Dynamic precision analysis of a redundant sliding manipulator

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Abstract. This paper deals with the dynamic precision analysis of a linear-type rail redundant sliding manipulator. Based on the analysis of the mechanism structure, kinematics, stiffness performance and dynamic characteristics of the redundant sliding manipulator, the dynamic error caused by elastic deformation of links is analyzed. A mathematical model of the dynamic precision of the redundant sliding manipulator system is established in this paper by the geometric method. The influence of parameters such as equivalent stiffness and moment of inertia on the dynamic error of the manipulator is analyzed and parameter optimization schemes are proposed to reduce the dynamic error. Simulation analysis and experiment verify the dynamic precision of the redundant sliding manipulator. This paper provides a theoretical guidance for the high-precision operations of the redundant sliding manipulator.

Keywords: Dynamic precision analysis, dynamic error, redundant manipulator, sliding manipulator

1 Introduction

For decades, as redundant manipulators have huge application prospects in aerospace, military industry, manufacturing, medical, civil and other fields, and will play an increasingly important role in the future [1–3]. Thus the study of their dynamic errors can improve the theoretical study of dynamic accuracy. With the progress of exploration and the change of experiment demand, more and more scientific experiments require the redundant manipulators to simultaneously possess the ability to carry out autonomous and precise operations to improve the efficiency and operational safety [4, 5]. Therefore, research on the dynamic accuracy of redundant manipulators has become an important part of robotic technology [6, 7].

Redundant manipulators are nonlinear and strongly coupled dynamic systems [8, 9]. Due to the limitation of mechanical manufacturing technology level and manufacturing cost, actual structural parameters and motion parameters inevitably have deviations [10]. In addition, during the high precision technical operation process of the redundant manipulator, some of their own error sources, elastic deformation and vibration caused by inertial forces during high-speed movement, and errors caused by external interference are uncertain and random [11]. Fang et al. [12] proposed a new set of kinematic indicators that can evaluate redundant parallel manipulators. Guo et al. [13] proposed a dynamic analysis method for spherical motors. Tong et al. [14] proposed a Gough-Stewart parallel manipulator with linear orthogonality to achieve dynamic decoupling. Tsai et al. [15] integrated the existing uncertainty models for the practical application of serial and parallel manipulators. Yang et al. [16] used a hypothetical mode method and Lagrange method to describe the dynamic model of a flexible manipulator with unknown interference. At present, most of the researchers' research on the dynamic precision of manipulators is for parallel manipulators or the flexible manipulators. However, there are few studies on the dynamic accuracy of the redundant rail-type sliding manipulator which is used for high-precision operations. Therefore, research on the dynamic accuracy of the redundant sliding manipulator is the top priority for the development of robotic technology.

In order to resolve the appeal issue, this paper deals with the dynamic precision analysis of a linear-type rail redundant sliding manipulator. The dynamic error of the redundant sliding manipulator caused by elastic deformation of links is analyzed, and parameter optimization schemes are proposed to reduce the dynamic error. This paper provides a theoretical guidance for the high-precision operations of the redundant sliding manipulator.

The remainder of this paper is organized as follows. Section 2 outlines the redundant sliding manipulator studied in this paper. Section 3 establishes the mathematical model of dynamic precision of the redundant sliding manipulator by the geometric method. Section 4 analyzes the influence of parameters on dynamic errors. Section 5 verifies the dynamic accuracy through simulation and experiment, and proposes parameter optimization schemes. Section 6 summarizes the innovation in this paper.

2 Overview of the redundant sliding manipulator

In this paper, we have carried out the development of a linear-type rail redundant sliding manipulator. The redundant sliding manipulator is mainly composed of macro manipulator, a micro manipulator platform, micro gripper, micro vision system and control system. Figure 1 shows the prototype of the macro-micro manipulator. The dexterous manipulator provides a wide range of accessible space and the dexterous posture of the end-effector, the micro manipulator performs the micro-operation, and the macro manipulator performs the macro-operation. The whole precision manipulator has good spatial accessibility and dexterity, as well as the characteristics of precise positioning, rapid response and easy oper-

Fig. 1. The linear-type rail redundant sliding manipulator

ation. Combined with the measuring instruments such as the microscope, the micro-operation task with high precision can be realized.

