

# Appendix of the paper: GAIA-SMAA-PROMETHEE for a hierarchy of interacting criteria

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In the manuscript we proposed an extension of the PROMETHEE I and II methods taking into account hierarchy and interaction between criteria as well as robustness concerns. Anyway, other PROMETHEE methods can be easily extended to deal with the same issues. In the following, we briefly recalled the main characteristics of the considered methods and we described their extensions to the hierarchical bipolar case.

**PROMETHEE III [3]**) PROMETHEE III defines a complete interval order by associating to each action  $a$  an interval  $[x_a, y_a]$  which is given by

$$\begin{cases} x_a = \bar{\phi}(a) - \alpha\sigma_a, \\ y_a = \bar{\phi}(a) + \alpha\sigma_a; \end{cases} \quad (39)$$

where:

- $\bar{\phi}(a) = \frac{1}{n} \sum_{x \in A} [\pi(a, x) - \pi(x, a)],$
- $\sigma_a^2 = \frac{1}{n} \sum_{x \in A} [\pi(a, x) - \pi(x, a) - \bar{\phi}(a)]^2,$
- $A$  is the set of all alternatives.

For  $\alpha > 0$  this method takes into account the variability of the net flows. The complete interval order  $(P^{III}, I^{III})$  is defined as follows:

$$\begin{aligned} aP^{III}b & \text{ (} a \text{ is preferred to } b \text{) iff } x_a > y_b; \\ aI^{III}b & \text{ (} a \text{ is indifferent to } b \text{) iff } x_a \leq y_b \text{ and } x_b \leq y_a. \end{aligned}$$

**Extension)** We observe that PROMETHEE III can be easily extended to the hierarchical bipolar case by considering an adaptation of the formulas provided in Section 3. Indeed, defining

$$\begin{aligned} \bar{\phi}_{\mathbf{r}}^B(a) &= \frac{1}{n} \sum_{x \in A} \frac{Ch^B(P_{\mathbf{r}}^B(a, x), \hat{\mu})}{\hat{\mu}(\{g_{\mathbf{t}} : \mathbf{t} \in E(g_{\mathbf{r}})\}, \emptyset)}, \\ \sigma_{a, \mathbf{r}}^2 &= \frac{1}{n} \sum_{x \in A} \left[ \frac{Ch^B(P_{\mathbf{r}}^B(a, x), \hat{\mu})}{\hat{\mu}(\{g_{\mathbf{t}} : \mathbf{t} \in E(g_{\mathbf{r}})\}, \emptyset)} - \bar{\phi}_{\mathbf{r}}^B(a) \right]^2 \end{aligned}$$

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and replacing  $\bar{\phi}(a)$  and  $\sigma_a$  with  $\bar{\phi}_{\mathbf{r}}^B(a)$  and  $\sigma_{a,\mathbf{r}}$  in Eq.(39), we obtain a complete interval order on the set of alternatives  $A$ . Note that it is possible to obtain the complete interval order w.r.t. each non-elementary criterion  $g_{\mathbf{r}}$  and w.r.t. each compatible bicapacity  $\hat{\mu}$ .

**PROMETHEE IV [3]**) PROMETHEE IV handles multicriteria decision problems with a continuous infinity of alternatives. Therefore denoting by  $X_j \subseteq \mathbb{R}$  the set of values that can be assumed by criterion  $g_j \in G$ , it is assumed that there exists a distribution  $\rho$  on  $X = X_1 \times \dots \times X_m$  such that, for all  $\mathbf{x} \in X$ ,  $\rho(\mathbf{x}) \geq 0$  and  $\int_X \rho(\mathbf{x}) d\mathbf{x} = 1$ . In this context the positive and the negative flows for each alternative  $a$  are defined as follows:

$$\phi^+(a) = \int_X \pi(a, \mathbf{x}) \rho(\mathbf{x}) d\mathbf{x} \quad (40)$$

$$\phi^-(a) = \int_X \pi(\mathbf{x}, a) \rho(\mathbf{x}) d\mathbf{x} \quad (41)$$

