to the two detectors (Fig. 1(b)). We emphasize that, since these two pairs of path completely overlap in space, the HBT effect is not sensitive to any relative phase delay. Indeed, no sinusoidal fringes can be observed in standard HBT experiments.

Recently, was proposed a novel interferometric scheme, depicted in Fig. 2(a), where, differently from the standard HBT scheme, two Mach-Zehnder interferometers are introduced at the two output ports of the beam splitter before a correlation measurement at approximately equal detection times is performed [1]. Furthermore, the difference between the long (dashed red) paths  $L = L_C, L_T$  and the short (dotted blue) paths  $S = S_C, S_T$  in each Mach-Zehnder interferometer is assumed to be larger than the coherence length of the source, while the differences in length between paths of the same type,  $L_C - L_T$  and  $S_C - S_T$ , are negligible with respect to the coherence length.

This interferometer has some similarities with the famous Franson interferometer [13], which however differs in two fundamental aspects:

- a source of two classically correlated beams obtained by beam splitting a thermal light beam is replaced in the Franson interferometer by a source of two photons entangled in energy and time;

- the time delay between the long and short paths in each Mach-Zehnder interferometer is required to be within the second-order coherence time of the two-photon source, which is usually given by the coherence time of the pump laser which generates the entangled pair of photons when interacting with an SPDC crystal.

These two conditions ensure the emission of two entangled photons simultaneously

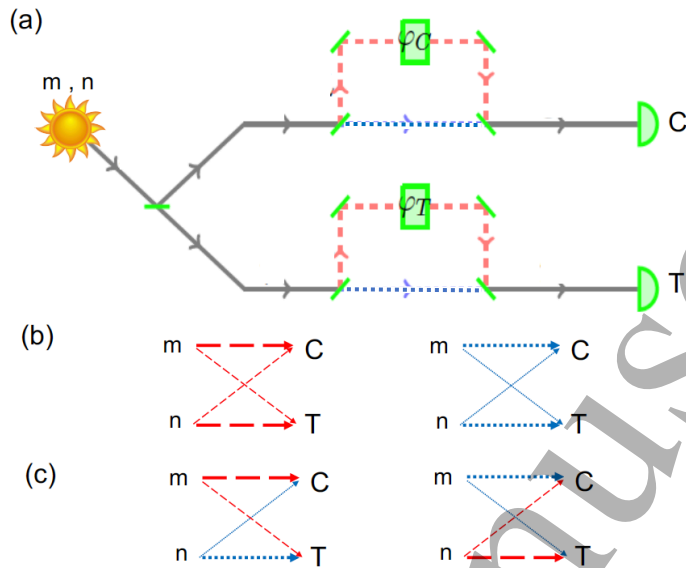


Figure 2: (a) Thermal light interferometer introduced in Ref. [1]: A source of classically correlated beams as in the HBT interferometer in Fig. 1(a) is used. However, correlated measurements are not performed directly at the two output ports of the beam splitter but after the light propagates through two unbalanced Mach-Zehnder interferometers with relative phases  $\varphi_C$  and  $\varphi_T$ , respectively. At detection time delays small compared with the coherence time of the source, second-order interference is observed between the pair of long (dashed red) paths  $L = L_C, L_T$  and the pair of short (dotted blue) paths  $S = S_C, S_T$ , which length difference  $L - S$  is beyond the coherence length of the source. This interference is a result of the collective contribution of all the possible ways two-photon interference can arise between two pairs of two-photon paths (indicated by bold and thin arrows, respectively) undertaken by any given pair  $(m, n)$  of detected photons emitted by the thermal source: (b) two-photon interference between the  $(L_C, L_T)$  and  $(L_T, L_C)$  (or  $(S_C, S_T)$  and  $(S_T, S_C)$ ) pairs of paths, which is phase-independent as in standard HBT experiments; (c) two-photon interference between the  $(L_C, S_T)$  and  $(L_T, S_C)$  (or  $(S_C, L_T)$  and  $(S_T, L_C)$ ) pairs of paths, which is instead sensitive to the phase difference  $\varphi_C - \varphi_T$ .

but with a “quantum uncertainty” in time (second-order coherence time) larger than the time delay  $(L - S)/c$  due to the unbalance between the long and short paths in each Mach-Zehnder interferometer, even if the usual first order coherence length of each single photon is much smaller than the path length difference  $L - S$ . This is at the very heart of the observation of the well known two-photon Franson interference between the two pairs  $(L_C, L_T)$  and  $(S_C, S_T)$  of two-photon paths.