The redundant sliding manipulator studied in this paper is composed of different types of sliding rails as a mobile base and a six degrees-of-freedom (DOF) redundant manipulator. Due to its unique structural features, it has the advantages and characteristics of being both a redundant manipulator and a mobile manipulator. On the one hand, as a redundant manipulator, the DOFs of the redundant manipulator is greater than the DOFs required to achieve the primary task defined by its end effector, so it has received great attention. The existence of redundancy allows the manipulator to complete secondary tasks and primary tasks simultaneously. Specifically, due to redundancy, the redundant manipulator has multiple control solutions for the tasks that can be completed, from which we can choose the best solution to complete the secondary tasks. On the other hand, with the development of complex technological society and the introduction of new concepts and innovative theoretical tools in the field of intelligent systems, the mobile manipulator has attracted great interest in the industrial, military and public service fields. Compared with fixed robots, they have large-scale maneuverability and control capability.

Therefore, the redundant sliding manipulator studied in this paper has the advantages of large working space, light weight, high fault tolerance, strong robustness, high operating accuracy, easy replacement of different types of rails to complete various operating tasks and so on. Compared with the traditional redundant manipulator, its configuration has larger workspace, more flexible task operation and higher fault tolerance. Moreover, compared with the traditional mobile manipulator, its controllability is stronger and the precision is higher.

Based on the above advantages, redundant sliding manipulators have huge application prospects in aerospace, military industry, manufacturing, medical, civil and other fields, and will play an increasingly important role in the fu-

ture. In the military field, it can meet the operational requirements of military operations such as mobile shooting, pursuit of the enemy, and material transmission. In the aerospace field, it can be used for high-precision operation of space on-orbit service, thereby improving astronauts' work efficiency and operating accuracy simultaneously. In terms of industry, redundant sliding manipulators can complete the production work of automated production lines, thus greatly liberating manpower. In terms of medical treatment, it can be applied in minimally invasive surgery and surgical surgery to improve the accuracy of surgery, thereby improving the success rate of the operation. In terms of civilian use, redundant sliding manipulators can be used as supernumerary robotic limbs. The wearer can wrap it around the waist or back, which can greatly increase the range of movement of the manipulator and thus better assist the people in daily life. Therefore, the redundant sliding manipulator in this paper has great application prospects, and plays an important role in practical applications. Therefore, the research on the precision of the redundant sliding manipulator is the most important part of the robot technology.

3 Dynamic precision modeling of the redundant sliding manipulator

3.1 Establish vibration differential equations

This paper assumes that the link of the manipulator is a cantilever beam fixed at one end, and the flexibility of the joint and the link itself merges into equal flexibility.

In general, linear vibration of mechanical system can be described by linear differential equation as follows:

$$m\ddot{x} + c\dot{x} + kx = f(t) \quad (1)$$

where m is the vibration mass, c is the damping coefficient, k is the stiffness, x is the displacement, $\dot{x}$ is the speed, $\ddot{x}$ is the acceleration, $f(t)$ is the disturbance force or excitation force, and t is time.

The Lagrangian equation method is to solve the vibration differential equation of the vibration system by establishing the kinetic energy T , potential energy U , and energy loss function D of the vibration system, i.e.,

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = F_i(t) \quad (i = 1, 2, 3, \dots, 7) \quad (2)$$

where q_i is generalized coordinate, $\dot{q}_i$ is a generalized velocity, and $F_i(t)$ is a generalized excitation force.

The vibration model discussed in this paper is the free vibration under the ideal state, so the energy dissipation function and the generalized excitation force are ignored, then the Laplace equation (2) can be simplified as:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = 0 \quad (i = 1, 2, 3, \dots, 7) \quad (3)$$

which gives

$$\begin{aligned} T &= \sum_{i=1}^7 \frac{1}{2} J_i \dot{\Delta\theta}_i^2 \\ V &= \sum_{i=1}^7 \frac{1}{2} k_i (l_i \Delta\theta_i)^2 \end{aligned} \quad (4)$$

where l_i , m_i , J_i and k_i are the length, the mass, moment of inertia and equivalent stiffness of the link i , respectively.