**Extension)** PROMETHEE IV can be extended to a hierarchy of interacting criteria by defining  $\phi_{\mathbf{r}}^{B+}(a)$  and  $\phi_{\mathbf{r}}^{B-}(a)$  as follows:

$$\phi_{\mathbf{r}}^{B+}(a) = \int_X \frac{Ch^{B+}(P_{\mathbf{r}}^{B+}(a, \mathbf{x}), \hat{\mu})}{\mu^+(\{g_{\mathbf{t}} : \mathbf{t} \in E(g_{\mathbf{r}})\}, \emptyset)} \rho(\mathbf{x}) d\mathbf{x} \quad (42)$$

$$\phi_{\mathbf{r}}^{B-}(a) = \int_X \frac{Ch^{B-}(P_{\mathbf{r}}^B(a, \mathbf{x}), \hat{\mu})}{\mu^-(\emptyset, \{g_{\mathbf{t}} : \mathbf{t} \in E(g_{\mathbf{r}})\})} \rho(\mathbf{x}) d\mathbf{x} \quad (43)$$

**PROMETHEE V [1]**) PROMETHEE V deals with problems in which alternatives have to be grouped in clusters or segments. After defining different clusters of alternatives  $C_1, \dots, C_k, \dots, C_R$ , and after computing the net flow of each alternative  $a \in A$ , the following 0-1 problem has to be solved

$$\begin{aligned} & \max \sum_{a \in A} x_a \phi(a), \\ & \sum_{a \in A} \alpha_a x_a \geq [\leq, =] \beta, [BeCl] \\ & \sum_{a \in C_k} \gamma_{a,k} x_a \geq [\leq, =] \delta_k, [WiCl], \end{aligned} \quad (44)$$

where  $x_a \in \{0, 1\}$  for all  $a \in A$ , while  $\alpha_a, \gamma_{a,k} \in \mathbb{R}$  are coefficients attached to the single alternatives. Inequalities  $[BeCl]$  are used to include constraints between the different clusters (for example, a maximum number of alternatives that should be selected), while inequalities  $[WiCl]$  are used to include constraints within the same cluster (for example, the maximum investment in a particular region can not be greater than a certain threshold  $\delta_k$ ). Of course, the chosen alternatives are those for which  $x_a = 1$  after solving the 0-1 program.

**Extension)** As already discussed in Section 3, for each alternative  $a \in A$  and for each criterion  $g_{\mathbf{r}}$ , one can compute the bipolar net flow of  $a$  on  $g_{\mathbf{r}}$ ,  $\phi_{\mathbf{r}}^B(a)$ , being the equivalent of  $\phi(a)$  in our case. As a consequence, the 0-1 programming problem in (44) can be easily extended to our methodology. Indeed, for each criterion  $g_{\mathbf{r}}$  in the hierarchy, after defining the clusters  $C_1^{\mathbf{r}}, \dots, C_k^{\mathbf{r}}, \dots, C_R^{\mathbf{r}}$ , one has to solve the following 0-1 program

$$\begin{aligned} & \max \sum_{a \in A} x_a^{\mathbf{r}} \phi_{\mathbf{r}}^B(a), \\ & \sum_{a \in A} \alpha_a^{\mathbf{r}} x_a^{\mathbf{r}} \geq [\leq, =] \beta^{\mathbf{r}}, [BeCl_{\mathbf{r}}] \\ & \sum_{a \in C_k^{\mathbf{r}}} \gamma_{a,k}^{\mathbf{r}} x_a^{\mathbf{r}} \geq [\leq, =] \delta_k^{\mathbf{r}}, [WiCl_{\mathbf{r}}], \end{aligned} \quad (45)$$

where constraints  $[BeCl_r]$  and  $[WiCl_r]$  are the equivalent of constraints  $[BeCl]$  and  $[WiCl]$ . The difference is related to the fact that they will take into account a criterion only. In this way, one can look at the best portfolio of alternatives when considering a particular criterion  $g_r$  in the hierarchy and, therefore, different subsets of alternatives can be chosen for different aspects of the problem at hand. For example, in a project selection problem in which economic, environmental and social aspects are considered, one can look at the best subset of alternatives that should be chosen when all three aspects are taken into account simultaneously but also the sets of alternatives that should be selected when the three different aspects are considered separately.

