Analytic solutions in non-linear massive gravity

Kazuya Koyama[1], Gustavo Niz[1], Gianmassimo Tasinato[1]
[1] *Institute of Cosmology & Gravitation, University of Portsmouth, Dennis Sciama Building, Portsmouth, PO1 3FX, United Kingdom*

We study spherically symmetric solutions in a covariant massive gravity model, which is a candidate for a ghost-free non-linear completion of the Fierz-Pauli massive gravity. We find a solution that exhibits the Vainshtein mechanism, recovering general relativity below a Vainshtein radius given by $(r_g m^2)^{1/3}$, where m is the graviton mass and r_g is the Schwarzschild radius of a matter source. We also found another exact solution corresponding to Schwarzschild-de Sitter spacetime, where the curvature scale of de Sitter space is proportional to the mass squared of the graviton.

PACS numbers: 04.50.-h

Introduction: It is a fundamental question whether there exists a consistent covariant theory for massive gravity models where the graviton acquires a mass and leads to a large distance modification of General Relativity (GR). The quest for massive gravity dates back to the work by Fierz and Pauli (FP) in 1939 [1]. They considered a mass term for linear gravitational perturbations, which explicitly breaks the gauge invariance of GR. As a result, there exist five degrees of freedom in the graviton, instead of the two found in GR. There have been intensive studies on what happens going beyond the linearised theory. In 1972, Boulware and Deser (BD) found that, at the non-linear level, there appears a sixth mode in the graviton that becomes a ghost in the FP model [2]. This problem was reexamined using the effective theory approach [3], where Stückelberg fields were introduced to restore the gauge invariance, and whose scalar part represents the helicity-0 mode of the graviton. In the FP model, the scalar acquires non-linear interaction terms that contain more than two time derivatives, signaling the existence of the ghost.

This approach also sheds light on the other puzzle in the FP gravity: if one linearises the system, the solutions in the FP theory do not go back to GR solutions even in the small mass limit. This is known as the van Dam, Veltman, Zakharov (vDVZ) discontinuity [4, 5]. On the other hand, in this limit the scalar mode becomes strongly coupled and one cannot linearise the system. Due to strong coupling, the scalar interaction is shielded and GR can be recovered. This is known as the Vainshtein mechanism [6]. The strong coupling scale in the FP model is identified as $\Lambda_5 = (m^4 M_{pl})^{1/5}$ where M_{pl} is the Planck scale and m is the graviton mass. This scale is tightly connected with the non-linear interactions of the scalar mode that contain more than two derivatives. In the decoupling limit, where $m \to 0, M_{pl} \to \infty$ while the strong coupling scale Λ_5 is kept fixed, one obtains an effective theory for the scalar mode, where it is possible to study the consistency of the theory in more detail.

Until recently, it was believed that there is no consistent way to extend the FP model [7, 8] to get a ghost free model at all orders. A breakthrough came with a 5D braneworld model known as Dvali-Gabadadze-Porrati (DGP) model [9]. In this model there appears a continuous tower of massive gravitons from a four dimensional perspective. The non-linear interactions of the helicity-0 mode of massive gravitons contain no more than two derivatives, which is crucial to avoid the BD ghost. Due to this fact, the strong coupling scale in this theory is given by $\Lambda_3 = (m^2 M_{pl})^{1/3}$ instead of Λ_5, where $m = r_c^{-1}$ and r_c is a cross-over scale between 5D and 4D gravity [10, 11]. Further studies have considered more general non-linear interactions which contain no more than two derivatives. In 4D, only a finite number of terms satisfy this condition; these are dubbed Galileon terms because of a symmetry under field transformations of the form $\partial_\mu \pi \to \partial_\mu \pi + c_\mu$ [12]. Ref. [13] constructed the extension of the FP theory that gives the Galileon terms in the decoupling limit, by choosing the correct parameters in the lagrangian up to quintic order in perturbations. Ref. [14] proposed a covariant non-linear action that automatically ensures this property to all orders, which we will discuss below.