For the link of the manipulator, only the bending stiffness and tensile stiffness are considered, and the stiffness of the joint and the link is approximated to a stiffness, which is called the equivalent bending stiffness. The equivalent bending stiffness and tensile stiffness are denoted by $k = 3EI/l^3$ and $k = EA/l$, where E is the elastic modulus, I is the moment of inertia, and l is the link length, A is the cross-sectional area.

The moment of inertia is a measure of inertia representing the rotation of an object and a property of the object itself. The moment of inertia of an object on any axis c is defined as $J_c = \sum_{i=1}^n m_i r_i^2$. It can be seen that the moment of inertia is determined by the mass and its distribution, and its unit is $kg \cdot m^2$.

By substituting the kinetic energy T and the potential energy V into (3), it can be obtained as follows:

$$0 = J_i \Delta\ddot{\theta}_i + k_i l_i^2 \Delta\theta_i \quad (5)$$

In the form of matrix, the following can be obtained:

$$\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} J_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & J_7 \end{bmatrix} \begin{bmatrix} \Delta\ddot{\theta}_1 \\ \vdots \\ \Delta\ddot{\theta}_7 \end{bmatrix} + \begin{bmatrix} k_1 l_1^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & k_7 l_7^2 \end{bmatrix} \begin{bmatrix} \Delta\theta_1 \\ \vdots \\ \Delta\theta_7 \end{bmatrix} \quad (6)$$

where $[\Delta\ddot{\theta}_1 \dots \Delta\ddot{\theta}_7]^T$ is the angular acceleration array and $[\Delta\theta_1 \dots \Delta\theta_7]^T$ is the angular displacement array. Further simplified (6) as the following equation:

$$[\mathbf{M}][\Delta\ddot{\theta}] + [\mathbf{K}][\Delta\theta] = [0] \quad (7)$$

where $[\mathbf{M}]$ is the mass matrix and $[\mathbf{K}]$ is the stiffness matrix. Equation (7) is the free vibration differential equation of the manipulator.

3.2 Natural frequency and main vibration of the system

The angular displacements $\Delta\theta_i (i \in [1, 2, 3, 4, 5, 6, 7])$ are used to describe the deviation between the deformed position and the ideal position of the links. Moreover, the angular displacements are used to qualitatively describe the dynamic precision caused by the elastic deformation of the manipulator. $\Delta\theta_i$ are

assumed as harmonic vibration at the same frequency and phase but different amplitude, i.e.,

$$\Delta\theta_i = A_i \sin(\omega t + \varphi) \quad (8)$$

where A_i are different amplitude.

Substitute (8) into (6), we can obtain:

$$\begin{bmatrix} k_1 l_1^2 - J_1 \omega^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & k_7 l_7^2 - J_7 \omega^2 \end{bmatrix} \begin{bmatrix} A_1 \\ \vdots \\ A_7 \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \quad (9)$$

The characteristic equation of (9) is as follows:

$$|B| = |[K] - \omega^2[M]| = \begin{vmatrix} k_1 l_1^2 - J_1 \omega^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & k_7 l_7^2 - J_7 \omega^2 \end{vmatrix} = 0 \quad (10)$$

Further extending the determinant in (10) can obtain that:

$$(k_1 l_1^2 - J_1 \omega^2) \dots (k_i l_i^2 - J_i \omega^2) = 0 \quad (11)$$

Then, (11) can be regarded as a quadratic equation with the frequency ω_i^2 as the unknown, and the solution can be expressed as:

$$\omega_i^2 = \frac{k_i l_i^2}{J_i} \quad (12)$$

It can be seen that the natural frequency problem can be solved by solving the eigenvalues. Theoretically, the vibration system has n DOFs, and n natural frequencies can be obtained. Since these frequencies are related only to the natural parameters of the vibration system (such as mass and stiffness) and not to the initial energy of the vibration, they are called the natural frequencies of the vibration system.