Can second order interference between the path configurations  $(L_C, L_T)$  and  $(S_C, S_T)$  be also observed in the thermal light interferometer in Fig. 2(a) where the path length difference  $|L - S|/c$  in each Mach-Zehnder interferometer is beyond the coherence

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4 length of the thermal source? Does it matter here how far beyond? Interestingly, as  
5 shown in Ref. [1], the answer is *yes* and it does *not* matter, in principle, how far beyond.  
6 This result is highly counter-intuitive since, differently from the Franson interferometer,  
7 no energy-time entanglement occurs to justify any two-path indistinguishability in terms  
8 of second-order coherence length. How does such two-path indistinguishability arise?  
9 How can we reconcile it with our common understanding of second-order coherence?  
10  
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12 Even more interestingly, it was predicted that this second-order interference  
13 phenomenon with thermal light, differently from the standard HBT effect, could  
14 manifest itself in sinusoidal fringes as a function of the *difference*  $\varphi_C - \varphi_T$  between  
15 the phase delays in the two Mach-Zehnder interferometer [1]. This differs from the  
16 Franson interferometer which is sensitive to the *sum*  $\varphi_C + \varphi_T$  of the interferometer  
17 relative phases.  
18  
19

20 But what is the physics behind the observation of these sinusoidal fringes? And  
21 why such a fundamental difference with respect to the Franson interferometer?  
22

23 All these questions have puzzled different scientists who were reluctant in believing  
24 that this interference effect was even correct when it was first predicted and no  
25 experimental realizations were yet performed. Fortunately, two main features were of  
26 substantial help in experimentally verifying this phenomenon: 1) a thermal source can  
27 be easily simulated in a laboratory by using laser light impinging on a fast rotating  
28 ground glass [45]; 2) given a coherence time for a thermal source which can range from  
29 the order of *ns* to  $\mu s$ , the calibration of the interferometric paths and of the detection  
30 times can be easily achieved. Furthermore, the same effect can be realised also in the  
31 spatial domain, by substituting the two Mach-Zehnder interferometers with two double  
32 slits and performing spatially correlated measurements at the output, as proposed in  
33 Ref. [40]. Indeed, it did not take too long until experimental demonstrations of this  
34 effect were first reported both in the temporal and spatial domain in Refs. [19, 41],  
35 respectively. Furthermore, as predicted in Ref. [1], interference fringes as a function  
36 of the phase difference  $\varphi_C - \varphi_T$  have been observed with almost constant visibility at  
37 the increasing of the path length difference  $L - S$  even if of the order of  $680m$  beyond  
38 the coherence length of the source [19]. This phenomenon therefore paves the way to  
39 applications in high precision metrology and sensing, including the characterization of  
40 remote objects as demonstrated first theoretically in Ref. [40] and then experimentally  
41 in Ref. [41].  
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49 In addition, the genuineness of the *correlations* between the long and the short paths  
50 in the observed second order interference is testified by the possibility to exploit them to  
51 simulate quantum logic gates in absence of entanglement [1]. Indeed, the simulation of a  
52 CNOT gate by using this interference effect, first theoretically proposed in the temporal  
53 domain in Ref. [1] and extended to the spatial domain in Ref. [40], was experimentally  
54 realised in Ref. [46]. This can lead to potential applications in the development of novel  
55 optical algorithms with thermal light [47, 48, 49, 50, 51].  
56  
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58 In this paper, we provide a deeper understanding of the physics behind this  
59 interference phenomenon in terms of interference of “two-photon detection amplitudes”  
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[9, 10]. We will demonstrate how, differently from standard HBT experiments, interference can actually occur in a phase-dependent manner between two-photon detection amplitudes corresponding to the propagation of a pair of detected photons  $(m, n)$  from the source to the detectors  $d = C, T$  through two possible pairs  $(L_C, S_T)$  and  $(L_T, S_C)$  of two different types of paths  $L$  and  $S$  (Fig. 2(c)). Further, standard HBT interfering amplitudes also arise from the propagation through pairs of the same type of path (either  $(L_C, L_T)$  interfering with  $(L_T, L_C)$  or  $(S_C, S_T)$  interfering with  $(S_T, S_C)$ ) (Fig. 2(b)).