A remaining crucial question is whether this property, holding in the decoupling limit, is sufficient to ensure the absence of the BD ghost or not. In Ref. [14], it was shown that there is no BD ghost in the decoupling limit to all orders in perturbation theory, but only up to and including quartic order away from this limit. However, it is very hard to show the absence of the BD ghost at all orders if one starts from Minkowski and studies non-linear interactions perturbatively. Therefore, it is important to obtain non-perturbative background solutions in this theory, and study fluctuations around them. Moreover, it is interesting to find solutions in this covariant non-linear theory, that can describe features of the observed universe. These are the topics of the present work.

Covariant non-linear massive gravity: We first construct the action for generalised FP model based on Ref. [13, 14]. We define the tensor $H_{\mu\nu}$ as covariantization of metric perturbations:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \equiv H_{\mu\nu} + \eta_{\alpha\beta}\partial_\mu \phi^\alpha \partial_\nu \phi^\beta. \qquad (1)$$

The Stückelberg fields $\phi^\alpha = (x^\alpha - \pi^\alpha)$ transform as scalars, while $\eta_{\alpha\beta}$ corresponds to a non-dynamical background metric that is needed to define the potential, which is assumed to be the Minkowski metric. The covariant tensor $H_{\mu\nu}$ can be rewritten as

$$H_{\mu\nu} = h_{\mu\nu} + \eta_{\beta\nu}\partial_\mu\pi^\beta + \eta_{\alpha\mu}\partial_\nu\pi^\alpha - \eta_{\alpha\beta}\partial_\mu\pi^\alpha\partial_\nu\pi^\beta, \\ \equiv h_{\mu\nu} - \mathcal{Q}_{\mu\nu}. \quad (2)$$

From now on, indices are raised/lowered with the dynamical metric $g_{\mu\nu}$; for example $H^\mu_\nu = g^{\mu\rho}H_{\rho\nu}$. Under the coordinate transformation $x^\mu \to x^\mu + \xi^\mu$, π^μ transforms as $\pi^\mu \to \pi^\mu + \xi^\mu$.

We define a new tensor $\mathcal{K}^\nu_{\ \mu}$ as

$$\mathcal{K}^\nu_{\ \mu} \equiv \delta^\nu_\mu - \sqrt{\delta^\nu_\mu - H^\mu_\nu} = \delta^\nu_\mu - \sqrt{g^{\nu\rho}\left[\eta_{\rho\mu} + Q_{\rho\mu}\right]}, \quad (3)$$

where we used $H_{\mu\nu} = g_{\mu\nu} - (\eta_{\mu\nu} + Q_{\mu\nu})$. This allows us to define the complete potential for gravitational interactions as

$$\mathcal{L} = \frac{M_{Pl}^2}{2}\sqrt{-g}\left(R - m^2\mathcal{U}\right), \ \ \mathcal{U} = \left[\mathrm{tr}\,(\mathcal{K}^2) - (\mathrm{tr}\,\mathcal{K})^2\right]. \quad (4)$$

Expanding the potential in $H_{\mu\nu}$, we get a sum of interaction terms for $H_{\mu\nu}$, with the FP term at lowest order.

In order to study exact solutions associated with the previous Lagrangian, it is convenient to express $\mathcal{K}$ in terms of matrices as

$$\mathcal{K} = \mathbb{I} - \sqrt{g^{-1}\left[\eta + \mathcal{Q}\right]}, \quad (5)$$

where $\mathbb{I}$ denotes the identity matrix. The potential in four dimension then reads

$$\mathcal{U} = \mathrm{tr}\, g^{-1}\left[\eta + \mathcal{Q}\right] - 12 \\ + \mathrm{tr}\sqrt{g^{-1}\left[\eta + \mathcal{Q}\right]}\left(6 - \mathrm{tr}\sqrt{g^{-1}\left[\eta + \mathcal{Q}\right]}\right). \quad (6)$$

