Centrifuge modelling in the undergraduate curriculum – a 5 year reflection

J.A. Black, S.M. Bayton & A. Cargill
Centre for Energy and Infrastructure Ground Research (CEIGR), University of Sheffield, Sheffield, UK

A. Tatari
University of Portsmouth, UK

ABSTRACT: A summary overview of 5 years working with a small-scale educational centrifuge at the University of Sheffield is presented. Various geotechnical design problems have been successfully performed throughout this period, including slope stability, bearing capacity failure and tunnelling. The opportunity to experimentally explore the theoretical content taught in lectures has had a positive impact on student learning in the undergraduate curriculum. The authors advocate there is an immediate need for greater adoption of experimental based observation/demonstration, either conducted at 1*g* or N*g*, to be embedded within the geotechnical undergraduate curriculum to enrich and deepen the student learning experience of geotechnical system performance.

1 INTRODUCTION

Civil Engineering undergraduate students often struggle to design with soil as an engineering material compared to steel or concrete owing to the variable nature of soil. Within the field of Geotechnical Engineering Burland (2006) describes the '*Soil Mechanics Triangle*' which identifies the interdependencies embedded within geotechnical design. Competency extends beyond understanding the soil material itself, but also requires knowledge of complex theory and analysis methods; many of which have evolved from empiricism, observation and experience, which undergraduate students lack in the embryonic stage of their career. The formative years of a degree programme typically focus purely on theoretical aspects and are void of opportunity to embed experience of actual geotechnical design in practice and learn through observation. Hence, active experimental, observation and reflection pedagogy described by Kolb (1984) via model testing, leading to the establishment of '*well-winnowed experience*' as referred by Burland (2006), represents an exciting opportunity to enhance comprehension and understanding of the design process in undergraduate students.

Laboratory based demonstrations form a valuable learning tool as they provide an opportunity to explore design scenarios, challenge and reinforce theory taught in lectures. Typically these demonstrations are limited to element tests used to assess soil properties such as compressibility and strength. While beneficial, these tests fail to provide any insight of how actual full-scale structures perform; for example, rotational instability of an embankment slope. Without observing failure of structures of their own design, students will not truly fully appreciate the impact of their design assumptions, design philosophy/concept and appreciate the consequence of inadequate design.

1.1 *Physical modelling in education*

Reduced-scale physical models at 1g can provide a basic insight of geotechnical performance with respect to indicative behavior, i.e. mode of failure. Quantifiable observations derived from small-scale model tests are limited as realistic prototype self-weight stresses are not preserved. Similitude of stress can be achieved by testing models in a high gravitational acceleration field produced by a centrifuge; hence, the stress and strain distributions in the model will reflect the field situation.

Craig (1989) was one of the first to formally discuss physical modelling for geotechnical engineering education. He described a modelling initiative that began in the mid-1970s at the University of Manchester where experiments were performed using an inexpensive "teaching centrifuge". Mitchell (1994), Collins et al. (1997), Newson et al. (2002) and Dewoolar et al. (2003) also demonstrated the applicability of centrifuge modelling for instructional purposes to illustrate concepts of slope stability, retaining walls, foundations, tunnel stability, and lateral earth pressure theory (Wartman 2006). A summary of several educational centrifuge facilities is reported in Table 1.

A small-scale educational centrifuge has been developed by the lead author at the University of Sheffield, and been in continuous operation since 2012. The educational centrifuge used to support a number of taught modules and dissertation projects. One specific module is the final year '*Advanced Geotechnics: CIV4501*' elective course. This seeks to enhance students understanding of geotechnical design through enquiry and problem based learning to promote critical/lateral thinking and reflective practice. This is achieved through the integration of advanced geotechnical theory relating to constitutive models to describe soil behaviour, small-scale physical model centrifuge tests, self-learning laboratories and complementary analytical and numerical analysis methods. The purpose of this paper is to highlight a number of projects that have been successfully completed during the last 5 years of operation to demonstrate the value this facility offers undergraduate students.

Table 1. Existing educational centrifuge facilities.