It is than the combination of all these interference contributions from all the possible pairs of photons emitted by the source that leads to the observed “collective” interference between the pair  $(L_C, L_T)$  of long paths and the pair  $(S_C, S_T)$  of short paths. Evidently, this does not mean that each pair  $(m, n)$  of two detected photons have either taken the  $(L_C, L_T)$  or the  $(S_C, S_T)$  paths in an indistinguishable manner. Indeed, the correlated paths  $(S_C, S_T)$  and  $(L_C, L_T)$  *cannot* be interpreted as *two-photon* paths. On the other hand, their interference is a result of the non trivial *collective* contribution of all the possible pairs  $(m, n)$  of detected photons associated with different interfering two-photon detection amplitudes within the statistics of the thermal source (Figs. 2(b) and 2(c)).

In Section 2, we will first analyze this interference phenomenon by using the standard description of thermal light in terms of the Glauber-Sudarshan probability distribution (Eq. (2)). In Section 3, we will give a detailed analysis of the fundamental two-photon interference physics underlying this effect. We will finally conclude the paper with conclusions and perspectives in Section 4.

## 2. Thermal light interference “beyond coherence”

We consider the interferometer in Fig. 1(a), with the input thermal state described by [5, 8]

$$\hat{\rho}_{ther} = \int \left[ \prod_{\omega} d^2\alpha_{\omega} \right] P_{\hat{\rho}_{ther}}(\{\alpha_{\omega}\}) \bigotimes_{\omega} |\alpha_{\omega}\rangle\langle\alpha_{\omega}|, \quad (1)$$

with the Glauber-Sudarshan probability distribution [9, 52]

$$P_{\hat{\rho}_{ther}}(\{\alpha_{\omega}\}) = \prod_{\omega} \frac{1}{\pi \bar{n}_{\omega}} \exp\left(-\frac{|\alpha_{\omega}|^2}{\bar{n}_{\omega}}\right), \quad (2)$$

and the average photon number [5]

$$\bar{n}_{\omega} = \bar{r} \frac{1}{\sqrt{2\pi}\Delta\omega} \exp\left\{-\frac{(\omega - \omega_0)^2}{2\Delta\omega^2}\right\},$$

at frequency  $\omega$ , with the mean photon rate  $\bar{r}$ , average frequency  $\omega_0$  and spectral width  $\Delta\omega$ .

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At the interferometer output the intensity fluctuations

$$\Delta I_d(t_d) = I_d(t_d) - \langle I_d(t_d) \rangle \quad (3)$$

at the detection time  $t_d$  around the mean value  $\langle I_d(t_d) \rangle$ , at each detector  $d = C, T$ , can be measured by using single-photon detectors with integration time  $\delta t \ll 1/\Delta\omega$  [53, 54]. One can then evaluate the correlation in the intensity fluctuations at the two output ports [53, 54, 5]

$$\langle \Delta I_C \Delta I_T \rangle = \langle I_C I_T \rangle - \langle I_C \rangle \langle I_T \rangle \quad (4)$$

by subtracting from the correlation in the intensities [5]

$$\langle I_C(t_C) I_T(t_T) \rangle \propto G^{(2)}(t_C, t_T) = \text{tr} \left[ \hat{\rho}_{ther} \hat{E}_C^{(-)}(t_C) \hat{E}_T^{(-)}(t_T) \hat{E}_C^{(+)}(t_C) \hat{E}_T^{(+)}(t_T) \right], \quad (5)$$

in terms of the field operators  $\hat{E}_d^{(+)}(t_d)$  and their Hermitian conjugate  $\hat{E}_d^{(-)}(t_d)$  at the detectors  $d = C, T$ , the background term [1]

$$\langle I_C(t_C) \rangle \langle I_T(t_T) \rangle \propto G^{(1)}(t_C, t_C) G^{(1)}(t_T, t_T) = 2a^2 \bar{r}^2, \quad (6)$$

associated with the product of the intensities

$$G^{(1)}(t_d, t_d) := \text{tr} \left[ \hat{\rho}_{ther} \hat{E}_d^{(-)}(t_d) \hat{E}_d^{(+)}(t_d) \right], \quad (7)$$

where  $a$  is a constant.