Differential equations cannot directly reveal the relationship between the amplitude of the link and the amplitude of the end-effector in the vibration process. Assuming that the initial condition of the vibration is: $t = 0, \Delta\theta_i = \Delta\theta_{i0}, \dot{\Delta\theta}_i = \Delta\dot{\theta}_{i0}$, then the expression for amplitude A_i is $A_i = \sqrt{\Delta\theta_{i0}^2 + \left(\frac{\Delta\dot{\theta}_{i0}}{\omega}\right)^2}$. Corresponding to different natural frequencies, the expression of the amplitude A_i is as follows:

$$A_i^{(n)} = \sqrt{\Delta\theta_{i0}^2 + \left(\frac{\Delta\dot{\theta}_{i0}}{\omega_n}\right)^2} \quad n \in (1, 2, \dots, 7) \quad (13)$$

where $A_i^{(n)}$ is the amplitude when the natural frequency is ω_n .

Fig. 2. Simplified model and simplified kinematics model of the redundant sliding manipulator

Thus, the main vibration of each order n of the system is:

$$\begin{cases} \Delta\theta_1^{(n)} = A_1^{(n)} \sin(\omega_n t + \varphi_n) \\ \vdots \\ \Delta\theta_7^{(n)} = A_7^{(n)} \sin(\omega_n t + \varphi_n) \end{cases} \quad (14)$$

In general, the free vibration of 7-DOF is synthesized by the main vibration of 7 different frequencies, and the result of the synthesis is not necessarily simple harmonic vibration. Therefore, the general solution of the differential equation (7) is the superposition of the above seven main vibrations, that is:

$$\Delta\theta_i = \Delta\theta_i^{(1)} + \cdots + \Delta\theta_i^{(7)} = A_i^{(1)} \sin(\omega_1 t + \varphi_1) + \cdots + A_i^{(7)} \sin(\omega_7 t + \varphi_7) \quad (15)$$

Equation (15) gives the general vibration solution of the 7-DOF free vibration model, which provides a theoretical basis for establishing the mathematical model of dynamic accuracy.

3.3 Establish mathematical model of dynamic precision

When discussing the error of the end-effector, the maximum value of the angular displacement is selected as the research object, so the maximum vibration error of 7-DOF is calculated as follows:

$$\Delta\theta_i = A_i^{(1)} + \cdots + A_i^{(7)} \quad (16)$$

The most commonly used maximum error in the error analysis selected in (16) can simplify the analysis process and make the results relatively accurate.

The linear sliding rail of the redundant sliding manipulator is considered as a revolute joint in kinematic modeling, so the manipulator is considered as a 7-DOF redundant sliding manipulator. The kinematic model of the redundant sliding manipulator is shown in Figure 2. According to the kinematics of the manipulator, the position of the end-effector can be obtained on the Cartesian coordinates, as shown below:

$$\begin{aligned}
p_x &= l_3 * \cos(\theta_1 + \theta_2 + \theta_3) * \sin(\theta_4 + \theta_5) - l_4 * \cos(\theta_7) * (\sin(\theta_1 + \theta_2 + \theta_3) * \\
&\quad \sin(\theta_6) - \cos(\theta_1 + \theta_2 + \theta_3) * \cos(\theta_4 + \theta_5) * \cos(\theta_6)) - l_0 * \cos(\theta_1 + \theta_2) + l_2 * \\
&\quad \cos(\theta_1 + \theta_2 + \theta_3) * \cos(\theta_4) - l_4 * \cos(\theta_1 + \theta_2 + \theta_3) * \sin(\theta_4 + \theta_5) * \sin(\theta_7) \\
p_y &= l_4 * \cos(\theta_7) * (\cos(\theta_1 + \theta_2 + \theta_3) * \sin(\theta_6) + \sin(\theta_1 + \theta_2 + \theta_3) * \cos(\theta_4 + \\
&\quad \theta_5) * \cos(\theta_6)) - l_0 * \sin(\theta_1 + \theta_2) + l_3 * \sin(\theta_1 + \theta_2 + \theta_3) * \sin(\theta_4 + \theta_5) + l_2 * \\
&\quad \sin(\theta_1 + \theta_2 + \theta_3) * \cos(\theta_4) - l_4 * \sin(\theta_1 + \theta_2 + \theta_3) * \sin(\theta_4 + \theta_5) * \sin(\theta_7) \\
p_z &= l_1 + l_3 * \cos(\theta_4 + \theta_5) - l_2 * \sin(\theta_4) - (l_4 * \cos(\theta_6 + \theta_7) * \sin(\theta_4 + \\
&\quad \theta_5))/2 - l_4 * \cos(\theta_4 + \theta_5) * \sin(\theta_7) - (l_4 * \cos(\theta_6 - \theta_7) * \sin(\theta_4 + \theta_5))/2
\end{aligned} \tag{17}$$