Reference	Gravity (*g*)	Radius (m)	Model size (mm LxHxW)
Newson et al. (2002)	400	0.325	80x80x80
Craig (1989)	500	0.30	125x70x25
Dewoolar et al. (2003)	400	0.61	223x165x25
Caicedo (2000)	500	0.565	140x120x70

2 UNIVERSITY OF SHEFFIELD CENTRIFUGE

A small-scale state-of-the-art beam centrifuge 1 m diameter was designed and is capable of rotating a payload up to 20 kg at 100 gravities (100*g*), referred to as UoS 2gT, and is shown in Figure 1. The maximum sample size that can be tested is 160 mm (L) x 100 mm (H) x 80 mm (W) which represents prototype dimensions of 16 m x 10 m x 8 m at 100*g*. This is sufficient to test a diverse range of reduced scale engineering structures such as slopes, retaining walls and foundations, while providing stress conditions that realistically duplicate prototype behaviour. The centrifuge is equipped with electrical power slip rings, dual port hydraulic rotating fluid union enabling the delivery of air and water in-flight, digital image capture, signal acquisition, onboard PC and real-time wireless data communication/transfer. Images of samples captured in-flight enable real-time observations of displacement and failure mechanisms. Detailed information about the centrifuge design and development is reported in Black et al. (2014). In the 5 years of operation from 09/2012 to 10/2017, the centrifuge has conducted excess of 500 tests and directly impacted on approximately 300 students via taught modules and dissertation research projects.

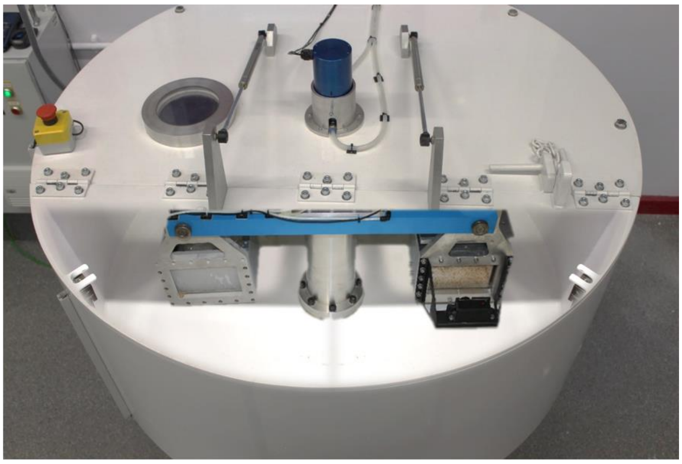

Figure 1. University of Sheffield educational centrifuge.

3 EXAMPLE PROJECTS

3.1 *Slope stability – gravity switch on*

The slope stability experiment is probably the most appealing of all centrifuge experiments because students can visually confirm the development of a failure surface driven by self-weight alone. Consider the case of a saturated clay slope of height (H) having a slope angle (α). The stability of the slope is dependent on the undrained shear strength of the soil (c_u), the slope height (H) and the unit weight (γ); such that it can be related to a dimensionless group referred to as the stability coefficient (N_s) (after Taylor, 1937) as shown in Equation 1:

$$N_s = \left(\frac{c_u}{\gamma g H} \right) \quad (1)$$

Block samples were prepared by consolidating Kaolin clay slurry mixed with de-aired water at 1.5 times the Liquid Limit (LL). Consolidation pressures were ramped up to 200 kN/m^2 to produce consolidated homogeneous blocks of clay from which model slopes would be prepared. The clay blocks are trimmed to the correct geometry with the aid of side templates and flocked with texture for digital image analysis. Model slopes tested in the centrifuge at N times the earth's gravitational field fail by increased self-weight forces; hence, gravity switch on allowed simple simulation of slope instability without the requirement for complex actuation.

As part of a complementary self-directed laboratory activity, students are required to evaluate the undrained shear strength of the soil block from which the model would be generated. The undrained shear strength for the samples was determined by triaxial compression to be approximately 20 kN/m^2. Using this data and design input parameters for the slope geometry, students are tasked with predicting the gravity at which the slope will fail based on the derived shear strength. The model slopes are taken to failure by increasing gravitational acceleration until collapse occurs. Real time observations of the deforming slope and shear plane are captured by the onboard cameras and post-processed using image analysis. Comparisons in the actual test performance with the pre-test predictions are considered in conjunction with back analysis of the failed slope to determine a revised estimate for the actual shear strength.