At approximately equally detection times with respect to the coherence time  $1/\Delta\omega$  of the source,

$$|t_C - t_T| \ll 1/\Delta\omega \quad (8)$$

and for path delays

$$|L_C - L_T|/c, |S_C - S_T|/c \ll 1/\Delta\omega, \quad |L_T - S_T|/c, |L_C - S_C|/c \gg 1/\Delta\omega, \quad (9)$$

the expression in Eq. (4) becomes [1]

$$\begin{aligned} \langle \Delta I_C \Delta I_T \rangle &\propto \left| G^{(L_C, L_T)}(t_C, t_T) + G^{(S_C, S_T)}(t_C, t_T) \right|^2 \\ &= 2a^2 \bar{r}^2 (1 + \cos(\varphi_C - \varphi_T)). \end{aligned} \quad (10)$$

Here, the interference between the two-path contributions [1]

$$G^{(l_C, l_T)}(t_C, t_T) = i a \bar{r} e^{i\omega_0[t_C - t_T]} e^{-i\omega_0[l_C - l_T]/c} e^{[l_C - l_T]^2 \Delta\omega^2 / 2c^2} e^{-[t_C - t_T]^2 \Delta\omega^2 / 2}, \quad (11)$$

with  $(l_C, l_T) = (L_C, L_T), (S_C, S_T)$ , leads to a sinusoidal dependence on the difference  $\varphi_C - \varphi_T$  between the relative phases  $\varphi_d = \omega_0(L_d - S_d)/c$ , with  $d = C, T$ , in the two Mach-Zehnder interferometers. These sinusoidal oscillations can be measured independently

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of how much the difference in length between the L-type paths and the S-type path is beyond the coherence length of the source, since each interfering contribution in Eq. (10) depends only on the relative distance between paths of the same type. Indeed, the other interfering terms  $G^{(S_C, L_T)}(t_C, t_T)$  and  $G^{(L_C, S_T)}(t_C, t_T)$  from path of different types give a negligible contributions to Eq. (10) in the conditions in Eq. (9).

We point out that the phase difference  $\varphi_C - \varphi_T = \omega_0[(L_C - L_T) - (S_C - S_T)]/c$  can be tuned by varying the difference in length between paths of the same type ( $0 \leq \omega_0(l_C - l_T)/c \leq 1$ ), within the coherence length of the source as in Eq. (9). This was obtained, for example, experimentally in Ref. [19] by using a thermal source of coherence length of about 120 m and central wavelength of 780 nm by varying the differences in the path lengths with piezoelectric actuators at the rate of 63 nm/s. We also emphasize that interference fringes can be observed also by measuring directly the correlation in the photon numbers in Eq. (5) but with a visibility of 1/3 due to the additional background term in Eq. (6).

How can we interpret these second-order interference fringes in terms of two-photon interference?

### 3. Two-photon interference physics

We describe here the observed interference effect in terms of interference of two-photon detection amplitudes [9, 10]. Toward this goal, it useful to mimic the thermal field emitted from the point-like source by a very large number of subfields  $m$  with frequency  $\omega_m$ , which state can be written in the coherent representation as [7]

$$|\Psi\rangle \doteq \prod_m |\alpha_m(\omega_m)\rangle, \quad (12)$$

where  $|\alpha_m(\omega_m)\rangle$  is an eigenstate of the annihilation operator  $\hat{a}_m(\omega_m)$  with eigenvalues  $\alpha_m(\omega_m)$  which contain a real-positive amplitude  $a_m(\omega_m)$  and a random phase  $\varphi_m(\omega_m)$  arising from the thermal nature of the subfields emitted by the source.