Furthermore, the position error of the end-effector of the redundant sliding manipulator can be obtained as follows:

$$\begin{aligned}
e_x &= l_3 * \cos(\Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3) * \sin(\Delta\theta_4 + \Delta\theta_5) - l_4 * \\
&\quad \cos(\Delta\theta_7) * (\sin(\Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3) * \sin(\Delta\theta_6) - \cos(\Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3) * \\
&\quad \cos(\Delta\theta_4 + \Delta\theta_5) * \cos(\Delta\theta_6)) - l_0 * \cos(\Delta\theta_1 + \Delta\theta_2) + l_2 * \cos(\Delta\theta_1 + \Delta\theta_2 + \\
&\quad \Delta\theta_3) * \cos(\Delta\theta_4) - l_4 * \cos(\Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3) * \sin(\Delta\theta_4 + \Delta\theta_5) * \sin(\Delta\theta_7) \\
e_y &= l_4 * \cos(\Delta\theta_7) * (\cos(\Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3) * \sin(\Delta\theta_6) + \sin(\Delta\theta_1 + \\
&\quad \Delta\theta_2 + \Delta\theta_3) * \cos(\Delta\theta_4 + \Delta\theta_5) * \cos(\Delta\theta_6)) - l_0 * \sin(\Delta\theta_1 + \Delta\theta_2) + \\
&\quad l_3 * \sin(\Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3) * \sin(\Delta\theta_4 + \Delta\theta_5) + l_2 * \sin(\Delta\theta_1 + \Delta\theta_2 + \\
&\quad \Delta\theta_3) * \cos(\Delta\theta_4) - l_4 * \sin(\Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3) * \sin(\Delta\theta_4 + \Delta\theta_5) * \sin(\Delta\theta_7) \\
e_z &= l_3 * \cos(\Delta\theta_4 + \Delta\theta_5) - l_2 * \sin(\Delta\theta_4) - (l_4 * \\
&\quad \cos(\Delta\theta_6 + \Delta\theta_7) * \sin(\Delta\theta_4 + \Delta\theta_5))/2 - l_4 * \cos(\Delta\theta_4 + \\
&\quad \Delta\theta_5) * \sin(\Delta\theta_7) - (l_4 * \cos(\Delta\theta_6 - \Delta\theta_7) * \sin(\Delta\theta_4 + \Delta\theta_5))/2
\end{aligned} \tag{18}$$

It can be seen from (18) that the errors e_x , e_y and e_z of the end-effector are functions of many parameters (such as equivalent stiffness k_i and moment of inertia J_i). Furthermore, the dynamic error can be minimized by analyzing the influence of these specific parameters on the dynamic errors, which is of guiding significance for the high-precision operation of redundant sliding manipulator.

4 Dynamic precision analysis of the redundant sliding manipulator

4.1 Preliminaries of dynamic precision analysis

Parametric analysis theory It is known that the dynamic errors $e_{x/y/z}$ are multi-independent and complex function variables. In the analysis process, dynamic errors $e_{x/y/z}$ are taken as the research objects. Taking only the research object as an independent variable and other variables as constants, we can obtain the following functions:

$$e_{x/y/z} = C_{x/y/z} \cdot f(\beta) \quad (19)$$

where β is the variable under study and $C_{x/y/z}$ is the constant quantity corresponding to $e_{x/y/z}$.