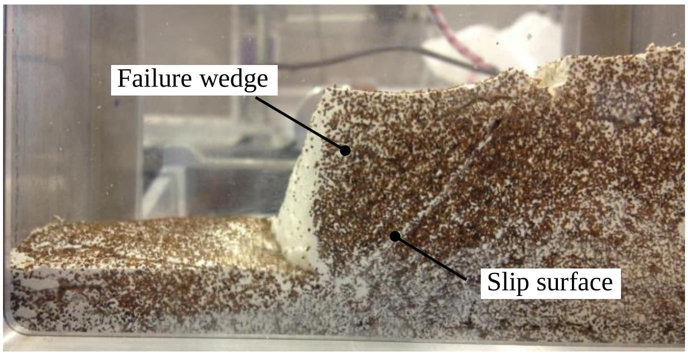

Figure 2. Slope failure by gravity switch on.

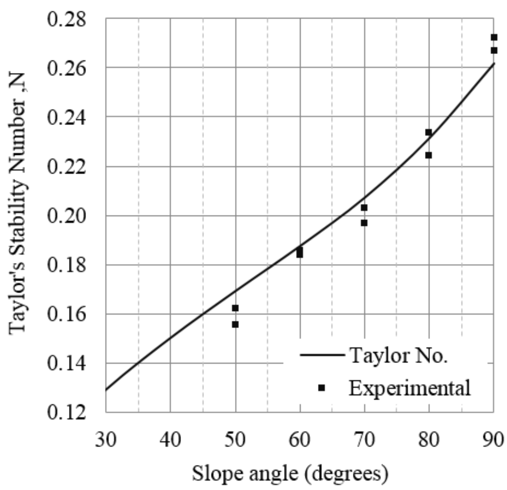

Figure 3. Theoretical Taylor stability curve compared to centrifuge experimental test data by gravity switch on.

By way of example, a failed model slope is shown in Figure 2. The model slope was 60 mm (0.06 m) in height with a slope angle of 90° (i.e. a vertical cut). The undrained strength was determined by the laboratory triaxial tests to be 22 kN/m^2. The saturated unit weight of soil was determined to be 17.2 kN/m^3. According to Taylor's stability chart, the stability coefficient (N_s) for this slope configuration is 0.26. The g-level (N) at which the model slope was expected to fail in the centrifuge was predicted using Equation 2 as follows:

$$0.26 = \left(\frac{22kN/m^2}{(N \times 17.2kN/m^3 \times 0.06m)} \right) \Rightarrow N \approx 81g \qquad (2)$$

The slope was expected to fail at 81*g*, whereas it failed instantaneously at 79*g*. In addition, Figure 3 presents the Taylor stability for a number of test case slopes whereby it is clear that the experimental results are in good agreement with theory; although noting that due to the reduced size of the payload, larger errors may exist than if using larger scale centrifuge systems due to boundary restrictions.

3.2 *Shallow bearing capacity*

Ultimate bearing capacity of strip footings resting on a single layer of homogeneous clay, described by Terzaghi (1943). Reality however is rarely this simple; soils are often non-uniform, layered and have varying strength/stiffness properties. Increased complexity such as layering, described by Davis & Booker (1973), is a significant departure from basic bearing capacity theory taught in undergraduate programmes and presents a significant challenge to students when faced with this uncertainty. A two layer, firm overlying soft soil, bearing capacity problem is considered (Fig. 4) that enabled students to evaluate aspects such as the impact on bearing capacity factor (N_c) and mode of failure.

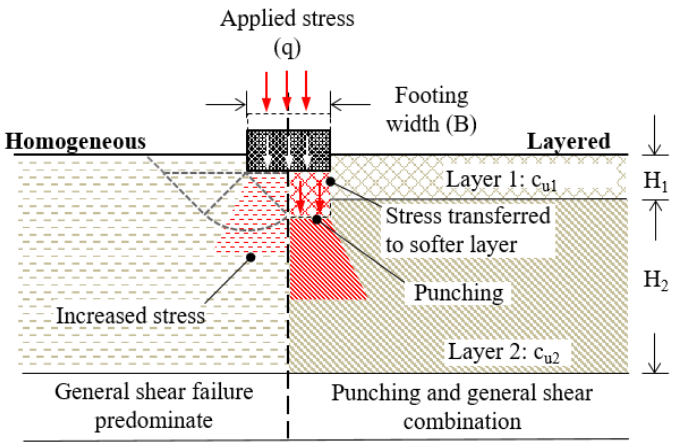

Figure 4. Bearing capacity of layered soil.