We can then evaluate the correlation in the intensity fluctuations

$$\begin{aligned} \langle \Delta I_C(t_C) \Delta I_T(t_T) \rangle &\propto \langle \langle \Psi | \hat{E}^{(-)}(t_C) \hat{E}^{(-)}(t_T) \hat{E}^{(+)}(t_C) \hat{E}^{(+)}(t_T) | \Psi \rangle \rangle_{E_s} \\ &\quad - \langle \langle \Psi | \hat{E}^{(-)}(t_C) \hat{E}^{(+)}(t_C) | \Psi \rangle \rangle_{E_s} \langle \langle \Psi | \hat{E}^{(-)}(t_T) \hat{E}^{(+)}(t_T) | \Psi \rangle \rangle_{E_s}, \end{aligned} \quad (13)$$

where  $\langle \cdot \cdot \cdot \rangle_{E_s}$  denotes the ensemble average over all the possible values of  $\alpha_m(\omega_m)$ . Here, the field operator  $\hat{E}^{(+)}(t_d)$  at the detectors  $d = C, T$  can be expressed as the sum

$$\hat{E}^{(+)}(t_d) = \sum_m \sum_{\rightarrow = \rightarrow, \rightarrow, \dots} \hat{E}_{m \rightarrow d}^{(+)}(t_d), \quad (14)$$

where the subfield operators

$$\hat{E}_{m \rightarrow d}^{(+)}(t_d) \propto \int d\omega e^{-i\omega[t_d - l_m \rightarrow d/c]} \hat{a}_m(\omega) \quad (15)$$



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4 take into account the propagation of each detected photon  $m$  from the point-like source  
5 where each corresponding subfields  $m$  is generated to the point-like detector  $d = C, T$   
6 through either the long path  $l_{m \rightarrow d} = L_d$  or the short path  $l_{m \dashrightarrow d} = S_d$ .  
7

8 We can now describe the correlation in the intensity fluctuations in Eq. (13) in  
9 terms of the interference of all the possible “two photon-detection amplitudes”  
10

$$11 \left[ \begin{array}{c} m \rightarrow C \\ n \rightarrow T \end{array} \right] \doteq \langle \alpha_m(\omega_m) | \hat{E}_{m \rightarrow C}^{(+)}(t_C) | \alpha_m(\omega_m) \rangle \langle \alpha_n(\omega_n) | \hat{E}_{n \rightarrow T}^{(+)}(t_T) | \alpha_n(\omega_n) \rangle, \quad (16)$$

12 with  $\rightarrow = \dashrightarrow, \dashrightarrow$ , associated with all the possible contributions of two different subfields  
13  $m$  and  $n$  to a two-photon detection, in analogy to a HBT experiment. However, since  
14 the time delay between the long (dashed red) and the short (dotted blue) paths (Fig.  
15 2(a)) is beyond the coherence time of the source, interference can occur only if each  
16 given subfield  $m, n$  undertakes the same type of path in the two-interfering two-photon  
17 amplitudes. There are therefore only four possible ways for these amplitudes to interfere  
18 with each other as in the expression  
19

$$20 \langle \Delta I_C(t_C) \Delta I_T(t_T) \rangle \propto \left\langle \sum_{m \neq n} \left[ \begin{array}{c} m \dashrightarrow C \\ n \dashrightarrow T \end{array} \right] \left[ \begin{array}{c} m \dashrightarrow T \\ n \dashrightarrow C \end{array} \right]^* + \left[ \begin{array}{c} m \dashrightarrow C \\ n \dashrightarrow T \end{array} \right] \left[ \begin{array}{c} m \dashrightarrow T \\ n \dashrightarrow C \end{array} \right]^* \right\rangle_{E_s}$$

$$21 + \left\langle \sum_{m \neq n} \left[ \begin{array}{c} m \dashrightarrow C \\ n \dashrightarrow T \end{array} \right] \left[ \begin{array}{c} m \dashrightarrow T \\ n \dashrightarrow C \end{array} \right]^* + \left[ \begin{array}{c} m \dashrightarrow C \\ n \dashrightarrow T \end{array} \right] \left[ \begin{array}{c} m \dashrightarrow T \\ n \dashrightarrow C \end{array} \right]^* \right\rangle_{E_s}$$