In general, the task is to calculate the trace of $\sqrt{g^{-1}\left[\eta + \mathcal{Q}\right]}$. If this matrix has non-vanishing determinant, it is diagonalizable, and can be expressed as $g^{-1}\left[\eta + \mathcal{Q}\right] = \mathcal{U}D\mathcal{U}^{-1}$, for some invertible matrix $\mathcal{U}$, and D is a diagonal matrix containing the eigenvalues of $g^{-1}\left[\eta + \mathcal{Q}\right]$. We call the eigenvalues $\lambda_1, \ldots, \lambda_4$. Then, since $\sqrt{g^{-1}\left[\eta + \mathcal{Q}\right]} = \mathcal{U}\sqrt{D}\mathcal{U}^{-1}$, one can easily express the traces in the formulae above in terms of eigenvalues

$$\mathrm{tr}\, g^{-1}\left[\eta + \mathcal{Q}\right] = \sum_i \lambda_i, \ \ \mathrm{tr}\sqrt{g^{-1}\left[\eta + \mathcal{Q}\right]} = \sum_i \sqrt{\lambda_i}.$$

Plugging these expressions into the potential, Eq. (6), we find an expression for $\mathcal{U}$ as a function of the eigenvalues:

$$\mathcal{U} = \sum_i \lambda_i + \left(\sum_j \sqrt{\lambda_j}\right)\left(6 - \sum_i \sqrt{\lambda_i}\right) - 12. \quad (7)$$

Asymptotically flat solutions: We first study asymptotically flat, spherically symmetric solutions in the unitary gauge $\pi^\mu = 0$. See Ref. [15] for spherical symmetric solutions in the FP theory. The non-dynamical Minkowski metric is $ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$, where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. First, we consider the following Ansatz for the metric

$$ds^2 = -N(r)^2 dt^2 + F(r)^{-1}dr^2 + r^2 H(r)^{-2} d\Omega^2. \quad (8)$$

Notice that in GR one can set $H(r) = 1$ by a coordinate transformation, but this is not possible in this theory, since we have already chosen the unitary gauge. The potential term is given by

$$\sqrt{-g}\,\mathcal{U} = -\frac{2r^2}{\sqrt{F}H^2}(-3 + \sqrt{F} + 2H + 6N \\ -3\sqrt{F}N - 6HN + 2\sqrt{F}HN + H^2 N). \quad (9)$$

The field equations are obtained by varying the action with respect to N, F and H. Let us study solutions in the weak field limit by expanding N, F and H as

$$N = 1 + n, \ \ F = 1 + f, \ \ H = 1 + h, \quad (10)$$

and truncating the field equations to first order in these perturbations. It is more convenient to introduce a new radial coordinate $\rho = r/H$, so that the linearised metric is expressed as

$$ds^2 = -(1 + 2n)dt^2 + (1 - \tilde{f})d\rho^2 + \rho^2 d\Omega^2, \quad (11)$$

where $\tilde{f} = f - 2h - 2\rho h'$ and a prime denotes a derivative with respect to ρ. The solutions for n and $\tilde{f}$ are then given by

$$2n = -\frac{8GM}{3\rho}e^{-m\rho}, \\ \tilde{f} = -\frac{4GM}{3\rho}(1 + m\rho)e^{-m\rho}, \quad (12)$$

where we fix the integration constant so that M is the mass of a point particle at the origin and $8\pi G = M_{pl}^{-2}$. These solutions exhibit the vDVZ discontinuity, *i.e.* they do not agree with GR solutions in the limit $m \to 0$. However, in order to understand what really happens in this limit, one should also include the behaviour of h as $m \to 0$. For this, let us consider scales below the Compton wavelength $m\rho \ll 1$, and at the same time ignore higher order terms in GM. Under these approximations, the equations of motion can still be truncated to linear order in f and n, but since h is not necessarily small, we will keep all non-linear terms in h. Therefore, we obtain the following equations

$$2\rho n' = \frac{2GM}{\rho} - (m\rho)^2 h, \\ \tilde{f} = -2\frac{GM}{\rho} - (m\rho)^2(h - h^2), \\ \frac{GM}{\rho} = -(m\rho)^2\left(\frac{3}{2}h - 3h^2 + h^3\right). \quad (13)$$