Samples were prepared by consolidating Kaolin clay slurry mixed with de-aired water at 1.5 times the Liquid Limit. Consolidation pressures were ramped up to 200 kN/m^2 and 400 kN/m^2 to produce consolidated homogeneous blocks of clay having undrained strength of 20 and 40 kN/m^2 respectively.

Layers of varying thickness of soft and firm soil were required to make composite specimens. Side cutting templates where placed alongside virgin soil blocks and the sample trimmed using a wire saw. Once configured the combined sample was then placed back into the consolidation press under a nominal 100 kN/m^2 for 24 hours to ensure 'knitting' of the interface boundary between the upper and lower layer. Centrifuge tests were conducted at 50g and considered footing tests on a homogeneous and layered combinations as detailed in Table 2.

The upper and lower layer properties are noted with the relevant subscript indicator, i.e. undrained shear strength of upper and lower layer are c_{u1} and c_{u2} respectively. The four upper layer thicknesses considered (10, 15, 20 and 40 mm) provided normalised thickness ratios, H_1/B, of 0.5, 0.75, 1 and 2 respectively, where B is the footing width (B = 20mm).

Figure 5 presents the bearing capacity against normalised settlement (s/B) response for the 20 mm wide strip footing at an accelerated gravity of 50g. Significant variation in the bearing resistance response is observed between the homogeneous soil bed (H_1/B = 4.0) and thinnest firm layer (H_1/B = 0.5) case.

Table 2. Layered footing tests.

	Layer 1		Layer 2	
Test No.	H_1 mm [*m]	c_{u1} kN/m^2	H_2 mm [*m]	c_{u2} kN/m^2
1	80 [4.0]	40	0	N/A
2	40 [2.0]	40	40 [2.0]	20
3	20 [1.0]	40	60 [3.0]	20
4	15 [0.75]	40	65 [3.25]	20
5	10 [0.5]	40	70 [3.5]	20

*square brackets denotes prototype at N=50g

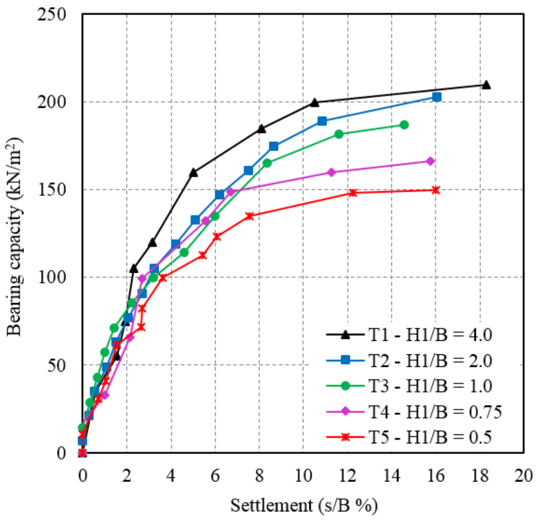

Figure 5. Experimental footing load-displacement.

In the case of H_1/B = 4.0 the maximum bearing capacity was 210 kN/m^2 at the point of failure compared to that of 150 kN/m^2 for H_1/B = 0.5. For H_1/B = 2.0 a similar maximum bearing capacity was recorded as that in the uniform bed, albeit with a slightly reduced stiffness response over the full displacement range. Tests H_1/B = 1.0 and 0.75 exhibit consistent responses up to s/B = 6% at which point the bearing capacity of the latter reduces quickly as the footing penetration advances. These results clearly demonstrate the complexities that exist in bearing capacity for layered soil configurations, emphasising the challenges faced by students in adapting their basic rudimentary understanding of bearing capacity on homogeneous soils to more diverse complex conditions.