$$22 \quad (17)$$

23 of the correlation in the intensity fluctuations, which follows from Eq. (13) by using Eqs.  
24 (12, 14, 16). Here, the large number of contributing subfields  $m, n$  spans in a continuous  
25 way all the possible eigenvalues  $\alpha_\omega = \alpha_m(\omega_m), \alpha_n(\omega_n)$  in Eq. (16), which contribute to  
26 the ensemble average in Eq. (17) according to the probability distribution associated  
27 with the P function in Eq. (2). One can therefore write the first two contributions in Eq.  
28 (17) in terms of the phase-independent terms  $|G^{(L_C, L_T)}(t_C, t_T)|^2$  and  $|G^{(S_C, S_T)}(t_C, t_T)|^2$   
29 (defined by Eqs. (11)) as  
30

$$31 \left\langle \sum_{m \neq n} \left[ \begin{array}{c} m \dashrightarrow C \\ n \dashrightarrow T \end{array} \right] \left[ \begin{array}{c} m \dashrightarrow T \\ n \dashrightarrow C \end{array} \right]^* + \left[ \begin{array}{c} m \dashrightarrow C \\ n \dashrightarrow T \end{array} \right] \left[ \begin{array}{c} m \dashrightarrow T \\ n \dashrightarrow C \end{array} \right]^* \right\rangle_{E_s}$$

$$32 \simeq |G^{(L_C, L_T)}(t_C, t_T)|^2 + |G^{(S_C, S_T)}(t_C, t_T)|^2 = 2a^2 \bar{r}^2. \quad (18)$$

33 These interference contributions arise from the interference of two-photon amplitudes  
34 associated with two paths of the same type (long or short) within the coherence length  
35 of the source. Evidently, interference terms of this type arise in an analogous way also  
36 in standard HBT interferometers (Fig. 2(b)) where  $(L_C, L_T)$  or  $(S_C, S_T)$  define the  
37 path lengths from the output ports of the beam splitter to the two detectors. Since,  
38 in this case, the paths associated with the two corresponding interfering amplitudes  
39 completely overlap spatially, neither of these two terms can be sensitive to any phase  
40 delay, as expected for standard HBT interference.  
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On the other hand, by again using Eqs. (2,11,16), the last two interference contributions in Eq. (17) can be expressed as (c.c. stands for complex conjugate)

$$\begin{aligned} \left\langle \sum_{m \neq n} \begin{bmatrix} m \rightarrow C \\ n \rightarrow T \end{bmatrix} \begin{bmatrix} m \rightarrow T \\ n \rightarrow C \end{bmatrix}^* + \begin{bmatrix} m \rightarrow C \\ n \rightarrow T \end{bmatrix} \begin{bmatrix} m \rightarrow T \\ n \rightarrow C \end{bmatrix}^* \right\rangle_{E_s} &= \left\langle \sum_{m \neq n} \begin{bmatrix} m \rightarrow C \\ n \rightarrow T \end{bmatrix} \begin{bmatrix} m \rightarrow T \\ n \rightarrow C \end{bmatrix}^* + c.c. \right\rangle_{E_s} \\ &\simeq [G^{(L_C, L_T)}]^* G^{(S_C, S_T)}(t_C, t_T) + c.c. = 2a^2 \bar{r}^2 \cos(\varphi_C - \varphi_T). \end{aligned} \quad (19)$$

Here, the two detected photons either have undertaken the  $(L_C, S_T)$  path or the  $(L_T, S_C)$  path, corresponding to pairs of two paths of different types (Fig. 2(c)), due to the absence of “which path information” in the propagation after the beam splitter. Therefore, these terms are sensitive to the phase difference  $\varphi_C - \varphi_T$  between the two unbalanced interferometers leading to the observation of sinusoidal fringes, which are unique to this interference phenomenon with respect to standard HBT interference. This is also very different from Franson-type interference where the two photons either take the  $(L_C, L_T)$  or the  $(S_C, S_T)$  path, leading to a dependence on the sum  $\varphi_C + \varphi_T$  of the two interferometer relative phases.