We should stress that these are exact equations in the limit $m\rho \ll 1$, $GM/\rho \ll 1$, *i.e.* there are no higher order corrections in h. If we linearise the equations for h, we recover the solution Eqs. (12). On the other hand, the Vainshtein mechanism applies, and below the so-called Vainshtein radius, $\rho_V = (GMm^{-2})^{1/3}$, h becomes larger than one, as expected. Actually, for $\rho \ll \rho_V$ the solution for h is simply given by $|h| = \rho_V/\rho \gg 1$. The latter solution for h and Eq. (13) imply

$$\begin{aligned} 2\rho n' &= \frac{2GM}{\rho}\left(1+\frac{1}{2}\left(\frac{\rho}{\rho_V}\right)^2\right), \\ \tilde{f} &= -\frac{2GM}{\rho}\left(1-\frac{1}{2}\left(\frac{\rho}{\rho_V}\right)\right). \end{aligned} \tag{14}$$

Therefore, the corrections to GR solutions are indeed small for $\rho < \rho_V$.

The Vainshtein mechanism is more transparent in the non-unitary gauge. Indeed by doing a coordinate transformation, $\rho = r/H$, we excite the ρ component of the Stückelberg field, $\pi^\rho = -\rho h$. Thus the strong coupling nature of h is encoded in π^ρ in this coordinate. It is possible to construct an effective theory for this Stückelberg field in the so-called decoupling limit [13]. First we introduce a scalar so that $\pi_\mu = \partial_\mu \pi/\Lambda_3^3$, where $\Lambda_3^3 = m^2 M_{pl}$. Then the covariantization of metric perturbations $H_{\mu\nu}$ is written as

$$H_{\mu\nu} = h_{\mu\nu} + \frac{2}{M_{pl}^2 m^2}\Pi_{\mu\nu} - \frac{1}{M_{pl}^2 m^4}\Pi^2_{\mu\nu}, \tag{15}$$

where $\Pi_{\mu\nu} = \partial_\mu\partial_\nu\pi$ and $\Pi^2_{\mu\nu} = \Pi_{\mu\alpha}\Pi^\alpha_\nu$. Formally, the decoupling limit is achieved by taking $m \to 0$ and $M_{pl} \to \infty$, but keeping Λ_3 fixed. By substituting Eq. (15) into the action, one can show that the kinetic terms of π become total derivatives and a mixing appears between $h_{\mu\nu}$ and π, which can be diagonalised using the definition

$$h_{\mu\nu} = \hat{h}_{\mu\nu} + \frac{\pi}{M_{pl}}\eta_{\mu\nu} - \frac{1}{\Lambda_3^3 M_{pl}}\partial_\mu\pi\partial_\nu\pi. \tag{16}$$

The Lagrangian then writes

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\rm GR}(\hat{h}_{\mu\nu}) + \frac{3}{2}\pi\Box\pi - \frac{3}{2\Lambda_3^3}(\partial\pi)^2\Box\pi \\ &+ \frac{1}{2\Lambda_3^6}(\partial\pi)^2([\Pi^2]-[\Pi]^2) \\ &+ \frac{5}{2\Lambda_3^9}(\partial\pi)^2([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]), \end{aligned} \tag{17}$$