In the absence of surcharge pressure, the ultimate bearing capacity (q_u) of a strip footing on an infinite uniform purely cohesive soil can be expressed as Equation 3:

$$q_u = N_c \times c_u \tag{3}$$

where c_u is the undrained shear strength and N_c is the bearing capacity factor. Equation 3 is valid for a homogeneous soil conditions; however, in practice non-homogeneous layered soil conditions are frequently encountered. Several authors have postulated modified bearing capacity factors to evaluate this more complex bearing problem (Merifield et al. 1999). A simplified modified bearing capacity approximation for N_c by Merifield et al. (1999), referred to as Nc*, was approached as the undrained shear strength divided by the strength of the soil in immediate contact with the footing (i.e. the upper soil layer). Using this approximation the bearing capacity factor for the current centrifuge model tests were determined and are presented in Figure 6. It is evident that the bearing capacity factor is influenced by the depth of the upper layer and its' relative thickness to the width of the footing.

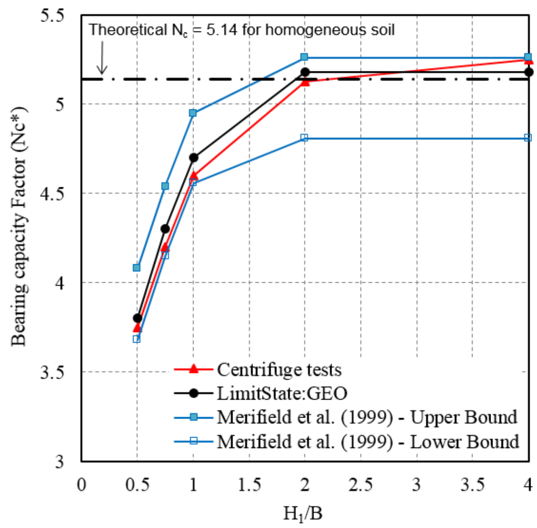

Figure 6. Bearing capacity factor from experiments compared with numerical limit analysis.

These values are also correlated with upper and lower solutions by Merifield et al. (1999) and yield good agreement. In addition, Test 1 (H_1/B = 4.0) represents a uniform soil strength sample and thus should conform to the classical theoretical bearing capacity factor π+2 (Terzaghi, 1943). The bearing capacity factor in Test 1 was determined to be 5.25, approximately 2% over this theoretical value which could be due to: (i) some residual interface friction at the soil-window boundary or; (ii) increased resistance being mobilised in the soil as the penetration advances due to increased self-weight stresses, as reported by Davis and Booker (1973).

Complementary numerical analysis was carried out using LimitState:GEO (Smith and Gilbert 2007) which uses linear programming to minimise internal energy dissipated along a potential slip planes to yield an upper bound solution and critical failure mechanism. The problem was modelled at prototype to represent the test configurations outlined previously using soil strength properties determined by triaxial tests. Numerical results for the reference test case (homogeneous soil) yielded a bearing capacity factor of 5.18, which compares favourably with theory. The bearing capacity results for the numerical study in Figure 6 and shows good agreement with the upper and lower bound solutions of Merifield et al. (1999) and the centrifuge test data. These observations serve to reinforce to students the importance of determining a suitable bearing capacity factor for complex layered soil conditions as failing to do so would have catastrophic consequences on the foundation stability if the classical value π+2 were inappropriately used.

3.3 *Tunneling*

All civil engineering works generate disturbance of the ground and great care should be exercised especially when developments are in a densely populated urban environment. As large cities continue to expand, interference of adjacent structures is unavoidable and hence the impact of tunnel-structure interactions must be fully considered and understood. The work reported here pertains to preliminary investigation conducted by Song & Black (2016) to assess the viability of the small-scale centrifuge environment to suitability model a tunnel interaction problem for undergraduate research studies.

The prediction of surface settlement in 'greenfield' conditions was first reported by Peck (1969), who presented a Gaussian based settlement equation (Eq. 4) which has been shown to provide good correlation with field measurement data.

$$S_v = S_{\max} \cdot \exp\left(\frac{-y^2}{2i^2}\right) \quad (4)$$

where vertical settlement is S_v, and the S_{max} is the maximum vertical settlement, occurring above the tunnel centre line. Horizontal offset distance from the tunnel centre line is X, and i is the location of the inflection point.

This approach forms the underlying principle of current design and key aspects are summarised in Figure 7 which indicates the maximum settlement (S_{max}), point of inflection (i) and the extent of the volume loss settlement trough.