Influence coefficient analysis theory The error influence coefficient is the ratio between the error increment and the increment of the independent variable causing the increment. Partial deviations of dynamic errors $e_{x/y/z}$ relative to each parameter are defined as the error influence coefficient of each parameter, that is:

$$Y_{x/y/z} = \frac{\partial e_{x/y/z}}{\partial \beta} \quad (20)$$

where β is the study variable, and $Y_{x/y/z}$ is the influence coefficient for $e_{x/y/z}$ corresponding to β .

The error influence coefficient is an important basic theory in the study of error and its compensation. The two-dimensional parametric analysis method (19) and the error influence coefficient method (20) are used to analyze the influence of each parameter on the dynamic error, which can not only make the two independent analysis results complement each other, but also make the overall analysis results more complete and accurate.

4.2 Stiffness and rotational inertia analysis of the redundant sliding manipulator

Influence of stiffness and rotational inertia analysis According to (19), the influence of stiffness k_i change and the moment of inertia J_i change on the dynamic errors $e_{x/y/z}$ is analyzed, and the functional relationships between $e_{x/y/z}$ and the independent variables k_i and J_i are as follows:

$$\begin{aligned} e_{x/y/z} &= C_{x/y/z} \cdot f(k_i) \\ e_{x/y/z} &= C_{x/y/z} \cdot f(J_i) \end{aligned} \quad (21)$$

Influence coefficients of stiffness and rotational inertia analysis The dynamic error influence coefficients of the equivalent stiffness k_i and the moment of inertia J_i are defined as the ratio of the increment of the dynamic errors $e_{x/y/z}$ to the increment of the equivalent stiffness k_i and the moment of inertia J_i that caused the increment. It can be obtained as follows:

$$\begin{aligned} Y_{k_i} &= \frac{\partial e_{x/y/z}}{\partial k_i} \\ Y_{J_i} &= \frac{\partial e_{x/y/z}}{\partial J_i} \end{aligned} \quad (22)$$

5 Simulation analysis and experiment

5.1 Simulation analysis

The links of the 7-DOF linear-type rail redundant sliding manipulator are made of alloy steel, whose cross section is a circle with radius $r = 20mm$. The preliminary dimensions of each link are $l_0 = 236mm$, $l_1 = 208mm$, $l_2 = 166mm$, $l_3 = 166mm$, and $l_4 = 47mm$. According to the equivalent stiffness theory and the configuration of the redundant sliding manipulator, it can be obtained that the link 1, link 3, link 4 and link 6 are equivalent bending stiffness, and the link 2, link 5 and link 7 are equivalent tensile stiffness.

Based on the known material and the design dimensions of each link, the required parameters are calculated as follows: $m_1 = 2.327kg$, $m_2 = 2.051kg$, $m_3 = 1.637kg$, $m_4 = m_5 = 1.637kg$, $m_6 = m_7 = 0.463kg$, $J_1 = 0.452kg \cdot m^2$, $J_2 = 2.315 \times 10^{-3}kg \cdot m^2$, $J_3 = 0.159kg \cdot m^2$, $J_4 = 5.700 \times 10^{-2}kg \cdot m^2$, $J_5 = 1.025 \times 10^{-3}kg \cdot m^2$, $J_6 = 6.261 \times 10^{-4}kg \cdot m^2$, $J_7 = 1.582 \times 10^{-4}kg \cdot m^2$, $k_1 = 5.905 \times 10^6 N/m$, $k_2 = 1.244 \times 10^9 N/m$, $k_3 = 8.626 \times 10^6 N/m$, $k_4 = 1.697 \times 10^7 N/m$, $k_5 = 1.559 \times 10^9 N/m$, $k_6 = 7.476 \times 10^8 N/m$, $k_7 = 5.505 \times 10^9 N/m$. The initial conditions of vibration are $A_1 = \dots = A_7 = 0.1mm$, $\Delta\theta_{10} = \dots = \Delta\theta_{70} = 0.1mm$, $\Delta\dot{\theta}_{10} = \dots = \Delta\dot{\theta}_{70} = 5mm/s$.