Interestingly, one can notice that it does not matter how far the long and short paths are apart from each other beyond the coherence length of the source since each subfields  $m, n$  always evolve through two possible paths of the same type (either  $L = L_C, L_T$  or  $S = S_C, S_T$ ) which length difference is within the coherence length of the source. Therefore the corresponding interfering two photon-amplitudes are always indistinguishable. Of course, the larger is the path unbalance in each Mach Zehnder interferometer the larger has to be the emission time delay between the photons  $m$  and  $n$  associated with the corresponding subfields triggering the two-fold detection at approximately equal detection times. However, given the continuous nature of the thermal field there is not, in principle, any constraint to such a time delay. This is evidently very different from the situation in the Franson interferometer where the time delay associated with the interferometer unbalance is required to be within the the coherence time of the pump laser used to generate the entangled pair of photons.

Remarkably, the “incoherent” sum in Eq. (17) of the two-photon interference contributions in Eqs. (18) and (19) leads to the interference of the two “effective” two-path amplitudes associated with the two pairs  $(L_C, L_T)$  and  $(S_C, S_T)$  of correlated paths as in Eq. (13). However, these are *not* interfering two-photon detection amplitudes, since no pairs of photon can have undertaken these two pairs of paths, with length difference  $L - S$  beyond the coherence length of the source, in an indistinguishable manner. On the other hand, such two-path interference is a “collective” result of all the possible two-photon interference contributions in Eq. (17) as depicted in Figs. 2(b) and 2(c).

#### 4. Conclusions and perspectives

We have described here how thermal light second-order interference beyond the coherence time arises, for any pair  $(m, n)$  of detected photons, from the interference

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4 of two-photon detection amplitudes without contradicting our understanding of two-  
5 photon coherence.  
6

7 In particular, for each pair of interfering two-photon detection amplitudes, each  
8 detected photon can undertake only paths of the same type (either L or S). Indeed,  
9 only in this case the corresponding path differences  $L_C - L_T$  or  $S_C, -S_T$  are within the  
10 coherence length of the source (Figs. 2(b) and 2(c)).  
11

12 If the m photon undertakes always the same type of path as the n photon before  
13 triggering a two-fold detection (Fig. 2 (b)) no sinusoidal fringes could be observed.  
14 These ‘‘HBT interference contributions’’ are usual in standard HBT experiments (Fig.  
15 1(b)). However, the novelty here is that the (m,n) pair of indistinguishable photons can  
16 also undertake indistinguishable pairs of different types of paths,  $(L_C, S_T)$  interfering  
17 with  $(L_T, S_C)$ . Remarkably, these ‘‘non-HBT interference contributions’’ are sensitive to  
18 difference  $\varphi_C - \varphi_T$  between the phases in the two unbalanced interferometer. Evidently,  
19 in this case, the emission time delay between the two photons is of the order of  $(L - S)/c$   
20 which is beyond the coherence time of the source. However, since the thermal field is  
21 a continuous field, there is no limit, in principle, to such a time delay and therefore to  
22 the path unbalance in each Mach-Zehnder interferometer.  
23  
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27 Interestingly, even if the described HBT and non-HBT contributions add  
28 incoherently, an effective interference between the two pairs  $(L_C, L_T)$  and  $(S_C, S_T)$   
29 of correlated paths arises from the collective contribution of all the possible pairs of  
30 photons emitted by the thermal source. This two-path interference is evidently beyond  
31 the thermal light coherence length. One can think that even if two-photon interference  
32 occurs within the coherence length of the source, a ‘‘collective’’ second-order interference  
33 can occur beyond such coherence length. In other words, the ‘‘incoherent’’ sum of all  
34 possible two-photon interference contributions can generate ‘‘coherence’’ in a collective  
35 manner according with the statistics of the thermal source.  
36  
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39 We emphasize that the same physics described here naturally applies also to  
40 experiments in the spatial domain where the two Mach Zehnder interferometers in Fig.  
41 2(a) are substituted by two double slits [40, 41].  
42  
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44

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59  
60

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