Ground disturbances were simulated at 100g using the conventional approach of tunnel volume loss by reducing the internal pressure of a thin latex membrane. The tunnel had a diameter of 19.05 mm, representative of an approximate 2 m prototype. Model tests were prepared from dry sand of D_{50} of 160μm, pluviated to 73% relative density. Three C/D ratios of 1.0, 1.6 and 2.0 were considered. Soil displacement measurement and quantification of interaction performance was achieved using image correlation methods. During ramp-up the pressure within the tunnel was balanced against the increased ground stress using the pressure volume controller system.

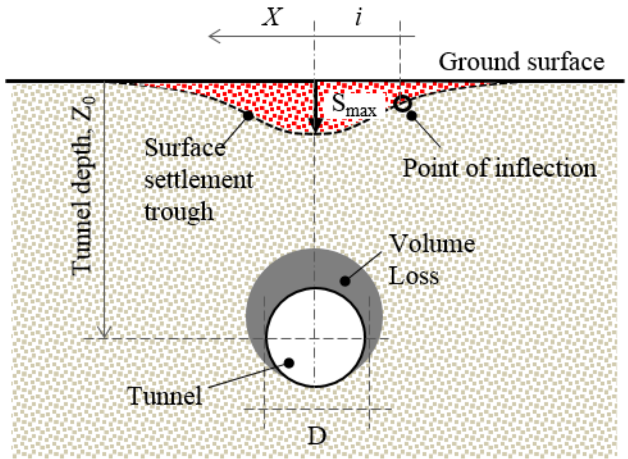

Figure 7. Test overview summary for tunnel experiments.

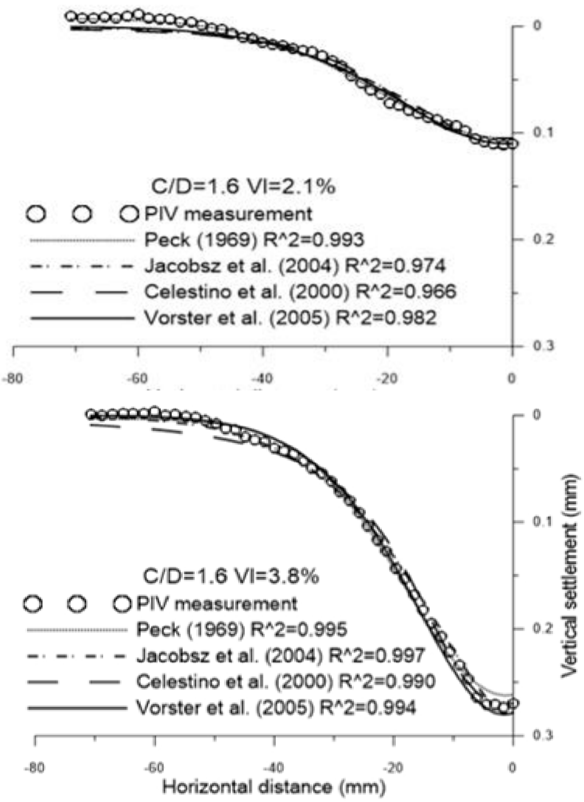

Figure 8. Surface settlement for a tunnel C/D = 1.6 at volume loss 2.1% and 3.8%.

Figure 8 presents settlement for C/D = 1.6 at a volume loss of 2.1% and 3.8% where the largest settlement displacements occur along the vertical centre line of the tunnel, diminishing with increased horizontal distance. Good agreement is observed with the classical Gaussian formulation of Peck (1969) and subsequent analytical solutions published by Jacobsz (2003) and Vorster (2006). Observations include: (i) increased levels of maximum settlement and; (ii) a changing point of inflection of the Gaussian settlement curve occur with increased volume loss. While only a preliminary study, the successful outcome of the tests to theoretical predictions confirm the potential impact to undergraduate research activities that extend beyond the scope of classic lecture design examples.

4 CONCLUSIONS

The purpose of this paper was to provide a summary overview of 5 years working with a small-scale centrifuge and demonstrate the impact it has provided to the student learning in the undergraduate curriculum. Classic slope stability by gravity switch on has been demonstrated and correlated stability theory. Greater test complexity involving in-flight actuation for simulating bearing capacity failure of layer soil systems demonstrates the broader range of functionality that the centrifuge offers. Finally the use of the small-scale centrifuge environment is demonstrated with a focus on undergraduate research projects. A tunnel example is presented that enabled the student to achieve a high quality parametric data set for investigation. In all cases good agreement with relevant design theory has been achieved confirming the success of the modelling techniques adopted. The impact on the undergraduate learning experience is unquestionable and the authors advocate there is an immediate need for greater adoption of experimental based observation/demonstration, either conducted at 1*g* or N*g*, to be embedded within the geotechnical undergraduate curriculum to enrich and deepen the student learning experience.