where $[\Pi] = \Pi^\mu_\mu$, $[\Pi^2] = \Pi^{\mu\nu}\Pi_{\mu\nu}$, $[\Pi^3] = \Pi^{\mu\nu}\Pi_{\nu\alpha}\Pi^\alpha_\mu$ and $\mathcal{L}_{\rm GR}$ is the linearised Einstein-Hilbert action for $\hat{h}_{\mu\nu}$. These terms are known as Galileon terms, which give rise to the second order differential equations for π. For the spherically symmetric case, the equation of motion for π is given by [12]

$$3\left(\frac{\pi'}{\rho}\right) + \frac{6}{\Lambda_3^3}\left(\frac{\pi'}{\rho}\right)^2 + \frac{2}{\Lambda_3^6}\left(\frac{\pi'}{\rho}\right)^3 = \frac{M}{4\pi M_{pl}\,\rho^3}, \tag{18}$$

where the integration constant is again chosen so that M is a mass of a particle at the origin. Using the relation between π and h, $h = -\pi'/\left(m^2 M_{pl}\rho\right)$, we can show that the solutions for $\tilde{f}$, n and h given by Eq. (13) agree with the solutions Eq. (16) and Eq. (18).

We have shown that the weak field solutions for the metric Eq. (8) have three phases. On the largest scales, $m^{-1} \ll \rho$, beyond the compton wavelength, the gravitational interactions decay exponentially due to the mass of graviton: see Eq. (12). In the intermediate region $\rho_V < \rho < m^{-1}$, we obtain the $1/r$ gravitational potential but the Newton constant is rescaled $G \to 4G/3$. Moreover, the post-Newtonian parameter γ is $\gamma = \tilde{f}/(2n) = (1/2)(1+m\rho)$, whic reduces to $\gamma = 1/2$ in the $m \to 0$ limit, instead of $\gamma = 1$ of GR, showing the vDVZ discontinuity. Finally, below the Vainshtein radius $\rho < \rho_V$, the GR solution is recovered due to the strong coupling of the π mode (see Eq. (14)). This background solution provides us a testing ground for the BD ghost. Instead of expanding the action in $H_{\mu\nu}$ around the Minkowski spacetime perturbatively, we can study *linear* perturbations around this non-perturbative solution using the complete potential Eq. (6). In order to obtain the fully non-linear solution, a numerical approach is necessary. In the next section, we consider a different kind of solution of this theory which can be obtained analytically.

Schwarzschild de Sitter solution: Unlike in GR, where the Birkhoff theorem holds, there is no uniqueness theorem for spherically symmetric solutions in this theory. Again in the unitary gauge, we consider the following metric Ansatz for the dynamical metric $g_{\mu\nu}$

$$ds^2 = -C(r)\,dt^2 + A(r)\,dr^2 + 2D(r)\,dtdr + B(r)d\Omega^2. \tag{19}$$

Even though this is the Ansatz adopted in [16] to obtain an exact solution for Fierz-Pauli massive gravity that asymptotes de Sitter spacetime, one should not expect *a priori* the same form of solution when non-linear terms are included. However, we find a solution similar to that in [16], up to numerical factors. By plugging the Ansatz into the field equations derived from the Lagrangian (4), it is straightforward to check that the following configuration is a solution

$$\begin{aligned} A(r) &= \frac{9\Delta_0}{4}(p(r)+\alpha+1), \quad B(r) = \frac{4}{9}r^2, \\ C(r) &= \frac{9\Delta_0}{4}(1-p(r)), \quad D(r) = \frac{9}{4}\Delta_0\sqrt{p(r)(p(r)+\alpha)}, \end{aligned} \tag{20}$$

where

$$p(r) = \frac{2\mu}{r} + \frac{m^2 r^2}{9}, \quad \alpha = \frac{16}{81\,\Delta_0} - 1, \tag{21}$$

with arbitrary μ and Δ_0. Consequently, this configuration depends on *two* integration constants. A sufficient condition to ensure that $D(r)$ is real, is to choose $\mu > 0$ and $0 < \sqrt{\Delta_0} < 4/9$.