**PROMETHEE VI [2])** In PROMETHEE VI, the DM does not provide exact values for the weights of criteria but intervals of possible values:

$$w = (w_1, \dots, w_n) : w_j^- \leq w_j \leq w_j^+, \text{ for all } j = 1, \dots, n.$$

Then, all allowable weight vectors are considered and their projections are represented on the GAIA plane in order to distinguish between hard and soft problems.

**Extension)** We are already taken into account this preference information in a direct or an indirect way by using the SMAA methodology as explained in Section 4. In order to consider a similar case, we added the possibility that the DM provides some preferences in terms of intervals of possible values for the weights of elementary criteria  $g_t$ . For example, he can state that the importance of elementary criterion  $g_t$  varies between a lower ( $l_t$ ) and an upper ( $u_t$ ) bound:

$$a_t \in [l_t, u_t].$$

Consequently, the application of SMAA permits to study the different ranking of the alternatives varying the weights in the considered intervals.

**PROMETHEE GDSS [4])** PROMETHEE GDSS deals with group decision making problems by using the classical PROMETHEE II method. Considering DMs  $DM_1, \dots, DM_r$ , and after a consensus on the set of considered alternatives and criteria has been reached by all the DMs, the net flow  $\phi^k(a)$  of each alternative  $a$  w.r.t.  $DM_k$  can be obtained. Therefore, an importance  $\omega_k$  has to be attached to each  $DM_k$  so that  $\sum_{k=1}^R \omega_k = 1$ . At this point, finding a compromise solution  $a \in A$  can be considered as a new multicriteria problem where the criteria are the net flows attached to each alternative by each DM (see Table 1) and, consequently, the best

Table 1: The new formulation

	$DM_1$	$\dots$	$DM_k$	$\dots$	$DM_R$
$a$	$\phi^1(a)$	$\dots$	$\phi^k(a)$	$\dots$	$\phi^R(a)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$z$	$\phi^1(z)$	$\dots$	$\phi^k(z)$	$\dots$	$\phi^R(z)$

alternative is  $a \in A$  such that  $\phi^{Gl}(a) = \max_{x \in A} \phi^{Gl}(x)$  where

$$\phi^{Gl}(x) = \sum_{k=1}^R \omega_k \phi^k(x).$$

**Extension)** The extension of the PROMETHEE GDSS to our methodology will be straightforward. Indeed, for each criterion in the hierarchy  $g_r$ , after computing the bipolar net flow  $\phi_r^{B,k}(a)$  for each alternative  $a$  and each  $DM_k$ , and after defining the importance  $\omega_1^r, \dots, \omega_R^r$  of the different DMs, the compromise alternative will be  $a \in A$  such that  $\phi_r^{B,Gl}(a) = \max_{x \in A} \phi_r^{B,Gl}(x)$ , where

$$\phi_r^{B,Gl}(x) = \sum_{k=1}^R \omega_k^r \phi_r^{B,k}(x).$$

In this context it is meaningful supposing that the importance attached to each DM is dependent on the considered criterion. For example, in the decision problem introduced above where economic, environmental and social aspects are taken into account, in looking at the best solution from an economic point of view, it is reasonable that experts in economic have a greater importance than those expert in environmental and social aspects.”

## References

- [1] J.P. Brans and B. Mareschal. PROMETHEE V: MCDM problems with segmentation constraints. *INFOR: Information Systems and Operational Research*, 30(2):85–96, 1992.
- [2] J.P. Brans and B. Mareschal. The PROMETHEE VI procedure: how to differentiate hard from soft multicriteria problems. *Journal of Decision Systems*, 4(3):213–223, 1995.
- [3] J.P. Brans, B. Mareschal, and Ph. Vincke. PROMETHEE: a new family of outranking methods in multicriteria analysis. In J.P. Brans, editor, *Operational Research, IFORS 84*, pages 477–490. North Holland, Amsterdam, 1984.
- [4] C. Macharis, J.P. Brans, and B. Mareschal. The GDSS PROMETHEE procedure. *Journal of decision systems*, 7(4):283–307, 1998.