5 ACKNOWLEDGEMENTS

The experiments reported in this paper were completed using the UoS2gT teaching centrifuge that was developed through funding by the National Higher Education STEM Programme. Continued support by Thomas Broadbent and Son Ltd. to the Centre for Energy & Infrastructure operation and large 4 m diameter centrifuge facility is gratefully acknowledged. The contribution by the Department of Civil & Structural Engineering technical staff Dr Paul Bentley and the post-doctoral staff within CEIGR is also acknowledged. Final acknowledgments go to the CIV4501 classes of 2012-2017 for their contribution in making this educational journey a success and the test data they have contributed to this paper.

6 REFERENCES

Black, J.A. Baker, N. and Ainsworth, A. 2014. Development of a small-scale teaching centrifuge. Physical Modelling in Geotechnics - Proceedings of the 8th International Conference on Physical Modelling in Geotechnics 2014, ICPMG 2014, 181 -186. Perth, Australia.

Burland, J. 2006. Interaction between Structural and Geotechnical engineers. *The Structural Engineer*, 84, 29-37.

Caicedo B. 2000. Geotechnical Centrifuge Applications to Foundations Engineering Teaching. First International Conference on Geotechnical Engineering Education and Training Geotechnical Engineering Education and Training (ISBN 9058091546)

Collins, B., Znidarcic, D., and Goddery, T. 1997. A New Instructional Geotechnical Centrifuge. Geotechnical News.

Craig, W. H. 1989. The Use of a Centrifuge in Geotechnical Engineering Education. *ASTM, Geotechnical Testing Journal*, 12, (4). 288–291.

Davis, E. H. and Booker, J. R. 1973. The effect of increasing strength with depth on the bearing capacity of clays. *Geotechnique*, 23, (4), 551-563.

Dewoolkar, M.M., Goddery, T. and Znidarcic, D. 2003. Centrifuge Modeling for Undergraduate Geotechnical Engineering Instruction. *ASTM, Geotechnical Testing Journal*, 26, (2), 201–209.

Jacobsz, S. W. 2003. The effects of tunnelling on piled foundations. PhD thesis, University of Cambridge Kolb, D. A. 1984. Experiential learning: Experience as the source of learning and development, Prentice-Hall, Englewood Cliffs, N.J.

Merifield, R. S., Sloan, S. W. and Yu, H. S. 1999. Rigorous plasticity solutions for the bearing capacity of two-layered clays. *Geotechnique*, 49, (4), 471-490.

Mitchell, R. J. 1994. Centrifuge Modeling as a Teaching Tool. Geotechnical News, 12, (3), September.

Newson, T. A., Bransby, M. F., and Kainourgiaki, G. 2002. The use of small centrifuges for geotechnical education." Proc., International Conference on Physical Modelling in Geotechnics: ICPMG '02, St. Johns, Canada. 215–220.

Peck, R. B. 1969. Deep excavations and tunnelling in soft ground. In Proc. 7th int. conf. on SMFE, 225-290.

Smith, C. and Gilbert, M. 2007. Application of discontinuity layout optimization to plane plasticity problems. Proceedings Royal Society A , 463,(2086), 2461-2484.

Song, G. and Black, J.A. 2016. Soil Displacement due to tunnelling using small-scale centrifuge technology. Proceedings of the 2nd European Conference on Physical Modelling in Geotechnics 2016, ECPMG 2016. Nantes, France.

Taylor, D,W. 1937. Stability of earth slopes. J. Boston Soc. Civil Eng., 243.

Terzaghi, K. 1943. Theoretical soil mechanics. New York: Wiley.

Vorster, T. E. B. 2006. The effects of tunnelling on buried pipes. PhD thesis, University of Cambridge.

Wartman, J. 2006. Geotechnical Physical Modelling for Education: Learning Theory Approach. ASCE, Journal of Professional Issues in Engineering Education and Practice. 288-296.