The previous form of the metric, Eq.(19), does not allow a manifest comparison with de Sitter spacetime:

a coordinate transformation of the time coordinate is not permitted since we have already adopted the unitary gauge. Therefore, if we allow for a vector π^μ, of the form $\pi^\mu = (\pi_0(r), 0, 0, 0)$, the metric can be rewritten in a diagonal form as

$$ds^2 = -C(r)\,dt^2 + \tilde{A}(r)\,dr^2 + B(r)d\Omega^2. \qquad (22)$$

Then we can write down the action in terms of $C, \tilde{A}, B$ and π_0, considering them as fields. It is possible to show that the following configuration solves the corresponding equations of motion

$$\tilde{A}(r) = \frac{4}{9}\frac{1}{1-p(r)}, \;\; \pi_0'(r) = -\frac{\sqrt{p(r)(p(r)+\alpha)}}{1-p(r)}, \quad (23)$$

while $C(r)$ and $B(r)$ are the same as in Eq. (20). The resulting metric has then a manifestly de Sitter-Schwarzschild form by making a time rescaling $t \to (4/9\Delta_0^{1/2})t$. However, we should note that this time rescaling cannot be done without introducing an additional time dependent contribution to π_0. As expected, the metric in Eq. (22) can be obtained by making the following transformation of the time coordinate $d\tilde{t} \equiv dt + \pi_0' dr$ to metric (19); this produces a non-zero time component of π^μ, that does not vanish even in the $m \to 0$ limit for any allowed value of Δ_0. There are two integration constants, μ and Δ_0, in this solution. In GR, μ corresponds to the mass of a source at the origin but a careful analysis including a matter source is necessary to fully understand the role of these integration constants. Note that there is an apparent singularity at the horizon $p(r) = 1$ both for the metric and π_0.

We can further make a coordinate transformation at the expense of exciting further components of π^μ. For example, by setting $\mu = 0$ and making the following coordinate transformations $t = F_t(\tau, \rho)$, $\; r = F_r(\tau, \rho)$ with

$$\begin{aligned} F_t(\tau,\rho) &= \frac{4}{3\Delta_0^{1/2} m}\,\mathrm{arctanh}\left(\frac{\sinh\left(\frac{m\tau}{2}\right) + \frac{m^2\rho^2}{8}e^{m\tau/2}}{\cosh\left(\frac{m\tau}{2}\right) - \frac{m^2\rho^2}{8}e^{m\tau/2}}\right), \\ F_r(\tau,\rho) &= \frac{3}{2}\rho e^{m\tau/2}, \end{aligned} \qquad (24)$$

the metric becomes that of flat slicing of de Sitter,

$$ds^2 = -d\tau^2 + e^{m\tau}(d\rho^2 + \rho^2 d\Omega^2), \qquad (25)$$

where the Hubble parameter is given by $m/2$. The Stückelberg fields π^μ are now given by $\pi^\mu = (\pi^\tau(\tau,\rho), \pi^\rho(\tau,\rho), 0, 0)$, $\pi^\tau = \pi_0 + F_t(\tau,\rho) - \tau$, $\;\pi^\rho = F_\rho(\tau,\rho) - \rho$. This is an interesting solution in which the acceleration of the universe is determined by the graviton mass and the Hubble parameter is given by $m/2$. Note that this "self-accelerating" solution was obtained in the decoupling limit in Ref. [17]. We find that this is an exact solution in this model.