It can be seen from Figure 3 that the change of equivalent stiffness k_i and the moment of inertia J_i will cause changes in dynamic errors and exhibit different curve relationships. The changes of equivalent stiffness k_7 and the moment of inertia J_3 have the greatest influence on the dynamic errors, while the changes of equivalent stiffness k_1 and the moment of inertia J_7 have the smallest influence on the dynamic errors. That means that the design of the redundant sliding manipulator has a great advantage for improving the dynamic accuracy due to the unique configuration of its slide rail. In other words, if the dynamic error is guaranteed to meet certain conditions, the equivalent stiffness of link 7 and the moment of inertia of link 3 are required the most in the design process, that is, the equivalent stiffness of link 7 and the moment of inertia of link 3 are first considered.

Figure 4 shows the stiffness and rotational inertia influence coefficient of the dynamic errors e_x , e_y and e_z . Comprehensive analysis of the dynamic error influence coefficient curves of each equivalent stiffness k_i and moment of inertia

Fig. 3. Dynamic errors e_x , e_y and e_z corresponding to different equivalent stiffness k_i and equivalent rotational inertia J_i .

Fig. 4. The stiffness influence coefficient and the rotation inertia influence coefficient of dynamic errors e_x , e_y and e_z .

J_i shows that the equivalent stiffness of link 6 and link 7 and the moment of inertia of link 3 and link 4 have the greatest influence on the dynamic errors of the manipulator in the whole working space. Generally speaking, when the equivalent stiffness and the moment of inertia are small, the dynamic errors are

not very stable and may change rapidly. When designing, we must pay attention to the instability of dynamic errors within this range.

Remark: From the above analysis, the parameter optimization scheme for reducing the dynamic errors can be obtained as follows:

- (1) Equivalent stiffness: The bending deformation of the link has a much greater influence on the dynamic error than the tensile deformation. Therefore, the adjustment of the equivalent bending stiffness is more important for reducing the dynamic errors of the system.
- (2) Moment of inertia: The size and distribution of the mass directly affect the magnitude of the moment of inertia, and thus directly affects the magnitude of the dynamic error of the manipulator.
- (3) When adjusting the moment of inertia, the cross-sectional area of each link may be changed, so the change of the equivalent stiffness must also be considered. Therefore, the moment of inertia and the equivalent stiffness can be adjusted at the same time, so as to reduce the dynamic errors of the system.

5.2 Implementation & verification experiment

Fig. 5. Configuration changes of the manipulator during operation.

The experimental platform in this paper is the linear-type rail redundant sliding manipulator introduced in Section 2, whose dimensions and D-H parameters are the same as the simulation. In this experiment, a set of joint angles is input to the manipulator, and the configuration changes of the manipulator during operation is as shown in Figure 5.

By changing the values of the angular displacements $\Delta\theta_i$ in real time during the movement of the redundant sliding manipulator, the corresponding errors are shown in Figure 6. We can find that the change of angular displacements $\Delta\theta_5$, $\Delta\theta_3$ and $\Delta\theta_4$ has the greatest impact on dynamic errors e_x , e_y and e_z ,

Fig. 6. Dynamic errors e_x , e_y and e_z corresponding to angular displacements $\Delta\theta_i$.

respectively. The rule of error variation in the experiment is consistent with the simulation, which further proves the accuracy and practicability of the dynamic error analysis method in this paper.

6 Conclusion

This paper deals with the dynamic precision analysis of a linear-type rail redundant sliding manipulator. Based on the analysis of the mechanism structure, kinematics, stiffness performance and dynamic characteristics of the redundant sliding manipulator, the dynamic error caused by elastic deformation of links is analyzed. A mathematical model of the dynamic precision of the redundant sliding manipulator system is established by the geometric method. The influence of parameters such as equivalent stiffness and moment of inertia on the dynamic error of the redundant sliding manipulator is analyzed and parameter optimization schemes are proposed to reduce the dynamic error. Simulation analysis and experiment verify the dynamic precision of the redundant sliding manipulator. This paper provides a theoretical guidance for the high-precision operations of the redundant sliding manipulator.

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