Conclusions: The solutions obtained in the non-linear covariant massive gravity are remarkably similar to those in the DGP braneworld model including the existence of the "self-accelerating" de Sitter solution without cosmological constant [18] although there are differences in detail. There are a number of important issues. Firstly, we should confirm that there is no BD ghost in this theory by studying perturbations around the non-perturbative solution obtained in this letter. In the DGP model, the self-accelerating solution suffers from a ghost instability [10, 11, 19], which is related to the ghost in the FP theory on a de Sitter background. It is crucial to study the stability of the de Sitter solution in this model. In fact Ref. [17] showed that there exists a ghost in this self-accelerating background in the decoupling limit. They argue that this ghost can be cured by adding higher order corrections in $\mathcal{K}$ to the potential. Our formalism is ready to apply for this extended model. Once these issues are clarified, this massive gravity model provides us with an interesting playground to study large distance modifications of general relativity.

Acknowledgments

We would like to thank Misao Sasaki for useful comments. KK and GN are supported by the ERC. KK is supported by STFC grant ST/H002774/1 and GT is supported by STFC advanced fellowship. KK acknowledges support from the Leverhulme trust and RCUK.

[1] M. Fierz, W. Pauli, Proc. Roy. Soc. Lond. **A173**, 211-232 (1939).
[2] D. G. Boulware, S. Deser, Phys. Rev. **D6**, 3368-3382 (1972).
[3] N. Arkani-Hamed, H. Georgi, M. D. Schwartz, Annals Phys. **305**, 96-118 (2003). [hep-th/0210184].
[4] H. van Dam, M. J. G. Veltman, Nucl. Phys. **B22**, 397-411 (1970).
[5] V. I. Zakharov, JETP Lett. **12**, 312 (1970).
[6] A. I. Vainshtein, Phys. Lett. **B39**, 393-394 (1972).
[7] P. Creminelli, A. Nicolis, M. Papucci, E. Trincherini, JHEP **0509**, 003 (2005). [hep-th/0505147].
[8] C. Deffayet, J. -W. Rombouts, Phys. Rev. **D72** (2005) 044003. [gr-qc/0505134].
[9] G. R. Dvali, G. Gabadadze, M. Porrati, Phys. Lett. **B485**, 208-214 (2000). [hep-th/0005016].
[10] M. A. Luty, M. Porrati, R. Rattazzi, JHEP **0309**, 029 (2003). [hep-th/0303116].

[11] A. Nicolis, R. Rattazzi, JHEP **0406**, 059 (2004). [hep-th/0404159].
[12] A. Nicolis, R. Rattazzi, E. Trincherini, Phys. Rev. **D79**, 064036 (2009). [arXiv:0811.2197 [hep-th]].
[13] C. de Rham, G. Gabadadze, Phys. Rev. **D82**, 044020 (2010). [arXiv:1007.0443 [hep-th]].
[14] C. de Rham, G. Gabadadze, A. J. Tolley, [arXiv:1011.1232 [hep-th]].
[15] T. Damour, I. I. Kogan, A. Papazoglou, Phys. Rev. **D67**, 064009 (2003). [hep-th/0212155]; E. Babichev, C. Deffayet, R. Ziour, Phys. Rev. Lett. **103**, 201102 (2009). [arXiv:0907.4103 [gr-qc]]; E. Babichev, C. Deffayet, R. Ziour, Phys. Rev. **D82**, 104008 (2010). [arXiv:1007.4506 [gr-qc]].
[16] A. Salam and J. A. Strathdee, Phys. Rev. D **16** (1977) 2668.
[17] C. de Rham, G. Gabadadze, L. Heisenberg, D. Pirtskhalava, [arXiv:1010.1780 [hep-th]].
[18] C. Deffayet, Phys. Lett. **B502**, 199-208 (2001). [hep-th/0010186].
[19] K. Koyama, Phys. Rev. **D72**, 123511 (2005). [hep-th/0503191]; D. Gorbunov, K. Koyama, S. Sibiryakov, Phys. Rev. **D73**, 044016 (2006). [hep-th/0512097]. C. Charmousis, R. Gregory, N. Kaloper, A. Padilla, JHEP **0610**, 066 (2006). [hep-th/0604086].